

Sum Rate Analysis and Dynamic Clustering for Multi-user MIMO Distributed Antenna Systems



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by

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Preface

With the ever-growing demand for higher data rates in wireless communication services, recently, various techniques have been proposed to increase system capacity. Among these techniques, multiple-input multiple-output (MIMO) transmission, as a method for multiplying the capacity of a radio link using multiple transmit and receive antennas to exploit multi-path propagation, has attracted considerable attention. It has been verified that, in an independent and identically distributed (i.i.d.) Rayleigh flat-fading channel, the MIMO capacity increases linearly with the smaller of the number of transmit and receive antennas. In practice, however, because of physical constraints, the number of antennas on a user cannot be made arbitrarily large that may prevent the realization of MIMO transmission.

Fortunately, using precoding techniques on a transmitter equipped with multiple centralized antennas, the required computational complexity and the number of antennas on each user can be reduced because multiple users can demodulate the signals without using any detection scheme while retaining the advantages of MIMO transmission. Because of these merits, the sum rate of the so-called multi-user (MU) MIMO transmission has been widely studied over some simple channel attenuation models, such as i.i.d. Rayleigh fading and i.i.d. shadowing models. However, since it is hard to average the sum rate over the shadowing distribution and spatial correlations further complicates the distribution, the theoretical analysis on MU-MIMO sum rate over correlated composite fading channels (i.e., with both correlated Rayleigh fading and correlated shadowing) has not been studied.

To bridge this gap, in the first part of our research, we concentrate on the sum rate distribution of MU-MIMO systems employing linear zero-forcing precoding, accounting for both Rayleigh fading and shadowing effects, as well as spatial correlation at the transmit and receiver sides. In particular, we consider the classical spatially correlated lognormal model and propose closed-form bounds on the distribution of the achievable sum rates in MU-MIMO systems. With the help of these bounds, we derive a relationship between the inter-user distance and sum rate corresponding to 10% of the cumulative distribution function. A practical conclusion, for instance, is that the effect of spatially correlated shadowing can be considered to be independent when the inter-user distance is approximately five times the shadowing correlation distance.

On the basis of the above analysis results, a distributed antenna system (DAS) consisting of multiple geographically distributed antennas has been proposed to further decrease the access distance and spatial correlations. To verify the sum rate improvement in MU-MIMO DASs, in the second part of our research, we investigate and compare the characteristics of sum rates in both centralized antenna systems (CASs) and DASs under the effects of correlated composite fading channels and inter-cell interference. In this part, we introduce two different types of functions to model the shadowing, auto-correlation and cross-correlation, and a typical exponential decay function to model the Rayleigh fading correlation. One of the computer simulation results indicates that the DAS can significantly improve the performance of the sum rate compared to the traditional CAS in the case under the consideration, therefore confirms the speculation.

Finally, combining MU-MIMO transmission and DAS with precoding techniques, the advantages of both MIMO and DAS can be achieved. However, by establishing a wide service area, i.e., a large-scale MU-MIMO DAS, to increase multiplexing gain, the use of a dynamic clustering scheme (CS) is necessary to reduce computation in precoding. In the last part of our research, we propose a simple multi-carrier based method for dynamic clustering to establish a large-scale MU-MIMO DAS and investigate its performance. We also compare the characteristics of the proposal to those of other schemes such as exhaustive search, traditional location-based adaptive CS, and improved norm-based CS in terms of sum rate improvement. Additionally, to make our results more universal, we further introduce spatial correlation to the considered system. Computer simulation results indicate that the proposed CS for the considered system provides better performance than the existing schemes and can achieve a sum rate close to that of exhaustive search but at a lower computational cost.

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Nomenclature

Acronyms

AOA: Angle-of-Arrival

BD: Block-Diagonalization

BS: Base Station

CAS: Centralized Antenna System

CC: Component Carrier

CCS: Completely Correlated Shadowing

CDF: Cumulative Distribution Function

c.f.: characteristic function

CS: Clustering Scheme

CoMP: Cooperative Multi-Point

CP: Central Processor

CSI: Channel State Information

CSIT: Channel State Information at the Transmitter

DAS: Distributed Antenna System

DOF: Degrees of Freedom

GLOBECOM: Global Telecommunication Conference

i.i.d.: independent and identically distributed

ICCS: International Conference on Communication Systems

IEEE: Institute of Electrical and Electronic Engineers

IEICE: Institute of Electronics Information and Communication Engineers

JASSO: Japan Student Services Organization

LOCA: Location- based Adaptive

LOS: Line of Sight
LTE: Long Term Evolution
MIMO: Multiple-Input Multiple-Output
MMSE: Minimum Mean Square Error
MU: Multi-user
OFDM: Orthogonal Frequency Division Multiple
P2P: Peer-to-Peer
PDF: Probability Density Function
PIMRC: Personal Indoor Mobile Radio Communications
RLN: Rayleigh/Lognormal
RV: Random Variable
SAC: Shadowing Auto-Correlation
SC: Single Carrier
SCC: Shadowing Cross-Correlation
SNR: Signal-to-Noise-Ratio
SRM: Sum Rate Maximization
SP: Sub-Processor
SU: Single User
VTC: Vehicular Technology Conference
ZF: Zero-Forcing

Roman Symbols

- D : Path loss
- $\mathbf{D}$: Path loss matrix
- S : Value of shadowing attenuation
- K : Number of users
- d : Distance
- $\mathbf{h}$: Channel attenuation matrix or vector
- $\mathbf{H}$: Channel attenuation matrix
- $\mathbf{W}$: Precoding matrix
- $\mathbf{s}$: Signal vector
- L : Inter-user distance
- $\mathbf{P}$: Transmit power scaling factor matrix
- C : Sum rate or capacity
- J : Number of cells or number of carriers
- f : Carrier frequency
- B : Number of base stations
- U : Number of users

Greek Symbols

- ζ : Path loss exponent
- σ_X : Standard deviation of random variable X
- σ_X^2 : Variance of random variable X
- μ_X : Average value (expectation) of random variable X
- Ω : Coefficient of shadowing auto-correlation
- ϵ : Coefficient of shadowing cross-correlation
- β : Coefficient of Rayleigh fading
- ρ : Transmit SNR
- γ : Receive SNR
- ξ : Normalized shadowing correlation distance
- $\theta_{\text{cor,t}}$: Threshold angle depending upon the shadowing correlation distance on the BS
- κ : Parameter of shadowing cross-correlation
- ψ : Digamma function

Subscripts

- R_e : Envelope random variable of Rayleigh fading
- R_p : Phase random variable of Rayleigh fading
- N_t : Number of transmit antennas
- N_r : Number of receiver antennas
- N_{cell} : Number of cells
- $\mathbf{S}_{\text{cor}}$: Correlated shadowing matrix
- $\mathbf{R}_{\text{cor}}$: Correlated Rayleigh fading matrix
- $\Theta_{\text{R,t}}$: Transmit correlation matrix in Rayleigh fading
- $\Theta_{\text{R,r}}$: Receiver correlation matrix in Rayleigh fading
- $\Theta_{\text{S,t}}$: Transmit correlation matrix in shadowing
- $\Theta_{\text{S,r}}$: Receiver correlation matrix in shadowing
- d_{cor} : Correlation distance of shadowing
- R_{ant} : Inter-antenna distance

Chapter 1

Introduction

1.1 Background

1.1.1 Radio Propagation

Radio propagation is the behavior of radio waves when they are transmitted, or propagated from one point on the Earth to another, or into various parts of the atmosphere. As a form of electromagnetic radiation, like light waves, radio waves are affected by the phenomena of reflection, refraction, diffraction, absorption, polarization and scattering.

Understanding the effects of varying conditions on radio propagation has many practical applications, from choosing frequencies for international shortwave broadcasters, to designing reliable mobile telephone systems, to radio navigation, to operation of radar systems. In our research, we first consider that a radio propagation between the transmit and receive antennas consists of path loss, shadowing and fading. Then, we simply introduce some concepts and propose some kinds of models for the radio propagation in wireless communications.

1.1.1.1 Path Loss

Path loss is the reduction in power density of an electromagnetic wave as it propagates through space. It is a major component in the analysis and design of the link budget of a telecommunication system. This term is commonly used in wireless communications

and signal propagation. Path loss may be due to many effects, such as free-space loss, refraction, diffraction, reflection, aperture-medium coupling loss, and absorption. Path loss is also influenced by terrain contours, environment (urban or rural, vegetation and foliage), propagation medium (dry or moist air), the distance between the transmitter and the receiver, and the height and location of antennas.

In the study of wireless communications, path loss can be represented by the path loss exponent, whose value is normally in the range of 2 to 4 (where 2 is for propagation in free space, 4 is for relatively lossy environments and for the case of full specular reflection from the earth surface, the so-called flat-earth model). In some environments, such as buildings, stadiums and other indoor environments, the path loss exponent can reach values in the range of 4 to 6; on the other hand, a tunnel may act as a waveguide, resulting in a path loss exponent less than 2. Path loss is usually modeled by the power law

$$D = d^{-\zeta}, \tag{1.1}$$

where D is the path loss, d is the distance between the transmit and the receive antennas, usually measured in meters, ζ is the path loss exponent [1].

1.1.1.2 Fading

In wireless communications, fading is deviation of the attenuation affecting a signal over certain propagation media. The fading may vary with time, geographical position or radio frequency, and is often modeled as a random process. A fading channel is a communication channel comprising fading. In wireless systems, fading may either be due to multi-path propagation, referred to as multi-path fading.

Multi-path fading channel models are often used to model the effects of electromagnetic transmission of information over the air in cellular networks and broadcast communication. Mathematically, this fading is usually modeled as a time-varying random

change in the amplitude and phase of the transmitted signal, for instance, the Rayleigh fading model which is most applicable when there is no dominant propagation along a line of sight (LOS) between the transmitter and receiver, and the Nakagami-Rician fading model which may be more applicable if there is a dominant LOS. In our research, we mainly employ the Rayleigh fading model because of its universality.

Rayleigh fading is a reasonable model when there are many objects in the environment that scatter the radio signal before it arrives at the receiver. The central limit theorem holds that, if there is sufficiently much scatter, the channel impulse response will be well modeled as a Gaussian process irrespective of the distribution of the individual components. If there is no dominant component to the scatter, then such a process will have zero mean and phase evenly distributed between 0 and 2π radians. The envelope of the channel response will therefore be Rayleigh distributed [1].

Calling the envelope and phase random variable R_e and R_p , their probability density function (PDF) can be expressed as

$$\text{PDF}_{R_e}(r_e) = \frac{r_e}{\sigma_{R_e}^2} \exp\left(-\frac{r_e^2}{2\sigma_{R_e}^2}\right) \quad (1.2)$$

and

$$\text{PDF}_{R_p}(r_p) = \frac{1}{2\pi}, \quad (1.3)$$

where $\sigma_{R_e}^2$ denotes the variance [1]. Often, the gain and phase elements of a channel distortion are conveniently represented as a complex number. In this case, Rayleigh fading is exhibited by the assumption that the real and imaginary parts of the response are modeled by independent and identically distributed (i.i.d.) Gaussian processes whose mean is zero variance is $\sigma_{R_e}^2$, so that the amplitude of the response is the sum of two such processes.

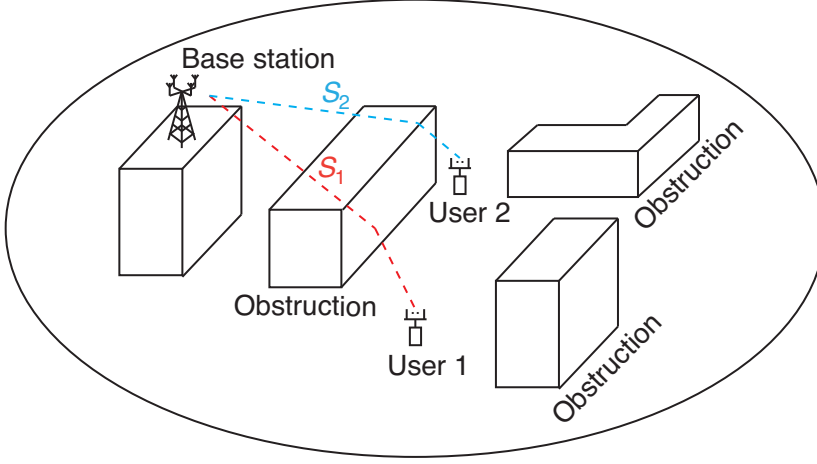


Figure 1.1: Shadowing by a building.

1.1.1.3 Shadowing

Fading, created by the superposition of different multi-path components, changes rapidly over the spatial scale of a few wavelengths. If the field strength is averaged over a small area (e.g., ten by ten wavelengths), we obtain the small scale average field strength, i.e., shadowing attenuation values. In a small spatial scale, the shadowing attenuation values can be treated as a constant. In fact, however, it varies when considered on a large spatial scale.

Fig. 1.1 presents two radio propagation paths that experience two different shadowing attenuation values S_1 and S_2 . Many experimental investigations have shown that the shadowing values S plotted on a logarithmic scale, shows a Gaussian distribution around a mean μ_S . Such a distribution is known as lognormal, and its PDF can be expressed as

$$\text{PDF}_S(s) = \frac{20/\ln 10}{s\sigma_S\sqrt{2\pi}} \exp\left(-\frac{(20\log_{10}(s) - \mu_S)^2}{2\sigma_S^2}\right), \quad (1.4)$$

where σ_S is the standard deviation of s , and μ_S is the mean of the values of s expressed in decibel. Typical values of σ_S are 4 to 10 dB [1].

