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MANIFOLDS WHICH DO NOT ADMIT EXPANSIVE HOMEOMORPHISMS WITH PSEUDO ORBIT TRACING PROPERTY

Koichi Hiraide 平出耕一

Abstract

In this paper we study some necessary conditions for a homeomorphism of compact manifold to be an expansive homeomorphism with pseudo orbit tracing property and give theorems which are extensions of K.Shiraiwa's result [3]. As their applications, we give some examples of compact manifolds which do not admit expansive homeomorphisms with pseudo orbit tracing property. The proofs will be treated in the future paper.

Let M be a compact connected topological manifold without boundary with a metric d and f a homeomorphism from M onto itself. We say that f is expansive if there exists $e > 0$ such that $d(f^n(x), f^n(y)) \leq e$ for all $n \in \mathbb{Z}$ implies $x = y$. We say that f has the pseudo orbit tracing property (abbrev. P.O.T.P.) if for any $\varepsilon > 0$ there exists $\delta > 0$ such that for every sequence $\{x_i\}_{i \in \mathbb{Z}}$ of M with $d(f(x_i), x_{i+1}) < \delta$ for all $i \in \mathbb{Z}$ there exists $x \in M$ such that $d(f^i(x), x_i) < \varepsilon$ for all $i \in \mathbb{Z}$. The author [1] defined the notion of orientability of local unstable sets of an expansive homeomorphism f with P.O.T.P..

Theorem 1. Let M be a compact connected topological manifold without boundary. If $f: M \to M$ is an expansive homeomorphism with P.O.T.P. and if local unstable sets of f are orientable, then for any $n > 0$ the induced homomorphism $f^n{}_*: H_*(M,\mathbb{R}) \to H_*(M,\mathbb{R})$ is not the identity map where $H_*(M,\mathbb{R}) = \bigoplus_{q \geqq 0} H_q(M,\mathbb{R})$ and $H_q(M,\mathbb{R})$ denotes the q-dimensional homology group of M with coefficients in the field $\mathbb{R}$ of real numbers.

Corollary 2. Any simply connected rational homology sphere does not admit expansive homeomorphisms with P.O.T.P.. Therefore any manifold whose universal covering space is a rational homology sphere does not admit expansive homeomorphisms with P.O.T.P..

We say that a manifold M satisfies condition (T) if the cohomology algebra $H^*(M,\mathbb{R})$ of M is a graded exterior algebra on generators of odd degree. An elemet of $H^*(M,\mathbb{R})$ is called decomposable if it is a sum of products of two elements of positive degree. The set D of all decomposable elements of $H^*(M,\mathbb{R})$ is an ideal in the algebra $H^*(M,\mathbb{R})$. Put $P = \bigoplus_{q \geqq 1} H^q(M,\mathbb{R}) / D$. Then P has naturally a graded $\mathbb{R}$-module structure. Let $f: M \to M$ be a continuous map. Then f induces homomorphisms on $H^*(M,\mathbb{R})$ and on D. Tnerefore it naturally induces a homomorphism $f_P: P \to P$.

Theorem 3. Let M be as in Theorem 1 and satisfy condition (T). If $f: M \to M$ be an expansive homeomorphism with P.O.T.P. and if local unstable sets of f are orientable, then $f_P : P \to P$ is hyperbolic.

Corollary 4. Let G be a compact connected Lie group. If f:

$G \to G$ is an expansive homeomorphism with P.O.T.P., then $f_P : P \to P$ is hyperbolic.

Corollary 5. SO(n), Spin(n), SU(n), SP(n) and the exceptional Lie groups G_2, F_4, E_6, E_7, E_8 do not admit expansive homeomorphisms with P.O.T.P..

Remark. It was proved in [2] that compact surfaces except a torus do not admit expansive homeomorphisms with P.O.T.P..

References

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[3] K.Shiraiwa, Some conditions on Anosov diffeomorphisms, Manifolds-Tokyo 1973, Univ. of Tokyo Press, 1975, 205-209.

Department of Mathematics
Tokyo Metropolitan University
Tokyo, Japan