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On the Unit Groups of Burnside Rings.

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1. Introduction.

Let G be a finite group and let $\underline{\underline{\mathrm{Set}}}_f^G$ be the category of finite (right) G-sets and G-maps. The Grothendieck ring of this category (with respect to coproduct $+$ and product $\times$) is called the Burnside ring of G and is denoted by $A(G)$.

A super class function is a map of the set of subgroups of G to $\underline{\underline{Z}}$ which is constant on each conjugate class of subgroups. Let $\tilde{A}(G) = \underline{\underline{Z}}^{Cl(G)}$ be the ring of super class functions. For any subgroup S of G, the map $[X] \longrightarrow |X^S|$ extends to a ring homomorphism $\varphi_S : A(G) \longrightarrow \underline{\underline{Z}}$, and so we have a ring homomorphism

$$\varphi = \prod_{(S)} \varphi_S : A(G) \longrightarrow \tilde{A}(G) = \underline{\underline{Z}}^{Cl(G)}.$$

This map is injective. Thus we can identify any element x of $A(G)$ with the super class function $\varphi(x)$, and so we simply write $x(S) := \varphi(x)(S) = \varphi_S(x)$ for a subgroup S. See tom Dieck [Di79] Chapter 1.

Now, by geometric methods, tom Dieck proved that for an $\underline{\underline{R}}G$-module V, the function

$$u(V) : S \longmapsto \operatorname{sgn} \dim V^S$$

belongs to the Burnside ring $A(G)$, where $\operatorname{sgn} n := (-1)^n$ ([Di79] Proposition 5.5.9). The first purpose of this paper is to prove this fact by purely algebraic methods. In fact we shall prove the following theorem in Section 2.

<u>Theorem A</u>. <u>Let</u> G <u>be a finite group and let</u> V <u>be a</u> $\underline{\underline{C}}G$-<u>module with real valued character</u>. <u>Then the function</u>

$$u(V) : S \longmapsto \operatorname{sgn} \dim_{\underline{\underline{C}}} V^S$$

<u>is a member of the Burnside ring</u> $A(G)$.

Since clearly $u(V)^2 = 1$ and $u(V \oplus W) = u(V)\, u(W)$, we have a group homomorphism into the unit group :

$$u = u_G : \overline{R}(G) \longrightarrow A(G)^*,$$

where $\overline{R}(G)$ is the ring of real valued virtual characters of G. We call this map u_G a <u>tom Dieck homomorphism</u>.

There are various maps between Burnside rings (and unit groups), and the assignment $A^* : H \longmapsto A(H)^*$ together with restrictions and multiplicative inductions forms a so called G-functor (= a Mackey functor from the

category of finite G-sets) and further that the tom Dieck homomorphism gives a morphism between G-functors. Since A^* is a G-functor, we have that $A(G)^*$ is a module over $A(G)$ (and also over $A(G)_{(2)}$, the localization at 2). In fact, the action $A(G)^* \times A(G) \longrightarrow A(G)^*$ is induced by the exponentaial map $(Y, X) \longrightarrow Y^X$ (the set of all maps of X to Y). From the theory of Burnside rings and G-functors, we can show some transfer theorems about $A(G)^*$. See Section 3. The proof will appear in another paper.

<u>Notation and terminology</u>. We always denote by G a finite group. The set of G-conjugate classes (H) of subgroups H of G is denoted by $Cl(G)$. For subsets A, B of G, we mean by $A =_G B$ (resp. $A \leq_G B$) that A and B are conjugate in G (resp. A is G-conjugate to a subgroup of B). We put $A^g := g^{-1}Ag$. When a group G acts on a set X, we denote by X^G the set of elements fixed by G. The ordinary character ring of G is denoted by $R(G)$. For a ring R, the unit group of R is denoted by R^*. The inner product of characters χ and θ is denoted by $\langle \chi, \theta \rangle$. Other notation and terminology are standard. See [Go68], [Di79].

2. Proof of Theorem A.

In this section, we prove Theorem A. As in the introduction, let G be a finite group and let $Cl(G)$ denote the set of conjugate classes of subgroups of G. We mean by (S) the class of a subgroup S. We set $WS := N_G(S)/S$ for a subgroup S of G.