In many studies, the spatial correlation between the two shadowing attenuations S_1 and S_2 be considered as independent, in fact, however, the correlation exists widely in our realistic environment and should not be ignored. Fig.1.2 and Fig.1.3 show an example of spatially independent shadowing and an example of correlated shadowing plotted on logarithmic scale. From these figures, we can clearly understand how different they are, that is why analyze the characteristics of correlated shadowing in wireless communications is also very important. In spite of this, theoretical analysis works about that is not so simple because the sum of correlated lognormal random variables dose not have a closed-form. Fortunately, an interesting property of the lognormal distribution is that the sum of log normally distributed values is also approximately log normally distributed. With the help of this property, in our research, we successfully presented some useful results described in other chapters.

1.1.2 Multi-user MIMO

1.1.2.1 System Description

In radio, multi-user (MU) multiple-input and multiple-output (MIMO) is a set of advanced MIMO technologies [2] where the available antennas are spread over a multitude of independent base stations (BS) and independent radio terminals each having one or multiple antennas. In contrast, single-user (SU) MIMO considers a single multi-antenna transmitter communicating with a single multi-antenna receiver. To enhance the communication capabilities of all terminals, MU-MIMO applies an extended version of space-division multiple access to allow multiple transmitters to send separate signals and multiple receivers to receive separate signals simultaneously in the same band.

MU-MIMO can be thought of as an extension of MIMO applied in various ways as a multiple access strategy. A significant difference is that the performance of MU-MIMO relies on precoding capability so that if the transmitter does not use precoding, the performance advantage of MU-MIMO is not achievable.

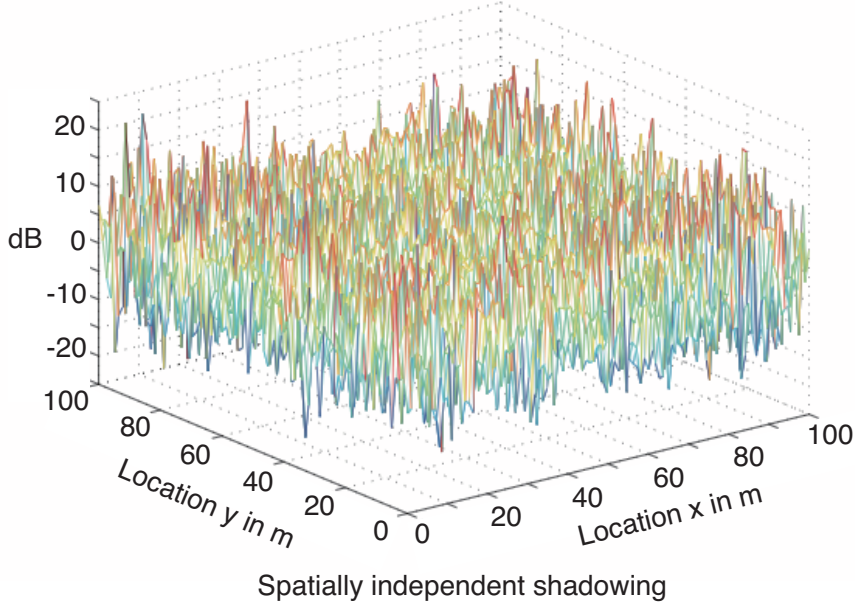


Figure 1.2: An example of spatially independent shadowing in decibel with $\mu_S = 0$ dB and $\sigma_S = 8$ dB.

In Fig. 1.4, we consider a classical downlink MU-MIMO communication system with N_t transmit antennas at the BS and K users each connected to N_r receive antennas. Through the radio propagation channels that we described before, the received signal at k th user can be expressed as

$$\mathbf{y}_k = \mathbf{h}_k \mathbf{x} + \mathbf{n}_k, \quad (1.5)$$

where $\mathbf{h}_k \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix between the BS and k th user. $\mathbf{n}_k \in \mathbb{R}^{N_r \times 1}$ is the noise vector at user k . The vector $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ is precoded signal transmitted from BS and can be expressed as

$$\mathbf{x} = \sum_{k=1}^K \sqrt{q_k} \mathbf{w}_k \mathbf{s}_k, \quad (1.6)$$

where $\mathbf{s}_k \in \mathbb{C}^{N_r \times 1}$, $\mathbf{w}_k \in \mathbb{C}^{N_t \times N_r}$ and p_k represent the desired data symbol, precoding weight and transmit power scaling factor for user k , respectively. Then put (1.6) into

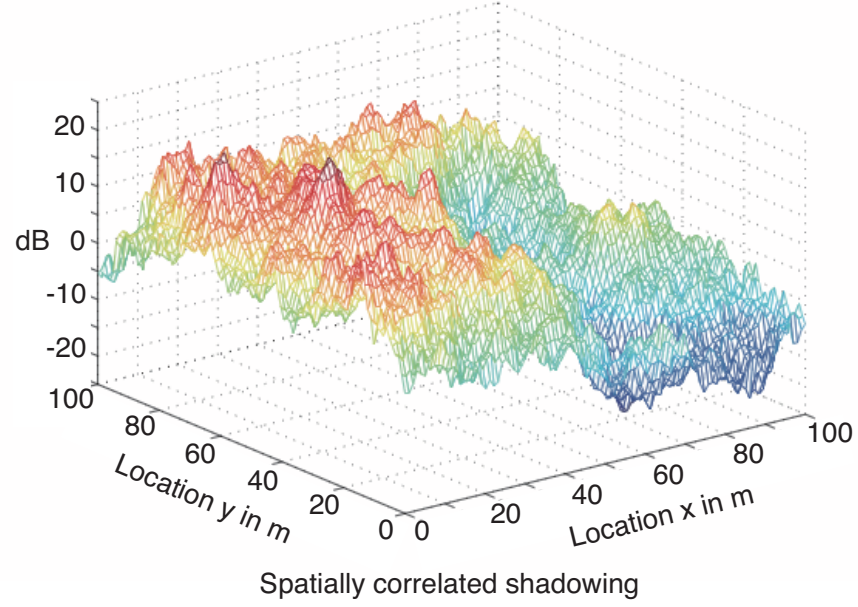


Figure 1.3: An example of spatially correlated shadowing in decibel with $\mu_S = 0$ dB and $\sigma_S = 8$ dB.

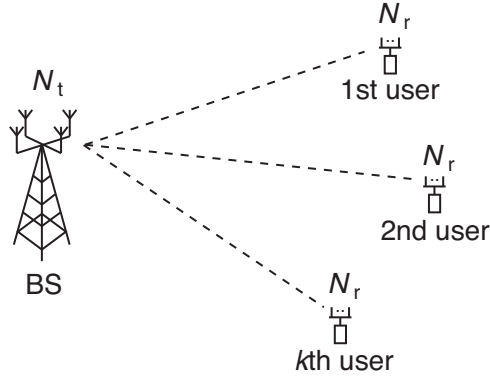


Figure 1.4: An example for MU-MIMO system.

(1.5), we can get another expression for $\mathbf{y}_k$ that is

$$\mathbf{y}_k = \sqrt{q_k} \mathbf{h}_k \mathbf{w}_k s_k + \sum_{i=1, i \neq k}^K \sqrt{q_i} \mathbf{h}_k \mathbf{w}_i s_i + \mathbf{n}_k, \quad (1.7)$$

where the second term on the right hand side of the equality is so-called inter-user

interference, that directly affects the strength of the received signal $\mathbf{y}_k$. Therefore, we should reduce or cancel these effects by selecting the suitable precoding matrix $\mathbf{w}_k$. That is the reason why the performance of MU-MIMO relies on precoding capability.

1.1.2.2 Zero-forcing Precoding

Zero-forcing (ZF) precoding is a spatial signal processing by which the multiple antenna transmitter can null inter-user interference signals (i.e., the second term of equality 1.7) in wireless communications. There is a key problem for applying the ZF precoding on transmit side, is acquisition of channel state information (CSI), i.e., $\mathbf{h}_k \forall k$.

If the transmitter knows the downlink CSI perfectly, ZF precoding can achieve almost the system capacity when the number of users is large; on the other hand, with limited channel state information at the transmitter (CSIT) the performance of ZF precoding decreases depending on the accuracy of CSIT. ZF precoding requires the significant feedback overhead with respect to signal-to-noise-ratio (SNR) so as to achieve the full multiplexing gain. Inaccurate CSIT results in the significant throughput or sum rate loss because of residual inter-user interferences. Inter-user interferences remain since they can not be nulled with beams generated by imperfect CSIT.

Here we assume the transmitter, i.e., BS, knows the downlink CSI $\mathbf{h}_k$ perfectly, and also consider that each user equipped with single antenna. Under the consideration case, the precoding matrix $\mathbf{W}$ can be obtained by using ZF precoding and expressed as

$$\mathbf{W} = \mathbf{H}^\dagger = \mathbf{H}^H (\mathbf{H} \mathbf{H}^H)^{-1}, \quad (1.8)$$

where the ZF precoding matrix $\mathbf{W} = (\mathbf{w}_1, \dots, \mathbf{w}_K)$, the channel matrix consists of total users $\mathbf{H} = (\mathbf{h}_1^H, \dots, \mathbf{h}_K^H)^H$. The symbols $(\cdot)^\dagger$ and $(\cdot)^H$ represent the pseudo-inverse and Hermitian transpose of a matrix, respectively,

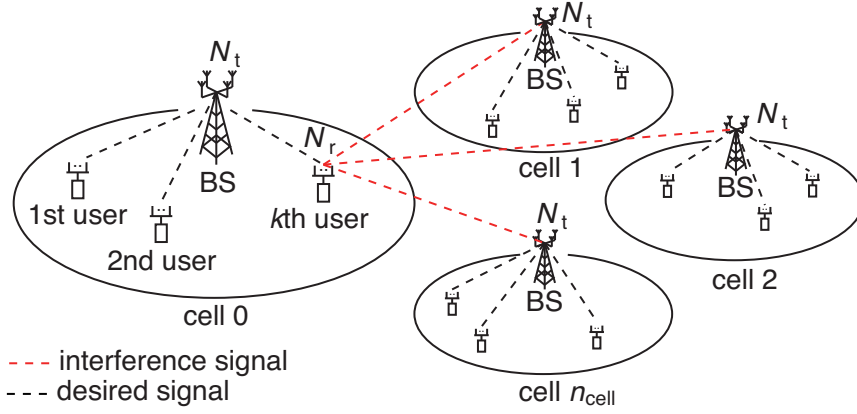


Figure 1.5: An example for MU-MIMO system with inter-cell interference.

By using (1.8), the inter-user interference can be nulled with perfect CSIT. In fact, several advanced precoding technology, for instance, block-diagonalization (BD) algorithm [3] has been applied for MU-MIMO systems in which the numbers of antenna at each user is not necessary to be limited to one. The details about other precoding technology can also be found in [1].

1.1.2.3 Inter-cell Interference

In previous section, we only considered a MU-MIMO system in a single cell and described the interferences come from total users. However, in cellular system, interference not only come from users who are distributed in the same cell, but also come from other cells, that degrades the performance of system more.

In Fig. 1.5, we consider a multi-cell MU-MIMO system communicates using the same frequency. The cell 0 is interested cell and cell $n_{\text{cell}} \in \{1, \dots, N_{\text{cell}}\}$ is interference cell. We use K , N_t and N_r to denote the number of users, BS antennas and user antennas, and assume that all of cell have the same number of these variables. Under this consideration

and invoking (1.7), the receive signal at user k in interested cell 0 can be expressed as

$$\mathbf{y}_k = \sqrt{q_k} \mathbf{h}_k \mathbf{w}_k \mathbf{s}_k + \sum_{i=1, i \neq k}^K \sqrt{q_i} \mathbf{h}_k \mathbf{w}_i \mathbf{s}_i + \sum_{n_{\text{cell}}=1}^{N_{\text{cell}}} \mathbf{h}_{k,n_{\text{cell}}} \sum_{k=1}^K \sqrt{q_{k,n_{\text{cell}}}} \mathbf{w}_{k,n_{\text{cell}}} \mathbf{s}_{k,n_{\text{cell}}} + \mathbf{n}_k, \quad (1.9)$$

where the matrix $\mathbf{h}_{k,n_{\text{cell}}} \in \mathbb{C}^{N_r \times N_t}$ is the interference channel matrix from cell n_{cell} to interested user k . $\mathbf{s}_{k,n_{\text{cell}}} \in \mathbb{C}^{N_r \times 1}$, $\mathbf{w}_{k,n_{\text{cell}}} \in \mathbb{C}^{N_t \times N_r}$ and $p_{k,n_{\text{cell}}}$ represent the desired data symbol, precoding weight and transmit power scaling factor for user k who belongs to cell n_{cell} , respectively.

The third term on the right hand side of the equality (1.9) is so-called inter-cell interference. In fact, our challenge is how to effectively reduce or cancel the effects of inter-cell interference, certainly, including the inter-user interference.

1.1.3 Distributed Antenna Systems

A distributed antenna system, or DAS, is a network of spatially separated antenna nodes connected to a common source via a transport medium that provides wireless service within a geographic area or structure. Fig. 1.6 shows a comparison of classical centralized antenna systems (CAS) and DAS in a single cell.

As illustrated in the figure, the idea is to split the transmitted power among several antenna elements, separated in space so as to provide coverage over the same area as a single antenna but with reduced total power and improved reliability. In Fig. 1.6, a CAS (a) is replaced by a group of less-number antennas to cover the same area (b). These antennas have recently been employed by several service providers in many areas. DAS is often used in scenarios where alternate technologies are infeasible due to terrain or zoning challenges.

The idea of DAS works because less power is wasted in overcoming penetration and shadowing losses (see 1.1.1.3), and because a LOS channel is present more frequently, leading to reduced fade depths (see 1.1.1.2) and reduced delay spread. Some discussions and comparisons of system performance about CAS and DAS can be also found in [4–8].

