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# Map 0L systems with markers

by

Akira Nakamura* (中村 昭)
Aristid Lindenmayer**
Kunio Aizawa* (会沢邦夫)

*Department of Applied Mathematics
Hiroshima University
Higashi-Hiroshima, 724
Japan
**Theoretical Biology Group
University of Utrecht
Utrecht 3508 TB
The Netherlands

## 0. Introduction

Recently, there have been published a number of interesting papers on map generating systems. (For example, see papers in [1] and [2].) They also include the cell division systems which are motivated from a biological point of view.

A map is defined combinatorially by Tutte [3].

In [4], we proposed two map generating systems based on string generation. The first one is binary, propagating map 0L system with makers (mBPM0L systems). The second one is binary, propagating map IL system with makers (mBPMIL systems). Then, we consider the following decision problems (1) and (2) on these systems:

(1) Whether or not an arbitrary mBPM0L system is deterministic ?
(2) Whether or not an arbitrary mBPMIL system is deterministic ?

These decision problems were solved in the paper [4]. Also, in the same paper the decision problem of stability of these two systems was solved. It is not so difficult to show that the membership problem of propagating systems is recursively solvable.

In this paper, we shall consider an extension of binary, propagating map 0L systems with makers. We propose map 0L systems with makers (mM0L systems). The systems are nondeterministic, and also neither binary nor propagating. Then, we shall examine the membership problem of the mM0L systems. As an interesting result, we show that this decision problem is recursively unsolvable. The main purpose of this paper is to prove this fact.

We assume that readers are familiar with the basic definitions and notations of map generating systems.

## 1. Definitions

We give here some definitions and notations. They are almost the same to those of [4].

Definition 1.1 A binary, propagating map 0L system with makers (mBPM0L system) is a quadruple ( $\Sigma$ , $\{\downarrow, \uparrow, +, -, (, )\}$ ,P,S), where

(1) $\Sigma$ is a finite edge alphabet,

(2) P is a finite set of edge productions,

(3) S is a starting map:

Here, edge productions are of the form A $\longrightarrow$ a, where A is a member of $\Sigma$ (an edge label) and $\alpha$ is a sequence of members of $\Sigma$ and signs +, -, and of markers $\downarrow, \uparrow$ and matched parentheses. In this case, only one symbol appears within a closed parentheses. For instance, the rewriting of an edge represented in Figure 1 is written as the edge production:

$$A \longrightarrow D^{+}C^{-} \downarrow (E^{-})B^{+}.$$

The makers $\downarrow, \uparrow$ indicate the place and the direction (to the left or right of the original edge according its orientation) in which a new edge can be inserted. The edge symbol and sign between parentheses associated with the maker indicates the label and orientation of the new edge ("+" if the orientation agree with the arrow, and "-" if it is opposite).

In these systems, a derivation step consists of the rewriting of all the edges surrounding a given wall (a wall means a cell in the two-dimensional case), of finding the makers pointing inwards into the wall, and of inserting one new edge if there are at least two makers with matching labels and orientation present. The insertion of a new edge is uniquely determined if there are exactly two such matching makers, and the insertion is nondeterministic if there are more than three such makers.

Definitions 1.2. An mBPM0L system is deterministic when all derivation steps are deterministic. Otherwise, an mBPM0L system is nondeterministic.

Deterministic systems are a special case of nondeterministic ones. In general, mBPM0L systems mean nondeterministic mBPM0L systems. When we need to distinguish those two systems, deterministic and nondeterministic ones are denoted by DmBPM0L and NmBPM0L systems, respectively.

We now extend mBPM0L systems as follows:

Definition 1.3. A map 0L system with markers (mM0L systems) G is a quintuple ( $\Sigma \cup \{\varepsilon\}$ , $\{\downarrow, \uparrow, +, -, (, )\}$, P, S, $\lambda$ ), where

(1) $\Sigma$ is a finite edge alphabet,

(2) $\varepsilon$ is the blank symbol which is not in ,

(3) P is a finite set of edge productions, which satisfies the following conditions:
Edge productions are of the form A $\longrightarrow \alpha$ , where A is a member of $\Sigma$ and $\alpha$ is a sequence of $\Sigma \cup \{\varepsilon\}$ and signs, of markers and matched parentheses,

(4) S is a starting map,

(5) $\lambda$ is the empty map.

