Non-linear Logit Model for High Frequency Currency Exchange

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円ドル為替市場における高頻度データの確率構造を捉える目的から、非線形のロジットモデルを提案し解析した。これは二値データの解析に対して有効なロジットモデルを、非線形に拡張したモデルである。AIC 規準でのモデル選択により、２つの独立した円ドル為替市場データにおける上下運動の確率構造が、５次のモデルによって最良に表現されることを示した。この非線形ロジットモデルにおける他の現象の時系列への適用可能性については、現在解析中である。

We analyze tick-by-tick data, the most high frequency data available, of yen-dollar currency exchange rates. There exists a non-trivial structure in conditional probabilities in binarized data indicating the direction of up and down movement of prices, which is not apparently seen from the price change itself. We propose here a non-linear logit model to capture this structure. This new model has overcome some of the shortcomings of the conventional analysis such as AR models and logit models, and can successfully show that the structure is such that it refers up to previous few minutes. The empirical result is consistent with dealers' perceptions that their strategies are slowly changing in the time scale of few minutes.

We use two data sets of tick-by-tick "trade" data for the yen-dollar exchange rate for the period of 10/26/1998 to 11/30/1998 (data set A) and 1/4/1999 to 3/12/1999 (data set B). The time series data sets are composed of values $Y(t)$ of yen value at "tick step" t. These are the same data sets we have studied earlier [1]. We here binarized the data to extract the direction of up down movement of price change. To focus on up down movement, we will disregard the cases, which the prices stay the same value between consecutive two ticks.

As we have studied in [1][2], these data show a characteristic structure in the conditional probabilities of high frequency binary data $X(t)$, which represents the dependency relation between a current step and several steps in the past. This is summarized in Table 1 of [1].

The logit model is known to be suitable for binary analysis. However, we have found that a conventional logit model was not sufficient for our data due to its non-linear behavior. This motivated us to develop a new extended non-linear logit model to

reproduce the binary probabilistic structure. The model of the order k is defined by the following equation.

$$\ln\left[\frac{P(x(t)=1|x(t-1),\ldots,x(t-k))}{1-P(x(t)=1|x(t-1),\ldots,x(t-k))}\right] = q_0 + \sum_{i_1=1}^{k} q_{i_1} x(t-i_1)$$

$$+ \cdots + \sum_{i_1,i_2\cdots,i_k=1}^{k} q_{i_1 i_2 \cdots i_k} x(t-i_1)x(t-i_2)\cdots x(t-i_k).$$

Then we select the appropriate model according to AIC (Akaike Information Criterion) which gives the smallest possible order by minimizing a penalty function defined by -2(Maximum log likelihood)$+2$(the number of the model parameters). AIC values of the model give a minimum value at the 5th order for both data sets and the function shapes of AIC values are similar, although conventional models do not have minimum AIC value at the same order for both of the data. Therefore the non-linear logit model with referring up to previous 5 states, equivalent to about few minutes, is appropriate according to AIC. The 5th order model can also capture the probabilistic structure very well.
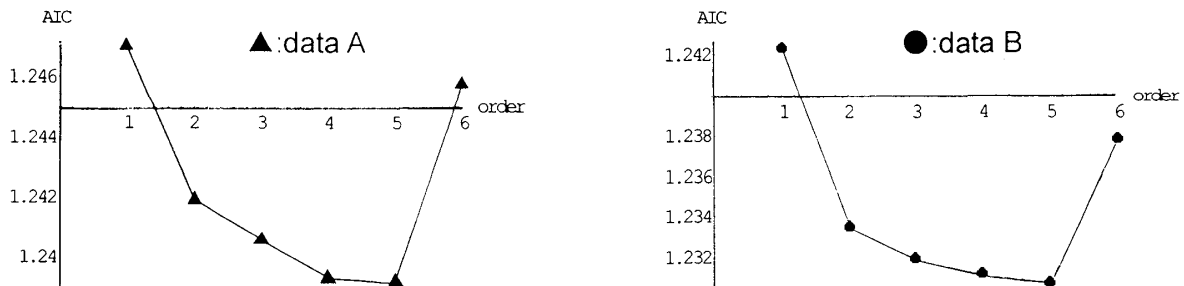
In this paper, we have analyzed tick-by-tick data, the most high frequency data available, of yen-dollar currency exchange market using a non-liner logit model. This new model has been able to capture the non-trivial probability structure in the binary currency exchange data, which was impossible using the conventional methods such as AR models or linear logit models. This is one indication that this new non-linear logit model can be a useful tool for analyzing wide range of binary time series with non-trivial dynamical structures. Applications to other time series to prove the model's potential are currently explored.

[1] T. Ohira, N. Sazuka, K. Marumo, T. Shimizu, M. Takayasu and H. Takayasu, Predictability of Currency Market Exchange, Physica A, 308, N1-4, pp. 368--374, 2002.
[2] N. Sazuka, T. Ohira, K. Marumo, T. Shimizu, M. Takayasu and H. Takayasu, Physica A, 324, N1-2, pp. 366--371, 2003.