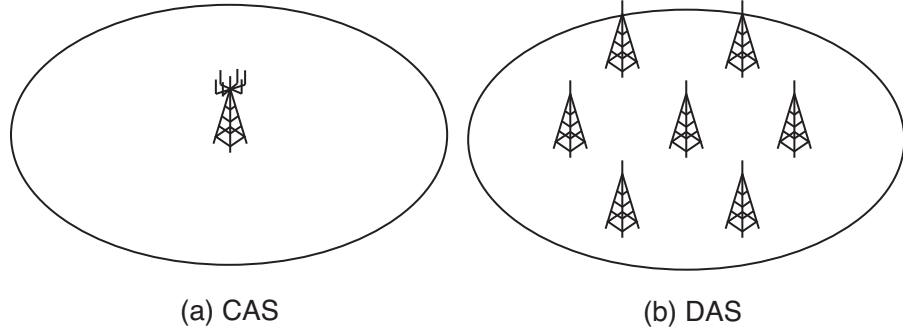


Figure 1.6: An example for centralized and distributed antenna systems.

1.1.4 Multi-carrier Based Clustering

Clustering using multiple carriers, or frequencies, can be considered arbitrary resource allocation among receivers, transmitters and carriers, so that a cluster is formatted by the selected receivers and transmitter communicating with the allocated carrier. Note that, the clusters can be overlapped, in other word, some receivers and transmitter can use multiple carriers for communication. In the present research, in order to reduce system complexity, we consider the case that the multiple clusters are not overlapped.

In general, clustering can be divided into two types, one is called the static clustering and the other one is called dynamic clustering. One of the characteristics of the static clustering is the clusters are not changed with channel realizations; on the other hand, the dynamic clustering strongly depends on each channel realization. For better understanding, in Fig. 1.7, we show an illustration of three different clustering methods.

In Fig. 1.7 (a), the static clustering which depends on the locations of receivers and transmitters, has been proven to improve the efficiency of the frequency spectrum [9,10]. In this method, the geographical area is divided into a fixed cluster structure with each cluster having a unique, fixed carrier. Any receiver and transmitter located in a given cluster will communicate on the carrier associated with that cluster, and are not changed

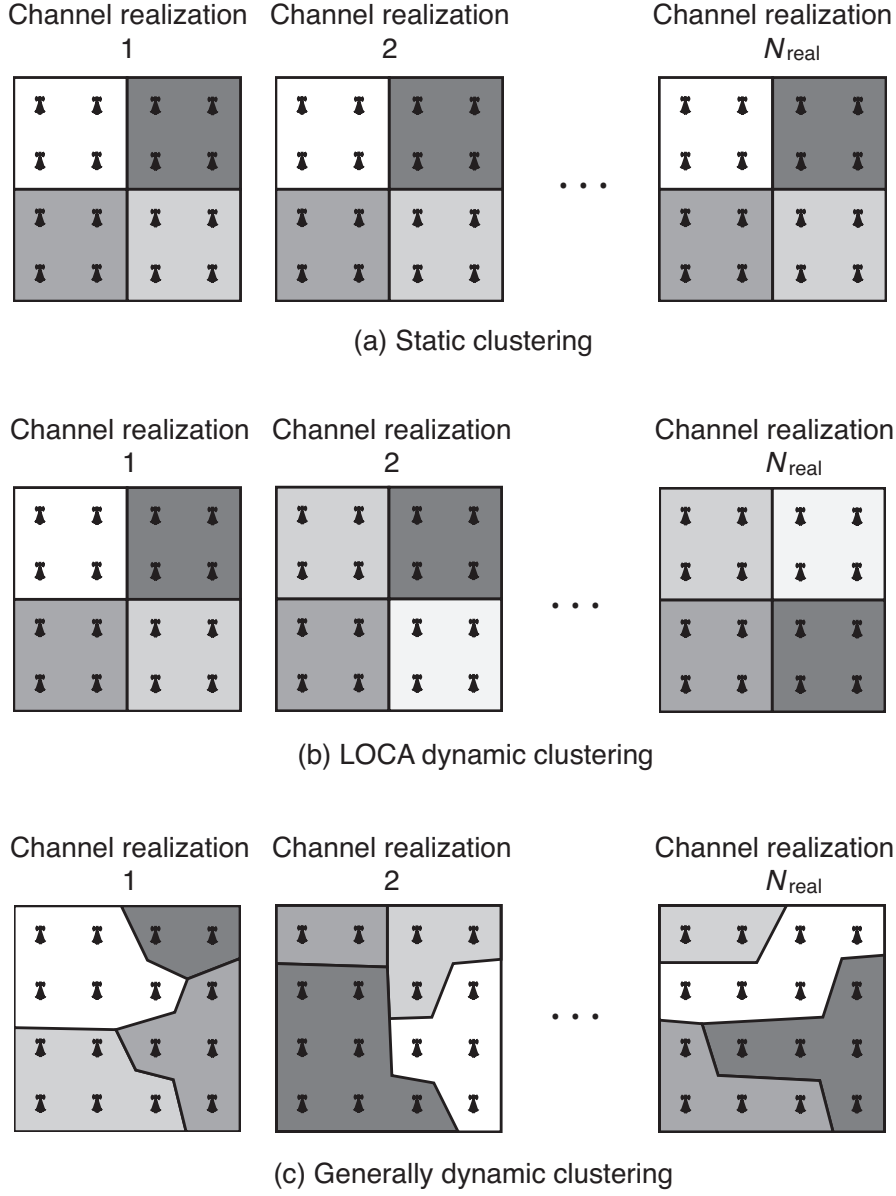


Figure 1.7: Illustration of three different clustering methods and each one is shown over N_{real} channel realizations. Clusters are represented by the shaded or blank areas. (a) Static clustering, (b) LOCA dynamic clustering, (c) Generally dynamic clustering.

with each channel realization. The limitation of this static clustering, however, is a lack of diversity with respect to changing channel conditions, because the clusters are static.

In order to improve the above-described method, an advanced so-called location-based adaptive (LOCA) dynamic clustering method shown in Fig. 1.7 (b) was considered in [11]. In this method, the carrier associated with its cluster may be changed for every channel realization, and can be dynamically allocated to the appropriate cluster. Although this method can achieve a frequency diversity gain, its limitation is also a lack of space diversity because of the fixed cluster structure.

The generally dynamic clustering is the one that the selections of receivers, transmitters and carriers are dynamic, are changed with channel realizations, as shown in Fig. 1.7 (c). Obviously, the optimal selection among the receivers, transmitters, and carriers for dynamic clustering is obtained by exhaustively searching all possible assignments for each channel realization and selecting the one that yields the maximum capacity, minimum bit error rate, or other objectives. However, it can be speculated that such a method is not practically feasible because the computational cost considerably increases as the number of receivers, transmitter and carriers increases.

Certainly, there are some algorithms to reduce the computational cost by dynamically selecting the receivers and transmitters, dynamically allocating the carriers to receivers or transmitters. Unfortunately, there have been few studies on achieving synchronously and dynamically allocating among all of receivers, transmitters, and carriers. Therefore, the topic on multi-carrier based clustering is a important issue in our research.

1.2 Outlook on Large-scale MU-MIMO DASs

With the ever-growing demand for higher data rates in wireless communication services, recently, various techniques have been proposed to increase system capacity. Among these techniques, MIMO transmission has attracted considerable attention because it can provide high data rates and link reliability without the need for additional bandwidth or power [12, 13]. In practice, however, the number of antennas on the user side cannot be

made arbitrarily large because of physical constraints that may prevent the realization of MIMO transmission. Another well-known technique is to use a DAS, which decreases the access distance and spatial correlation by geographically distributing the antennas, thereby providing macro-diversity and enhanced system capacity [4, 5, 14, 15].

By combining MU-MIMO transmission and DAS with a precoding technique [16, 17], the required computational complexity and the number of antennas on the user side can be reduced because users can demodulate the signals without using any MIMO detection scheme while retaining the advantages of DAS and MIMO. However, to establish a wide service area, i.e., a large-scale MU-MIMO DAS, the transmit side requires a large amount of CSI [18]. More importantly, the complexity of the process of obtaining precoding weight matrices, for instance, running the pseudo-inverse transpose of the channel attenuation matrix under the ZF precoding scheme, increases with an increase in the number of users and BSs. Therefore, creating a large-scale MU-MIMO DAS service in a realistic environment remains a challenging problem.

As an example for our research, consider a DAS covering a wide range of urban area through multiple cells. In each cell, multiple BSs are spatially separated and try to simultaneously communicate with multiple users by employing a MU-MIMO transmission scheme. However, according to the previous discussion, it is hard to establish these transmissions when the number of users and BSs becomes large. In addition, to suppress the effect of inter-cell interference caused by the use of the same carrier, a problem of coordinated precoding design [19] for sum rate optimization with BS transmit power constraints in this system cannot be also avoided.

Fortunately, clustering schemes (CSs) are considered to be effective approaches for solving existing problems, mainly because the precoding techniques can be performed independently for each orthogonal carrier assigned to each cluster [20, 21]; in this way, for each carrier in each cell, the precoding weight matrix can be obtained at low complexity.

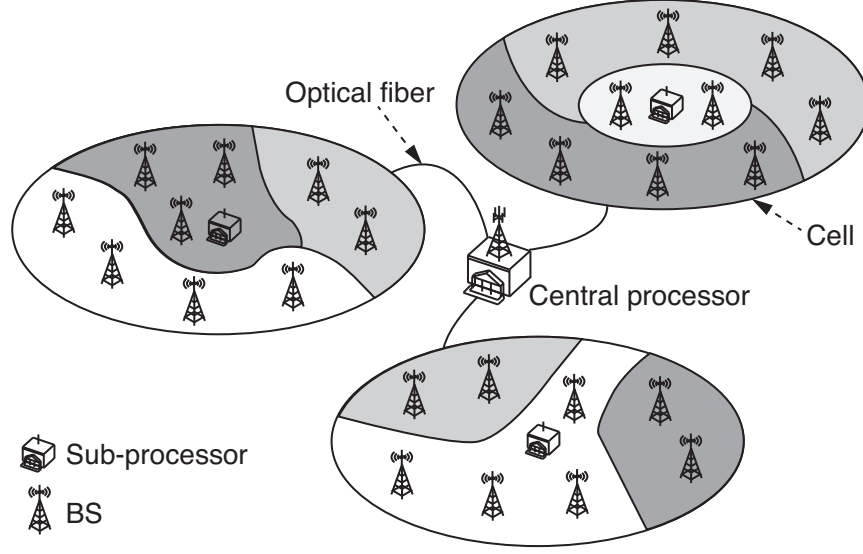


Figure 1.8: An illustration of the multi-cell MU-MIMO DAS using orthogonal carriers based CS. Clusters are represented by the shaded or blank areas, and in each cluster, the MU-MIMO transmission is performed.

An illustration of the considered multi-cell MU-MIMO DAS using orthogonal carriers based CS is presented in Fig. 1.8. In this system, each cell is separated into different clusters based on multiple carriers, and MU-MIMO transmissions are performed in each cluster. BSs in each cell are connected to a local sub-processor (SP) and the SPs are connected to a central processor (CP), which works as a centralized controller and is in charge of the resource management task of the system.

To successfully establish the MU-MIMO transmissions in the large-scale DAS with multi-carrier, such as in Fig. 1.8, there are still some technical problems waiting to be verified and solved. For instance,

- The effects of composite fading channel with correlation on the performance of sum rate in MU-MIMO systems should be theoretically analyzed because transmit and receive antennas are located in a quite complicate attenuation environment where the correlation cannot be ignored in most cases;

- Compared to MU-MIMO transmission used in traditional CASs, it should be verified and clarified that if the MU-MIMO DASs can significantly improve the performance of sum rate even if we employ the correlated composite fading model and consider the inter-cell interference;
- A scheme to format multiple clusters with low computational cost and satisfactory performance for each cell in the considered system as shown in Fig. 1.8 should be explored.

In addition, because the effect of inter-cell interference cannot be ignored due to the same carrier may be used in different cells, studies on the coordinated precoding for the optimization of system sum rate becomes also necessary. Moreover, since the large amounts of CSI can be only transferred over limited rate channel, how to reduce the feedback rate while maintaining good performance of system is also one of the valuable topics in future works.

As the most important part of the big picture, in this research, we try to solve the above listed topics. Here, the orthogonal carriers can be (more concretely) considered as, for instance, multiple subcarriers in orthogonal frequency division multiple (OFDM) systems or multiple component carriers (CCs) in long term evolution (LTE) advance systems [20–24].

Note that, in the latter systems, the CCs may be full overlapped, partial overlapped or orthogonal. Although orthogonal CCs cannot realize maximum capacity under a limited frequency bandwidth (as compared to other overlapped CCs), the transmission performances of users who are located at the boundary of clusters can be preferably improved because there is no inter-cluster interference [22]. Because each user may be positioned at the cluster edges (because clustering is dynamic), the orthogonality of CCs should be used to solve the existing problems. In our research, for better understanding, the orthogonal carriers are defined more broadly, without any loss of generality.

1.3 Thesis Outline and Contributions

In order to solve the existing problems, in particular, the above listed ones, in Chapter 2, we conduct a detailed theoretical analysis of ZF precoding over Rayleigh/lognormal (RLN) fading channels in MU-MIMO systems considering the effects of spatial correlation in both shadowing and Rayleigh fading at the transmit and receiver sides. This scenario can occur when, for instance, a multi-antenna BS transmits data to users who are each equipped with a single antenna.

In Chapter 3, we investigate and compare the characteristics of system sum rates in both CASs and DASs under the effects of path loss, spatially correlated shadowing, correlated Rayleigh fading, and inter-cell interference. In this chapter, we introduce two different types of functions to model the shadowing, auto-correlation and cross-correlation, and a typical exponential decay function to model the Rayleigh fading correlation. Thus, we obtain the distribution of the system sum rate and investigate the characteristics in MU-MIMO DASs.

