Games as dynamical systems

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本稿の概要

我々は他者との係わりの中、様々な状況で様々な意思決定を行う。我々が選択した行動は他人に影響を与え、逆に我々自身も他人の行動から影響を受ける。こういった、個人間、企業間、国家間、（あるいは生物間）など複数の意志決定主体が関わる状況を数学的言語で表現し、そして「意思決定を行う複数の主体の合理的な行動とは何か」を問うのがゲーム理論である。

生態系型システムの進化ゲームの理論も含めた従来のゲーム理論のモデルでは、設定としてのゲームは固定的（静的）なものが使われることが多かった。しかし、現実の人間社会や生物集団での相互作用は時間構造を含むことが普通で、時間的な側面が重要な役割を果たすことは珍しいことではない。例えば現実世界のプレーヤーは、限られた情報の下でリアルタイムに意思決定を行い、相互作用を通して認知構造を進化させ、社会のルールを形成・発展させていく。利得行列などによる相互作用の代数的・静的記述だけでこういった現象を取り扱うのは困難である。

本稿では、まず、ゲームにおける時間構造を考察することの意義について議論する。次に、ゲームの時間構造を自然な形で記述することを目的とした枠組み ——「力学系ゲーム(dynamical systems game)」を紹介し、その具体的な適用として社会的ジレンマ問題の分析を行う。そして最後に、社会的ジレンマ状況における協力の発生とその維持の問題について、従来のゲーム理論の枠組みでは原理的に理解し得ないメカニズムについて議論する。

Formalization of games and description of players

The main topic of this paper is to discuss formalization of temporal structures in games, which become essential especially when we consider the formation and the development of rules among players through learning or evolution.

As is known well, Neumann and Morgenstern, in their book "Theory of Games and Economic Behavior (1944)", introduced the theory of games, a powerful framework to deal with the situation of multiple conflicting decision-makers. In this book, they referred to the structural and conceptual difference between the problems a participant in a social exchange economy meets and ordinary optimization problems (those that a isolated person, such as Robinson Crusoe, meets). They discussed the mathematical structure for the participants in a social exchange economy: *"If two or more persons exchange goods with each other, then the result for each one will depend in general not merely upon his own actions but on those of the others as well."* And in this case, *"all maxima are desired at once."* Every individual *"can determine the variables which describe his own actions but not those of the others,"* while, in addition, *"those **alien variables** cannot, from his point of view, be described by statistical assumptions."*

In game theory, algebraic payoff matrices or game trees are adopted for the formalization of game-like interactions, in which the above "alien variables" are called *strategies* (complete plans). At this point, we should note that there is no description for decision-making mechanisms of players in game theory, which allows us to center on the investigation of the relation between decision makers. Most works on game theoretical models investigate the solution of games on the assumption that players are *rational.* Rational players are supposed to have informational and computational power unlimitedly, and assume that others are also rational [Simon 1957]. And then, the rational solution of games are supposed to be Nash Equilibria, in which every player cannot make his profit higher by changing his strategy.

Solution concept vs. description of players and games

Solution concept cannot be discussed without the description or assumption of players. As players in the real world cannot be rational, if *economics* is the one for the *real economies*, an important question is how real agents behave in game-like situations. Another important question is whether bounded rational players can reach Nash equilibria ultimately by *learning* or *evolution*.

Discussions for the first question are given in the next section. If the answer for the second question is positive, we can safely say Nash equilibrium is a good estimation of the limit of mutual learning process. Actually, the answer should vary according to what kind of game structure we are supposed to deal with. (1) If there are (strong) pure Nash equilibria in a game, what is the result of learning process? According to [Hofbauer 1988] and etc., we can say they are basins of attraction of mutual learning process within Nash equilibria. (2) What if there are only mixed Nash equilibria? Unfortunately, we do not have clear answer for this question at this point, but the conjecture of [Sato 2001] indicates that either chaotic behaviors or Nash equilibria are the only possibility.

Bounded rationality.

Regarding the first question about the behavior of bounded-rational agents, there are two issues to be investigated, which is also discussed in [Arthur 1994]; (1) our informational / computational abilities, and (2) our inherent way of reasoning. For example, there exists a theoretical solution for the game of chess from the viewpoint of game theory. Game theory can derive this conclusion on the assumption that both players consider all the possible cases caused by all the combinations of both players' actions deductively, and that players can choose the complete, best plan of this game. However, possible moves in chess are said to be 100^{120}, which is impractical to trace. In the first place, this kind of deductive approach does not always seem to be our daily decision-making process.

