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Semantics of Joins of Knowledge Bases

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Abstract

In this paper we propose a more natural model of updates of knowledge bases, where we represent a knowledge as a closed formula of first-order logic and consider a knowledge base as a theory. To do this we extend the model of updates of theories proposed by Fagin et al., and define joins of theories. Furthermore we extend the concept of join to treat logical databases of Fagin et al. and show that we can formulate the insertion of theories of Fagin et al. as a special case of the join of logical databases.

1. Introduction

Recently many researches have been devoted to theoretical studies of knowledge bases, particularly, the formal semantics of updating knowledge bases. They naturally consider that a knowledge can be represented as a sentence, i.e., a closed formula of the first-order logic and a knowledge base as a theory, i.e., a consistent set of sentences.

Fagin et al. [1,2] studied the semantics of updates in databases, and introduced a partial order in the possible new theories that accomplish the update to define the concept of the minimal theory accomplishing it. In particular they required that for an insertion of a sentence σ into a theory S, the sentence σ should belong to the theory that accomplishes the insertion.

The main results of Fagin et al. [1,2] are summarized as follows:

(1) Let S, T be theories, and let σ be a sentence. $T \cup \{\sigma\}$ accomplishes the insertion of σ into S minimally if and only if T is a maximal subset of S that is consistent with σ.

(2) Let S, T be theories, and let Σ be a set of sentences. $T \cup \Sigma$ accomplishes the insertion of Σ into S minimally if and only if T is a maximal subset of S that is consistent with Σ.

On the other hand, the authors [3] independently defined a different model of insertions to discuss a dynamic behavior of knowledge bases. They considered that for an insertion of a sentence σ into a theory S the theories that accomplish the insertion should be the maximal consistent subsets of $S \cup \{\sigma\}$. These theories include not only the theories that accomplish the insertion in the sense of the model of Fagin et al., but also the theory S itself. We can regard the latter case as the rejection of the insertion because of arising of inconsistency.

In this paper, we extend the model of Fagin et al., to define a more natural model of updates of theories, that is, joins of theories instead of insertions. This enables us to treat the models of Fagin et al. and the authors in the same frame work. Furthermore we extend the concept of join to treat logical databases of Fagin et al. [1] and show that we can formulate the concept of insertion of Fagin et al. as a special case of the join of logical databases.

We use the similar notions and notations to Fagin et al. [1,2]. We assume that a sentence is neither inconsistent nor valid. We shall use the letters such as $\sigma, \tau, \cdots$ to denote a sentence, and S, T, $\cdots$ to denote a theory.

2. Joins of Theories

We define the semantics of joins of theories like Fagin et al.

Definition 2.1 Assume that $S_1 \cup S_2 \neq \phi$. A theory T accomplishes the join of S_1 and S_2 if $(S_1 \cup S_2) \cap T \neq \phi$. When $S_1 \cup S_2 = \phi$, we define that any theory T accomplishes the join of S_1 and S_2. □

Here we introduce a partial order in the theories that accomplish the join. This enables us to discuss the minimal changes for joins of theories.

Definition 2.2 Let T_1 and T_2 be two theories that accomplish the join of S_1 and S_2, and let S be $S_1 \cup S_2$.
(1) T_1 has fewer insertions than T_2 with respect to S if $T_1 - S \subset T_2 - S$.
(2) T_1 has the same insertions as T_2 with respect to S if $T_1 - S = T_2 - S$.
(3) T_1 has fewer deletions than T_2 with respect to S if $S - T_1 \subset S - T_2$.
(4) T_1 has the same deletions as T_2 with respect to S if $S - T_1 = S - T_2$. □

We shall omit reference to S when it is clear from the context.

Lemma 2.1 For each theory T that accomplishes the join of S_1 and S_2, there is a theory T' such that
(1) T' accomplishes the join of S_1 and S_2,
(2) $T' \subseteq T$, and
(3) T' has the same deletions as T. □

In fact, $T' = (S_1 \cup S_2) \cap T$ satisfies the above conditions (1)～(3).