In Chapter 4, we explore and propose a simple scheme to dynamically format multiple clusters at a reduced computational cost for the DASs with correlated composite fading attenuations. In contrast to the previous studies, in each cluster, MU-MIMO transmission is performed using a carrier selected from among several alternative carriers. Moreover, the proposal can further achieve dynamic allocation for all users (receivers), BSs (transmitters), and carriers at the same time so as to maximize the system sum rate. Our method starts with several empty sets over all carriers and then adds one user and one BS per step to the set corresponding to the carrier used. In each step, the user and BS with the highest contribution to the sum rate from among all clusters is added to the appropriate set. We expand this method and use it to solve the existing clustering problems.

The contributions of our research can now be summarized as follows:

- We use generic bounding techniques to derive closed-form bounds on the sum rate distribution of the traditional MU-MIMO CASs with ZF precoding operating over RLN fading channels and experiencing correlation in both shadowing and fading [25]. The proposed bounds apply for any finite number of antennas, are particularly tractable, and allow for fast and efficient computation. With the help of the proposed bounds, for instance, we obtain a relationship between the inter-user distance and sum rate corresponding to 10% of the cumulative distribution function (CDF) under different environmental conditions.
- The proposed bounds suggest implications for the model parameters on the achievable sum rate distribution. For instance, the effects of the number of antennas, SNR, and spatial correlation are assessed in detail. Our analytical results are also informative and insightful; for example, they allow the characterization of the effects of spatially correlated shadowing and correlated fading, as well as path loss, on the sum rate distribution [25].
- Based on the above theoretical analysis results, we verify and clarify the improvement on the performance of sum rates in MU-MIMO DASs by investigating and comparing the characteristics in both CASs and DASs under the effects of composite fading and inter-cell interference [4]. We also introduce spatial correlation to the well-known channel attenuation models, e.g., i.i.d. shadowing and/or i.i.d. Rayleigh fading on both transmit and receiver sides, thus improve the understanding of channel attenuation in a realistic environment.
- With the help of the previous results, we propose a clustering method using multiple orthogonal carriers to establish the large-scale MU-MIMO DASs with considering

the effects of correlated composite fading [26]. To verify its validity, we also investigate the performance of proposal by observing the CDF and the expectation value of the system sum rate, and compare the characteristics of the proposal to those of other methods such as exhaustive search.

Chapter 2

Theoretical Analysis on Sum Rate in MU-MIMO Systems

In this chapter, as the beginning of our research, we theoretically analyze the sum rate distributions of MU-MIMO transmissions in traditional CASs. We also conduct a detailed analysis of ZF precoding over Rayleigh/lognormal fading channels in this kind of systems with considering the effects of spatial correlation in both shadowing and Rayleigh fading at the transmit and receiver sides, to further understand the effects of correlated composite fading channel on the performance of system sum rates.

The introduction of this chapter is described in Sect. 2.1. The MU-MIMO systems under consideration are described in Sect. 2.2. In Sect. 2.3 and 2.4, we provide new analytical distribution bounds on the sum rate of MU-MIMO systems with ZF precoding. In Sect. 2.5, we present the analysis results and summarize the key findings. Concluding remarks are found in Sect. 2.6.

Notations: In this study, we use uppercase and lowercase boldface to denote matrices and vectors, respectively. The $n \times n$ identity matrix is denoted by $\mathbf{I}_n$, and the $n \times m$ matrix whose elements are all 0 or 1 is written as $\mathbf{0}_{n \times m}$ or $\mathbf{1}_{n \times m}$. The (i, j) th element of a matrix is denoted by $[\mathbf{A}]_{ij}$. We write the (i, j) th minor of a matrix as $\mathbf{A}_{ij}$, and $\mathbf{A}_i$ is $\mathbf{A}$ with the i th row removed. We let $\mathbb{E}\{\cdot\}$ represent the expectation and use

subscript R to identify the expectation of the Rayleigh fading random variables (RVs). The matrix determinant is expressed as $\det(\cdot)$. The symbols $(\cdot)^\dagger$ and $(\cdot)^H$ represent the pseudoinverse and Hermitian transpose of a matrix, respectively, and $\otimes$ denotes the Kronecker product. The symbol $\sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma})$ denotes a complex Gaussian matrix with mean $\mathbf{M}$ and covariance $\mathbf{\Sigma}$. The notation $\psi(\cdot)$ represents the well-known digamma function by Euler [27].

2.1 Introduction

MU-MIMO technique with ZF precoding allows for simultaneous transmission of multiple data streams when each user or terminal has a single antenna. By locating multiple antennas with independent radio users or terminals that are geographically separated, these systems can combine the well-known benefits of MIMO spatial multiplexing with macrodiversity gains [28]. Contrary to SU-MIMO systems, each radio link experiences different path loss and shadowing effects due to the different propagation paths, which makes the performance analysis of MU-MIMO mathematically difficult. The problem of averaging the channel eigen-statistics over the shadowing distribution means that scant analytical work has investigated the effects of composite channels (i.e., with both Rayleigh fading and shadowing) on the performance of MU-MIMO systems.

In [29], the capacity upper and lower bounds of sem-icorrelated¹ MIMO Rayleigh channels were proposed, and a characteristic function (c.f.) of the capacity for MIMO Rayleigh fading channels in concise closed form with arbitrary correlation among the transmitting elements or among the receiving elements was documented in [30]. In [31], three novel bounding techniques for the achievable sum rate of MIMO ZF receivers were presented. The derived bounds are generic because they apply to several fading models of

¹The terminology “semi-correlated” was used in [29,32] for the scenario in which correlation exists at either the transmitter or receiver.

interest. In general, the capacity bounds of Rayleigh correlated or uncorrelated channels have been widely studied. As previously described, however, it is difficult to average the channel capacity over the shadowing distribution; thus, the effect of shadowing was not considered in these papers.

Fortunately, there have been several works that study the ergodic capacity or sum rate bounds over shadowing and multipath fading or other channel models, mainly motivated by the need to quantify the effect of composite channels [8, 32–35]. In [8], a large system capacity analysis was performed that took into account the effects of spatially correlated fading and i.i.d. shadowing, and closed-form expressions for the mean ergodic capacity bounds over this composite channel attenuation were recently formulated in [33]. In [34], the authors considered a SU-MIMO system operating in Rayleigh/lognormal (RLN) fading and approximated the sum rate numerically via Gauss-Hermite polynomials. In [32, 35], the ergodic uplink capacity of MU-MIMO system was explored with the aid of majorization theory by deriving several upper and lower capacity bounds. However, a common characteristic of these works is that they did not consider the effect of spatially correlated shadowing.

In fact, the effect of spatially correlated shadowing can be considered as the sum of correlated lognormal RVs, and this generally appears in wireless communications [33, 36]. Although using moment matching or cumulant matching can approximate the sum of correlated lognormal RVs and hence obtain the value of correlated shadowing, a closed-form expression for the distribution of a lognormal sum does not exist. This highlights the difficulty of the task facing us. To the best knowledge of the authors, there has been no previous study on the bounds of the sum rate distribution for MU-MIMO systems under correlated shadowing and correlated Rayleigh fading. In light of this, we have tried to bridge this gap by analytically investigating the sum rate distribution of MU-MIMO systems with ZF precoding.

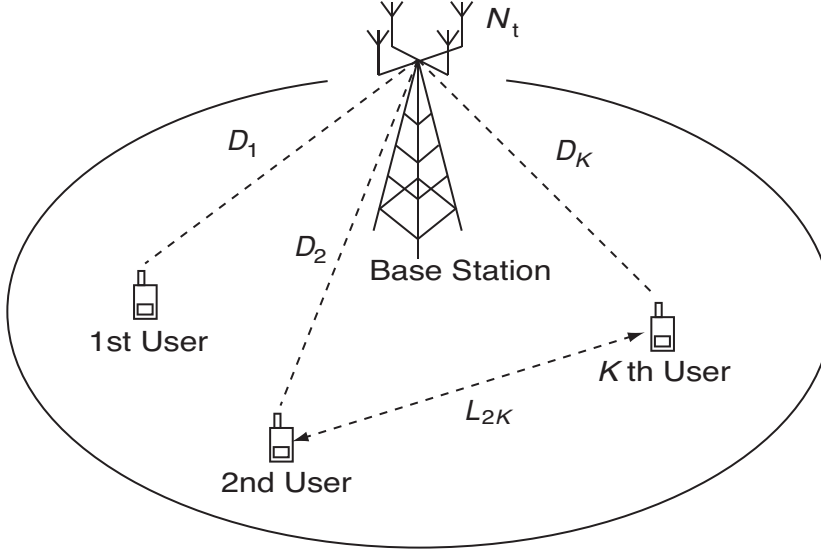


Figure 2.1: Schematic illustration of a MU-MIMO system with a multi-antenna BS simultaneously transmitting data to K single antenna users using the same carrier frequency.

2.2 MU-MIMO Systems with ZF Precoding

We consider a downlink single cell MU-MIMO communication system with N_t transmit antennas at the BS and K users, each equipped with one receiver antenna, which simultaneously communicates with the BS using the same carrier frequency. Further, users are uniformly distributed in the cell. In this system, to achieve a total number of spatial degrees of freedom (DOF) of K using ZF precoding, we assume that $N_t \geq K$ [37]. The radius of this cell is R_{cell} , and the distance between the i th and j th users is defined by L_{ij} . A schematic illustration of the system under consideration is depicted in Fig. 2.1.

The channel matrix from the BS to K users is expressed as

$$\mathbf{H} = \mathbf{D}^{\frac{1}{2}} \mathbf{S}_{\text{cor}}^{\frac{1}{2}} \mathbf{R}_{\text{cor}}, \quad (2.1)$$

where the entries of the diagonal matrix $\mathbf{D} \in \mathbb{R}^{K \times K}$ represent the distance-dependent path loss effects. Thus, $\mathbf{D} = \text{diag}\{D_k^\zeta\}_{k=1}^K$, where D_k denotes the distance between the k th user and the BS, and ζ is the path loss exponent.

The small scale multi-path fading is captured by the random matrix $\mathbf{R}_{\text{cor}} \in \mathbb{C}^{K \times N_t}$, which follows a complex zero mean Gaussian distribution with correlation among every row and column. Hence, we have $\mathbf{R}_{\text{cor}} = \mathbf{\Theta}_{\text{R},r}^{\frac{1}{2}} \mathbf{R}_{\text{i.i.d.}} \mathbf{\Theta}_{\text{R},t}^{\frac{1}{2}}$, where the entries of $\mathbf{R}_{\text{i.i.d.}} \in \mathbb{C}^{K \times N_t}$ are modeled as i.i.d. $\mathcal{CN}(\mathbf{0}, \mathbf{1})$ RVs, and $\mathbf{\Theta}_{\text{R},t} \in \mathbb{R}^{N_t \times N_t}$ and $\mathbf{\Theta}_{\text{R},r} \in \mathbb{R}^{K \times K}$ represent the transmit and receiver correlation matrices, respectively. Note that the fading correlation does not usually occur between antennas of different users, as the K users are, in general, geographically separated. In special cases, however, when users are distributed closely such as in the same vehicle, the fading correlation should not be ignored when the inter-antenna distance is smaller than ten times the operating wavelength [38]. Furthermore, as previously mentioned, in such a scenario, the fading correlation can also occur when BS antennas are distributed closely [30, 39, 40]. Therefore, to distinctly analyze the effects of spatial correlation on the sum rate distribution, we consider that in some cases, fading correlation can occur in both the BS and user sides.

The entries of the diagonal matrix $\mathbf{S}_{\text{cor}} \in \mathbb{R}^{K \times K}$ represent the spatially correlated shadowing matrix. Thus, $\mathbf{S}_{\text{cor}} = \text{diag}\{10^{0.1S_k}\}_{k=1}^K$, where $S_k \in \mathcal{N}(\mu_{S_k}, \sigma_{S_k}^2) \forall k$ is the corresponding Gaussian RV. We use $\mathbf{\Theta}_S \in \mathbb{R}^{K \times K}$ to represent an arbitrary auto-correlation matrix of RVs S_k with entries $[\mathbf{\Theta}_S]_{ij} = \Omega_{ij}, (i, j \in \{1, \dots, K\})$. In previous experiments reported in [41], it was found that the spatial auto-correlated function of the shadow process can be modeled using an exponential decay function, which is represented by

$$\Omega_{ij} = \exp\left(-\frac{L_{ij}}{d_{\text{cor}}} \ln 2\right), \quad (2.2)$$

where d_{cor} represents the shadowing correlation distance, which depends on the environment, and corresponds to the distance at which the correlation drops to 0.5 [41]. Note that the shadowing correlation that occurs between antennas of the BS can be considered completely because these antennas are, in general, closely distributed. Moreover, our model embodies the assumption that the fading and shadowing can be considered

as two independent random processes, as we can then investigate their mathematical properties. This assumption was widely used in [8, 33] and other research.

Through these composite fading channels, the received signal at all users is expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{W}\mathbf{P}^{\frac{1}{2}}\mathbf{x} + \mathbf{n}, \quad (2.3)$$

where $\mathbf{y} \in \mathbb{C}^{K \times 1}$ is the received signal vector, and $\mathbf{x} \in \mathbb{C}^{K \times 1}$ is the vector of the transmitted symbols drawn from a zero-mean Gaussian codebook with unit average power. The complex noise term is zero mean with $\mathbb{E}\{\mathbf{n}\mathbf{n}^H\} = N_0\mathbf{I}_K$, where N_0 is the noise power. $\mathbf{W} = \{\mathbf{w}_1, \dots, \mathbf{w}_K\}$ is the precoding weight matrix with element $\mathbf{w}_k \in \mathbb{C}^{N_t \times 1} \forall k$, and $\mathbf{P} = \text{diag}\{p_1, \dots, p_K\}$ is the transmit power scaling factor matrix.