Lemma 2.1. There is an exact sequence of abelian groups:

$$0 \longrightarrow A(G) \xrightarrow{\varphi} \underline{\underline{Z}}^{Cl(G)} \xrightarrow{\psi} \prod_{(S)} (\underline{\underline{Z}}/|WS|\underline{\underline{Z}}) \longrightarrow 0,$$

where φ is the injective ring homomorphism given in the introduction, and for a super class function x, the S-component of $\psi(x)$ is defined by

$$(x)_S := \sum_{gS \in WS} x(\langle g \rangle S) \mod |WS| .$$

This lemma is well-known and its proof is found in, for example, tom Dieck [Di79] 1.3. We can now prove Theorem A.

Proof of Theorem A. Let χ be the real valued character afforded by the $\underline{\underline{C}}G$-module V, and let $u(V)$ be the present super class function:

$$u(V) : (S) \longmapsto \operatorname{sgn} \dim V^S.$$

By Lemma 2.1, we must show that for each subgroup S of G,

$$\sum_{gS \in WS} u(V)(\langle g \rangle S) \equiv 0 \mod |WS|. \tag{1}$$

Let χ' be the character afforded by the $\underline{\underline{C}}WS$-module V^H, so that by an easy representation theory, we have that

$$\chi'(gS) = \frac{1}{|S|} \sum_{h \in S} \chi(gh), \qquad gS \in WS,$$

and so χ' is also a real valued character. Thus in order to prove (1), we may assume that $S = 1$. Set $u_\chi(g) := u(V)(\langle g \rangle)$, then it has the value

$$u_\chi(g) = \operatorname{sgn} \langle \chi_{|\langle g \rangle}, 1_{\langle g \rangle} \rangle,$$

where $\langle\ ,\ \rangle$ stands for the inner product of characters. Now when $S = 1$, (1) becomes

$$\sum_{g \in G} u_\chi(g) \equiv 0 \mod |G|. \tag{2}$$

In order to prove (2), it will suffice to show that u is a virtual character of G. In fact, we can show that

$$u_\chi = (-1)^{\chi(1)} \det \chi, \tag{3}$$

where $\det \chi$ is the linear character of G defined by the

composition

$$\det \chi : G \longrightarrow GL(V) \xrightarrow{\det} \underline{\underline{C}}^{*}.$$

See Yoshida [Yo78]. In order to prove (3), we may assume that G is cyclic. Since $u_{\chi+\varphi} = u_\chi u_\varphi$ and $\det(\chi+\varphi) = \det\chi \cdot \det\varphi$, we may further assume that either χ is a real valued linear character or $\chi = \lambda + \bar{\lambda}$ for some nonreal linear character λ. In the first case, we have that $\langle \chi_{|\langle g\rangle}, 1_{\langle g\rangle}\rangle = 1$ if g is in the kernel of and $= 0$ otherwise, and so $u = -1$ or $+1$, respectively. Since $\det\chi = \chi$, (3) holds in this case. Next assume that $\chi = \lambda + \bar{\lambda}$, so that $\langle \chi_{|\langle g\rangle}, 1_{\langle g\rangle}\rangle = 0$ or 2 and $\det\chi = 1_G$. Thus (3) holds also in this case. The theorem is proved.

3. Some transfer theorems for the unit groups.

Let p be a prime. We put

$$\underline{\underline{Z}}_{(p)} := \{ a/b \in \underline{\underline{Q}} \mid a \in \underline{\underline{Z}},\ b \in \underline{\underline{Z}} - p\underline{\underline{Z}} \},$$
$$A(G)_{(p)} := \underline{\underline{Z}}_{(p)} \otimes_{\underline{\underline{Z}}} A(G).$$

For a finite group H, the subgroup generated by all p'-elements of H is denoted by $O^p(H)$. When $O^p(Q) = Q$ (that is, Q has no normal subgroup of index p), the group

Q is called to be p-<u>perfect</u>. Let $Cl_p(G) \subseteq Cl(G)$ denote the classes of p-perfect subgroups.

There is a one-to-one correspondence between primitive idempotents of $A(G)_{(p)}$ and $Cl_p(G)$ (cf. [Di79] 1.4). An explicit formula of primitive idempotents was obtained by Gluck [Gl81] and Yoshida [Yo83]. Let μ be the Mobius function of the subgroup lattice of G and δ_G the function defined by

$$\delta_G(H, K) := \begin{cases} 1 & \text{if } H =_G K \\ 0 & \text{otherwise.} \end{cases}$$

