数理解析研究所講究録
第515巻 1984年 89-98

Adjunction of Semifunctors: Categorical Structures in Non-extensional Lambda Calculus

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Introduction

Some connections between λ-calculus and category theory have been known. Among them, it has been known by Lambek that cartesian closed categories (ccc in short) can be identified with extensional typed λ-calculus (cf. Lambek [5], Lambek & Scott [6]). In this note , we introduce the notion of adjunction of semifunctors (for simplicity , we refer this as "semi adjunction") and ,by the aid of this notion, we define the notion of semi cartesian closed category (semi ccc in short). Some categorical and algebraic systems introduced incorporating with λ-calculus will turn special cases of semi ccc.

Another interesting connection between ccc and λ-calculus is Scott's embedding of λ-theory into a ccc (cf. Scott [9]). (This will be referred as Scott embedding.) We will show that any semiadjunction is embedded in an adjunction (of functors) and Scott embedding is its special case.

1. Adjunction of semifunctors.

In this section, the notions of semifunctors and adjunction of them are introduced, and some basic facts are shown.

1.1. Definition. Let $\mathbf{A}$ and $\mathbf{B}$ be categories. A *semifunctor* from $\mathbf{A}$ to $\mathbf{B}$ is a pair of *object function* from $Obj(\mathbf{A})$ to $Obj(\mathbf{B})$, where $Obj(\mathbf{A})$ is the set of objects of $\mathbf{A}$, and *morphism functions* $F_{X,Y}:\mathbf{A}(X,Y)\rightarrow\mathbf{B}(X,Y)$ *preserving compositions* ,i.e. $F(f\circ g)=F(f)\circ F(g)$. Note that semifunctors may not preserve identity morphisms. Let G be a semifunctor from $\mathbf{A}$ to $\mathbf{B}$ and let F be a semifunctor from $\mathbf{B}$ to $\mathbf{A}$. A quadruple pair $(F,G,\{\alpha_{X,Y}\}_{X,Y},\{\beta_{X,Y}\}_{X,Y})$ is an *adjunction of semifunctors F and G (or semiadjunction of F and G)* ,if and only if, four squares in the following diagram are commutative:

$$\begin{tikzcd}
\mathbf{A}(F(Y),X) \arrow[r, shift left, "\alpha_{X,Y}"] \arrow[d, "\phi"'] & \mathbf{B}(Y,G(X)) \arrow[l, shift left, "\beta_{X,Y}"] \arrow[d, "\psi"] \\
\mathbf{A}(F(Y'),X') \arrow[r, shift left, "\alpha_{X',Y'}"] & \mathbf{B}(Y',G(X')) \arrow[l, shift left, "\beta_{X',Y'}"]
\end{tikzcd}$$

where $f \in \mathbf{B}(Y', Y)$, $g \in \mathbf{A}(X, X')$, $\phi = \mathbf{A}(F(f), g)$ and $\psi = \mathbf{B}(f, G(g))$. Namely, the following equations hold:

$$\psi \circ \alpha_{X,Y} = \alpha_{X',Y'} \circ \phi,\ \phi \circ \beta_{X,Y} = \beta_{X',Y'} \circ \psi,\ \phi = \beta_{X',Y'} \circ \psi \circ \alpha_{X,Y},\ \psi = \alpha_{X',Y'} \circ \phi \circ \beta_{X,Y}.$$

Note that the first and second equations mean the *naturality of* α *and* β. (For simplicity, we will denote $\{\alpha_{X,Y}\}_{X,Y}$ and $\{\beta_{X,Y}\}_{X,Y}$ by α and β ,respectively.)

Let F and G be functors and let (F, G, α, β) be an adjunction of *semifunctors* F and G. Set f be id_X and g be id_Y. Then ϕ and ψ are identical functions, for $F(id_X)$ and $G(id_Y)$ are identical morphisms. So α and β are inverse functions each other and (F, G, α, β) gives an *adjunction of functors* F *and* G in the usual sense. This justifies the terminology "adjunction of semifunctors". But this terminology is sometimes confusing, so we will often say *semiadjunction* instead of "adjunction of semifunctors".

