Quantum Transport in Classically-Chaotic Quantum Billiards

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Dynamics of billiard balls and its role in physics have come to receive a very wide attention since the monumental lecture by Lord Kelvin at the turn of the 19-th century. On April 27(Fri.), 1900, at Royal Institution of Great Britain, he delivered a lecture entitled as "The 19-th century clouds over the dynamical theory of heat and light". The first cloud was a question on the existence of ether propagating the light. He denied a possibility of the earth to move through the ether. The second one was a question on the validity of Maxwell-Boltzmann (MB) distribution leading to the equi-partition of energy and he ultimately doubted the ergodicity hypothesis behind MB distribution. Five years later after Kelvin's lecture, the first cloud was swept away by Einstein's "special theory of relativity". By the way, how did the second cloud disappear?

The ergodicity hypothesis means an assumption that a long time average of a given physical quantity should accord with its phase-space average. Choosing as an example the ideal gas consisting of atoms with no internal degree of freedom, Lord Kelvin addressed a discrepancy of the ratio of two kinds of its specific heats(at constant pressure and at
constant volume) between the theoretical issue predicted by the equi-partition of energy and the experimentally-observed value. Noting further this discrepancy to be enhanced for molecules with rotational degrees of freedom as well as translational ones, he insisted on a breakdown of the ergodicity ansatz.

To demonstrate more explicitly the breakdown of ergodicity hypothesis, Kelvin investigated a point-particle motion bouncing with the hard wall of a triangular billiard. Measuring each line segment between successive bouncings and each reflection angle at the wall repeatedly, he showed the breakdown of equi-partition of energy, i.e., inequivalence between long-time averages of transverse and perpendicular components of kinetic energy. Next, he chose a flower-like billiard, carried out a similar pursuit, and again showed the long-time averages of radial and angular parts of kinetic energy not to satisfy the equi-partition of energy. This investigation implies a birth of physics of billiards. Physics of billiards was thus launched on April 27, 1900.

Hence, in order to sweep the 19-th cloud over ergodicity hypothesis, it had become indispensable to envisage complex features of nonlinear dynamics of a particle in billiards. In particular, an accumulation of studies on billiards (by Birkhoff, Krylov, Sinai and others) during the 20-th century since Kelvin's lecture were devoted to those on nonintegrable and chaotic billiards with the shape like the flower-like billiard. In fact, concave and convex billiards as prototypes of conservative chaotic systems have received a growing theoretical and experimental interest in the fields of nonlinear dynamics and statistical mechanics [1]. Dynamics of a billiard ball is chaotic, i.e., extremely sensitive to initial conditions: a very slight variation in initial coordinates or momenta yields a thoroughly different orbit. The sensitivity to initial conditions causes a cluster of initial points with similar initial conditions to exhibit mixing in phase space as time elapses, and thereby to show an ergodic property. In this way, chaotic billiards have resolved the second cloud of Kelvin, and provide an essential playground by which to consolidate the foundation of statistical mechanics.

Since new turning years around 1990, the physics of billiards has developed in every direction of science and technology. Billiards are nowadays fabricated as quantum dots or antidots in ballistic microstructures where the system size is much less than the mean free path $\ell(\sim 20\mu m)$ and larger than the Fermi wavelength ($\lambda \sim 50nm$)[2]. One can envisage quantum-mechanical manifestations of chaos of billiard balls (: electrons)[3]. Many puzzling experiments on resistance fluctuations in these quantum billiards are raising a fancy of exploring the effect of billiard-ball dynamics on ballistic quantum transport.

In the following, we show two interesting themes bridging between nonlinear dynamics and quantum transport in these mesoscopic billiards: (i) For antidot lattices, the experimentally-observed anomalous fluctuations in the magneto-resistivity are attributed to orbit bifurcations; (ii) for 3-dimensional quantum dots, the Arnold diffusion is pointed out to have a possibility to yield the enhanced magneto-resistance beyond the weak localization correction. To mention in detail,

(i) Within a semiclassical framework, we have first analyzed two parts of conductivity of fully-chaotic triangular antidots in the low but intermediate magnetic field. Taking
into account both the smooth classical part evaluated by mean density of states and
the oscillation part evaluated by periodic orbits, we find that resistivity of the system
yields a monotonic decrease with respect to magnetic field. But when including the
effect of orbit bifurcation due to the overlapping between a couple of periodic orbits,
several distinguished peaks of resistivity appear. The theoretical results accord with the
interesting issue of the recent experiment of NEC group;

(ii) We have also investigated the semiclassical conductance for three-dimensional (3-d)
ballistic open billiards. For partially or completely broken-ergodic 3-d billiards such as
SO(2) symmetric billiards, the dependence of the conductance on the Fermi wavenumber
is dramatically changed by the lead orientation. Application of a symmetry-breaking weak
magnetic field brings about mixed phase-space structures of 3-d billiards which ensures a
novel Arnold diffusion that cannot be seen in 2-d billiards. In contrast to the 2-d case, the
anomalous increment of the conductance should inevitably include a contribution arising
from Arnold diffusion as well as a weak localization correction.

Thus, while classical billiards launched by Kelvin are means by which to verify the
foundation of statistical mechanics, quantum billiards (quantum dots and antidots) fabri-
cated by nanotechnology provide stages where to capture via quantum transport quantal
signatures of orbital bifurcations, Arnold diffusions, and other interesting phenomena in
nonlinear dynamics. Details of the present talk are given in Refs. [4].

This review talk has emerged from joint works with my former student, Dr. Jun Ma
with whom I had enjoyed a very fruitful period.

References

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