

On the Uniformization of Analytic Sets  
with countable sections

(Abstract)

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In (Ya 1) we announced the following results:

(I) Every analytic set in the plane with countable sections can be uniformized by the difference of two analytic sets.

(II) There is an analytic set in the plane with countable sections which cannot be uniformized by an analytic set.

Clearly we have

(III) There is an analytic set in the plane with countable sections which cannot be uniformized by a co-analytic set.

But we cannot decide the following proposition:

(\*) There is an analytic set in the plane with countable sections which cannot be uniformized by the sum of an analytic set and a co-analytic set.

In my visiting Berkeley at the March of 1984, Professor Addison pointed out me the following more weak proposition:

(\*\*) There is an analytic set in the plane with countable sections which cannot be uniformized by either an analytic set or a co-analytic set.

For (\*\*) we can show

THEOREM 1. There is a  $\sum_1^1$  set  $A$  in the plane with the following properties:

- (i) For each real  $\alpha$   $A^{<\alpha>} = \{\beta : A(\alpha, \beta)\}$  is nonempty and at most two elements.
- (ii)  $A$  cannot be uniformized by either an analytic set or a co-analytic set.

Assuming projective determinacy, we can extend the theorem to

THEOREM 2. There is a  $\sum_{2n+1}^1$  set  $A$  in the plane with the following properties:

- (i) For each real  $\alpha$   $A^{<\alpha>}$  is nonempty and has at most two elements.
- (ii)  $A$  cannot be uniformized by either a  $\sum_{2n+1}^1$  set or a  $\prod_{2n+1}^1$  set.

For the proofs of these, see (Ya 2). Also full uniformization problem of analytic sets the reader can be found in (Mo) and (Ya 3).

#### References

- (Mo) Y.N. Moschovakis, Descriptive Set Theory, North-Holland, 1980.
- (Ya 1) Y. Yasuda, On the uniformization of analytic sets with countable sections and related results, RIMS Kokuroku, 480(1983), 204-208.

- (Ya 2) Y.Yasuda, An answer of a question of Addison concerning the uniformization of analytic sets, to appear in Comment. Math. St. Paul. in 1986.
- (Ya 3) Y.Yasuda, Some properties of thin  $\prod_{2n+1}^1$  sets, to appear in Z. Math. Logik Grundlag. Math. in 1987.