The notions of semifunctor and semiadjunction are very similar to the notions of functors and adjunctions. So the notions such as covariant or contravariant semifunctors, right or left semiadjoints etc. are defined as the corresponding notions on functors and adjunctions. We will use such notions without any explicit definitions. (Consult MacLane [7] for the terminologies on functors and adjunctions.) Adjoint of a functor is unique up to isomorphism. Opposite to this, a semiadjoint of a semifunctor is *not* unique up to isomorphism. *This means semiadjunction is not extensional in a sense.*

1.2. Completion of semiadjunction

In this section, we embed adjunctions of *semifunctors* into adjunction of *functors*. For this aim, we will use the notion of Karoubi envelope.

1.2.1. Definition (Karoubi envelope). Let $\mathbf{A}$ be a category. Then its Karoubi envelope $\tilde{\mathbf{A}}$ is the category defined as follows:

$$Obj(\tilde{\mathbf{A}}) = \{f \mid f \circ f = f\}$$

A morphism f such that $dom(f) = codom(f)$ and $f \circ f = f$ will be called *idempontent*, and the object $dom(f)$ is denoted by ∂f. Let f and g be objects of $\tilde{\mathbf{A}}$. Then hom-sets are defined by

$$\tilde{\mathbf{A}}(X, Y) = \{h \in \mathbf{A}(\partial f, \partial g) \mid g \circ h \circ f = h\}.$$

The canonical embedding functor $\epsilon_{\mathbf{A}}: \mathbf{A} \to \tilde{\mathbf{A}}$ is defined by

$$\epsilon_{\mathbf{A}}(X) = id_X \quad (X \in Obj(\mathbf{A})),$$
$$\epsilon_{\mathbf{A}}(f) = f \quad (f \in \mathbf{A}(X, Y)).$$

This was first introduced by M. Kroubi [4] for an entirely different purpose. Scott [9] used the same idea (independently from Karoubi) to embed λ-theory into a ccc, and Lambek and Scott [9] pointed out Scott's construction can be regarded as a Karoubi envelope.

1.2.2. Definition. For clarity, we will denote morphism function and object function of a functor G by G_m and G_0, respectively. Assume that $f \in \tilde{\mathbf{A}}(X, Y)$.

Set

$$\tilde{F}_o(X)=F_m(X) \quad (X\in Obj(\tilde{\mathbf{A}})),$$
$$\tilde{F}_m(f)=F_m(f) \quad (f\in\tilde{\mathbf{A}}(X,Y)).$$

Obviously, $\tilde{F}$ is a semifunctor. For each object X of $\tilde{\mathbf{A}}$, its identity morphism id_X is X itself. So

$$\tilde{F}_m(id_X)=F_m(X)=id_{\tilde{F}_o(X)}.$$

Hence $\tilde{F}$ is a functor. This functor $\tilde{F}$ is called the *completion of F.*

1.2.3. Proposition. *If G is a functor from $\tilde{\mathbf{A}}$ to $\tilde{\mathbf{B}}$, then there is a unique functor F such that $\tilde{F}=G$.*

Proof. Trivial. Left for readers.

1.2.4. Definition. Let f be an idemponent of a category and let A be the object ∂f. An object X is called a *quotient* of A by f iff there are two morphisms e and m such that

$$\begin{tikzcd} X \arrow[r, shift left, "m"] & A \arrow[l, shift left, "e"] \arrow[loop right, "f"] \end{tikzcd}$$

satisfying $m\circ e=f$, $e\circ m=id_X$ and $e\circ f\circ m=id_X$. The morphisms e and m are called the *retraction* and *coretraction* of the quotient, respectively. It is easy to check that two quotient of A by f are isomorphic. Each object X of $\tilde{\mathbf{A}}$ is a quotient of $\epsilon_{\mathbf{A}}(A)$ by $\epsilon_{\mathbf{A}}(f)$, where f is an idemponent of A and $A=\partial f$. Thus every idemponent in a Karoubi envelope $\tilde{\mathbf{A}}$ splits. (Cf. Lambek & Scott [6], Adachi [1].)