The above lemma claims that when dealing with joins it suffices to consider the set of deleted sentences. Thus we can make the following definition.

Definition 2.3 T_1 accomplishes the join of S_1 and S_2 with a smaller change than T_2 if both T_1 and T_2 accomplishes the join, and T_1 has fewer deletions than T_2. □

Definition 2.4 T accomplishes the join of S_1 and S_2 minimally if there is no theory that accomplishes this join with a smaller change than T. □

Theorem 2.1 T accomplishes the join of S_1 and S_2 minimally if and only if T is a maximal consistent subset of $S=S_1 \cup S_2$. □

[Proof] This theorem trivially holds when $S=\phi$. So we assume that $S\neq \phi$.

(1) Sufficiency. Assume that T does not accomplish the join minimally. Then there is a theory T' such that T' accomplishes the join and $S-T' \subset S-T$. Note that Lemma 2.1 allows us to take T' as a subset of S. So $T \subset T'$. For a sentence σ in $T'-T$, $T \cup \{\sigma\}$ is consistent. This is a contradiction.

(2) Necessity. Assume that T is not a maximal subset. Then there is a sentence σ in S-T, and $T \cup \{\sigma\}$ is consistent. $T \cup \{\sigma\}$ accomplishes the join, and $S-(T \cup \{\sigma\}) \subset S-T$. This is a contradiction. □

Example 2.1 Let S_1 and S_2 be the propositional theories $\{A, A \Rightarrow B\}$ and $\{\neg B\}$, respectively. Then the theories that accomplish the join of S_1 and S_2 minimally are $T_1=\{A, \neg B\}$, $T_2=\{A \Rightarrow B, \neg B\}$, and $T_3=\{A, A \Rightarrow B\}$. □

Example 2.2 Let S_1 and S_2 be the propositional theories $\{A,B\}$ and $\{\neg A, \neg B\}$, respectively. Then the theories that accomplish the join of S_1 and S_2 minimally are $T_1=\{A,B\}$, $T_2=$

$\{\neg A, \neg B\}$, $T_3=\{A, \neg B\}$, and $T_4=\{\neg A, B\}$. □

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be sets of theories that accomplish the insertion of S_2 into S_1 minimally, and S_1 into S_2 minimally, respectively. Then $\mathcal{T}_1=\{T_1, T_2\}$, $\mathcal{T}_2=\{T_3\}$ for Example 2.1, and $\mathcal{T}_1=\{T_2\}$, $\mathcal{T}_2=\{T_1\}$ for Example 2.2.

Let $\mathcal{T}$ be a set of theories that accomplish the join of S_1 and S_2 minimally. Then you may expect that $\mathcal{T}_1 \cup \mathcal{T}_2=\mathcal{T}$, but this is not generally true. In fact $\mathcal{T}_1 \cup \mathcal{T}_2=\mathcal{T}$ in the case of Example 2.1, but not so in the case of Example 2.2. We can prove the following theorem.

<u>Theorem 2.2</u> $\mathcal{T}_1 \cup \mathcal{T}_2 \subseteq \mathcal{T}$. □

[Proof] Suppose that $T \in \mathcal{T}_1$, and let S_1' be a maximal subset of S_1 that is consistent with S_2. Then $T=S_1' \cup S_2$. Thus T is a maximal consistent subset of $S_1 \cup S_2$. That is, $T \in \mathcal{T}$. By the similar way, when $T \in \mathcal{T}_2$, we can show $T \in \mathcal{T}$. Thus $\mathcal{T}_1 \cup \mathcal{T}_2 \subseteq \mathcal{T}$. □

But under a certain condition $\mathcal{T}_1 \cup \mathcal{T}_2=\mathcal{T}$ holds. The next theorem gives a sufficient condition that $\mathcal{T}_1 \cup \mathcal{T}_2=\mathcal{T}$ holds.

<u>Theorem 2.3</u> $\mathcal{T}_1 \cup \mathcal{T}_2=\mathcal{T}$ if S_1 or S_2 is a singleton set. □

Before we prove this theorem, we give the next lemma.