The meaning of productions is the same to that of

Definition 1.1, except the case containing $\varepsilon$. Productions containing $\varepsilon$ are as follows: Let the production be $A \longrightarrow \varepsilon$. Then, A is erased and two end points of A become a point. If an edge labelled A has only one vertex, then this vertex remains (if it is a part of another boundary), or it goes to $\lambda$. For two or more A's having the same end points, this process is done nondeterministically. We give some examples of applications of $A \longrightarrow \varepsilon$.

By applications of $A \longrightarrow \varepsilon$, $B \longrightarrow D^+$, $C \longrightarrow E^-$ to the map of Figure 2a, we get a new map of Figure 2b. Also, by applications of the same rules to the map of Figure 3a, we get nondeterministically one of the maps shown as in Figure 3b-c. That is, one of A's is erased and both of its ends become a vertex, and then the A disappears.

For a submap surrounding with edges labelled with A, the same situation occurs. For example (a case of existing tree edges having label A), by applications of $A \longrightarrow \varepsilon$, $B \longrightarrow G^+$, $C \longrightarrow H^+$, $D \longrightarrow K^+$ to the map in Figure 4, we get nondeterministically one of the maps shown in Figure 5a-c.

Derivation steps of mM0L systems are also the same to mBPM0L's. A derivation step of mM0L systems consists of the rewriting of all the edges surrounding a given wall, of finding the makers pointing inwards into the wall, and of inserting new edges, if there are at least two markers with matched labels and orientation present. In this case, there may be more than one inserted if there are four or more matching markers. For instance, let Figure 6 be a configuration obtained after rewriting of all edges. From this configuration, we get one of the maps in Figure 7.

Deterministic and nondeterministic cases of mM0L systems are defined by the same way to the mBPM0L systems.

In mM0L systems, it happens that a wall disappears. An example is given as follows:

By applying production rules $A \longrightarrow A^+$, $B \longrightarrow B^+$, $C \longrightarrow \varepsilon$, we have the derivation represented in Figure 8. Thus, mM0L systems are generally nondeterministic and also neither binary nor propagating.

## 2. Membership problem

In this section, we consider the membership problem of the mMOL systems. it has been known that the usual 0L system has the decidability result for its membership problem. In contrast with this fact, the menbership problem of our map 0L systems with markers is recursively unsolvable. As the usual technique to prove the unsolvability, the idea used here is based on the halting problem of Turing machine. Exactly speaking, we use the result of the meeting problem of the finite causal $\omega^2$-systems (see [5]).

The meeting problem (A) of finite causal $\omega^2$-system has been given as follows:

(A) To decide, for an arbitrary given finite causal $\omega^2$-system with one input symbol, whether or not it will meet with a given special state under starting from a given initial state.

In [5], we proved that this decision problem (A) is unsolvable. We use this result to prove that the membership problem of mMOL's is recursively unsolvable.

Before the proof, we give meanings of some notations used below. Let us assume that $|Q|=t$. In finite causal $\omega^2$-systems, all squares in the first quadrant have been given in advance. An mBPMIL system could build these squares by making use of interaction I, as mentioned in [4]. However, an mMOL system can also build those squares nondeterministically, as described below. In the following notations, h, v, u, and r correspond to "horizontal", "vertical", "upper", and "right", respectively. Also, D, E mean "odd" and "even" in the horizontal direction from the (1,1)-square. As edge labels, we use quadruple $(q_i,u,w,n)$, $(q_i,r,w,n)$, $((q_i,q_j),u,w,n)$, where $i=1,\dots,t$, $w=D,E$, and $n=0,1,2$. As seen in the following explanation, $q_i$ of $(q_i,r,w,n)$ corresponds to a state $q_i$ (of the finite causal $\omega^2$-system) of the under square of this edge label. Similarly, $q_i$ of $(q_i,r,w,n)$ corresponds to a state $q_i$ of the left square of this edge label.