1.2.5. Proposition. *Let F be a semifunctor from $\mathbf{A}$ to $\mathbf{B}$. Then the following hold:*

(1) *$\tilde{F}\circ\epsilon_{\mathbf{A}}(X)$ is a quotient of $\epsilon_{\mathbf{B}}\circ F(X)$ by $\epsilon_{\mathbf{B}}\circ F(id_X)$.*

(2) *For any $f\in\mathbf{A}(X,Y)$, the following diagram is commutative:*

$$\begin{tikzcd} \epsilon_{\mathbf{B}}\circ F(X) \arrow[r, "\epsilon_{\mathbf{B}}\circ F(f)"] \arrow[d, "e"] & \epsilon_{\mathbf{B}}\circ F(Y) \arrow[d, "e'"] \\ \tilde{F}\circ\epsilon_{\mathbf{A}}(X) \arrow[r, "\tilde{F}\circ\epsilon_{\mathbf{A}}"] & \tilde{F}\circ\epsilon_{\mathbf{A}}(Y) \end{tikzcd}$$

where e and e' are the retractions of the quotients assured in (1).

(3) *$\tilde{F}$ is uniquely determined from F (up to isomorphism) by these two conditions.*

(4) *F is a functor, iff $\epsilon_{\mathbf{B}}\circ F=G\circ\epsilon_{\mathbf{A}}$ holds.*

1.2.6. Remark. It is possible to characterize $\tilde{\mathbf{A}}$ by an universal property as

Karoubi [4, 6.10]. Thus $\tilde{\mathbf{A}}$ and $\tilde{F}$ determines a functor from the category of categories and *semifunctors* to the categories of categories, in which idemponents split, and *functors*. See Adachi [1] for a detailed description of the essentially same functor.

1.2.7. Definition. Let F and G be semifunctors and let α be a natural transformation (of semifunctors) as follows:

$$\mathbf{A}(F(B),A) \xrightarrow{\alpha} \mathbf{B}(B,G(A)).$$

Set

$$\tilde{\alpha}(f)=\alpha(f)$$

for $f\in\tilde{\mathbf{A}}(\tilde{F}(Y),X)$. This $\tilde{\alpha}$ is called *the completion of* α.

By the naturality of α and the assumption $f\in\tilde{\mathbf{A}}(\tilde{F}(Y),X)$,

$$\begin{aligned}\tilde{G}(X)\circ\tilde{\alpha}(f)\circ Y &= G(X)\circ\alpha(f)\circ Y\\ &= \alpha(X\circ f\circ F(Y))\\ &= \alpha(f)\\ &= \tilde{\alpha}(f).\end{aligned}$$

Hence, $\tilde{\alpha}(f)$ belongs to $\tilde{\mathbf{B}}(Y,\tilde{G}(X))$. So $\tilde{\alpha}$ is a natural transformation from $\tilde{\mathbf{A}}(\tilde{F}(Y),X)$ to $\tilde{\mathbf{B}}(Y,\tilde{G}(X))$.

1.2.8. Theorem (generalized Scott embedding). *Let* (F,G,α,β) *be an adjunction of semifunctors. Then* $(\tilde{F},\tilde{G},\tilde{\alpha},\tilde{\beta})$ *is an adjunction of functors. This adjunction is called the completion of* (F,G,α,β).

Proof. Obvious from the definitions of $\tilde{F}$ and $\tilde{\alpha}$.

1.2.9. Proposition (the inverse of 1.2.8). *Let* β *be a natural transformation from* $\tilde{\mathbf{A}}(\tilde{F}(X),Y)$ *to* $\tilde{\mathbf{B}}(X,\tilde{G}(Y))$. *Then there is a natural transformation* α *from* $\mathbf{A}(F(X),Y)$ *to* $\mathbf{B}(X,G(Y))$ *such that* $\tilde{\alpha}=\beta$.

Proof. Trivial. Left for readers.

1.2.10. Proposition. *The completion* $\tilde{\alpha}$ *is the unique natural transformation commuting the following diagram:*

$$\begin{tikzcd}
\tilde{\mathbf{A}}(\tilde{F}(\epsilon_{\mathbf{B}}(B)),\epsilon_{\mathbf{A}}(A)) \arrow[r, "\tilde{\alpha}"] & \tilde{\mathbf{B}}(\epsilon_{\mathbf{B}}(B),\tilde{G}(\epsilon_{\mathbf{A}}(A))) \\
\mathbf{A}(F(B),A) \arrow[r, "\alpha"] \arrow[u, "\phi"] & \mathbf{B}(B,G(A)) \arrow[u, "\psi"']
\end{tikzcd}$$

where $\phi=\tilde{\mathbf{A}}(m,id)\circ\epsilon_{\mathbf{A}}$, $\psi=\tilde{\mathbf{B}}(id,e')\circ\epsilon_{\mathbf{B}}$, m *is the coretraction of the quotient* $\tilde{F}\circ\epsilon_{\mathbf{B}}(B)$ *of* $\epsilon_{\mathbf{A}}\circ F(B)$ *and* e' *is the retraction of the quotient* $\tilde{G}\circ\epsilon_{\mathbf{A}}(A)$ *of* $\epsilon_{\mathbf{B}}\circ G(A)$.

