# A New Model of Labor Dynamics: Ultrametrics, Okun's Law, and Transient Dynamics

Masanao Aoki
Department of Economics, University of California, Los Angeles
and
Hiroshi Yoshikawa
Facluty of Economics, University of Tokyo

## 1 Model

The model is a continous-time Markov chain model of output and growth in Aoki (2992, Sec. 8.6), augmented by layed-off worker pools.

Gross Domestic Product is the sum of outputs of all the sectors

$$Y = \sum_{i=1}^{K} Y_i,$$

with

$$Y_i = c_i n_i, \text{ if } v_i = 0,$$

$$Y_i = c_i(n_i + 1), \text{if } v_i = 1,$$

$$i = 1, 2, \dots, K,$$

where $c_i$ is the productivity of sector $i$, $i = 1, 2, \dots, K$. Arrange $c$s as $c_1 \geq c_2 \geq \cdots, c_K$. The variable $v_i$ indicates if the sector is on normal time or overtime.

Demand for sector $i$ goods:
$s_i Y$, $\sum_i s_i = 1$, where $s_i$ is the share of demand for sector $i$ goods, $\sum_i s_i = 1$. We regard the patterns of shares of demand as proxy for macroeconomic policy.

$$\text{Excess demand for sector } i := f_i := s_i Y - Y_i.$$

The set of sectors with positive excess demands

$$I_+ = \{i; f_i \geq 0, \}$$

and the set with the negative excess demands similarly defined.

All sectors with non-zero excess demands wants to adjust their outputs. However, the sector with the shortest holding time jumps first. Due to externality through GDP, the problem starts anew after the jump.

## 2 Transition rates

Sector $a$ jumps first.

If $f_a < 0$,

$n_a \to n_a - 1$, $u_a \to u_a + 1$.

If $f_a > 0$,

$(n_a, u_a, v_a = 0) \to (n_a, u_a, v_a = 1)$, and

$(n_a, U_a, v_a = 0) \to (n_a + 1, U_a - 1, v_a = 1)$,

where $U_a = u_a + \sum_{j \neq a} u_j/[1 + d(a, j)]$, where $U_a$ is the actual pool of unemployed from which sector a hires one employee, where $d(.,.)$ is the ultrametric distance between the pools of layed off workers of individual sectors. Here, the assumption is that the economy is in high unemployment state, there is no labor hoarding, zero costs for firing or hiring workers. Firms must first post vacancy sign and go on overtime before it can hire one unit of labor some time later.

## 3 Novel Features

Three novel features and results

1. Use of ultrametric distance for subgroups of unemployed
2. Okun's law; $\Delta Y/Y = -x\Delta U/N$, is observed by simulation, with $x$ in the range of 4 to 9 per cent, depending on the demand patters,(inspite of seemingly liner production functions)
3. Effects of higher demands falling on more efficient sectors
   (a) Higher GDP
   (b) Larger coefficient values in the Okun's law
   (c) Faster transients, larger amplitudes of stationary excursions

## 4 Remarks

The main purpose of this paper is to show the effectiveness of demand policy in boosting GDP, but it also shows the effective demand policy also affects the coefficients of Okun's law as well as business cycles. These are new.

It is believed that the use of ultrametrics to formally model distance between clusters of unemployed with different job experienc, human capital, and geographic distance and so on to reflect different probabilities with which new hires are drawn.

This model can be used to measure effectivenss of such projects as retraining of unemployed, although it has not been done in this paper.

Use of continuous-time Markov chain is effective in this model. By dividing state space into the closed subset of states and transient states, expected values spent in transient states before being captured by the closed set, and its variance etc can be calculated.

# Appendix: Okun's law

Okun's law in the economic literature usually refers to changes in gross domestic products (GDP) and unemployment rates measured at two different time instants, such as one year apart. There may therefore be growth or decline in the economies.

To avoid confusing the issues about the relations between GDP and unemployment rates during business cycle fluctuations without growth of GDP, and those with growth, we run our simulations in statioanry states assuming no change in the numbers of sectors, productivity coefficients, or the total numbers of labor force in the model.