We hereafter focus on the sum rate of an MU-MIMO system with ZF precoding. We assume that the BS knows the channel state information (CSI) perfectly, and the available average power P_t is distributed uniformly among all users. For the case under consideration, the ZF precoding weight matrix is expressed as $\mathbf{W} = \mathbf{H}^\dagger$ [16]. The instantaneous received SNR at the k th ($k \in \{1, \dots, K\}$) user is equal to [25]:

$$\gamma_k = \frac{\rho [\mathbf{D}\mathbf{S}_{\text{cor}}]_{kk}}{K \left[(\mathbf{R}_{\text{cor}}\mathbf{R}_{\text{cor}}^H)^{-1} \right]_{kk}}, \quad (2.4)$$

where $\rho = P_t/N_0$ represents the total transmit SNR. The achievable sum rate is essentially the sum of the throughputs contributed from all users and is expressed as

$$C = \sum_{k=1}^K \log_2(1 + \gamma_k), \quad (2.5)$$

where the probability distribution of sum rate C is obtained over all channel realizations of $\mathbf{R}_{\text{cor}}$, $\mathbf{S}_{\text{cor}}$, and the channel is assumed to be ergodic. Our main challenge in analytically evaluating (2.5) is the lack of closed-form expressions for the CDF of C in the case of correlation in both fading and shadowing.

2.3 Closed-form Bounds on Sum Rate CDF in MU-MIMO Systems with ZF Precoding

In this section, on the basis of the results of [31,33], we present closed-form bounds on the sum rate distribution of MU-MIMO systems with ZF precoding. Although the results of [31] apply for all types of SU-MIMO systems, their extension to MU-MIMO systems is not straightforward because of the presence of shadowing. Although the results of [33] can be extended to MU-MIMO uplink systems, shadowing was considered as an i.i.d. random process. We begin by proposing the following bounds for correlated RLN MIMO composite fading channels.

From (2.4) and (2.5), the sum rate C consists of fading and shadowing RVs $\mathbf{R}_{\text{cor}}$ and $\mathbf{S}_{\text{cor}}$. In [33], it was shown that the effect of fading is asymptotically averaged out, and the sum rate is only affected by the significantly more slowly varying shadowing. On the basis of this result and previous assumptions described in Sect.2.2, we first bound the fading expectation of the sum rate C and then formulate the CDF of the sum rate with shadowing RVs.

2.3.1 Rayleigh Fading Expectation of the Sum Rate

Theorem 1 *The Rayleigh fading expectation of the achievable sum rate in MU-MIMO systems with ZF precoding is bounded by $C_L \leq \mathbb{E}_R\{C\} \leq C_U$, where C_U and C_L have the same form, which is expressed as*

$$C_{U(L)} = \sum_{k=1}^K \log_2 (A_{1,k} + A_{2,k} [\mathbf{D}\mathbf{S}_{\text{cor}}]_{kk}) - A_3. \quad (2.6)$$

For the upper bound C_U , elements $A_{1,k}$, $A_{2,k}$, and A_3 are determined by

$$\begin{cases} A_{1,k} = \mathbb{E} \{ \det (\mathbf{R}_{\text{cor},k} \mathbf{R}_{\text{cor},k}^H) \} \\ A_{2,k} = \frac{\rho}{K} \mathbb{E} \{ \det (\mathbf{R}_{\text{cor}} \mathbf{R}_{\text{cor}}^H) \} \\ A_3 = \frac{1}{\ln 2} \sum_{k=1}^K \mathbb{E} \{ \ln (\det (\mathbf{R}_{\text{cor},k} \mathbf{R}_{\text{cor},k}^H)) \}; \end{cases} \quad (2.7)$$

for the lower bound C_L , elements $A_{1,k}$, $A_{2,k}$, and A_3 are determined by

$$\begin{cases} A_{1,k} = 1, A_3 = 0 \\ A_{2,k} = \frac{\rho}{K} \exp \left(\mathbb{E} \left\{ \ln (\det (\mathbf{R}_{\text{cor}} \mathbf{R}_{\text{cor}}^H)) - \ln (\det (\mathbf{R}_{\text{cor},k} \mathbf{R}_{\text{cor},k}^H)) \right\} \right) \end{cases}. \quad (2.8)$$

Proof: Starting from (2.5), the expectation of the Rayleigh fading process is expressed

as

$$\mathbb{E}_R \{C\} = \mathbb{E}_R \left\{ \sum_{k=1}^K \log_2 \left(1 + \frac{\rho [\mathbf{D}\mathbf{S}_{\text{cor}}]_{kk}}{K [\mathbf{W}^{-1}]_{kk}} \right) \right\} \quad (2.9)$$

$$\begin{aligned} &= \sum_{k=1}^K \mathbb{E}_R \left\{ \log_2 \left(\det (\mathbf{W}_{kk}) + \frac{\rho}{K} \det (\mathbf{W}) [\mathbf{D}\mathbf{S}_{\text{cor}}]_{kk} \right) - \right. \\ &\quad \left. \log_2 (\det (\mathbf{W}_{kk})) \right\}, \end{aligned} \quad (2.10)$$

where we have used

$$\mathbf{W} = \mathbf{R}_{\text{cor}} \mathbf{R}_{\text{cor}}^H, \quad (2.11)$$

and

$$[\mathbf{W}^{-1}]_{kk} = \frac{\det (\mathbf{W}_{kk})}{\det (\mathbf{W})} = \frac{\det (\mathbf{R}_{\text{cor},k} \mathbf{R}_{\text{cor},k}^H)}{\det (\mathbf{R}_{\text{cor}} \mathbf{R}_{\text{cor}}^H)}. \quad (2.12)$$

Once more, as $\log_2(\cdot)$ is a concave function, we apply the well-known Jensen's inequality on (2.10) to obtain the upper bound C_U , which is expressed as

$$\begin{aligned} C_U &= \sum_{k=1}^K \log_2 \left(\mathbb{E} \{ \det (\mathbf{R}_{\text{cor},k} \mathbf{R}_{\text{cor},k}^H) \} + \frac{\rho}{K} \mathbb{E} \{ \det (\mathbf{R}_{\text{cor}} \mathbf{R}_{\text{cor}}^H) \} [\mathbf{D}\mathbf{S}_{\text{cor}}]_{kk} \right) - \\ &\quad \frac{1}{\ln 2} \sum_{k=1}^K \mathbb{E} \{ \ln (\det (\mathbf{R}_{\text{cor},k} \mathbf{R}_{\text{cor},k}^H)) \}. \end{aligned} \quad (2.13)$$

For the lower bound, the proof relies on the general bounding technique, originally proposed in [42] and later adopted by [31–33, 35], for lower bounding the ergodic MIMO capacity with optimal receivers. In particular, we can rewrite the Rayleigh expectation of (2.5) according to

$$\mathbb{E}_R \{C\} = \sum_{k=1}^K \mathbb{E}_R \left\{ \log_2 \left(1 + \frac{\rho [\mathbf{D}\mathbf{S}_{\text{cor}}]_{kk}}{K} \exp \left(\ln \left(\frac{1}{[\mathbf{W}^{-1}]_{kk}} \right) \right) \right) \right\}. \quad (2.14)$$

By exploiting the convexity of $\log_2(1 + \alpha \exp(x))$ in x for $\alpha > 0$, and thereafter applying Jensen's inequality, we obtain the lower bound (2.15) via (2.12), which is expressed as

$$C_L = \sum_{k=1}^K \log_2 \left(1 + \frac{\rho [\mathbf{D}\mathbf{S}_{\text{cor}}]_{kk}}{K} \exp \left(\mathbb{E} \left\{ \ln (\det(\mathbf{R}_{\text{cor}} \mathbf{R}_{\text{cor}}^H)) - \ln (\det(\mathbf{R}_{\text{cor},k} \mathbf{R}_{\text{cor},k}^H)) \right\} \right) \right). \quad (2.15)$$

Comparing (2.13) and (2.15), we conclude the proof. $\blacksquare$

2.3.2 CDF of the Sum Rate with Shadowing RVs

In this subsection, we present a general method that uses the Fenton-Wilkinson (FW) method as a tool to approximate the CDF of the sum rate. The FW method computes the mean and variance by exactly matching the first and second central moments of the logarithm function. From the result of Theorem 1, (2.6) can be simplified to

$$\begin{aligned} C_{U(L)} &= \sum_{k=1}^K \log_2 \left(1 + \frac{A_{2,k}}{A_{1,k}} [\mathbf{D}\mathbf{S}_{\text{cor}}]_{kk} \right) + \sum_{k=1}^K \log_2 A_{1,k} - A_3 \\ &= \log_2 \prod_{k=1}^K \left(1 + \frac{A_{2,k}}{A_{1,k}} D_k^\zeta 10^{\frac{S_k}{10}} \right) + \sum_{k=1}^K \log_2 A_{1,k} - A_3 \\ &= \log_2 \prod_{k=1}^K \left(1 + 10^{\left(\frac{\lambda}{10} \left(\zeta \ln D_k + \ln \frac{A_{2,k}}{A_{1,k}} \right) + \frac{S_k}{10} \right)} \right) + \sum_{k=1}^K \log_2 A_{1,k} - A_3 \end{aligned} \quad (2.16)$$

$$= \log_2 \prod_{k=1}^K \left(1 + 10^{\frac{X_k}{10}} \right) + \sum_{k=1}^K \log_2 A_{1,k} - A_3, \quad (2.17)$$

where $\lambda = 10/\ln 10$. From (2.16) to (2.17), we clearly observed that the exponents $\lambda \left(\zeta \ln D_k + \ln \frac{A_{2,k}}{A_{1,k}} \right) + S_k$ in the first term of (2.16) can be considered as correlated Gaussian RVs $X_k \in \mathcal{N}(\mu_{X_k}, \sigma_{X_k}^2)$ with

$$\begin{cases} \mu_{X_k} = \lambda \left(\zeta \ln D_k + \ln \frac{A_{2,k}}{A_{1,k}} \right) + \mu_{S_k} \\ \sigma_{X_k}^2 = \sigma_{S_k}^2, \end{cases} \quad (2.18)$$

and have an arbitrary correlation matrix $\mathbf{\Theta}_S$ that can be determined by (2.2).

Thereafter, we define a set $\mathcal{G} := \{X_k | k \in \{1, \dots, K\}\}$, and the set of all k -combinations chosen from $\mathcal{G}$ is defined as $\mathcal{G}_k := \{\mathcal{G}_{ki} | i \in \{1, \dots, \binom{K}{k}\}\}$, where $\mathcal{G}_{ki}$ is the subset of $\mathcal{G}_k$,

corresponding a k -combination of set $\mathcal{G}$, and $\binom{K}{k} = \frac{K!}{k!(K-k)!}$. For instance, if $K = 3$, $\mathcal{G} = \{X_1, X_2, X_3\}$, $\mathcal{G}_1 = \{\mathcal{G}_{11}, \mathcal{G}_{12}, \mathcal{G}_{13}\} = \{\{X_1\}, \{X_2\}, \{X_3\}\}$, $\mathcal{G}_2 = \{\mathcal{G}_{21}, \mathcal{G}_{22}, \mathcal{G}_{23}\} = \{\{X_1, X_2\}, \{X_1, X_3\}, \{X_2, X_3\}\}$, and $\mathcal{G}_3 = \{\mathcal{G}_{31}\} = \{\{X_1, X_2, X_3\}\}$. Then in (2.17), the product in the first term on the right-hand side is rewritten as

$$\prod_{k=1}^K \left(1 + 10^{\frac{X_k}{10}}\right) = 1 + \sum_{k=1}^K \sum_{i=1}^{\binom{K}{k}} 10^{\frac{\text{sum}(\mathcal{G}_{ki})}{10}}, \quad (2.19)$$

where $\text{sum}(\mathcal{G}_{ki})$ denotes a sum operation over the elements of $\mathcal{G}_{ki}$.

The second term on the right-hand side of (2.19) can be considered as the sum of correlated lognormal RVs. In fact, no exact closed-form expression for the distribution of (2.19) is known. In [43], a simple closed-form expression for a lower bound of (2.19) on the outage sum rate was evaluated. In [44], a limit distribution of the correlated lognormal RVs was presented when the number of antennas was infinity. In [45], the authors derived bounds for the CDF of a sum of two or three correlated lognormal RVs with arbitrary correlation and of a sum of any number of equally correlated² lognormal RVs. Obviously, it is difficult to formulate a closed-form expression for the distribution of (2.17). Fortunately, in order to solve this problem, several approximations have been developed, such as moment matching [46] and cumulant matching [47], enabling us to bound the distribution.

Theorem 2 *The CDF of the sum rate C in MU-MIMO systems with ZF precoding is bounded by $F_{C_L} \leq F_C \leq F_{C_U}$, where F_{C_U} and F_{C_L} have the same form, which is expressed as*

$$F_{C_{U(L)}}(c) = F_{\tilde{X}} \left(\lambda \ln \left(2^{c - \sum_{k=1}^K \log_2 A_{1,k} + A_3} - 1 \right) \right). \quad (2.20)$$

²The terminology “equally correlated” means the correlation coefficient between any two summands is equal. Note that this model is not exactly valid for many scenarios encountered in wireless communications. However, it can be used to describe the correlation among equally spaced (close) antennas.

$F_{\tilde{X}}(x)$ is the CDF of the Gaussian distribution and takes the form of

$$F_{\tilde{X}}(x) = \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{x - \mu_{\tilde{X}}}{\sqrt{2}\sigma_{\tilde{X}}} \right) \right), \quad (2.21)$$

where $\operatorname{erf}(x)$ is the well-known error function. $\mu_{\tilde{X}}$ and $\sigma_{\tilde{X}}$ are functions of $A_{1,k}$ and $A_{2,k}$ and can be calculated using **Proposition 1**. In addition, we can use (2.7) and (2.8) to obtain F_{C_U} and F_{C_L} , respectively.