Proof. Easy. Left for readers.

1.2.11. Remark. If the natural transformation α of 1.2.7 satisfies

$$G(id_A)\circ\alpha(f)=\alpha(f), \tag{1}$$

then not only the commutativity in 1.2.10 but also the following equation holds:

$$\tilde{\mathbf{B}}(id, m')\circ\tilde{\alpha}\circ\phi=\epsilon_{\mathbf{B}}\circ\alpha,$$

where ϕ is the same as in 1.2.10 and m' is the coretraction of the quotient of $\tilde{G}\circ\epsilon_{\mathbf{A}}(A)$ of $\epsilon_{\mathbf{B}}\circ G(A)$. This means that α is determined by $\tilde{\alpha}$. The equation (1) means that α is natural with respect to A (*not* with respect to A and B). Such an α will be said *normal*. Let α be a natural transformation as in 1.2.7. Set

$$\alpha'=\alpha(f\circ F(id_B)).$$

Then α' is a normal natural transformation, and its completion is identical to $\tilde{\alpha}$. Let (F, G, α, β) be a semiadjunction. Then it is easy to see that (F, G, α', β) , (F, G, α, β') and (F, G, α', β') are semiadjunctions and their completions are identical to $(F, G, \tilde{\alpha}, \tilde{\beta})$ In this sense, we may assume that the two natural transformations in a semiadjunction are normal without loss of generality.

2. Semi cartesian closed category.

A ccc is a category $\mathbf{A}$ equipped with the following three adjunctions (cf. MacLane [7, IV,6]) :

$$\begin{gathered} 0 \dashv 1_{(-)}, \\ \Delta(-) \dashv - \times -, \\ - \times b \dashv (-)^b. \end{gathered}$$

A semi cartesian closed category is defined through replacing these adjunctions by semi adjunctions.

2.1. Definition. A *semi cartesian closed category (semi ccc)* category equipped with the following three semiadjunctions:

$$\begin{gathered} 1 \rightleftarrows \mathbf{A}(X,1) \\ \mathbf{A}^2(\Delta(X),(Y,Z)) \rightleftarrows \mathbf{A}(X, Y\times Z), \\ \mathbf{A}(X\times Y, Z) \rightleftarrows \mathbf{A}(X, Z^Y). \end{gathered}$$

Such adjunctions will be called a *semi ccc structure on bold A* Note that there may be many different semi ccc structures on a category. (Contrary to this, ccc structure on a category is unique up to isomorphism.) A *morphism* from a semi ccc $\mathbf{A}$ to a semi ccc $\mathbf{B}$ is a semifunctor from $\mathbf{A}$ to $\mathbf{B}$ which is a *map* of the each of the above three semiadjunctions. (See MacLane [7] for the definition of a map of adjunctions.)

2.1.1. Remark. The second semiadjunction in the definition of semi ccc is a *semiadjunction with a parameter Y.* (Cf. MacLane [7] for adjunction with a parameter.) By a semiadjunction version of MacLane[7, IV, 7, Theorem 3], a canonical semiadjunction with a parameter Y exists, if there is a semiadjunction for each Y.

2.2. Algebraic description of a semi ccc.

How to describe a ccc *algebraically* by the aid of pairing operators and evaluation morphisms is well-known. An algebraic description is also given by the following theorem.

