カオス系量子ビリアードにおける量子輸送
Quantum Transport in Classically-Chaotic Quantum Billiards

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ポアンカレにより指摘されケルヴィン卿によりエルゴード性の根拠として考察されたカオスを示すビリアードの物理学は、ナノテクノロジーの出現により新局面を迎えている。ここでは、電子のド・ブロイ波長がビリアードの特徴的長さ（サブミクロンスケール）より充分小さい場合、つまり、半古典領域の電子の量子カオス（カオスの量子論的兆候）を考察する。

まず最初に、アンチドット超格子（シナイビリアードの量子版）をとりあげる。半古典論を用いて弱磁場中の充分カオス的な3角格子（超格子）の伝導率を解析する。孤立した周期軌道理論（最も単純な鞍点近似）を採用すると、電気抵抗は磁場の関数として単調な減少を示す。しかし、磁場変調により生じる軌道分岐（あるいは軌道融合）の効果を取り入れると、スパイク状の顕著な抵抗ピークが生まれ、これはピーク位置においても、ピークの高さにおいても最近のNECグループの実験結果と一致する。

続いて、リード線のついた3次元量子ドット（量子ビリアード）のコンダクタンスを考察する。位相空間の混合性を反映して、3次元ビリアードではアーノルド拡散が期待できる。まず、この系に対するコンダクタンスを半古典論で導く。その表式を見ると、部分的にあるいは完全にエルゴード性の破れた3次元ビリアード（たとえば、SU(2)対称性を持つビリアード）では、弱磁場をかけると、コンダクタンスのフェルミエネルギーへの依存性のベキ指数が増えることがわかる。このため、3次元量子ドットでは弱磁場によるコンダクタンスの増加は、弱局在効果だけで無くアーノルド拡散にも起因する。

Dynamics of billiard balls and its role in physics have come to receive a very wide attention since the monumental lecture by Lord Kelvin at the turn of the 19-th century. On April 27(Fri.), 1900, at Royal Institution of Great Britain, he delivered a lecture entitled as "The 19-th century clouds over the dynamical theory of heat and light" . The first cloud was a question on the existence of **ether** propagating the light. He denied a possibility of the earth to move through the ether. The second one was a question on the validity of Maxwell-Boltzmann (MB) distribution leading to the equi-partition of energy and he ultimately doubted the **ergodicity hypothesis** behind MB distribution. Five years later after Kelvin's lecture, the first cloud was swept away by Einstein's "special theory of relativity". By the way, how did the second cloud disappear?

The ergodicity hypothesis means an assumption that a long time average of a given physical quantity should accord with its phase-space average. Choosing as an example the ideal gas consisting of atoms with no internal degree of freedom, Lord Kelvin addressed a discrepancy of the ratio of two kinds of its specific heats(at constant pressure and at constant volume) between the theoretical issue predicted by the equi-partition of energy and the experimentally-observed value. Noting further this discrepancy to be enhanced for molecules with rotational degrees of freedom as well as translational ones, he insisted on a breakdown of the ergodicity ansatz.

To demonstrate more explicitly the breakdown of ergodicity hypothesis, Kelvin investigated a point-particle motion bouncing with the hard wall of a triangular billiard. Measuring each line segment between successive bouncings and each reflection angle at the wall repeatedly, he showed the breakdown of equi-partition of energy, i.e., inequivalence between long-time averages of transverse and perpendicular components of kinetic energy. Next, he chose a flower-like billiard, carried out a similar pursuit, and again showed the long-time averages of radial and angular parts of kinetic energy not to satisfy the equi-partition of energy. This investigation implies a birth of physics of billiards. Physics of billiards was thus launched on April 27, 1900.

Hence, in order to sweep the 19-th cloud over ergodicity hypothesis, it had become indispensable to envisage complex features of nonlinear dynamics of a particle in billiards. In particular, an accumulation of studies on billiards (by Birkhoff, Krylov, Sinai and others) during the 20-th century since Kelvin's lecture were devoted to those on nonintegrable and chaotic billiards with the shape like the flower-like billiard. In fact, concave and convex billiards as prototypes of conservative chaotic systems have received a growing theoretical and experimental interest in the fields of nonlinear dynamics and statistical mechanics [1]. Dynamics of a billiard ball is chaotic, i.e., extremely sensitive to initial conditions: a very slight variation in initial coordinates or momenta yields a thoroughly different orbit. The sensitivity to initial conditions causes a cluster of initial points with similar initial conditions to exhibit mixing in phase space as time elapses, and thereby to show an ergodic property. In this way, chaotic billiards have resolved the second cloud of Kelvin, and provide an essential playground by which to consolidate the foundation of statistical mechanics.

Since new turning years around 1990, the physics of billiards has developed in every direction of science and technology. Billiards are nowadays fabricated as **quantum dots** or **antidots** in ballistic microstructures where the system size is much less than the mean free path $\ell(\sim 20\mu\mathrm{m})$ and larger than the Fermi wavelength $(\lambda \sim 50\mathrm{nm})$[2]. One can envisage quantum-mechanical manifestations of chaos of billiard balls (: electrons)[3]. Many puzzling experiments on resistance fluctuations in these quantum billiards are raising a fancy of exploring the effect of billiard-ball dynamics on ballistic quantum transport.

In the following, we show two interesting themes bridging between nonlinear dynamics and quantum transport in these mesoscopic billiards: (i)For antidot lattices, the experimentally-observed anomalous fluctuations in the magneto-resistivity are attributed to orbit bifurcations; (ii)for 3-dimensional quantum dots, the Arnold diffusion is pointed out to have a possibility to yield the enhanced magneto-resistance beyond the weak localization correction. To mention in detail,

(i)Within a semiclassical framework, we have first analyzed two parts of conductivity of fully-chaotic triangular antidots in the low but intermediate magnetic field. Taking into account both the smooth classical part evaluated by mean density of states and the oscillation part evaluated by periodic orbits, we find that resistivity of the system yields a monotonic decrease with respect to magnetic field. But when including the effect of orbit bifurcation due to the overlapping between a couple of periodic orbits, several distinguished peaks of resistivity appear. The theoretical results accord with the interesting issue of the recent experiment of NEC group;

(ii)We have also investigated the semiclassical conductance for three-dimensional (3-d) ballistic open billiards. For partially or completely broken-ergodic 3-d billiards such as SO(2) symmetric billiards, the dependence of the conductance on the Fermi wavenumber is dramatically changed by the lead orientation. Application of a symmetry-breaking weak magnetic field brings about mixed phase-space structures of 3-d billiards which ensures a novel Arnold diffusion that cannot be seen in 2-d billiards. In contrast to the 2-d case, the anomalous increment of the conductance should inevitably include a contribution arising from Arnold diffusion as well as a weak localization correction.

Thus, while classical billiards launched by Kelvin are means by which to verify the foundation of statistical mechanics, quantum billiards (quantum dots and antidots) fabricated by nanotechnology provide stages where to capture via quantum transport quantal signatures of orbital bifurcations, Arnold diffusions, and other interesting phenomena in nonlinear dynamics. Details of the present talk are given in Refs. [4].

This review talk has emerged from joint works with my former student, Dr. Jun Ma with whom I had enjoyed a very fruitful period.

References

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