Proof: According to [46], the sum of lognormal RVs, i.e., the second term on the right-hand side of (2.19) are accurately approximated by a new lognormal RV $10^{0.1\tilde{X}}$, where $\tilde{X}$ is a Gaussian RV with mean $\mu_{\tilde{X}}$ and variance $\sigma_{\tilde{X}}^2$. The probability distribution function (PDF) of $\tilde{X}$ takes the form of

$$f_{\tilde{X}}(x) = \frac{1}{\sigma_{\tilde{X}}\sqrt{2\pi}} \exp \left(-\frac{(x - \mu_{\tilde{X}})^2}{2\sigma_{\tilde{X}}^2} \right). \quad (2.22)$$

On the basis of this approximation, (2.17) can be successively approximated to

$$C_{U(L)} \approx \log_2 \left(1 + 10^{\frac{\tilde{X}}{10}} \right) + \sum_{k=1}^K \log_2 A_{1,k} - A_3. \quad (2.23)$$

From (2.23), we can use (2.22) to easily conclude the proof after several simplifications. ■

Proposition 1 $\mu_{\tilde{X}}$ and $\sigma_{\tilde{X}}$ are calculated by solving

$$\begin{cases} E_1 = \sum_{n=1}^N \frac{w_n}{\sqrt{\pi}} \log_2 \left(1 + 10^{0.1(\sqrt{2}\sigma_{\tilde{X}}a_n + \mu_{\tilde{X}})} \right) \\ E_2 = \sum_{n=1}^N \frac{w_n}{\sqrt{\pi}} \left(\log_2 \left(1 + 10^{0.1(\sqrt{2}\sigma_{\tilde{X}}a_n + \mu_{\tilde{X}})} \right) \right)^2, \end{cases} \quad (2.24)$$

where E_1 and E_2 represent the first and second central moments of the first term on the right-hand side of (2.17), respectively. N is the Hermite integration order (the weights w_n and abscissas a_n for N up to 20 are tabulated in [48]). These nonlinear equations in $\mu_{\tilde{X}}$ and $\sigma_{\tilde{X}}$ are solved numerically using standard functions such as `fsolve` in Matlab and

NSolve in *Mathematica*. A detailed method for calculating the ν th central moment E_ν is provided in Appendix. Note that, however, the solution of (2.24) does not always exist because these equations are derived from the Gauss-Hermite formula (see Proof), which is an approximation method, and the initial values, for example, those used in *fsolve*, are uncertain, which may further lead to undesired solutions. Given this information, we suggest increasing N and using the results of **Proposition 2** as initial values of $\mu_{\tilde{X}}$ and $\sigma_{\tilde{X}}$ to iteratively obtain an approximate solution for (2.24) with the help of standard functions.

Proof: Our aim is to match the first and second central moments of the first term in (2.17) and (2.23) according to [46], i.e., solving the following equations:

$$\begin{cases} E_1 = \int_{-\infty}^{\infty} \log_2(1 + 10^{0.1x}) f_{\tilde{X}}(x) dx \\ E_2 = \int_{-\infty}^{\infty} (\log_2(1 + 10^{0.1x}))^2 f_{\tilde{X}}(x) dx. \end{cases} \quad (2.25)$$

We can use the Gauss-Hermite formula to solve the expressions in (2.25) with an integration interval of $(-\infty, \infty)$. Improved estimates of $\mu_{\tilde{X}}$ and $\sigma_{\tilde{X}}$ are obtained by increasing the Hermite integration order N ; on the other hand, reducing N decreases the computational complexity. We have found $N = 12$ to be sufficient to determine the values of $\mu_{\tilde{X}}$ and $\sigma_{\tilde{X}}$. ■

In this subsection, we further consider the high-SNR regime, i.e., $\rho \rightarrow \infty$. Under this condition, we can obtain the following straightforward result:

Proposition 2 *In the high-SNR case (i.e., $\rho \rightarrow \infty$), $\mu_{\tilde{X}}$ and $\sigma_{\tilde{X}}$ are calculated by*

$$\begin{cases} \mu_{\tilde{X}} = \sum_{k=1}^K (\lambda \zeta \ln D_k + \mu_{S_k}) + \lambda \sum_{k=1}^K \ln \frac{A_{2,k}}{A_{1,k}} \\ \sigma_{\tilde{X}}^2 = \sum_{i=1}^K \sum_{j=1}^K \sigma_{S_i} \sigma_{S_j} \Omega_{ij}. \end{cases} \quad (2.26)$$

Proof: The proof for (2.26) follows by assuming a large value for ρ , and then (2.17) and (2.23) are given asymptotically by

$$C_{U(L)} = \log_2 \prod_{k=1}^K \left(10^{\frac{X_k}{10}} \right) + \sum_{k=1}^K \log_2 A_{1,k} - A_3 + \mathcal{O} \left(\sum_{k=1}^K 10^{-\frac{X_k}{10}} \right) \quad (2.27)$$

and

$$C_{U(L)} \approx \log_2 \left(10^{\frac{\tilde{X}}{10}} \right) + \sum_{k=1}^K \log_2 A_{1,k} - A_3 + \mathcal{O} \left(10^{-\frac{\tilde{X}}{10}} \right) \quad (2.28)$$

after noting that $X_k \forall k$ and $\tilde{X}$ are also large because they approach infinity as $\rho \rightarrow \infty$. $\mathcal{O}$ signifies big O notation. Thereafter, by eliminating the identity term, the matching of the first term in (2.27) and (2.28) are rewritten as

$$\log_2 \left(10^{0.1 \sum_{k=1}^K X_k} \right) = \log_2 \left(10^{0.1 \tilde{X}} \right). \quad (2.29)$$

In the next step, we match the average and variance of $\sum_{k=1}^K X_k$ and $\tilde{X}$, i.e.,

$$\begin{cases} \mu_{\tilde{X}} = \sum_{k=1}^K \mu_{X_k} \\ \sigma_{\tilde{X}}^2 = \sum_{i=1}^K \sum_{j=1}^K \text{Cov}(X_i, X_j), \end{cases} \quad (2.30)$$

where $\text{Cov}(\cdot)$ is the covariance. The second formula states that the variance of a sum is equal to the sum of all elements in the covariance matrix of the components. This formula is used in the theory of Cronbach's alpha in classical test theory. Thereafter, we perform several basic simplifications and conclude the proof. $\blacksquare$

In Fig. 2.2, the simulated achievable CDF of the sum rate in (2.6) along with the proposed approximation of using (2.24) and (2.26) are plotted. We consider different values of the shadowing correlation coefficient Ω_{ij} determined in (2.2) by keeping $A_{1,k} = 1 \forall k$ and $A_3 = 0$, using several MIMO configurations, and increasing only $A_{2,k} \forall k$.

This figure indicates that, as anticipated, both Propositions 1 and 2 become exact in the high-SNR case. However, Proposition 2 cannot successively estimate the CDF in the

in this section, we mathematically analyze the sum rate CDF under different Rayleigh fading cases.

2.4.1 Semi-correlated Rayleigh Fading

This is a scenario in which fading correlation only occurs between the users. This can occur because of either limited receive angular spread or small interval spacings in the user antenna. Under these circumstances, the channel matrix $\mathbf{R}_{\text{cor}}\mathbf{R}_{\text{cor}}^H$ is rewritten as

$$\mathbf{R}_{\text{cor}}\mathbf{R}_{\text{cor}}^H = \mathbf{\Theta}_{\text{R,r}}^{\frac{1}{2}} \mathbf{R}_{\text{i.i.d.}} \mathbf{R}_{\text{i.i.d.}}^H \mathbf{\Theta}_{\text{R,r}}^{\frac{1}{2}} \quad (2.31)$$

because $\mathbf{\Theta}_{\text{R,t}} = \mathbf{I}_{N_t}$.

Proposition 3 *In the semi-correlated Rayleigh fading case, for the distribution of the upper bound F_{C_U} , elements $A_{1,k}$, $A_{2,k}$, and A_3 are calculated by*

$$\begin{cases} A_{1,k} = \frac{\det(\mathbf{\Theta}_{\text{R,r},kk}) N_t!}{(N_t - K + 1)!} \\ A_{2,k} = \frac{\rho \det(\mathbf{\Theta}_{\text{R,r}}) N_t!}{K(N_t - K)!} \\ A_3 = \frac{K}{\ln 2} \sum_{k=1}^{K-1} \psi(N_t - k + 1) + \sum_{k=1}^K \log_2 \det(\mathbf{\Theta}_{\text{R,r},kk}); \end{cases} \quad (2.32)$$

for the distribution of the lower bound F_{C_L} , the element $A_{2,k}$ is calculated by

$$A_{2,k} = \frac{\rho}{K} \exp \left(\psi(N_t - K + 1) - \ln \left[\mathbf{\Theta}_{\text{R,r}}^{-1} \right]_{kk} \right). \quad (2.33)$$

Proof: The proof starts by analyzing the expectation of $\det(\mathbf{R}_{\text{cor}}\mathbf{R}_{\text{cor}}^H)$, which is expressed as

$$\mathbb{E} \left\{ \det(\mathbf{R}_{\text{cor}}\mathbf{R}_{\text{cor}}^H) \right\} = \det(\mathbf{\Theta}_{\text{R,r}}) \mathbb{E} \left\{ \det(\mathbf{R}_{\text{i.i.d.}}\mathbf{R}_{\text{i.i.d.}}^H) \right\}, \quad (2.34)$$

where we successively use the property of square matrices $\det(\mathbf{B}_1\mathbf{B}_2) = \det(\mathbf{B}_1)\det(\mathbf{B}_2)$ to obtain this equality. Thereafter, we invoke the following results for an $K \times N_t$ (with $N_t \geq K$) central (zero-mean) Wishart matrix

$$\mathbb{E} \left\{ \det(\mathbf{R}_{\text{i.i.d.}}\mathbf{R}_{\text{i.i.d.}}^H) \right\} = \frac{N_t!}{(N_t - K)!} \quad (2.35)$$

and

$$\mathbb{E} \left\{ \ln \left(\det \left(\mathbf{R}_{\text{i.i.d.}} \mathbf{R}_{\text{i.i.d.}}^H \right) \right) \right\} = \sum_{k=0}^{K-1} \psi(N_t - k) \quad (2.36)$$

which are obtained from [12, 31]. The elements in (2.7) for the upper bound and the elements in (2.8) for the lower bound are directly inferred with the aid of (2.35) and (2.36) after noting that

$$\mathbf{R}_{\text{cor},k} \sim \mathcal{CN}(\mathbf{0}_{(K-1) \times N_t}, \boldsymbol{\Theta}_{\text{R},r,kk} \otimes \mathbf{I}_{N_t}), \quad (2.37)$$

and further simplifications. ■

2.4.2 Doubly Correlated Rayleigh Fading

This is a scenario in which fading correlation occurs on both the BS and user sides. This can occur because of either limited transmit and receive angular spread or small interval spacings both in the BS antenna array and user distribution. Under these cases, the $\mathbf{R}_{\text{cor}} \mathbf{R}_{\text{cor}}^H$ matrix is rewritten as

$$\mathbf{R}_{\text{cor}} \mathbf{R}_{\text{cor}}^H = \boldsymbol{\Theta}_{\text{R},r}^{\frac{1}{2}} \mathbf{R}_{\text{i.i.d.}} \boldsymbol{\Theta}_{\text{R},t} \mathbf{R}_{\text{i.i.d.}}^H \boldsymbol{\Theta}_{\text{R},r}^{\frac{1}{2}}. \quad (2.38)$$

In general, this scenario of doubly correlated Rayleigh fading is not amenable to tractable manipulations because the determinant of the random term (2.38) cannot be further decomposed [49, 50]. That is the main reason for this scenario not being thoroughly addressed in the context of MIMO systems with ZF precoding. However, by using several recent advances in random matrix theory as a starting point, from [31], we obtain several results for the average value of $\det(\mathbf{R}_{\text{i.i.d.}} \boldsymbol{\Theta}_{\text{R},t} \mathbf{R}_{\text{i.i.d.}}^H)$ and its logarithm, allowing us to then obtain the elements $A_{1,k}$, $A_{2,k}$, and A_3 .

