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Theoretical Background in Experimental Data Analysis

June, 1984

Research Institute for Mathematical Sciences

Kyoto University, Kyoto, Japan

SYMPOSIUM ON THEORETICAL BACKGROUND IN EXPERIMENTAL DATA ANALYSIS

Place : Research Institute for Mathematical Sciences, Kyoto University, Kyoto, Japan

Date : June 7 - 9, 1982

Organizer : Sanpei Kageyama, Department of Mathematics, Faculty of School Education, Hiroshima Univ.

PROGRAM AND ABSTRACT

1. Kei Takeuchi (University of Tokyo)

On the types of factors in a design of experiment and the methods of inference

Abstract: Through discussions among Japanese applied statisticians, various kinds or types of factors in factorial experimentations have been defined and clarified. Main types are : Controrable factors, which can and should be controled to attain optimum conditions; Descriptive factors, which affect the results and should be taken into consideration when we seek the optimum condition, and also whose levels can reproducible and identifiable; Block factors, which although may affect the results but not reproducible; Auxiliary factors which although identifiable ex poste but not before the experiment is performed. When we analyyze the data it is necessary to take into consideration the different types of the factors involved. Especially, the

meaning of interactions among factors is different when one of the factors is descriptive, from the case when both are controllable. It is, however, to be noted that the distinction of the different types of factors is not always fixed but it varies according to the purpose of analysis.

2. Yasuo Ohashi (University of Tokyo)

On the multiplicative model

Abstract:

For the analysis of a two-way table $\underset{\sim}{X}=(X_{ij})$, Fisher and Mackenzie (1923) proposed the multiplicative model in which the (i,j) element is expressed as

$$X_{ij} = \alpha_i \beta_j + \varepsilon_{ij} \quad ,$$

where α and β represent the row and the column effect, respectively, and ε is a random variable with variance σ^2. They used the least squares method for estimating α's and β's and checked the adequacy of the model by compair--ing the residual sum of squares

$$R = \Sigma(x_{ij} - a_i b_j)^2$$

with an estimate of σ^2 independent of $\underset{\sim}{X}$. But their z-test is approximate in the sense that the distribution of R/σ^2 is not exactly χ^2.

The accuracy of χ^2-approximation is examined and, by using Fisher's data, the comparison of the above model is made with alternative ones such as

$$X_{ij} = \alpha_i \beta_j (1 + \varepsilon_{ij}) \quad ,$$
$$X_{ij} = \alpha_i + \beta_j + \varepsilon_{ij} \quad .$$

3. Chihiro Hirotsu (University of Tokyo) and Hiroe Tsubaki (University of Tokyo)

A statistical model for clinical data from Cross-over design

Abstract:

We introduce a statistical model for "the m-drug and 2-period cross-over clinical experiments (abb. (m,2)C.O.D.)",where observations are assumed to be binary. In the (m,2)C.O.D. the subjects are randomly partitioned into ${}_mP_2$ groups and each subject within the same "$(i \to j)$ group" receives the i-th drug at the first period and sucessively the j-th drug at the second period,where $1 \leq i,j \leq m$ and $i \neq j$.

At first we review the linear logistic model approach to the most widely used (2,2)C.O.D. by Gart(1969),which involves parameters of individual-effects,drug-effects and period-effects. Then the Gart's model is extended to the case of $m \geq 3$,where we newly introduce parameters for carry-over-effects,mixture-effects and order-effects. The mixture-effects and order-effects may also be considered to be a sort of the interactions between drug-effects and period-effects. The hypotheses on those introduced parameters become testable by the weighted least square methods, if we assume the Bradley-Terry model(Bradley and Terry(1952)) for the drug-effects. An example is given in Section 4 to give some of ideas.

We next discuss how to use the information from tied-pairs which is usually ignored in the conditional inference.

First approach to this problem is to use the Davidson-Beaver model which is a genelalization of the Bradley-Terry model to incorporate ties and within-pair order effects.(Davidson and Beaver(1977)) It is shown that a researcher can appreciate the data obtained from (m,2)C.O.D. by the Davidson-Beaver approach if individual-effects are negligible and goodness-of-fit of the model can be testified by likelihood ratio test or weighted least square method as usual.

Second approach is to use random-effect model like Grizzle's model. (Grizzle(1965)) It is shown that it is possible to test the hypothesis on order-effects even in the case of (2,2)C.O.D. if individual-effects can be regarded as random effects.