2.2.1. Theorem. *A category A is a semi ccc if and only if there is an algebraic structure on B*

$$(<*,*>,\ *\times*,\ p,\ q,\ 1,\ 1_{(*)},\ *^{*},\ \Lambda(*),\ ev).$$

satisfying the following conditions:

(1) *For morphisms $a:x\to y$ and $b:x\to z$, $<a,b>$ is a morphism from x to $y\times z$.*

(2) *For objects y and z, there are morphisms*

$$p:y\times z\to y,\ q:y\times z\to z$$

such that

$$p\circ<a,b>=a,\ q\circ<a,b>=b.$$

(3) *For a morphism $h:x\times y\to z$, $\Lambda(h)$ is a morphism from x to z^y.*

(4) *For objects y and z, there is a morphism*

$$ev:z^y\times y\to z$$

such that

$$ev\circ<\Lambda(h)\circ u,v>=h\circ<u,v>,$$

$$\Lambda(h)\circ u=\Lambda(h\circ<u\circ p,q>),$$

where

$$u:a\to x,\ v:a\to y,\ p:a\times y\to a,\ q:a\times y\to y.$$

(5) *For each object a, 1_a is a morphism from a to 1 such that $f\circ 1_{codom(f)}=1_{dom(f)}$ holds.*

(6) *$ev\circ<p,q>=ev$ holds.*

Proof. Assume h is a morphism from $x\times y$ to z. Then $\Lambda(h)$ is the image of $h\circ id\times id$ by the natural transformation $\mathbf{A}(x\times y,z)\to\mathbf{A}(x,z^y)$. The definitions of the others and the details of proof is left for readers. (The proofs of 6.5 and 6.6 of Koymans [3] serve as good references.)

2.2.2. Definition A category $\mathbf{A}$ equipped with such an algebraic structure is called an *algebraic semi ccc.*

2.2.3. Remark The conditions (5) and (6) of the algebraic semi ccc are superfluous in a sense. In fact, they are not necessary to prove "if part" of Theorem 2.2.1. If $\mathbf{A}$ has an object A_0 and satisfies (1)-(4), then set $1=A_0$ and set $1_B=\Lambda(q)$, where $q:B\times A_0\to A_0$. Then they satisfy the condition (5). Let

$S_0=(<*,*>, \ldots, ev)$ be an algebraic structure on **A** satisfying (1)-(5). Set $ev_1=ev\circ<p,q>$. Then $S_1=(<*,*>, \ldots, ev_1)$ satisfies the conditions (1)-(6) and S_0 and S_1 give the same ccc. Furthermore, the same equations on λ-terms hold in S_0 and S_1 in the sense of the semantics of section 3.

2.3. Theorem *Let* **A** *be a semi ccc. Then* $\tilde{\mathbf{A}}$ *is a ccc* ~~*and the embedding functor* $\epsilon_{\mathbf{A}}$ *is a morphism of semi ccc*~~. *Namely* **A** *is embedded into a ccc* $\tilde{\mathbf{A}}$ *by* $\epsilon_{\mathbf{A}}$. *The ccc structure on* $\tilde{\mathbf{A}}$ *will be called the completion of the semi ccc structure on* **A**.

2.4. Theorem *Let* **A** *be a category whose Karoubi envelope* $\tilde{\mathbf{A}}$ *is a ccc. Then there is a canonical semi ccc structure on* **A** *such that the ccc* $\tilde{\mathbf{A}}$ *is its completion.*

Proof. This is a direct consequence of 1.2.8.

2.5. Examples of semi ccc.

In this subsection, we will examine some categorical or algebraic systems introduced to characterize type free λ-calculus.

2.5.1. CCM, weak cartesian closed monoid and C-domain.

Koymans [3], Lambek & Scott [6] and Yokouchi [10] introduced a sort of monoid which corresponds to $\lambda\beta$-calculus. Their definitions are different but they are essentially the same.

Koyman's CCM is an algebraic semi ccc with just one object which may not satisfy the condition (5). But the condition is superfluous as was noted in 2.2.3. His version of Scott embedding ("if part" of Koymans [3 ,Theorem 6.6]) is a direct consequence of Theorem 2.2.1 and Theorem 2.3. He also proved the inverse of Scott embedding ("only if" part of Koymans [3, Theorem 6.6]). Theorem 2.4 generalizes it. If a CCM is regarded as a semi ccc, then the natural transformation corresponding to λ-abstraction is normal. Hence the interpretation of λ-terms in a CCM can be achieved through its Karoubi envelope as was remarked in 1.2.11. (See Scott [9] and Koymans [3].)

Weak cartesian closed monoid of Lambek & Scott [6] can be defined as a CCM may not satisfying the condition (6) of Theorem 2.2.1. But the condition is superfluous as was noted in 2.2.3. So the notion of weak cartesian closed category is essentially equivalent to the notion of CCM.