Proposition 4 *In the doubly correlated Rayleigh fading case, for the distribution of the upper bound F_{C_U} , elements $A_{1,k}$, $A_{2,k}$, and A_3 are calculated by*

$$\begin{cases} A_{1,k} = \frac{\det(\Theta_{R,r,kk})\det(\hat{\Xi})(K-1)!}{\prod_{i<j}^{N_t}(\eta_j - \eta_i)} \\ A_{2,k} = \frac{\rho \det(\Theta_{R,r})\det(\Xi)(K-1)!}{\prod_{i<j}^{N_t}(\eta_j - \eta_i)} \\ A_3 = \frac{K}{\ln 2} \left(\frac{\sum_{m=N_t-K+2}^{N_t} \det(\Psi_m)}{\prod_{i<j}^{N_t}(\eta_j - \eta_i)} + \sum_{k=1}^{K-1} \psi(k) \right) + \sum_{k=1}^K \log_2 \det(\Theta_{R,r,kk}); \end{cases} \quad (2.39)$$

for the distribution of the lower bound F_{C_L} , the element $A_{2,k}$ is calculated by

$$A_{2,k} = \frac{\rho}{K} \exp \left(\psi(K) + \frac{\det(\Psi_{N_t-K+1})}{\prod_{i<j}^{N_t}(\eta_j - \eta_i)} - \ln [\Theta_{R,r}^{-1}]_{kk} \right), \quad (2.40)$$

where $\eta_i, i \in \{1, \dots, N_t\}$ are the real, positive eigenvalues of $\Theta_{R,t}$, and Ξ is an $N_t \times N_t$ matrix with entries

$$[\Xi]_{ij} = \begin{cases} \eta_i^{j-1} & j = 1, \dots, N_t - K \\ \eta_i^j & j = N_t - K + 1, \dots, N_t. \end{cases} \quad (2.41)$$

The matrix $\hat{\Xi}$ is directly related to Ξ and has entries

$$[\hat{\Xi}]_{ij} = \begin{cases} \eta_i^{j-1} & j = 1, \dots, N_t - K + 1 \\ \eta_i^j & j = N_t - K + 2, \dots, N_t, \end{cases} \quad (2.42)$$

whereas Ψ_m is an $N_t \times N_t$ matrix with entries

$$[\Psi_m]_{ij} = \begin{cases} \eta_i^{j-1} & j \neq m \\ \eta_i^{j-1} \ln \eta_i & j = m. \end{cases} \quad (2.43)$$

Proof: This proof invokes results for semi-correlated Rayleigh fading with correlation on the transmit side. The expectations of $\det(\mathbf{R}_{i.i.d.} \Theta_{R,t} \mathbf{R}_{i.i.d.}^H)$ and $\ln(\det(\mathbf{R}_{i.i.d.} \Theta_{R,t} \mathbf{R}_{i.i.d.}^H))$ are

$$\mathbb{E}\{\det(\mathbf{R}_{i.i.d.} \Theta_{R,t} \mathbf{R}_{i.i.d.}^H)\} = \frac{K! \det(\Xi)}{\prod_{i<j}^{N_t}(\eta_j - \eta_i)} \quad (2.44)$$

and

$$\mathbb{E}\{\ln(\det(\mathbf{R}_{i.i.d.} \Theta_{R,t} \mathbf{R}_{i.i.d.}^H))\} = \sum_{k=1}^K \psi(k) + \frac{\sum_{m=N_t-K+1}^{N_t} \det(\Psi_m)}{\prod_{i<j}^{N_t}(\eta_j - \eta_i)} \quad (2.45)$$

which are obtained from [12, 31]. To evaluate $\mathbf{R}_{\text{cor},k} \mathbf{R}_{\text{cor},k}^H$, we notice that

$$\mathbf{R}_{\text{cor},k} \sim \mathcal{CN}(\mathbf{0}_{(K-1) \times N_t}, \boldsymbol{\Theta}_{\text{R},\text{r},kk} \otimes \boldsymbol{\Theta}_{\text{R},\text{t}}), \quad (2.46)$$

which implies that we can directly employ (2.44) and (2.45). Combining these results and using (2.7) and (2.8), after several simplifications, we conclude the proof. ■

2.5 Numerical Results

In this section, we present the CDF obtained from Monte Carlo simulations and the proposed bounds. The CDF is necessary because we can investigate the expectation and sum rate corresponding to a given value of the CDF. The results in this section illustrate the effects of spatial correlation on the distribution of the sum rate. Another goal is to verify that (2.23) is a good approximation to the exact sum rate distribution.

2.5.1 Effect of Spatially Correlated Shadowing

In this subsection, we consider that spatial correlation occurs only in shadowing. This scenario exists when the inter-antenna distance of the BS and inter-user distance are ten times greater than the operating wavelength, i.e., fading can be considered to be an independent process ($\boldsymbol{\Theta}_{\text{R},\text{t}} = \mathbf{I}_{N_t}$ and $\boldsymbol{\Theta}_{\text{R},\text{r}} = \mathbf{I}_K$). Hereafter, we first concentrate on the effect of the number of transmit antennas and then summarize the effect of correlated shadowing by analytically investigating the results.

In Fig. 2.3, the simulated CDF of the achievable sum rate and the proposed analytical bounds are plotted. We normalize the correlation distance of shadowing by defining $\xi = d_{\text{cor}}/R_{\text{cell}}$ ranging from 0 to ∞ . We also consider different MIMO configurations by keeping the user number $K = 5$ constant and increasing only N_t .

The figure indicates that, as expected, both the bounds remain sufficiently tight across the entire sum rate range, which implies that the proposed methods can explicitly

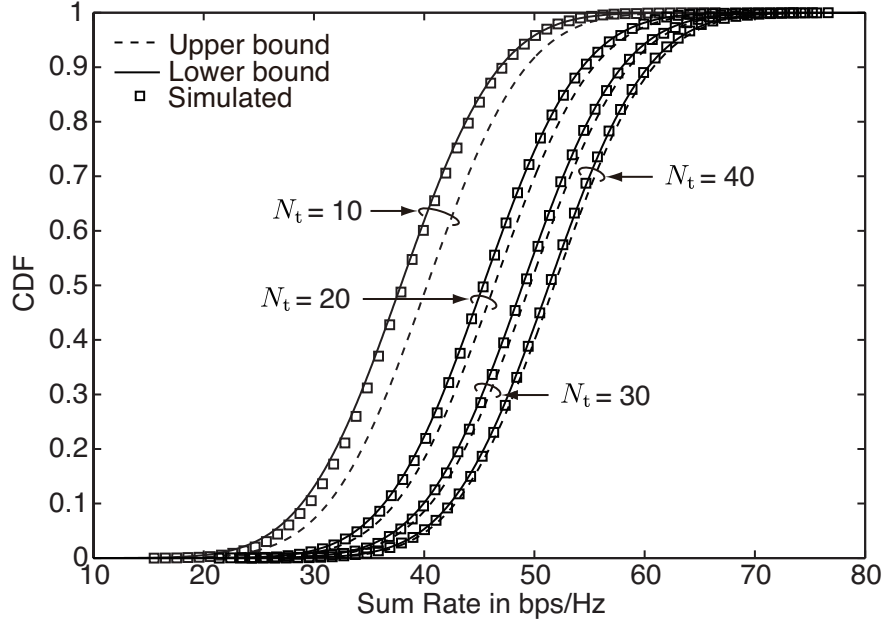


Figure 2.3: Simulated CDF of sum rate and the proposed analytical bounds with $R_{\text{cell}} = 1$, $\zeta = -4$, $K = 5$, $S_k \in \mathcal{N}(0, 8) \forall k$, $\xi = 0.2$, and $\text{SNR} = 10$ dB.

predict the exact sum rate distribution with a lower computational load. Furthermore, the tightness of the upper bound improves with increasing N_t . In fact, at the limit of a large number of transmit antenna, the bound becomes exact because of the law of large numbers, which states that $\lim_{N_t \rightarrow \infty} \frac{\mathbf{R}_{\text{i.i.d.}} \mathbf{R}_{\text{i.i.d.}}^H}{N_t} \rightarrow \mathbf{I}_K$. These results are consistent with those in [31].

To investigate the relationship between the inter-user distance L and sum rate corresponding to a given value of the CDF and hence analyze the effect of spatially correlated shadowing, in Fig. 2.4, the simulated sum rate corresponding to 10% of the CDF and the proposed analytical bound are plotted against L over the range of 0 to $2R_{\text{cell}}$. In this case, we remove the effect of path loss, enabling the effect of correlated shadowing to be clearly observed. Note that the sum rate corresponding to a low value of a CDF, for instance, 10%, is not commonly used to evaluate communications systems. However,

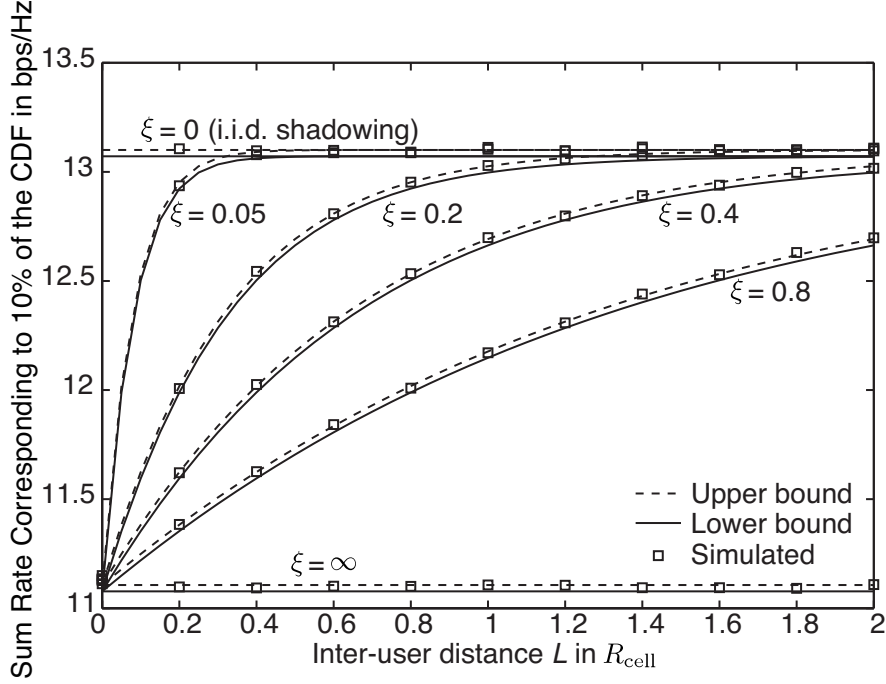


Figure 2.4: The simulated sum rate corresponding to 10% of the CDF and the proposed analytical bounds against L with $K = 2$, $N_t = 100$, $S_k \in \mathcal{N}(0, 8) \forall k$, and $\text{SNR} = 10$ dB. The effect of path loss has been removed.

in this study, we can use it as a performance measure mainly because it is worth understanding how the shadowing auto-correlation affects the sum rate distribution when the MIMO channel is poor. This measure also seems more intuitive when investigated as a function of the inter-user distance or SNR, which can help us analyze the effects of shadowing correlation more efficiently.

Figure 2.4 shows that the sum rate corresponding to 10% of the CDF approaches higher values as L increases because the correlation of shadowing decreases. Moreover, this sum rate decreases as the normalized correlation distance ξ increases. The reason for this is the decrease in the number of DOFs in shadowing RVs. Thus, if the spatial area can be considered completely correlated, the effect of shadowing will disappear. This result is confirmed by the curve of $\xi = \infty$ in Fig. 2.4.

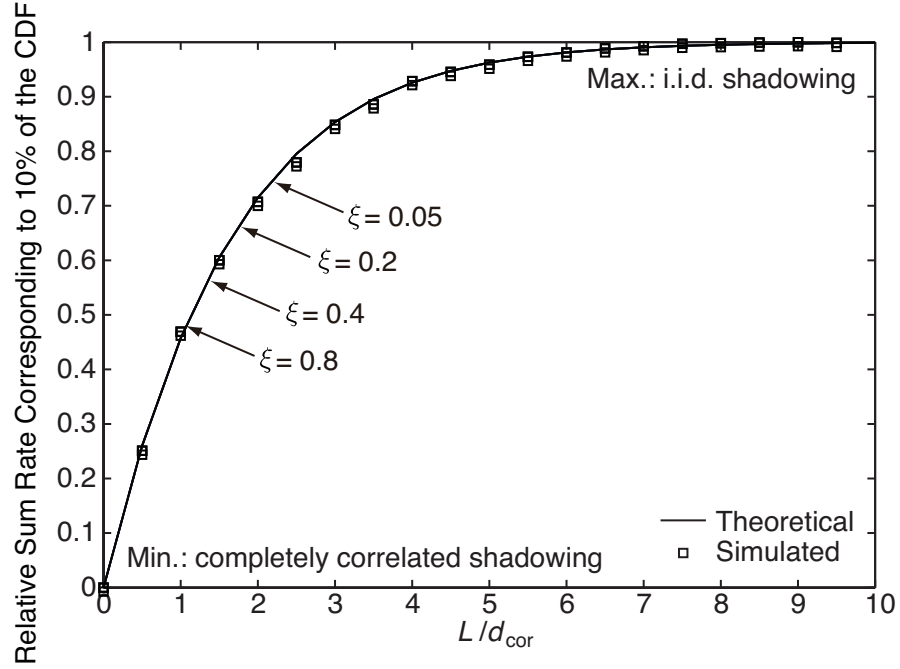


Figure 2.5: Simulated and theoretical relative sum rate corresponding to 10% of the CDF against L/d_{cor} with $K = 2$ and $S_k \in \mathcal{N}(0, 8) \forall k$. The effect of path loss has been removed.

Fig. 2.5 presents the relative sum rate corresponding to 10% of the CDF against the ratio of the inter-user distance L to the shadowing correlation distance d_{cor} with different ξ , and we let this ratio range from 0 to 10. The relative sum rate corresponding to 10% of the CDF $\hat{C}$ is defined as

$$\hat{C} = \frac{C - C_{\text{CCS}}}{C_{\text{i.i.d.}} - C_{\text{CCS}}}, \quad (2.47)$$

where $C_{\text{i.i.d.}}$ and C_{CCS} are the sum rate corresponding to 10% of the CDF in the i.i.d. and completely correlated shadowing (CCS) models, respectively, i.e., in the $\xi = 0$ and $\xi = \infty$ cases, respectively. Note that the effect of the path loss has also been removed.

From Fig. 2.5, we conclude that the shadowing is essentially independent when the inter-user distance L is approximately five times greater than the shadowing correlation

distance d_{cor} . This validates recent results in the MIMO literature [51]. It is worth mentioning that shadowing attenuation and its spatial correlation depend on the parameters of a realistic environment. In this study, however, we demonstrated a relationship between the i.i.d. and spatially correlated shadowing employing the lognormal function to model the shadowing attenuation and exponential decay function to model its autocorrelation. Therefore, this result obtained from Fig. 2.5 is applicable only for the case under consideration and may be possibly used in a realistic environment with indispensable experimental support.

2.5.2 Effect of Correlated Rayleigh Fading

In this subsection, we consider that spatial correlation can occur in both shadowing and fading. This scenario exists when the antenna array is sited at the BS, and users are distributed closely. Hereafter, we first investigate the CDF of the sum rate under the effects of composite fading and then concentrate on analyzing the sum rate corresponding to 10% of the CDF. Note that, in this case, shadowing can be considered to be completely correlated because fading correlation occurred in a small-scale spatial area.