Each primitive idempotent of $A(G)_{(p)}$ is then written in the form

$$e^p_{G,Q} = \sum_{(D)\in Cl(G)} \lambda_{G,Q}(D) \; [D \backslash G],$$

where $(Q) \in Cl_p(G)$ and

$$\lambda_{G,Q}(D) := \frac{1}{|N_G(D)|} \sum_{K \leq G} \mu(D, K)\, \delta_G(O^p(K), Q).$$

As a super class function, $e^p_{G,Q}$ has the value

$$e^p_{G,Q}(S) = \begin{cases} 1 & \text{if } O^p(S) =_G Q \\ 0 & \text{otherwise.} \end{cases}$$

For the finite group G, let $\underline{\underline{Set}}_f^G$ denote the

category of finite (right) G-sets and G-maps. For two G-sets X and Y, let Y^X be the G-set consisting of all mappings of X to Y with G-action defined by $\alpha^g(x) := \alpha(xg^{-1})\cdot g$. For an element $a = [A] - [B]$ of $A(G)$, we furthermore define the exponential map

$$(-)^a : A(G)^* \longrightarrow A(G)^* \; ; \; u \longmapsto u^{A+B}.$$

We often write $u \uparrow a$ for u^a. By this action, $A(G)^*$ is an $A(G)_{(2)}$-module whose annihilator contains $2A(G)_{(2)}$.

<u>Theorem B</u>. (i) <u>There is a decomposition</u>

$$A(G)^* = \prod_{(Q)} A(G)^* \uparrow e^2_{G,Q},$$

<u>where</u> (Q) <u>runs over</u> $Cl_2(G)$, <u>classes of</u> 2-<u>perfect subgroups</u>.

(ii) <u>Let</u> Q <u>be a</u> 2-<u>perfect subgroup of</u> G <u>and let</u> P <u>be a subgroup of</u> $N := N_G(Q)$ <u>such that</u> P/Q <u>is a Sylow</u> 2-<u>subgroup of</u> N/Q. <u>Then there are group isomorphisms</u>:

$$A(G)^* \uparrow e^2_{G,Q} \cong A(N)^* \uparrow e^2_{N,Q} \cong (A(P)^*)^N \uparrow e^2_{P,Q}.$$

<u>where the last group is the subgroup of</u> $A(P)^* \uparrow e^p_{P,Q}$ <u>consisting of all elements</u> x <u>such that</u>

$$\mathrm{res}^P_{P^n \cap P}\, \mathrm{con}^n_P\,(x) = \mathrm{res}^P_{P^n \cap P}\,(x)$$

for any element n of N.

Theorem C. Let N be a finite group with 2-perfect normal subgroup Q. Put $W := N/Q$. Let $\bar{P} = P/Q$ be a Sylow 2-subgroup of W. Then the following groups are isomorphic:

(a) $A(N)^* \uparrow e^2_{N,Q}$,

(b) $\{\bar{u} \in A(W)^* \uparrow e^2_{W,1} \mid \bar{u}(S/Q) = 1 \text{ if } O^{2'}(S) \not\geq Q\}$,

(c) $\{\bar{v} \in A(\bar{P})^{*W} \mid \bar{v}(S/Q) = 1 \text{ if } O^{2'}(S) \not\geq Q\}$,

where $O^{2'}(S)$ is the subgroup of S generated by all 2-elements and $A(\bar{P})^{*W}$ is the set of elements v of $A(\bar{P})^*$ such that

$$\mathrm{res}_{\bar{P}^w \cap \bar{P}}\, \mathrm{con}^g(x) = \mathrm{res}_{\bar{P}^w \cap \bar{P}}(x) \quad \text{for all } w \text{ in } W.$$

Theorem D. Assume that the finite group G has an abelian Sylow 2-subgroup. Let Q be a 2-perfect subgroup of G. Put $N := N_G(Q)$, $W := N/Q$, and let P/Q be an (abelian) Sylow 2-subgroup of W. Put $L := N_G(P)$ $(\leq N)$, $\bar{L} := L/Q'$, $\bar{Q} := Q/Q'$, and $\bar{P} := P/Q'$, where Q' is the intersection of subgroups of Q of odd prime index. Then the following hold :

(i) $A(G)^* \uparrow e^2_{GQ} \cong A(\bar{L})^* \uparrow e^2_{\bar{L}\bar{Q}}$.