In addition to these edge labels, we use synbols such as $q_0$, $v_d$, $v_r$, $h_d$, $h_u$, $(v_d,n)$, $(h_d,n)$. These meaning is possibly understood from Figure 9-14.

<u>Lemma 2.1</u> One can define an mM0L system that constructs a map corresponding to the two-dimensional cell space spread over the first quadrant.

<u>Sketch of the proof</u>:

As exact proof of this lemma is complicated and tedious. So, we give a sketch of the proof. As mentioned before, an idea is based on simulation of a finite causal $\omega^2$-system in which the two-dimensional cell space is constructed. Since an mM0L system cannot get the neighboring information, it proceeds by guess.

Now, we consider an mM0L system $G=(\Sigma \cup \{\varepsilon\}, \{\downarrow, \uparrow, +, =, (, )\}, P, S, \lambda)$ satisfying the conditions:

(1) $\Sigma$ consists of all symbols which appear in Figure 9-14, and in the following P.

(2) P consists of all rules which are necessary for derivations as shown in Figure 10-14.

(3) S is the map shown in Figure 9.

Derivations by G are very lengthy and for this reason omitted here. The first step is shown in Figure 10a-b. Notice that, in Figure 7, $q_1$ of $\underset{\dashrightarrow}{(q_1,u,D,2)}$ and $q_2$ of $\downarrow\!\vdots\,(q_2,u,E,2)$ are nondeterministically chosen. But, $q_1$ of $\downarrow\!\vdots\,(q_1,u,D,2)$ is determined from $q_0$ of $\underset{\longrightarrow}{\underline{(q_0,u,D,1)}}$ by the corresponding finite causal $\omega^2$-system and also $q_2$ of $\underset{\dashrightarrow}{(q_2,u,E,2)}$ is similarly determined. The next step is shown in Figure 11. Then, we get the maps in Figure 12 and 13. Notice here that, in Figure 13, $q_4$ is determined from $(q_2,q_3)$ by the corresponding finite causal $\omega^2$-system. By repeated these processes, we get the map shown in Figure 14. Therefore, we get this lemma.

The G of Lemma 2.1 could construct a map which corresponds to the two-dimensional cell space. In this case, labelling of edges similates the state transition of the corresponding finite causal $\omega^2$-system. This is done by guessing. By this guessing, however, G generates also the map shown in Figure 15. We want to exclude such a map. By making use of this illegal map, we prove the following theorem.

Theorem 2.2. The membership problem for mM0L systems is recursively unsolvable.

Proof: Let us consider an mM0L system which can simulate a finite causal $\omega^2$-system. This is definable by making use of the mM0L system of Lemma 2.1. Because the mM0L system of Lemma 2.1 was defined such a way that its edge labels correspond to states of finite causal $\omega^2$-system. Here, we notice again that the mM0L system generates illegal maps. to characterize this illegal map, we extend the mM0L system of Lemma 2.1 in such a way that it generated illegal edge $i_e$. This illegal edge $i_e$ is obtained by connecting between a "vertical" odd edge and a "horizontal" odd edge (or a "vertical" even edge and a " horizontal" even edge). In this case, to obtain this illegal edge the makers are put for consecutive two steps sfter generation of edge labelled with 0. Thus, we get an illegal edge as shown in Figure 16. Further, we extend this mM0L system in such a way that it satisfies the fillowing conditions:

(1) All edges except illegal edges and the special edge corresponding to the special state of the problem (A) of a finite causal $\omega^2$-system are erased after three steps from generation.

(2) The special edge generates a special map and the illegal edges generate the extra maps.

This extension of the mM0L system is always possible, although the definition is complicated. Here, we consider legal map. Then, we ask if a map which contains the extra map and all other edges are erased is generated by the extended mM0L system.

Obviously, this question is equivalentto the meeting problem (A) of finite causal $\omega^2$-system. Therefore, we get this theorem.