We assume that economies fluctuate about its equilibrium state, and call the relation

$$\frac{\Delta Y}{Y_e} = -x\frac{\Delta U}{N},$$

Okun's law, where $Y_e$ is the equilibrium level of GDP, approximated by the central value of the oscillations in Y. Similarly, $\Delta U$ is the amplitude of the business cycle oscilaltion in the unemployed labor force, and $N$ is taken to be $L_e + U_e$, where $U_e$ is approximated by the central value of the oscillations in $U$, and $Y_e$ and $L_e$ are related by the model structure to be

$$Y_e\{\sum_i \frac{s_i}{c_i}\} = L_e.$$

This relation is obtained by summing the relation $Y_i = s_i Y_e = c_i l_i$, where $l_i$ is the equilibrium level of employed in sector $i$, and $L_e := \sum_i l_i$.

The changes $\Delta Y/Y$ and $\Delta U/U$ are read off from the scatter diagrams in simulation after allowing for sufficient number of time to ensure that the model is in "stationary " state.

We report on three cases, Case 1, 3, and 5, with demand share vectors $\mathbf{s} = (5,4,3,2,1,1,1,1)/18$ in Case 1 and 3, and $\mathbf{s} = (3,3,4,4,1,1,2,2)/20$ in Case 5. In both cases the productivity coefficients are $c_1 = 1$, and $c_8 = .225$ with equally spaced decrease in between. In Case 1 and 3, about 78 per cent of the demands fall on the top 4 productive sectors. In Case 5 total of 70 per cent of demand falls on the top 4 sectors. The sum$\sum_i s_i/c_i = 1.3$ in Case 1. In Case 5 , the sum is 1.83. Case 1 and Case 3 uses different initial conditions. They appear to settle in different basins since Case 1 has a larger $Y_e$ values than Case 3.

The value of $x$ is 3.5 in Case 1, 5.9 in Case 3 and 6.0 in Case 5. The values of $x$ clearly depends on the basins of attractions, if the different starting points lead the model to different $Y_e$ values. GDP values are in the decreasing order from ase 1, 3, and 5. The number of unemployed is the largest in Case 3, then Case 5 and Case 1 in that order. The ratio of $U_e/L_e$ are slightly higer in the Case 5. It is 2 per cent vs. 2.8 per cent.