(ii) *If* Q *is perfect, then*

$$A(\bar{L})^* \uparrow e^2_{\bar{L}\bar{Q}} \cong A(P/Q)^{*L/Q} \cong C_{(P/QP^2)}(L/Q),$$

where C *stands for the centralizer group.*

(iii) *Assume that* Q *is not perfect. If* $\bar{P}$ *is generated by elements* t *with* $C_{\bar{Q}}(t) \neq 1$, *then* $A(\bar{L})^* \uparrow e^2_{\bar{L}\bar{Q}} = 1$, *and otherwise it is of order* 2.

References.

[Ar82] Araki, S. : Equivariant stable homotopy theory and idempotents of Burnside rings, Publ. RIMS, Kyoto Univ. 18 (1982), 1193-1212.

[CR81] Curtis, C.W. and Reiner, I. : "Methods of Representation Theory", Wiley-Interscience publ., New York, 1981.

[Di79] tom Dieck, T. : "Transformation Groups and Representation Theory", Lecture Notes in Math., 766, Springer-Verlag, Berlin-New York, 1979.

[Dr71] Dress, A. : Operations in representation rings, Proc. Symposia in Pure Math. XXI (1971), 39-45.

[Dr73] Dress, A. : Contributions to the theory of induced representations. Algebraic K-Theory II, Proc. Battle Institute Conference 1972, Springer Lecture Notes in

Math., 342, 183-240, Springer-Verlag, Berlin-New York, 1973.

[Gl83] Gluck, D. : Idempotent formula for the Burnside algebra with applications to the p-subgroup simplicial complex, Illinois J. Math., 25 (1981), 63-67.

[Go68] Gorenstein, D. : "Finite Groups", Harper and Row, New York, 1968.

[Gr71] Green, J.A. : Axiomatic representation theory for finite groups, J. Pure and Applied Algebra, 1 (1971), 41-77.

[Ma82] Matsuda, T. : On the unit groups of Burnside rings, Japanese J. Math., New series, 8 (1982), 71-93.

[MM83] Matsuda, T. and Miyata, T., On the unit groups of the Burnside rings of finite groups, J. Math. Soc. Japan 35 (1983), 345-354.

[Sa82] Sasaki, H. : Green correspondence and transfer theorems of Wielandt type for G-functors, J. Algebra, 79 (1982), 98-120.

[Yo78] Yoshida, T. : Character-theoretic transfer, J. Algebra, 52 (1978), 1-38.

[Yo80] Yoshida, T. : On G-functors I : Transfer theorems for cohomological G-functors, Hokkaido Math. J., 9 (1980), 222-257.

[Yo83] Yoshida, T. : Idempotents of Burnside rings and Dress induction theorem, J. Algebra, 80 (1983), 90-105.

[Yo85] Yoshida, T. : Idempotents of Burnside rings and character rings, to appear.

Appendix

(References on Burnside rings)

[Ada.1979] Adams J.F., Graeme Segal's Burnside ring conjecture, (Proc. Symp. Siegen 1979), 378-395, Lecture Notes in Math.788, Springer, Berlin, 1979.

[Ara.1982] Shoro Araki, Equivariant stable homotopy theory and idempotents of Burnside rings, Publ. RIMS, Kyoto Univ. 18 (1982), 1193-1212.

[Bla.1979] Andreas Blass, Natural endomorphisms of Burnside rings, Trans. Amer. Math. Soc. 253 (1979),121-137.

[Boo.1975] Boorman, E.H., S-operations in representation theory, Trans. Amer. Math. Soc. 205 (1975), 127-149.

[Car.1984] Carlsson, G., Equivariant stable homotopy and Segal's Burnside ring conjecture, Ann. Math. 120 (1984), 189-224.

[Con.1968] S. B. Conlon, Decompositions induced from the Burnside algebra, J. Algebra 10 (1968), 102-122.

[Die.1975] Tammo tom Dieck, The Burnside ring and equivariant stable homotopy, Lecture Notes by Michael C.

Bix, Dept. Math., Univ. of Chicago, Chicago, Ill., 1975. MR 54/11368.

[Die.1975] Tammo tom Dieck, The Burnside ring of a compact Lie group.I, Math. Ann. 215 (1975), 235-250. MR 52/15510.

[Die.1977] Tammo tom Dieck, A finiteness theorem for the Burnside ring of a compact Lie group, Compositio Math. 35 (1977), 91-97. MR 57/13990.

[Die.1977] Tammo tom Dieck, Idempotent elements in the Burnside ring, J. Pure Appl. Algebra 10 (1977), 239-247.

[DiePet.1978] Tammo tom Dieck and Ted Petrie, Geometric modules over the Burnside ring, Invent. Math. 47 (1978), 273-287.