How humans in the real world reason has been discussed in psychological studies [De Groot 1965][Clark 1993]. According to these studies, we are moderately good at deductive logic, and superb at seeing or recognizing or matching patterns -- behaviors that confer obvious evolutionary benefits. When we encounter complicated problems, we look for some patterns; simplify the problem by using patterns to construct temporary internal models or hypotheses to work with. And then we carry out localized deductions based on our current hypotheses and act on them. Through the feedback from the environment, we strengthen or weaken our beliefs in our current hypotheses, discard ones that do not work anymore, and replace them with new ones. This is rather inductive than deductive.

Recently, there have been a lot of attempts to create new economics based on players with inductive reasoning. However, because bounded rationality has arbitrary level in boundedness, there often exists a fear that a model using such agents may become ad hoc one. One of the most successful frameworks based on non-rational agents is the Evolutionary Game by Maynard Smith [Smith 1973]. In a sense, Evolutionary Game is an extreme framework where inductive reasoning process completely dominates deductive one. It has been developed up to now by many researchers both in economics and in biology (e.g. [Kandori 1995]).

Formalization of players and games

As attempts of formalization of games, there have been two directions of researches developed these days. One direction is to describe players' decision-making process by logic [M. Kaneko 1996], which enables us to discuss players' deductive / inductive reasoning process from a viewpoint of multi-modal logical process. The other direction is to describe games as dynamical systems, in which players are also dynamical systems that autonomously learn or evolve. Games / players as dynamical systems are first considered in [Rapoport 1947] and [Rashevsky 1947], and recently have been developed in [Ikegami 1999], [Rossler 1994] and [Akiyama 2000], where players' decision-making process are given by dynamical systems, and players' decisions affect the dynamics in game environment, which will affect the players decision making.

Dynamical systems game

In [Akiyama 2000], K. Kaneko and I introduced "dynamical systems (DS) game framework" to investigate games. A merit of the use of DS game framework is that it can naturally deal with space-time structures that should exist in the real games. Another merit is that DS game enables us to investigate, from viewpoints of dynamical systems, the developmental process of social phenomena, which exist everywhere in our world but whose mathematical structure has not been investigated well.

For example, fishers of Valenca a village in Brazil shared a single fishery early in this century. They gradually made rules to divide their estuary into various areas and allocated those areas through mutual communications. They succeeded in avoiding tragedy by improving the rules they used. Another example involves farmers of rice paddies in Miyawaki in Japan. For hundreds of years, the farmers have succeeded in developing their rules to allocate water. Under the rules, dozens of farmers change their own rice paddies every 10 years. In both cases, people used the information on the physical environment, and succeeded in forming and maintaining cooperative states by time and area arrangement of their roles.

For games in our real world the space time structure, such as the geography of the estuary, the rise and fall of the water, the growth of the pasture, and so on, is also important. And we sometimes base cooperation on strategies involving space-time arrangements, for example allocating the divided areas to fishermen, or using the common resources alternately, and so on.

Example of DS game

In [Akiyama 2002], K. Kaneko and I discussed the formation and the development of rules in communities under social dilemma, by introducing an example of DS games, which we call the "Lumberjacks' Dilemma, or LD, game."

Its schematic story is the following. There is a wooded hill where several lumberjacks live. The lumberjacks can maximize their collective profit if they cooperate by waiting until the trees are grown fully before felling them and sharing the profits. However, the lumberjack who fells a given tree earlier than the others takes the entire profit for that tree. Thus each lumberjack can maximize his personal profit by cutting trees earlier, and theoretically this is the rational behavior. If all the lumberjacks do this, however, the hill will go bare, and there will eventually be no profit. These circumstances inevitably bring about a dilemma.

Evolutionary simulations of this LD game, where game environment is described by a certain dynamical system and players are described by evolving dynamical systems, showed that there can be several types of cooperation rules to manage the dynamics of the tree. Presented in Fig. 1 is the transition among such cooperation rules. As is seen in the fitness chart of the LD game, which is placed at upper left, the fitness value of the generation begins to rise step-by-step. At each epoch A, B, C and D with stepped plateaus, one type of game dynamics specific to each epoch arises. We name these game dynamics as type A, B, C and D, respectively. In these diagrams, the horizontal axis shows the round (time). The white tile represents the action of "cutting," while the black tile represents "waiting." As is shown in the lower-left figure of epoch A, the players assume an action cycle with a period of 5 – "wait, cut, wait, cut and wait." The lumberjacks collect the lumber while allowing the tree to grow appropriately. As generations pass, the dominant action dynamics shift to type B and to type C, and then to type D, as shown in this figure to develop the rule to cooperate in a community. These rules of cooperation are formed and shift with generations in spite of the dilemma underlying our LD game. Stability of the dynamics of games formed by certain cooperation rules can be analyzed by the introduction of a kind of bifurcation diagram as seen in Fig. 2, whose control parameters are players' strategies that change through mutual interactions.

Why dynamical systems approach?

