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Tsallis statistics and Astrophysics dark matter distribution in galaxies and clusters of galaxies

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Abstract

We study the differential energy distribution of dark matter halos, carrying out cosmological N-body simulation. We give an analytical formula of the differential energy distribution of dark matter in the halos obtained by the numerical simulation. From the analytical formula we reconstruct the density profile described by the Navarro, Frenk, & White (NFW) profile. The NFW profile is consistent with the analytical formula of our fractional mass distribution. We find that a parameter in our analytical formula of differential energy distribution which is related with the slope of inner cusp of dark halo. We obtain the distribution function for the NFW profile which has sharp cut off at the high binding energy. We discuss physical reason of form of the analytical formula.

1 Introduction

Tsallis' non-extensive generalized statistics (Tsallis 1988) is paid attention in the area of statistics of a multi-fractal system. In non-extensive system (long-range microscopy memory, long range forces, fractral space time) the following generalized entropy has been proposed :

$$S_q = k\frac{1-\sum_i p_i^q}{q-1} \quad \left(\sum_i p_i = 1; \quad q \in \Re\right), \tag{1}$$

where k is a positive constant. Optimization of S_q yields, for the canonical ensemble,

$$p_i = Z_q^{-1}[1-(1-q)\varepsilon_i/T']^{1/(1-q)}, \tag{2}$$

$$Z_q \equiv \sum_i [1-(1-q)\varepsilon_i/T']^{1/(1-q)} \tag{3}$$

and, when $q \longrightarrow 1$, the Boltzmann-Gibbs result is recovered. In this statistics an expected value of any physical variable is given by the Tsallis' escort distribution :

$$\langle A\rangle_q = \frac{\sum_i p_i^q A_i}{\sum_j p_j^q}, \tag{4}$$

where $\{A_i\}$ are the eigenvalues of an arbitrary observable A.

Lavagno *et al.* (1998) have recently shown that fraction of peculiar velocity of cluster of galaxies (Bahcall & Oh 1996) is well explained by the Tsallis escort integral,

$$P(>v) = \frac{\int_v^{v_{max}}(1-(1-q)(v/v_0)^2)^{q/(1-q)}dv}{\int_0^{v_{max}}(1-(1-q)(v/v_0)^2)^{q/(1-q)}dv}. \tag{5}$$

They obtained $q = 0.23$ to fit the fraction of peculiar velocity of cluster of galaxies and is smaller than in our case. There is a conjecture that q of system approaches unity when the system proceeds relaxation (Tsallis 1999).

It is interesting that density profiles of galaxies and clusters of galaxies formed in cosmological, numerical simulations have self-similar structure. Navarro, Frenk and White (1995, 1996, 1997; NFW) have shown in their N-body simulations of Cold Dark Matter (CDM) in the standard biased CDM and four power law spectra with indices $n = 0, -0.5$, and -1, open CDM ($\Omega_0 = 0.1$) with power-law spectra ($n = 0$ and -1), and ΛCDM cosmology that density profiles of dark halos have an universal profile described as

$$\rho(r) \propto \frac{1}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}. \tag{6}$$

Several investigators have shown that the formula provides a good fit to their numerical results (Cole and Lacey 1996, Tormen, Bouchet & White 1997, Huss, Jain & Steinmmetz 1999, Thomas *et al.* 1998).

In this paper we give our recent study on the differential energy distribution of clusters in our numerical results (Hanyu and Habe 2001). We find an analytical formula of the fractional mass distribution is fitted by an analytical formula, which is similar to Tsallis escote distribution. Using the iteration method, we construct the density profile from the analytical formula to show how the slope of the cusp of the density changes with parameters in our analytical formula. We give some discussion on our results.

2 Numerical simulation

We simulate SCDM model (e.g. Davis *et al.* 1985) ($\Omega = 1$, $\sigma_8 = 0.67$, $H_0 = 100h$ km s^{-1} Mpc^{-1},and $h = 0.5$).

Numerical simulations are carried out using GRAPESPH code. GRAPE is a special purpose hardware to calculate gravitation between N-body particles (Sugimoto *et al.* 1990). We combined Smoothed Particle Hydrodynamics (SPH) (Monaghan, 1992) with GRAPE. We select massive halos of which mass is as large as that of cluster of galaxies and calculate their density profile and the fractional mass distribution.