[Die.1978] Tammo tom Dieck, Homotopy equivalent group representations and Picard groups of the Burnside ring and the character ring, Manuscripta Math. 26 (1978), 179-200.

[Die.1978] Tammo tom Dieck, Semi-linear group action on spheres: Dimension functions, Proc. Symp. of Algebraic topology, Aarhus 1978, pp. 448-457, Springer Lecture Notes in Math. 763, Springer-Verlag, Berlin-New York, 1979.

[Die.1979] Tammo tom Dieck, "Transformation groups and representation theory", Lecture Notes in Math., 766, Springer-Verlag, Berlin-New York, 1979.

[Dre.1969] Andreas Dress, A characterization of solvable

groups, Math. Z. 110 (1969),213-217.

[Dre.1970] Andreas Dress, The Burnside ring of a finite group and applications towards classical representation theory, Papers from Open House for Algebraists, Aarhus University 1970. Various publ. ser. 17 (1970), 47-52.

[Dre.1971] Andreas Dress, Notes on the theory of representations of finite groups. I, Lecture Notes, Bielefeld, 1971.

[Dre.1971a] Andreas Dress, Operations in representation rings, Proc. Symposia in Pure Math. XXI (1971), 39-45.

[Dre.1971b] Andreas Dress, The ring of monomial representations, I. Structure theory, J. Algebra 18 (1971), 137-157.

[DreKuc.1970] Andreas Dress and M. Kuchler, Zur Darstellungstheorie Endlicher Gruppen I (preliminary edition), Bielefeld, 1970.

[GayMorMor.1983] Gay, C.D., Morris, G.C. and Morris, I., Computing Adams operations on the Burnside ring of a finite group, J. Reine Angew Math. 341 (1983), 87-97.

[Glu.1981] David Gluck, Idempotent formula for the Burnside algebra with applications to the p-subgroup simlicial complex, Illinois J. Math. 25 (1981), 63-67.

[Gor.1975]? Robin A. Gordon, Contributions to the theory of the Burnside ring. Doctorial Thesis, Saarbrucken 1975.

[Gor.1977] Robin A. Gordon, The Burnside ring of a cyclic

extension of a torus, Math. Z. 153 (1977),149-153.

[Gor.1978] Robin A. Gordon, On the Burnside ring of a compact group, Proc. Amer. Math. Soc. 68 (1978),101-107.

[HasKak81] Shin Hashimoto and Shin-ichiro Kakutani, A note on elements of the Burnside ring and a finite group, J. Math. Kyoto Univ., 21 (1981), 619-623.

[Hus.1979] D. Husemoller, Burnside ring of a Galois group and the relations between zeta functions of intermediate fields,The Santa Cruz Conference on Finite Groups (Univ. California, Santa Cruz, Calif., 1979), pp. 603-610, Proc. Symp. in Pure Math., 37, Amer. Math. Soc., Providence, R. I., 1980.

[Kos.1973] Czes Kosniouski, Localizing the Burnside ring, Math. Ann. 204 (1973) 93-96.

[Kos.1974] Kosniowski, C., Equivariant cohomology and stable cohomotopy, Math. Ann. 210 (1974), 83-104.

[Kra.1970] Helmut Krämer, On the singularities of the Burnside ring of a finite group, Papers from Open House for Algebraists, Aarhus University 1970. Various publ. ser. 17 (1970), 53-71.

[Kra.1974a] Helmut Krämer, Uber die Automorphismengruppe des Burnsideringes endlicher abelscher Gruppen, J. Algebra 30 (1974), 279-293.

[Kra.1974b] Helmut Krämer, Uber die injective Dimension des Burnsideringes einer endlichen Gruppe, J. Algebra 30

(1974), 294-304.

[Kra.1981] Helmut Krämer, Zur Idealtheorie des Burnsideringes von p-Gruppen, Arch. Math. 37 (1981), 129-139.

[KraThe.1983] Charles Kratzer and Jacques Thévenaz, Anneau de Burnside et fonction de Moebius d'un groupe fini, Comment. Math. Helvetici 59 (1984), 425-438.

[Lai.1979] Erkki Laitinen, On the Burnside ring and stable cohomotopy of a finite group, Math. Scand. 44 (1979), 37-72.

[Mat.1979] Toshimitsu Matsuda, On the equivariant self-homotopy equivalences of spheres, J. Math. Soc. Japan, 31 (1979), 69-89.