4. Tadakazu Okuno (University of Tokyo)

Construction of fractional factorial designs by using tables of orthogonal arrays and their applications

Abstract:

Many agricultural, biological and industrial experiments are of a factorial nature —— that is, each treatment consists of a number of factors, each at two or more levels. Generally, interest of experimenters is in estimating all the main-effects and some of the two-factor interactions. For high design efficiency, the total number of parameters to be estimated should be as large as possible under a given number of observations. In general, this requirement is satisfied with a suitably selected "fractional factorial design" through the assignment of each factor to one or more arrays in a specially devised Table of Orthogonal arrays.

The method for assigning effects to the appropriate arrays is given with a few actual examples.

5. Chihiro Hirotsu (University of Tokyo)

Multiple comparisons for interaction effects

Abstract: A method is given to classify rows and columns into subgroups so that additivity holds within each of the subtables made of the grouped rows or the grouped columns. The least squares estimators of the cell means are easily obtained for the resulting linear model together with their variances. An estimator of the error variance σ^2 is given when there is only one observation per cell. A treatment of an ordered table is also given.

6. T. Miwa (National Institute of Agricultural Sciences)

Multiple range methods in multiple comparisons

Abstract: The common F test, in an analysis of variance, which has rejected the homoginity hypothesis of treatment means, has still not indicated the source of the significance. Multiple comparisons methods are used to investigate the significance in detail. Here we consider two problems. One is that of comparisons with a control, and the other, that of paired comparisons. Dunnett's method and Tukey's studentized range method are widely used. However both methods become conservative as the number of treatments increaces. Some multiple range methods have been proposed to increace power. We state the present situation of multiple range methods and point out the question at issue.

7. Masahide Kuwada (Hiroshima University)

On a balanced fractional 2^m factorial design of resolution $(2\ell+1)^*$ with block effects.

Abstract: Consider a 2^m-FF design T with N assemblies such that $T' = [T_1'; \cdots ; T_r']$, where T_k $(k=1,\cdots,r)$ are BAs$(N_k, m, 2, 2\ell)\{\mu_0^{(k)}, \mu_1^{(k)}, \cdots, \mu_{2\ell}^{(k)}\}$. Suppose that each vector $\underline{y}(T_k)$ of N_k observations based on T_k has its proper effect (say, block effect) and further has the covariance matrix $\sigma_k^2 I_{N_k}$, where σ_k^2 is unknown but σ_1^2/σ_k^2 is known. Then, after eliminating block effects, we present an explicit expression of the characteristic polynomial of the covariance matrix based on T.

8. Teruhiro Shirakura (Kobe University)

Optimum block plan for a fractional 2^m factorial design

Abstract : (see the full paper)

9. Junjiro Ogawa (University of Calgary) and Sadao Ikeda (Soka University)

Connectedness of PBIB designs

Abstract: A necessary and sufficient condition for the connectedness of an m-class PBIB design is discussed. Conditions are obtained concretely for 2, 3 and 4 class PBIB designs, which extend the results by Mohan (1981) and Kageyama (1981) in case of 2-class PBIB designs.

10. Ryoh Fuji-Hara (University of Waterloo)

Kirkman problems and their applications

Abstract:

In 1955, the following problem was given by T.G. Room (called Room square):

Let A be a square array such that

(i) each cell of A is empty or a 2-subset of n-set N,

(ii) every element of N appears in each row and each column exactly once,

(iii) every 2-subset appears in A precisely once.

This problem was already mentioned in a paper by T.P. Kirkman in 1850. Hence, a more appropriate term might be "Kirkman square". We use the term "Kirkman square" for the more general square defined by replacing 2-subset in (i) by k-subset.

In this paper, we discuss an existence of Kirkman squares, applications to statistics and related problems.

No. 11

Inspection of a lot of products in which some of the defectives may be overlooked

Yoritake Fujino

Technological University of Nagaoka

Abstract

In the inspection of a lot of products, it occurs rather often that all of the defective items cannot be detected for some reason or other. If this is the case, some of the items in the lot must be inspected at least two times in order to estimate the number of defectives in the lot and the probability of overlooking.

The method of maximum likelihood and that of conditional maximum likelihood are applied for two cases:

(1) In the first place, inspect all of the items; then re-inspect the items which were once looked on as non-defective.

(2) Inspect all of the items twice.

It is concluded that, fairly better estimates can be obtained in the second case than in the first case.

実験データ解析の理論的背景

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