If we try to obtain a certain amount of resources without fail, we actually need to manage the dynamics of the resources. In this case, it is necessary to have a certain agreement for cooperation. As a result, we will begin to take actions such as "raising the resources jointly and then consuming them together" or "raising the resources and consuming them alternately." In many real cases, it is essential to consider the space-time structure in the environment, to avoid tragedy within social dilemmas.

One merit of the introduction of the dynamical-systems approach into the analysis of games (and players) is that we can definitely describe the nature of dynamics and can cope with the issues that are relevant to the management of dynamical resources. In the traditional game theory, however, we cannot discuss such cooperation in the form of dynamics, let alone explain the stability of the cooperative state. In the first place, it neither can describe, in principle, the temporal change in resources nor can it show the effect of the dynamics of the game environment.

As long as our world includes space-time structure by nature, there must be cases where the dynamical systems approach for games becomes a powerful theoretical tool to study various types of problems produced by multiple decision-makers.

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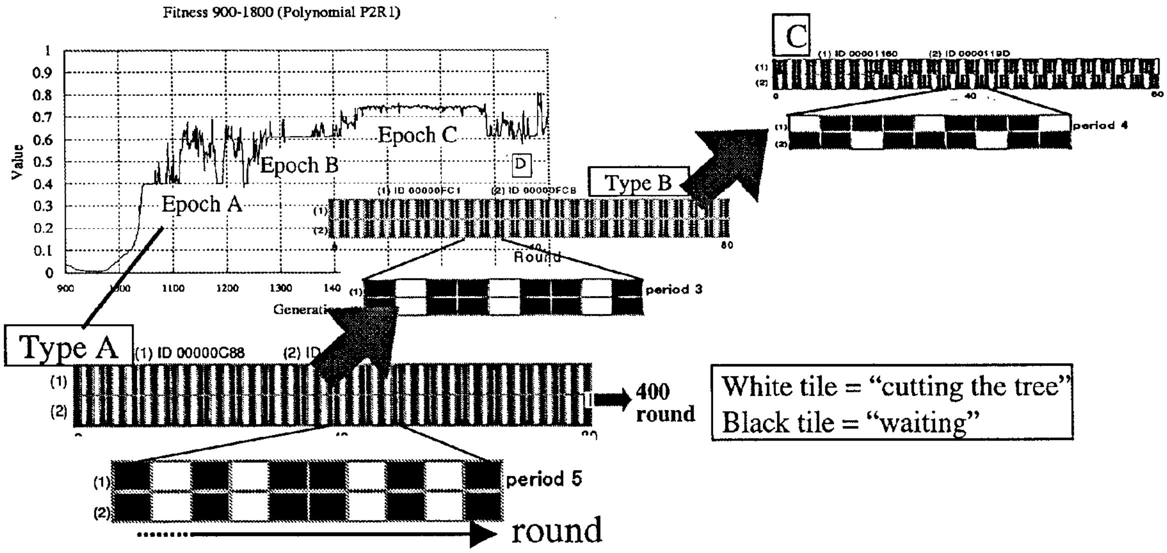

Fig. 1: Formation and development of cooperation rules

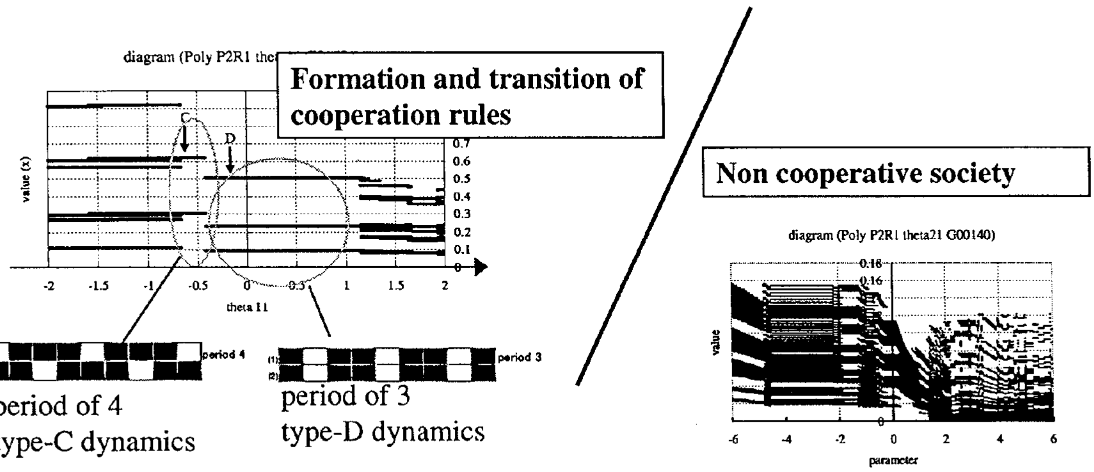

Fig. 2: Bifurcation diagram with players' strategies as control parameters