[Mat.1982] Toshimitsu Matsuda, On the unit groups of Burnside rings, Japanese J. Math., New series, 8 (1982), 71-93.

[MatMiy.1983] Toshimitsu Matsuda and Takehiko Miyata, On the unit groups of the Burnside rings of finite groups, J. Math. Soc. Japan 35 (1983), 345-354.

[Min.1984] Minami, Norihiko, On the I(G)-adic topology of the Burnside ring of compact Lie groups, Publ. RIMS, Kyoto Univ. 20 (1984), 447-460.

[MorWen.1984] Morris, I. and Wensley, C.B., Adams operations and -operations in -rings, Discrete Math. 50 (1984), 253-270.

[Nic.1978a] D. M. Nicolson, The orbit of the regular G-set under the full automorphism group of the Burnside ring of a finite group G, J. Algebra 51 (1978), 288-299.

[Nic.1978b] D. M. Nicolson, On the graph of prime ideals of the Burnside ring of a finite group, J. Algebra 51 (1978), 335-353.

[Oli.1975] Robert Oliver, Fixed point set of group actions on finite acyclic complexes, Comment. Math. Helv. 50 (1975), 155-177.

[Oli.1977] Robert Oliver, G-actions on disks and permutation representations II, Math. Z. 157 (1977), 237-263.

[Oli.1978a] Robert Oliver, G-actions on disks and permutation representations, J. Algebra 50 (1978), 44-62.

[Oli.1978b] Robert Oliver, Group actions on disks, integral permutation representations and the Burnside ring, In Proc. Symp. Pure Math. 32 vol. 1: Algebraic and Geometric topology, Amer Math. Soc. 1978, 339-346.

[Pet.1978] Ted Petrie, Three theorems in transformation groups,Algebraic topology, Aarhus 1978 (Proc. Sympos., Univ. Aarhus, Aarhus, 1978), pp 549-572, Lecture NOtes in Math., 763, Springer, Berlin, 1979.

[Pet.1979] Ted Petrie, Transformation groups and representation theory, The Santa Cruz Conference on

Finite Groups (Univ. California, Santa Cruz, Calif., 1979), pp. 621-631, Proc. Symp. in Pure Math., 37, Amer. Math. Soc., Providence, R. I., 1980.

[Rit.1972] J. Ritter, Ein Induktionssatz fur rationale Charaktere von nilpotenten Gruppen, J. f. d. reine u. angew. Math. 254 (1972), 1330151.

[Rym.1975] N. W. Rymer, Burnside ring and the Euler characteristic of a symmetric power, (preprint).

[Rym.1977] N. W. Rymer, Power operations on the Burnside ring, J. London Math. Soc. (2) 15 (1977), 75-80.

[Sch.1975] Roland Schwanzl, Der Burnsidering der speziellen orthogonalen Gruppe der Dimension drei, Diplomarbeit, Saarbrucken 1975.

[Sch.1977] Roland Schwanzl, Koeffizienten im Burnsidering, Arch. Math. 29 (1977), 621-622.

[Sch.1979] Roland Schwanzl, On the spectrum of the Burnside ring, J. Pure Applied Algebra 15 (1979), 181-185.

[Seg.1970] G. B. Segal, Equivariant stable homotopy theory, Actes Congres international Math. (1970), Tome 2, 59-63.

[Seg.1972] G. B. Segal, Permutation representations of finite p-groups, Quart. J. Math. Oxford (2), 23 (1972), 375-381.

[Sie.1976] Christian Siebeneicher, -Ringstrukturen auf dem Burnsidering der Permutationsdarstellungen einer endlichen Gruppe, Math. Z. 146 (1976), 223-238.

[Sol.1967] Louis Solomon, The Burnside algebra of a finite group, J. Combinatorial Theory 2 (1967), 603-615.

[Wag.1980] Bernd Wagner, A permutation representation theoretical version of a theorem of Frobenius, Bayreuther Math. Schur. 6 (1980), 23-32.

[Yos.1983] Tomoyuki Yoshida, Idempotents of Burnside rings and Dress induction theorem, J. Algebra 80 (1983), 90-105.

[Yos.1984] Tomoyuki Yoshida, The Mobius algebra as a Burnside ring, Hokkaido Math. J., 13 (1984), 362-376.

[Yos.1985] Tomoyuki Yoshida, Idempotents of Burnside rings and character rings, to appear.

[Yos.198?] Tomoyuki Yoshida, On the unit groups of Burnside rings, to appear.