**3. Remark**

We have proposed mM0L systems which are neither propagating nor binary. These systems seem to be interesting since they include the possibility of developmental sequences with certain walls or edges disappearance. There are further problems on relationships between the mM0L and map 0L systems with other controle devices than markers (e.g., those in [6]) for division determination.

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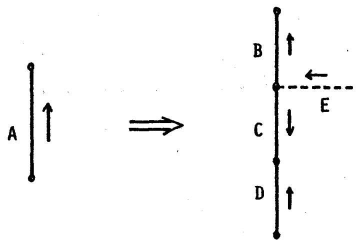


Figure 1.

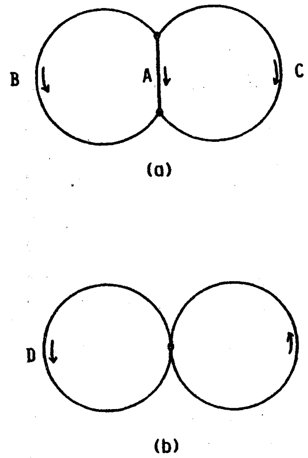


Figure 2.

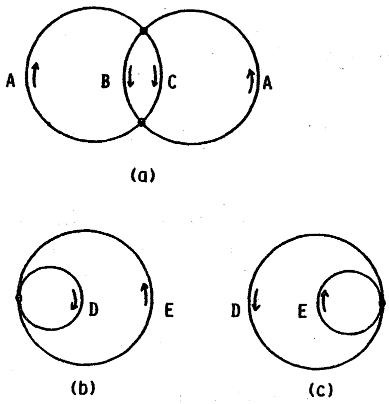


Figure 3.

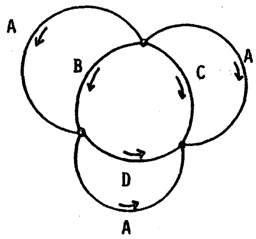


Figure 4.

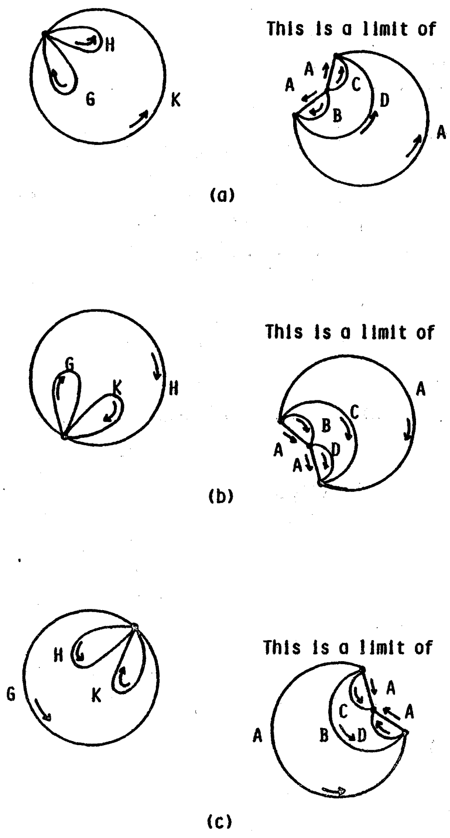


Figure 5.

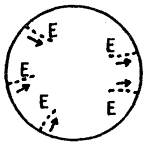


Figure 6.

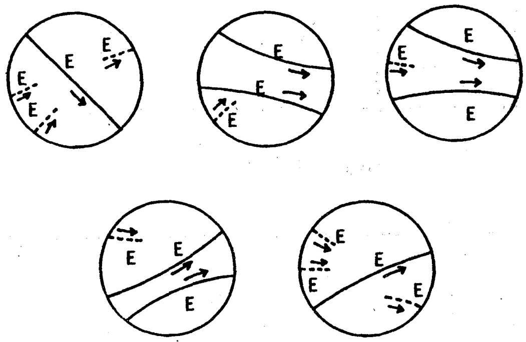


Figure 7.

