MANIFOLDS WHICH DO NOT ADMIT EXPANSIVE HOMEOMORPHISMS
WITH PSEUDO ORBIT TRACING PROPERTY

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Abstract

In this paper we study some necessary conditions for a homeomorphism of compact manifold to be an expansive homeomorphism with pseudo orbit tracing property and give theorems which are extensions of K.Shiraia's result [3]. As their applications, we give some examples of compact manifolds which do not admit expansive homeomorphisms with pseudo orbit tracing property.

The proofs will be treated in the future paper.

Let M be a compact connected topological manifold without boundary with a metric d and f a homeomorphism from M onto itself. We say that f is expansive if there exists ε > 0 such that d(f^n(x), f^n(y)) ≥ ε for all n ∈ Z implies x = y. We say that f has the pseudo orbit tracing property (abbrev. P.O.T.P.) if for any ε > 0 there exists δ > 0 such that for every sequence \{x_i\}, i ∈ Z of M with d(f(x_i), x_{i+1}) < δ for all i ∈ Z there exists x ∈ M such that d(f^i(x), x_i) < ε for all i ∈ Z. The author [1] defined the notion of orientability of local unstable sets of an expansive homeomorphism f with P.O.T.P..

Theorem 1. Let M be a compact connected topological manifold
without boundary. If \( f: M \to M \) is an expansive homeomorphism with P.O.T.P. and if local unstable sets of \( f \) are orientable, then for any \( n > 0 \) the induced homomorphism \( f^*_n: H_*(M, \mathbb{R}) \to H_*(M, \mathbb{R}) \) is not the identity map where \( H_*(M, \mathbb{R}) = \bigoplus_{q \geq 0} H_q(M, \mathbb{R}) \) and \( H_q(M, \mathbb{R}) \) denotes the \( q \)-dimensional homology group of \( M \) with coefficients in the field \( \mathbb{R} \) of real numbers.

**Corollary 2.** Any simply connected rational homology sphere does not admit expansive homeomorphisms with P.O.T.P.. Therefore any manifold whose universal covering space is a rational homology sphere does not admit expansive homeomorphisms with P.O.T.P..

We say that a manifold \( M \) satisfies condition (T) if the cohomology algebra \( H^*(M, \mathbb{R}) \) of \( M \) is a graded exterior algebra on generators of odd degree. An element of \( H^*(M, \mathbb{R}) \) is called decomposable if it is a sum of products of two elements of positive degree. The set \( D \) of all decomposable elements of \( H^*(M, \mathbb{R}) \) is an ideal in the algebra \( H^*(M, \mathbb{R}) \). Put \( P = \bigoplus_{q \geq 1} H^q(M, \mathbb{R}) / D \). Then \( P \) has naturally a graded \( \mathbb{R} \)-module structure. Let \( f: M \to M \) be a continuous map. Then \( f \) induces homomorphisms on \( H^*(M, \mathbb{R}) \) and on \( D \). Therefore it naturally induces a homomorphism \( f_P: P \to P \).

**Theorem 3.** Let \( M \) be as in Theorem 1 and satisfy condition (T). If \( f: M \to M \) be an expansive homeomorphism with P.O.T.P. and if local unstable sets of \( f \) are orientable, then \( f_P: P \to P \) is hyperbolic.

**Corollary 4.** Let \( G \) be a compact connected Lie group. If \( f:
G → G is an expansive homeomorphism with P.O.T.P., then $f_p : P → P$ is hyperbolic.

Corollary 5. SO(n), Spin(n), SU(n), SP(n) and the exceptional Lie groups $G_2$, $F_4$, $E_6$, $E_7$, $E_8$ do not admit expansive homeomorphisms with P.O.T.P..

Remark. It was proved in [2] that compact surfaces except a torus do not admit expansive homeomorphisms with P.O.T.P..

References

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