<u>Lemma 2.2</u> For two theories S_1 and S_2, let S_1' be a maximal subset of S_1 that is consistent with S_2. Let T be a maximal consistent subset of $S_1 \cup S_2$. Then $S_1' \cup S_2=T$ if and only if $S_2 \subseteq T$. □

[Proof] The necessity is trivial. We prove only the sufficiency.

Since $S_2 \subseteq T$, we know that $T=(T \cap S_1) \cup S_2$. Thus it suffices to show that $T \cap S_1=S_1'$, i.e., $T \cap S_1$ is a maximal subset of S_1

that is consistent with S_2. Assume that for some sentence σ in $S_1-(T\cap S_1)$, $(T\cap S_1)\cup\{\sigma\}$ be consistent with S_2. Then $T\cup\{\sigma\}=(T\cap S_1)\cup S_2\cup\{\sigma\}$ is consistent. Since T is a maximal consistent subset of $S_1\cup S_2$, we know that $T\cup\{\sigma\}$ is inconsistent. This is a contradiction. □

[Proof of Theorem 2.3] By Theorem 2.2 it suffices only to show $\mathcal{T}\subseteq\mathcal{T}_1\cup\mathcal{T}_2$. Suppose $T\in\mathcal{T}$. Then T is a maximal consistent subset of $S_1\cup S_2$. Now suppose $S_2=\{\sigma\}$. If $\sigma\in T$, then $T=S_1'\cup S_2$ by Lemma 2.2. Thus $T\in\mathcal{T}_1\cup\mathcal{T}_2$. On the other hand, if $\sigma\notin T$, then $T=S_1$ and $S_2'=\phi$, where S_2' is a maximal subset of S_2 that is consistent of S_1. Since $T=S_1\cup S_2'$, $T\in\mathcal{T}_1\cup\mathcal{T}_2$. The proof for the case that S_1 is a singleton is similar to the above. □

3. Joins of Logical Databases

Fagin et al. [1] also studied updates of theories where different sentences can carry different priorities. The main results are as follows:

A pair $\langle i,\sigma\rangle$ of a non-negative integer i and a sentence σ is called a tagged sentence, and a non-negative integer i a tag. A smaller value of a tag means a higher priority of the tagged sentence. A logical database is a consistent set of tagged sentences. We shall use D, E, ⋯ to denote a logical database. D^i is the set of tagged sentences in D whose tag is smaller or equal to i, i.e., $D^i=\{\langle j,\tau\rangle \mid \langle j,\tau\rangle\in D,\ j\leq i\}$. Then $D\cup\{\langle j,\sigma\rangle\}$ accomplishes the insertion of σ into E if and only if D^i is a maximal subset of E^i that is consistent with σ for $i=0,\cdots,n$, where n is the highest tag in E.

Next we consider the join of logical databases.

Definition 3.1 Assume that $E_1\cup E_2\neq\phi$. A logical database D accomplishes the join of E_1 and E_2 if $(E_1\cup E_2)\cap D\neq\phi$. When $E_1\cup E_2=\phi$, we define that any logical database D accomplishes the join of E_1 and E_2. □

Definition 3.2 Let D_1 and D_2 be two logical databases that accomplish the join of E_1 and E_2, and let E be $E_1 \cup E_2$ with n as the highest tag in it. D_1 accomplishes the join with a smaller change than D_2 if for some i, $0 \leq i \leq n$, $E^{i-1}-D_1{}^{i-1}= E^{i-1}-D_2{}^{i-1}$, $E^i-D_1{}^i \subset E^i-D_2{}^i$. □

Definition 3.3 D accomplishes the join of E_1 and E_2 minimally if there is no logical database that accomplishes this join with a smaller change than D. □

Theorem 3.1 Let E be $E_1 \cup E_2$ with n as the highest tag in it. D accomplishes the join of E_1 and E_2 minimally if and only if D^i is a maximal consistent subset of E^i for $i=0,\cdots,n$. □

We can prove Theorem 3.1 by a similar way to Theorem 2.1.