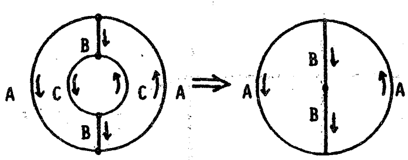


Figure 8.

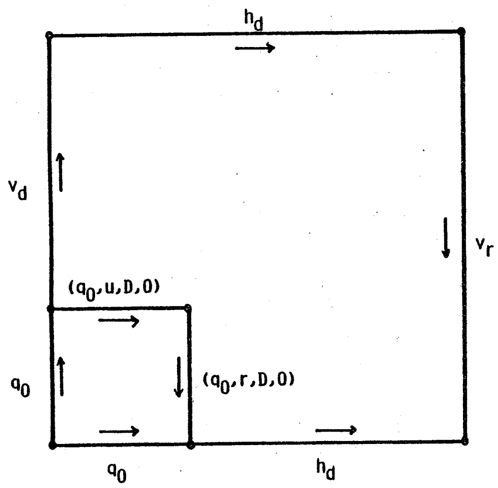


Figure 9.

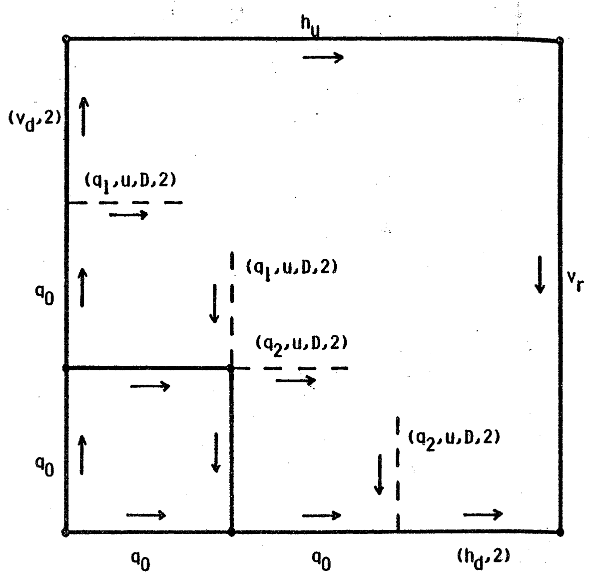


Figure 10a.

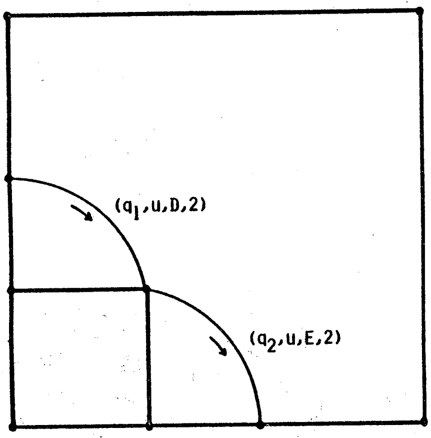


Figure 10b.

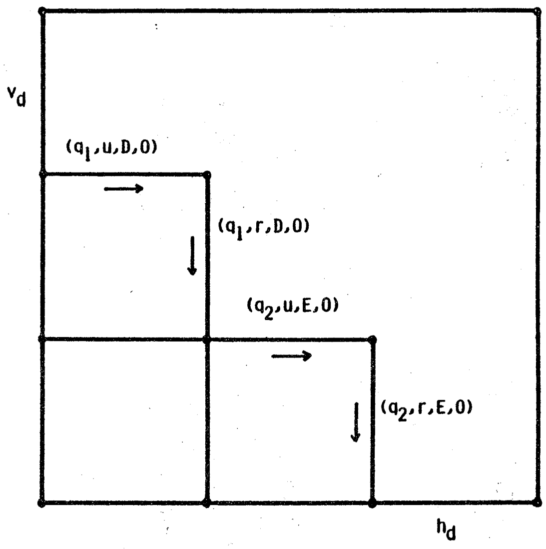


Figure 11.