## Case 1

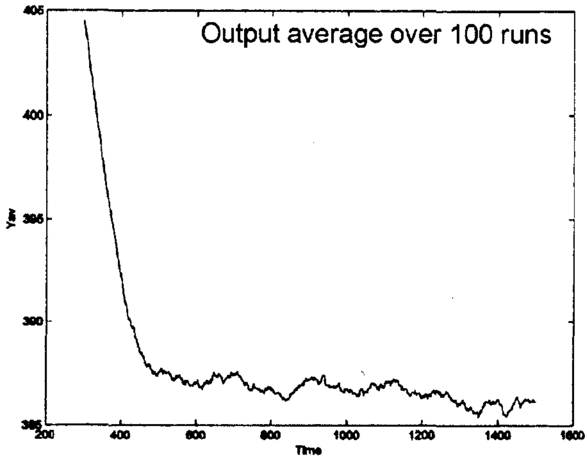


Fig1.1

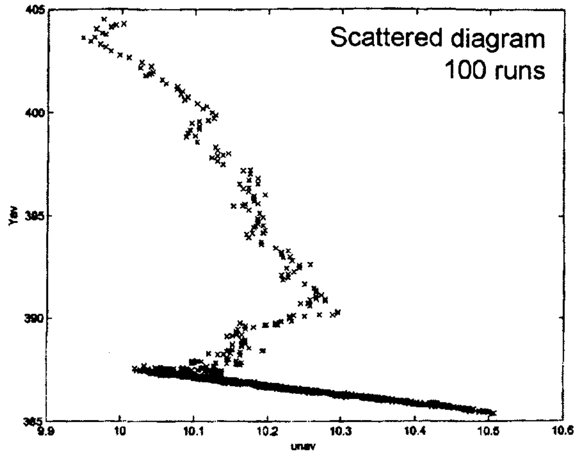


Fig1.2

## Case 3

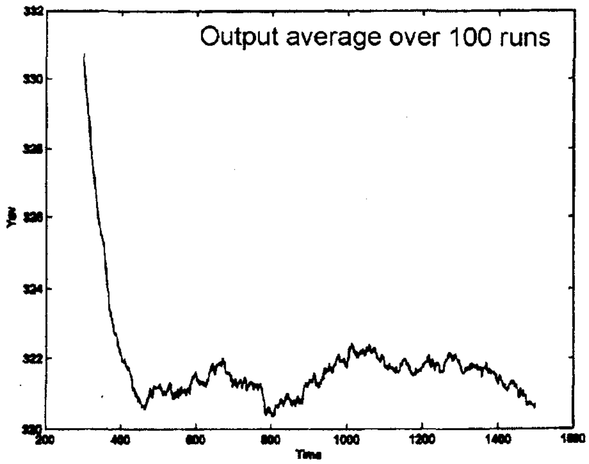


Fig3.1

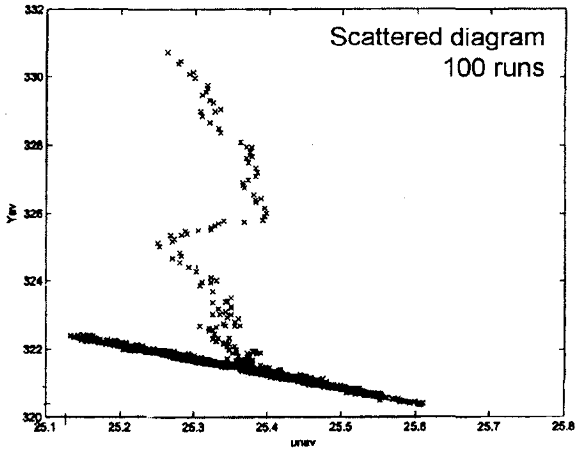


Fig3.2

## Case 5

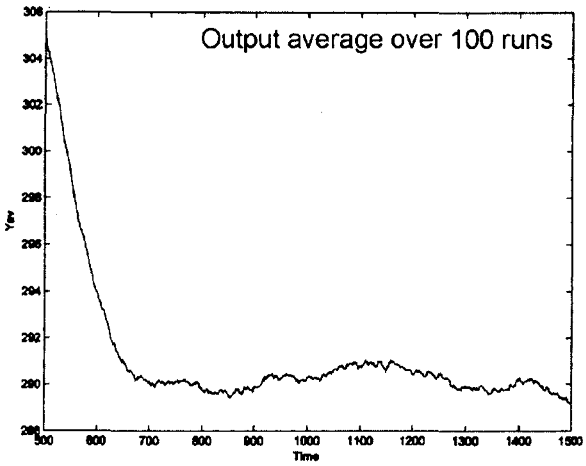


Fig5.1

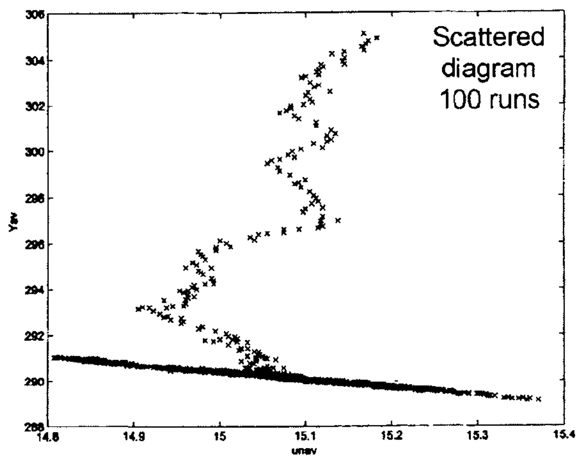


Fig5.2