Mass of a CDM particle and a SPH particle are $5.89\times10^{11} M_\odot$ and $3.10\times10^{10} M_\odot$, respectively. Gravitational softening length is 100 kpc. Both number of CDM and SPH particles are 29855, respectively. Size of the simulation box is 80 Mpc.

3 Numerical results

3.1 Density distribution

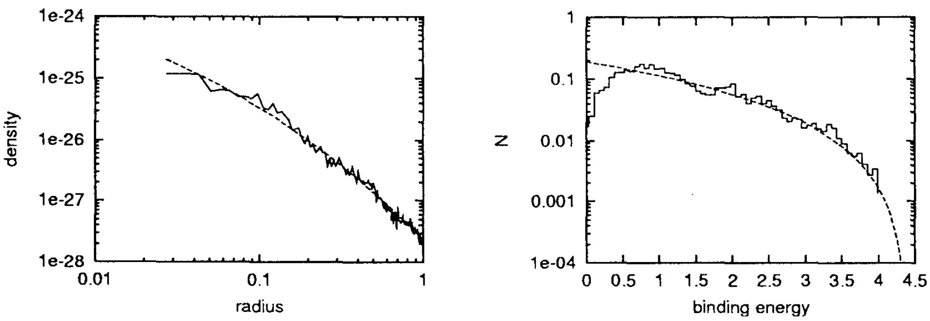


Figure 1 (left) shows a density profile (solid line) of our simulated typical rich cluster and the NFW profile (dasshed line) with $c = 4.4$ which fits well the numerical result. Density profiles of dark halos obtained by us agree well with the NFW profile in the range from the gravitational softening length to r_{200}.

3.2 The differential energy distribution

We introduce the differential energy distribution, $dM/d\varepsilon$ which gives the mass of dark matter in the dark halo with binding energy between ε and $\varepsilon + d\varepsilon$, where ε is the specific binding energy,

$$\varepsilon = \Psi(r) - \frac{1}{2}v^2, \tag{7}$$

and a relative potential, $\Psi \equiv -\Phi + \Phi_0$. Φ is gravitational potential and we choose Φ_0 to be such that a distribution function, f, is $f > 0$ for $\varepsilon > 0$ and $f = 0$ for $\varepsilon \leq 0$. In our analysis, ε is normalized by

GM_{200}/r_{200}. And we also introduce the fractional mass distribution as the differential energy distribution divided by the total mass of the dark halo, $N(\varepsilon) = dM/d\varepsilon/M$.

Figure 1 (right) shows the fractional mass distribution (solid line) of the cluster. In figure 1 (right), we also show $N(\varepsilon)$ (dashed line) given by

$$N(\varepsilon) = N_0 \left[1 - (1-q)\left(\frac{\varepsilon}{\varepsilon_0}\right)\right]^{q/(1-q)}, \tag{8}$$

with $q = 0.667$ and $\varepsilon_0 = 1.47$. Figure 1 (right) shows that equation (8) agrees well with our numerical results in the range of $0.5 < \varepsilon < 4$. There is cut off near $\varepsilon \simeq 4$. We find the fractional distribution $N(\varepsilon)$ can be fitted by following formula, for $q \simeq 0.6 - 0.7$ and $\varepsilon_0 \simeq 1.2 - 1.5 GM_{200}/r_{200}$ for rich clusters in our numerical results.

We assume that phase-space distribution function $f(\mathbf{x}, \mathbf{v})$ depends ε. At a radius r, velocity of a dark matter particle of the binding energy, ε, is $v = \sqrt{2(\Psi - \varepsilon)}$. The density profile may be given as follows (Binney and Tremaine 1987)

$$\rho(r) = 4\pi \int_{\Psi(r_g)}^{\Psi(r)} f(\varepsilon)[2(\Psi - \varepsilon)]^{1/2} d\varepsilon, \tag{9}$$

where r_g is the edge of the dark halo. From this equation, we may give $f(\varepsilon)$ as

$$f(\varepsilon) = \frac{1}{\sqrt{8}\pi^2} \frac{d}{d\varepsilon} \int_{\varepsilon_{min}}^{\varepsilon} \frac{d\rho/d\Psi}{[\varepsilon - \Psi]^{1/2}} d\Psi, \tag{10}$$

where $\varepsilon_{min} = \Psi(r_g)$.