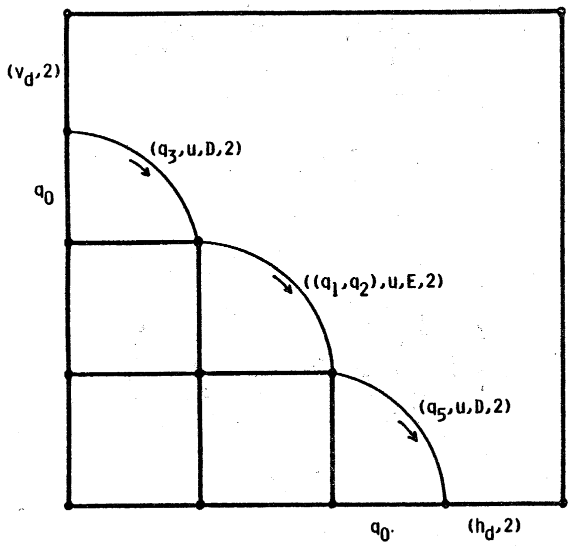


Figure 12.

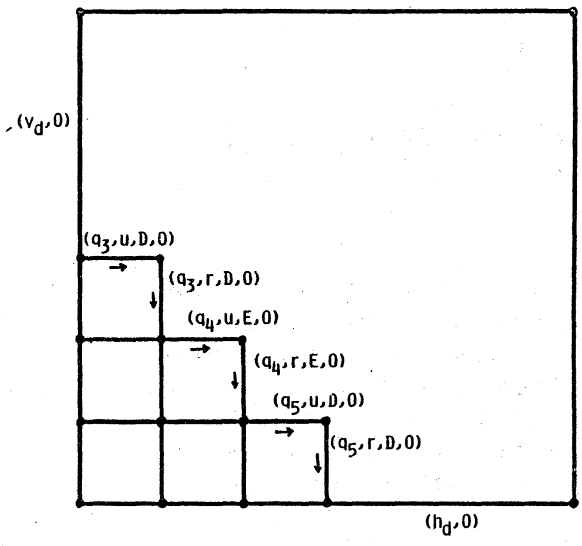


Figure 13.

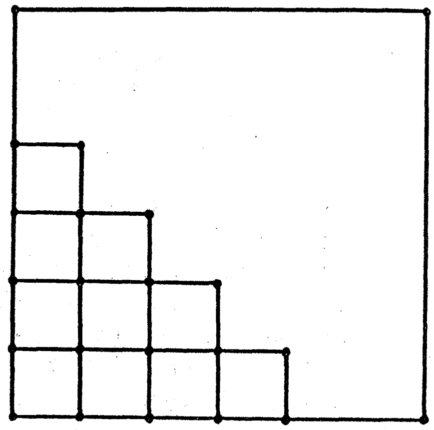

Figure 14.

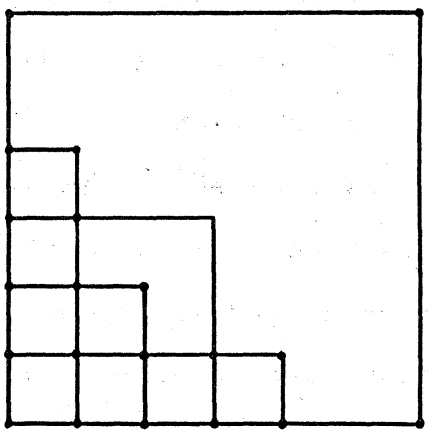

Figure 15.

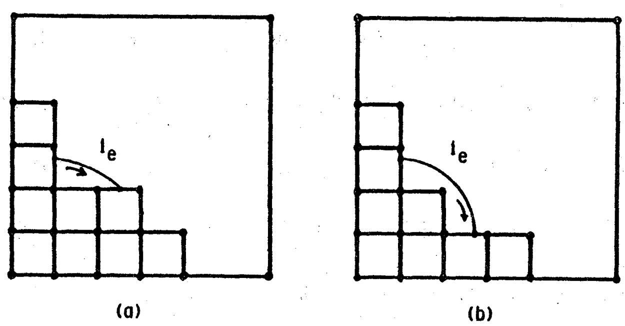


Figure 16.