Let Th(D) be a theory obtained from D by stripping the tags, i.e., $Th(D)=\{\tau \mid \langle i,\tau\rangle \in D\}$. Let $\mathcal{D}$ be a set of logical databases and let $Th(\mathcal{D})$ be a set of theories obtained from logical databases in $\mathcal{D}$, i.e., $Th(\mathcal{D})=\{Th(D) \mid D \in \mathcal{D}\}$.

The next theorem shows that the definition of joins is an extension of that of insertions.

Theorem 3.2 Let all the tags of tagged sentences in E_1 are larger than those of E_2. Let $\mathcal{D}$ be the set of logical databases that accomplish the join of E_1 and E_2 minimally. Let $\mathcal{T}_1$ be the set of theories that accomplish the insertion of $Th(E_2)$ into $Th(E_1)$ minimally. Then $Th(\mathcal{D})=\mathcal{T}_1$. □

[Proof] Put $E=E_1 \cup E_2$. Let $Th(E_1')$ be a maximal subset of $Th(E_1)$ that is consistent with $Th(E_2)$.

(1) Suppose that $Th(D) \in Th(\mathcal{D})$. Since $Th(D^i)$ is a maximal consistent subset of $Th(E^i)$ for all i, $0 \leq i \leq t_1$, where t_1 is the highest tag in E_1, we know that $Th(E_2) \subseteq Th(D)$. Since Th(D) is a maximal consistent subset of Th(E), by Lemma 2.2,

$Th(D)=Th(E_1{}')\cup Th(E_2)$. Thus $Th(D)\in\mathcal{T}_1$.
(2) Suppose that $T\in\mathcal{T}_1$. Then $T=Th(E_1{}')\cup Th(E_2)$. Thus T is a maximal consistent subset of Th(E). That is, $T\in Th(\mathcal{D})$. □

The theories that accomplish the insertion minimally are the same as the theories that accomplish the join minimally with the condition about tags of sentences. Intuitively, in the case of insertions of theories, the inserted sentences are treated as if they have the highest tags, but in the case of joins of theories, all sentences are treated equivalently.

Example 3.1 Let E_1 and E_2 be logical databases $\{\langle 0,A\rangle,\langle 0,A\Rightarrow B\rangle\}$ and $\{\langle 1,\neg B\rangle\}$, respectively, and consider the join of E_1 and E_2. Now we construct a logical database D that accomplishes the join minimally. Let E be $E_1\cup E_2$, then D^0, the maximal consistent subset of E^0, is $\{\langle 0,A\rangle,\langle 0,A\Rightarrow B\rangle\}$. And $D^1=\{\langle 0,A\rangle,\langle 0,A\Rightarrow B\rangle\}$, that is, we can not add tagged sentences to D^0 anymore. Thus $D=\{\langle 0,A\rangle,\langle 0,A\Rightarrow B\rangle\}$. □

Example 3.2 Let E_1 and E_2 be logical databases $\{\langle 1,A\rangle,\langle 1,A\Rightarrow B\rangle\}$ and $\{\langle 0,\neg B\rangle\}$, respectively. In this case $D^0=\{\langle 0,\neg B\rangle\}$. And D^1 is $\{\langle 0,\neg B\rangle,\langle 1,A\rangle\}$ or $\{\langle 0,\neg B\rangle,\langle 1,A\Rightarrow B\rangle\}$. Thus D is $\{\langle 0,\neg B\rangle,\langle 1,A\rangle\}$ or $\{\langle 0,\neg B\rangle,\langle 1,A\Rightarrow B\rangle\}$. □

In Example 3.1, $Th(D)=\{A,A\Rightarrow B\}$ is the same as the theory that accomplish the insertion of $Th(E_1)=\{A,A\Rightarrow B\}$ into $Th(E_2)=\{\neg B\}$ minimally, that is, $Th(\mathcal{D})=\mathcal{T}_1$. And in Example 3.2 we can also make it sure that $Th(\mathcal{D})=\mathcal{T}_1$ holds.

4. Conclusion

We studied joins of theories as an extension of insertions, which can model the insertion of knowledges into a knowledge base more naturally.

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