Equation (9) gives mass M as

$$\begin{aligned} M(r) = \quad & 16\pi^2 \int_0^r r^2 dr \\ & \times \int_0^{\Psi(r)} f(\varepsilon)[2(\Psi - \varepsilon)]^{1/2} d\varepsilon. \end{aligned} \tag{11}$$

From equation (11), the differential energy distribution is

$$\frac{dM(\varepsilon)}{d\varepsilon} = f(\varepsilon) g(\varepsilon), \tag{12}$$

where

$$g(\varepsilon) = 16\pi^2 \int_0^{r_m(\varepsilon)} [2(\Psi - \varepsilon)]^{1/2} r^2 dr, \tag{13}$$

and $r_m(\varepsilon)$ is maximum radius that can reached by a particle of the binding energy ε.

If we assume the density profile is the NFW profile, we get $dM/d\varepsilon$ from the equations (10), (12), and (13) for the NFW profile.

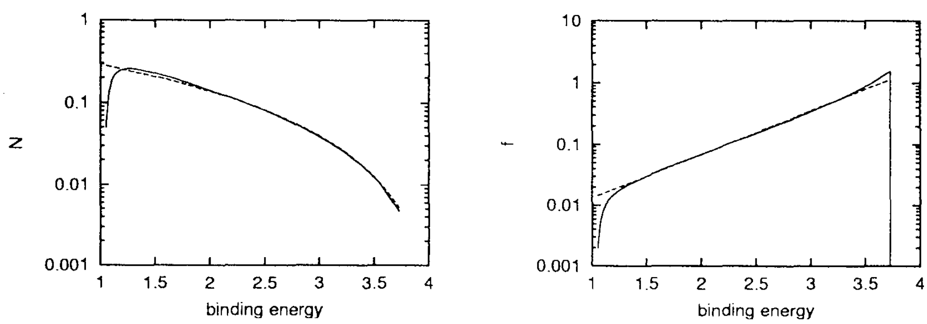


Figure 2 (left) shows the fractional mass distribution of NFW (solid line) obtained in this way and $N(\varepsilon)$ (dashed line) given by equation (8). $N(\varepsilon)$ given by equation (8) is consistent with the NFW profile.

4 The fractional mass distribution, density profile, and the distribution function

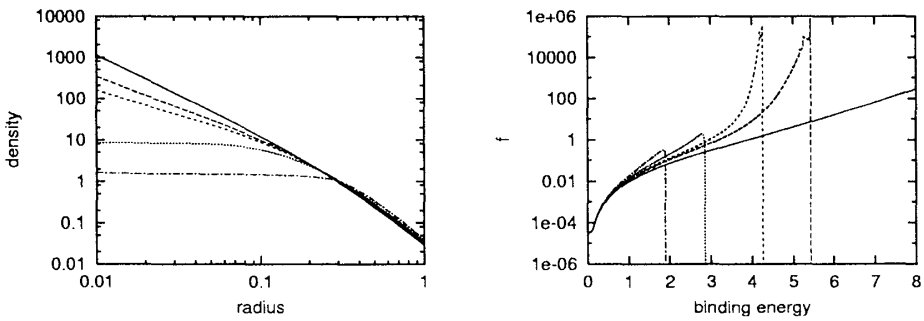
We find that the NFW profile satisfies the fractional mass distribution given by equation (8) with $q \simeq 0.6 - 0.7$. We study how the density profile changes when we change the parameter q and ε_0 in equation (8). In this study, we use an iteration method as shown in the next subsections.

4.1 The iteration method

Binney (1982, and see also Binney and Tremaine 1987) studied the phase space structure of galaxies of which surface brightness is the de Vaucouleurs' $r^{1/4}$ law. We apply his method to our study of the phase space structure of dark halo with the NFW profile. We obtain the density profile and the phase space distribution which are consistent with equation (8), using the iteration method.