Yokouchi's C-domain is another description of weak cartesian closed monoid with the condition (5) but without the condition (6). See Yokouchi [10] for a discussion on the equivalence of C-domain and CCM.

2.5.2. Semi ccat and Church algebraic theory.

A semi cartesian closed algebraic theory (semi ccat) is an algebraic theory **A** in the sense of Lawvere with the following semiadjunction (with a parameter n):

$$\mathbf{A}(m+n,p) \underset{\epsilon_n}{\overset{\lambda_n}{\rightleftarrows}} \mathbf{A}(m,p^n)$$

satisfying

$$p^n = p,$$
$$\lambda_m \circ \lambda_n = \lambda_{m+n},\ \lambda_0 = id,$$
$$\epsilon_m \circ \epsilon_n = \epsilon_{m+n},\ \epsilon_0 = id$$

Hence a semi ccat is a semi ccc. It is easy to check that the notion of semi ccat is essentially equivalent to the notion of Church algebraic theory of Obtulowicz & Wiweger [8]. A semi ccat or a Church algebraic theory is a categorical description of a $\lambda\beta$-theory (in the sense of Barentregt [2].)

Let $(\mathbf{C}, U, i, j)$ be a categorical model of λ-calculus in the sense of Koymans [3, 3]. Then the full subcategory $\{U^m | m \in \mathbf{N}\}$ of $\mathbf{C}$ is an algebraic theory. By the aid of the morphism i and j, the algebraic theory turns to be a semi ccat, say $\mathbf{T}(\mathbf{C}, U, i, j)$. A *model* of a semi ccat $\mathbf{A}$ in $(\mathbf{C}, U, i, j)$ is a semi ccc morphism from $\mathbf{A}$ to $\mathbf{T}(\mathbf{C}, U, i, j)$. Then, by Theorem 2.3. and Proposition 1.2.10, there is an identical model in $\tilde{\mathbf{A}}$ for any semi ccat $\mathbf{A}$ (completeness theorem of semi ccat).

3. Typed $\lambda\beta$-theory and semi ccc.

As was shown in the previous section semi ccc is a generalization of some categorical or algebraic systems corresponding to non-extensional λ-calculus. We will introduce the notion of *typed $\lambda\beta$-theory (with pairing)* and relate it to semi ccc. A similar but extensional typed λ-theory can be found in Lambek & Scott [6] and Yokouchi [10]. We will follow the way of Lambek & Scott[6].

3.1. Definition. A typed $\lambda\beta$-theory is a typed equational theory equipped with the following data:

(1) The set of types is closed under cartesian product $A \times B$ and exponential A^B. There is a special type 1.

(2) If t_1 and t_2 are terms of types A and B,respectively, then $<t_1, t_2>$ is a term of the type $A \times B$. If t is a term of type $A \times B$ then $\pi(t)$ and $\pi'(t)$ are terms of types A and B, respectively. There is a constant $*$ of the type 1.

(3) If x is a variable of a type A and t is a term of a type B, then $\lambda x. t(x)$ is a term of the type B^A. If t_1 and t_2 are terms of types A^B and B, respectively, then $t_1 t_2$ is a term of the type A.

(4) Substitution $t_1[x := t_2]$ is defined as usual. Note that

$$<t_1, t_2>[x := t_3] = <t_1[x := t_3], t_2[x := t_3]>.$$

(5) The following equations are the postulates.

$$\pi(<t_1, t_2>) = t_1,$$
$$\pi'(<t_1, t_2>) = t_2,$$
$$(\lambda x. t_1(x))\, t_2 = t_1[x := t_2].$$

3.2. Interpretation of the typed $\lambda\beta$-theory.

Koymans [3] gives an interpretation of $\lambda\beta$-theory in a reflexive domain in a ccc. By a similar method, an interpretation of our typed $\lambda\beta$-theory in a semi ccc. The point is how to interpret a constant and a variable in an environment (assignment). This problem is solved to fix a product of n objects in a systematic way. (See Koymans [3, 3.1-3.4, 7.3-7.5] and Yokouchi [10, 2.2] for the type-free case.) By Lindenbaum-Tarski construction, a typed $\lambda\beta$-theory has a semi ccc with an identical interpretation. (See Barentregt [2, 5.3.13] for the type-free case.) Hence the notion of typed $\lambda\beta$-theory is essentially equivalent to the notion of semi ccc.

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