Simulations of financial markets in spin models

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概 要

Bornholdtによって提唱された経済市場モデルを利用してさまざまなパラメータについて研究を行なった。磁化の時間差によって提議されたリターンのヒストグラムはファットテイルとなっており、そのテイルは小さな格子サイズほど顕著である。また、リターンの絶対値の自動相関関数は長時間相関を示す一方、リターン自身の相関は短時間相関でしかも負の相関となっていることがわかった。

Bornholdt [1] proposed a simple spin model based on the Ising model. The model includes two interactions: the nearest neighbor one which exists in the Ising model and the global one which couples to the magnetization. He demonstrated that the model exhibits interesting behavior similar to the real economic markets. The model has been further studied by others[2, 3].

Let us consider an $L \times L$ square lattice. Each spin $S_i(t)$ is assigned to a site of the lattice. The spins $S_i(t)$ have two states ± 1 and are updated according to the following probability p.

$$S_i(t+1) = 1 \quad \text{with} \quad p = \frac{1}{1+\exp(-2\beta h_i(t))} \tag{1}$$

$$S_i(t+1) = -1 \quad \text{with} \quad 1-p \tag{2}$$

where

$$h_i(t) = \sum_{j=1}^{N} J_{ij} S_j - \alpha S_i |M(t)| \tag{3}$$

and

$$M(t) = \frac{1}{N} \sum_{i=1}^{N} S_i(t). \tag{4}$$

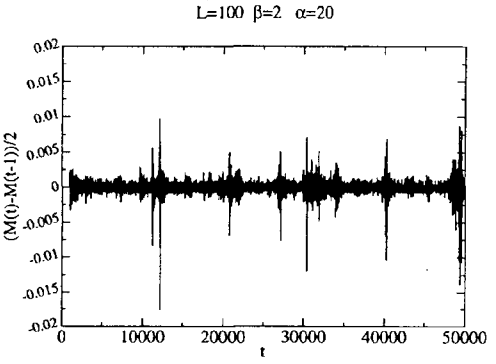

図 1: Time history of the return $r(t)$ for $L = 100$ at $\alpha = 20$ and $\beta = 2$.

We take the nearest neighbor interactions $J_{ij} = J$ and $J_{ij} = 0$ for all other pairs. The first term of eq.(3) is considered to be a local interaction. On the other hand the second term is a global interaction coupling to the magnetization. Following to [3] we define the return $r(t)$ by the time different of the magnetization:$(M(t) - M(t-1))/2$. We performed 10^6 updates for each parameter set. Figure 1 shows the time history of $r(t)$ for $L = 100$ at $\alpha = 20$ and $\beta = 2$. The coupling J is set to one. We can see intermittent phases similar to the ones observed also in the real financial markets.

Figure 2 shows the histograms of return for different lattice sizes. We see that the fat tail becomes more dominant for a small lattice.

Figure 3 compares histograms for $\alpha = 5$ and 30. We tried to fit the histogram by the q-Gaussian function[4]

$$e_q^{-a_q x^2} = \frac{1}{[1+(q-1)a_q x^2]^{\frac{1}{q-1}}} \tag{5}$$

which is derived from the nonextensive statistical

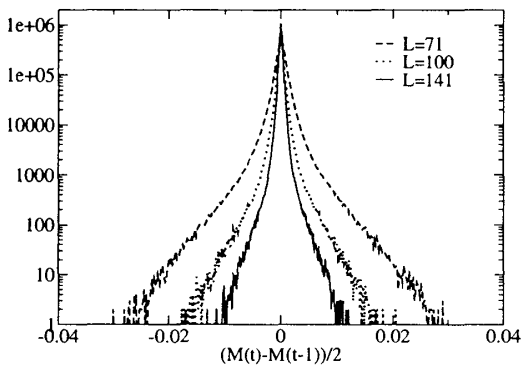

図 2: Histograms of return for $L = 71, 100$ and 141.

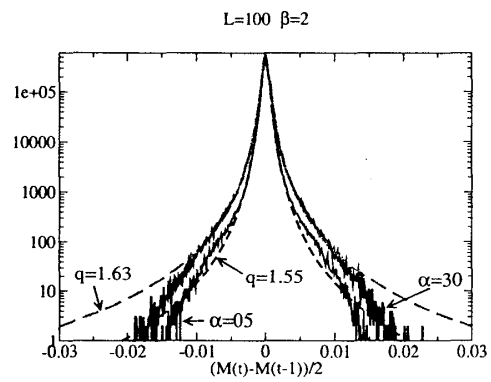

図 3: Histograms of return for $\alpha = 5$ and 30. The lines are the results fitted to the q-Gaussian function $e_q^{-a_q x^2}$ defined by eq.(5) .

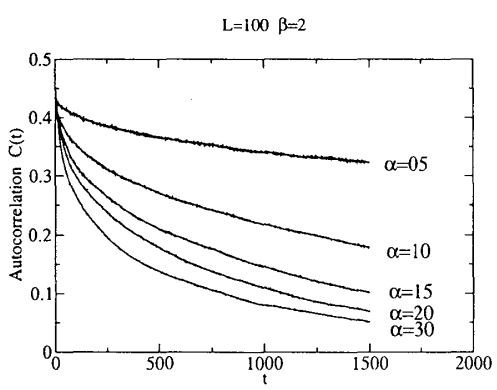

図 4: Autocorrelation of the absolute return $|r(t)|$. The lines correspond to $\alpha = 5$ to 30 from top to bottom.

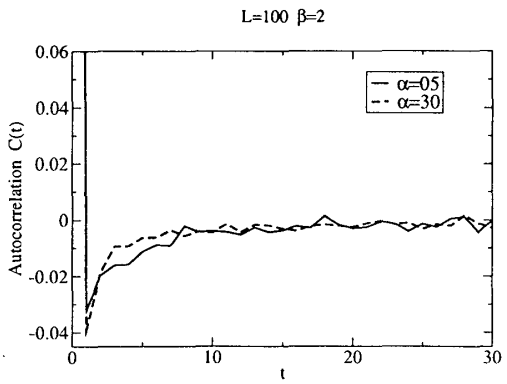

図 5: Autocorrelation of the return $r(t)$. Here we show the results for $\alpha = 5$ and 30. But we observed similar results for other parameters we studied ($5 < \alpha < 30$).

mechanics[5] and can be considered to generalize the Gaussian function. For $q > 1$ this function shows the fat tail. We obtained $q = 1.55(1.63)$ for $\alpha = 5(30)$.

Figure 4 shows the autocorrelation function $C(t)$ of the absolute return $|r(t)|$. We find that the autocorrelation shows the long correlation. The correlation increases as α decreases. We also calculated the autocorrelation for different lattice sizes. The correlation increases as the lattice size increases.

Figure 5 shows the autocorrelation of the return itself for $\alpha = 5$ and 30. Contrast to the autocorrelation of the absolute return, that of the return itself shows no long correlation but short negative correlation, which is actually observed in the real economic markets. We calculated the autocorrelation for other α and found the similar results.

参考文献

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