4.2 The density profile and the distribution function

Using the Binney's iteration method, we reconstruct mass density profile . We confirm that $dM(\varepsilon)/d\varepsilon$ characterizes well the NFW profile.



In figure 3 (left), we show density profiles for different q but $\varepsilon = 1.4$. Smaller q (e.g. $q = 0.5$) results in shallower core in the inner region. On the other hand, larger q ($q > 0.67$) makes a cusp steeper than the NFW profile, density profile approaches $\rho \propto r^{-2}$ in the inner part, for $q \longrightarrow 1$.

For various values of ε_0, the density profiles are similar to the NFW for $q = 0.6$. Absolute value of the density depends on ε_0. Therefore, the slope of the cusp depends on only q, not ε_0.

Figure 3 (right) shows the distribution function, f, obtained by the iteration method for various values of q . These curves show the same dependence on ε in $0 < \varepsilon < 1$. Peak values of f are different each other. We also show the Boltzmannian distribution for comparison in figure 3 (right). For large q, peak value of $f(\varepsilon)$ and the maximum binding energy of the distribution become large. We have shown that the density profile with large q have the steep cusp. Therefore, sharp peak of $f(\varepsilon)$ corresponds to the steep cusp.

5 Summary and Discussion

We analyze the universal density profile of dark halo proposed by NFW and its differential energy distribution. Our main results are summarized as follows.

1. We study the fractional mass function $N(\varepsilon)$ for dark halo obtained by our numerical simulation and find its analytical formula which is the equation (8).
2. We show that the NFW profile is given by the equation (8).
3. We show that the slope of the cusp in the density profile changes with a value of the parameter q in the analytical formula.

We can regard that $N(\varepsilon)$ shows the statistical property of the NFW profile. If the NFW profile is universal, $q = 0.6 - 0.7$ in equation (8). Different q makes slope of a cusp different. Since q plays an important role, we should make clear what physical process determines q. Recent high resolution numerical simulation (Okamoto

& Habe 1999, 2000) shows the steeper cusp, $\rho \propto r^{-1.5}$, than the the NFW profile. This profile corresponds to $q = 0.75 - 0.8$. Isothermal profile, $\rho \propto r^{-2}$, corresponds to $q = 1$.

We study $f(\varepsilon)$ for the NFW profile. The formula of this is not isothermal one nor the King formula $f_K \propto e^{\varepsilon/\sigma^2} - 1$. $f(\varepsilon)$ for the NFW profile have the energy cut off at the high end of ε. We should study the reason why $f(\varepsilon)$ has such a form. Lynden-Bell (1967) studied distribution function $f(\varepsilon)$ of a virialized system. Maximizing the Boltzmann entropy of the system, resulting distribution is isothermal profile, $\rho \propto r^{-2}$. In this case the system has infinite extend and infinite mass. This is not realistic for astronomical objects. Cosmological simulations have shown that galaxies and clusters of galaxies formed in these simulations have more rapid radial decline than isothermal in the outer part.

We note that the form of equations (8) is similar to the Tsallis' escort distribution,

$$\frac{P(E,T')}{P(0,T')} = \left[1 - (1-q)\frac{E}{T'}\right]^{q/(1-q)}, \tag{14}$$

where T' is temperature parameter and q is entropic index (Tsallis, Mendes, & Plastino, 1998).

We should study the reason why dark halo has the value of $q = 0.6 - 0.7$ in the hierarchical clustering scenarios. It is interesting to study the differential energy distribution of self-gravitational system formed in a circumstance without hierarchical clustering to make clear mechanism what determines q of the gravitational system.

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宇宙 N 体数値実験を実施し，ダークマター・ホールの微分エネルギー分布を調べる。数値実験で得られたホール内のダークマター微分エネルギー分布に対する解析的公式を与える。その解析的な表式から，ナヴァロ・フレンク・ホワイト(NFW)のプロファイルで記述される密度プロファイルを再構成する。NFW プロファイルは，我々の微分エネルギー分布の解析的公式と矛盾しない。微分エネルギー分布の我々の解析的表式が有するパラメータが，ダークホールの内部先端の傾きと関連していることを発見した。高結合エネルギーに鋭いカットオフのある NFW プロファイル分布関数が得られる。解析的公式の物理的理由付けを議論をする。