In Figs. 2.6 and 2.7, the effects of composite fading on the CDF and sum rate corresponding to 10% of the CDF are addressed, respectively. In this case, the transmit fading correlation matrix $\Theta_{\text{R,t}}$ and receiver fading correlation matrix $\Theta_{\text{R,r}}$ are modeled via the well-known exponential correlation model, and thus $[\Theta_{\text{R,t}}]_{ij} = \beta_{\text{t}}^{|i-j|}$ and $[\Theta_{\text{R,r}}]_{ij} = \beta_{\text{r}}^{|i-j|}$ with $\beta_{\text{t}}, \beta_{\text{r}} \in [0, 1)$ as the transmit and receiver correlation coefficient, respectively [52].

From these figures, we observe that the spatially correlated shadowing equally changes the distribution by increasing the sum rate corresponding to high values of the CDF and decreasing the sum rate corresponding to low values of the CDF. For average values of the CDF, however, spatially correlated shadowing does not have any effect. We also easily notice the diminishing effects of Rayleigh fading correlation on the CDF, which validates

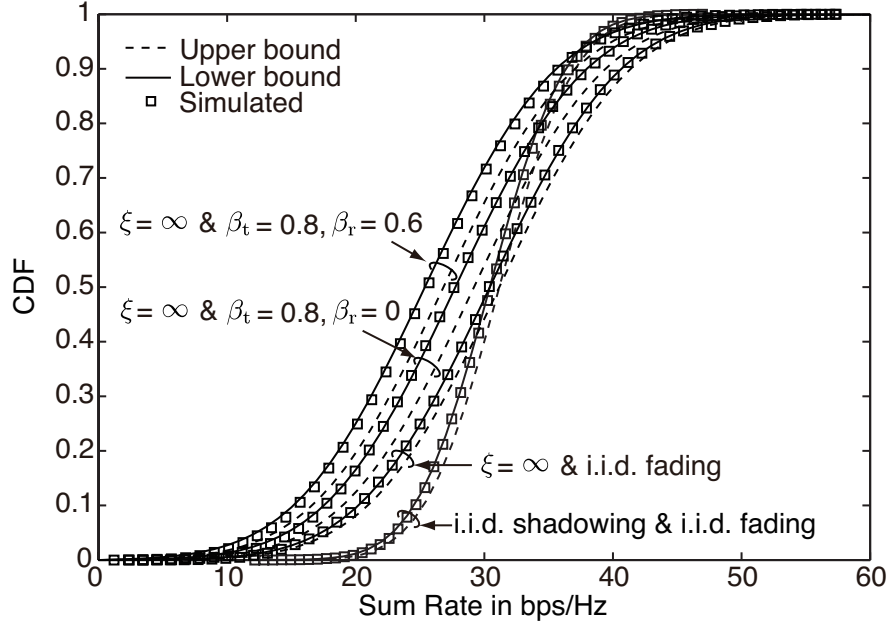


Figure 2.6: Simulated CDF of the sum rate and the proposed analytical bounds under the effects of composite fading with $R_{\text{cell}} = 1$, $K = 3$, $N_t = 10$, $S_k \in \mathcal{N}(0, 8) \forall k$, and $\text{SNR} = 10$ dB.

several well-known results in the MIMO literature [8, 29, 30, 39]. This is due to the reduced transmit and receiver diversity because the spatial signatures of the wavefronts become identical with a higher β_t and β_r , thereby decreasing the number of DOFs in the MIMO channel.

In Fig. 2.8, comparisons of the effects of spatial correlation using the proposed bounds and simulated methods for the sum rate corresponding to 10% of the CDF against the SNR are evaluated. Three different pairs of Rayleigh fading correlation coefficients are considered under the effect of CCS. This figure shows an interesting result, namely, the loss of the sum rate corresponding to 10% of the CDF is more severe when fading correlation occurs at the user side rather than at the BS side. In fact, from the third expression in (2.39) and (2.40), the effect of $\Theta_{R,t}$ is purely through its eigenvalues. However, the relative effect of the transmit correlation not only depends on N_t but also

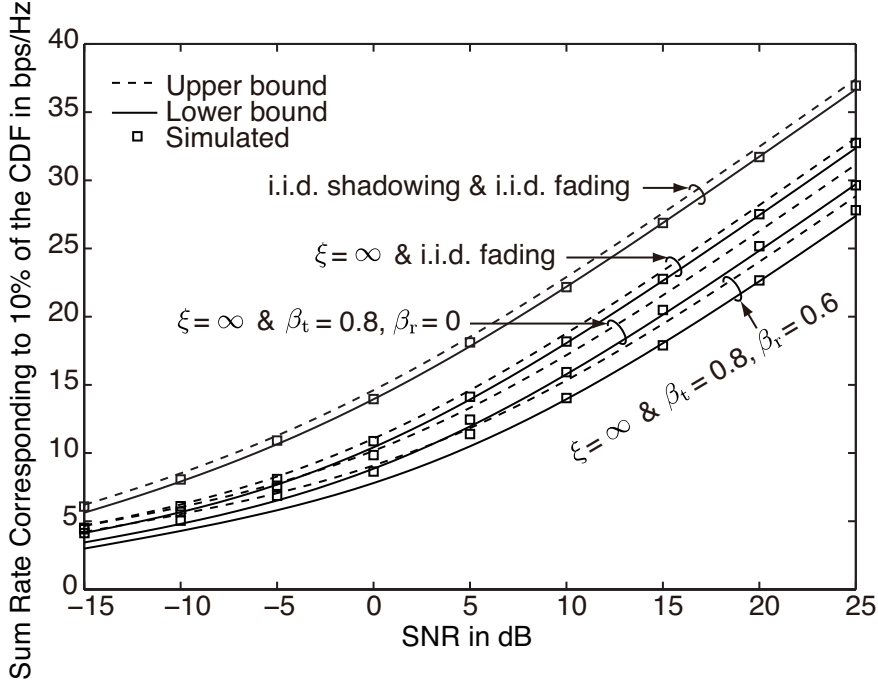


Figure 2.7: Simulated sum rate corresponding to 10% of the CDF and the proposed analytical bounds against the SNR under the effects of composite fading with $R_{\text{cell}} = 1$, $K = 3$, $N_t = 10$, and $S_k \in \mathcal{N}(0, 8) \forall k$.

on K . This is in contrast to the effect of receiver correlation, which depends only on K . A detailed theoretical analysis of the SNR power offset due to Rayleigh fading correlation in MIMO minimum mean square error (MMSE) receivers is found in [53]; however, it should be noted that the authors in [53] employed an SU-MIMO system using MMSE detection, which is an inverse process to the ZF precoding technology employed in our work.

2.6 Summary

In this study, a detailed analysis of the sum rate distribution in MU-MIMO systems with ZF precoding was presented. Specifically, novel upper and lower CDF bounds were devised, which can be applied for an arbitrary number of antennas and remain tight across

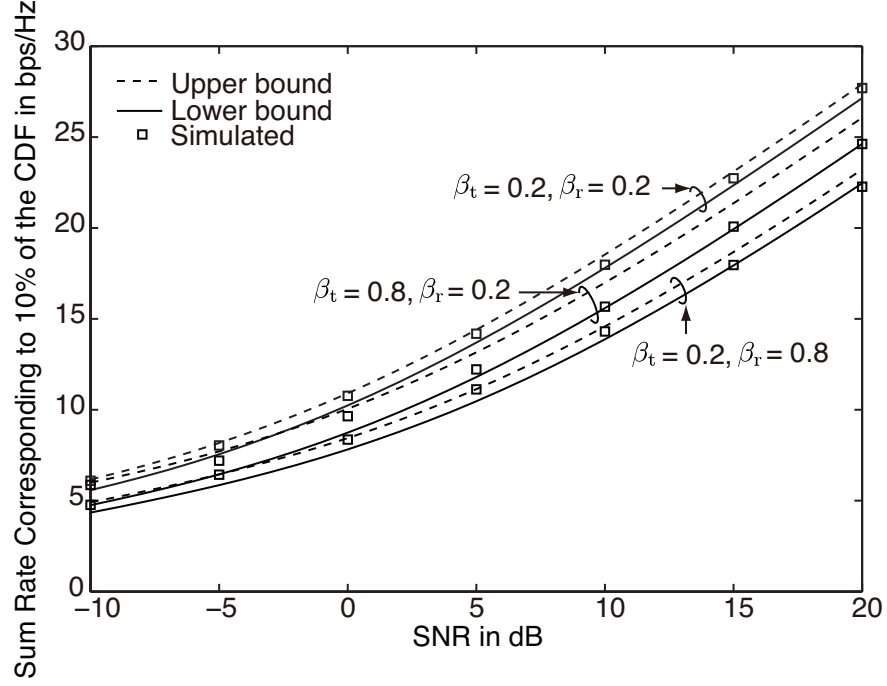


Figure 2.8: Comparisons of effects of spatial correlation by using the proposed bounds and simulated methods for the sum rate corresponding to 10% of the CDF against SNR, $R_{\text{cell}} = 1$, $K = 3$, $N_t = 10$, $S_k \in \mathcal{N}(0, 8) \forall k$, and $\xi = \infty$.

the entire SNR range. In fact, the proposed lower bound becomes exact at high SNRs. More importantly, the proposed bounds are generic because they encompass the composite fading channel model consisting of spatially correlated shadowing and correlated Rayleigh fading, which have practical interest, and can be very easily evaluated. With the help of these bounds, we gained valuable insights into the effects of the composite fading parameters on the performance of MU-MIMO systems. For instance, we derived a relationship between the inter-user distance and sum rate corresponding to 10% of the CDF. A practical conclusion from our results based on the considered system is that the effect of spatially correlated shadowing can be considered to be independent when the inter-user distance is more than five times the correlation distance of shadowing. Moreover, we examined in detail the effects of spatial correlation under shadowing and

fading, the number of antennas, and the SNR.

Chapter 3

Performance Evaluation for MU-MIMO DASs

In chapter 2, we mathematically analyzed the sum rate distribution in MU-MIMO CASs and summarized that the increasing of inter-antenna distance on user side can decrease the spatial correlations existing both in shadowing and Rayleigh fading, and thus improves performances of system sum rates. In this chapter, in order to further verified and clarified that the increasing of inter-antenna distance on BS side, i.e., a MU-MIMO DAS, can also significantly improve the performances of sum rates under the effects of correlated composite fading and inter-cell interference, we investigate and compare the characteristics of system sum rates in both CASs and DASs under the effects of the considered attenuation channels.

The introduction of this chapter is described in Sect. 3.1. In Sect. 3.2, we define the system model for the CAS and DAS and derive an expression that can calculate the system sum rate with inter-cell interference. In Sect. 3.3, we introduce and describe the mathematical background for the channel model in which the spatial correlation is considered. In Sect. 3.4, we present the simulation results and analyze the effect of spatial correlation on the sum rate performance in a CAS and DAS, respectively. Concluding remarks can be found in Sect. 3.5.

Notations: In this study, we use upper- and lower-case boldface to denote matrices and vectors, respectively. The $n \times n$ identity matrix is denoted by $\mathbf{I}_n$, and the (i, j) th element of a matrix is denoted by $[\mathbf{A}]_{ij}$. The expectation is given by $\mathbb{E}\{\cdot\}$, and the matrix determinant is given by $\det(\cdot)$. The symbols $(\cdot)^\dagger$ and $(\cdot)^H$ represent the pseudo-inverse and Hermitian transpose of a matrix, respectively, and “ $\circ$ ” denotes the Hadamard product, i.e., $[\mathbf{A} \circ \mathbf{B}]_{ij} = [\mathbf{A}]_{ij}[\mathbf{B}]_{ij}$. To distinguish the square root of a matrix $\mathbf{A}^{\frac{1}{2}}$, we use $\mathbf{A}^{\frac{1}{2}\circ}$ to denote the positive square root of $\mathbf{A}$ with respect to the Hadamard product such that $\mathbf{A}^{\frac{1}{2}\circ} \circ \mathbf{A}^{\frac{1}{2}\circ} = \mathbf{A}$. The symbol $\sim \mathcal{CN}(M, \Sigma)$ denotes a complex Gaussian RV with a mean M and covariance Σ , and $\text{diag}\{\cdot\}$ and $\text{vec}\{\cdot\}$ represent a diagonal matrix and a column vector consisting of their inside elements, respectively.

3.1 Introduction

Theoretical research has shown that a DAS has great advantages in capacity and coverage improvement, and a comparison of CAS and DAS downlink channel capacities has been also discussed in detail in many papers [5–8, 14, 15, 54, 55]. To exploit the performance potential of a DAS, the authors in [5, 7, 14, 15, 54] studied the channel capacity of a DAS considering the effects of path loss and fading that combines both the multi-path fading and lognormal shadowing. In [8], the advantages of a DAS in correlated fading scenarios were demonstrated via an asymptotic analysis, whereas the channel capacity of DAS over-shadowed Nakagami- m fading channels was analyzed in [6]. In [55], the authors derived a capacity expression for DAS over-shadowing and multi-path fading attenuation in a frequency-selective channel using a Monte Carlo numerical computation method. Although many studies consider these channel capacities assuming path loss, shadowing, correlated or uncorrelated Rayleigh fading, and a frequency-selective or non-selective channel, the shadowing attenuation was considered as i.i.d. lognormal RVs.

In a well-known i.i.d. shadowing model, multiple antennas, as in a CAS, sited in the same locale are assumed to experience the same shadowing attenuation; on the other hand, multiple antennas, as in a DAS, are distributed among multiple widely separated radio ports and experience mutually independent shadowing for each link between a user–port pair. This independence yields an improvement in the channel ergodic capacity. In particular, there have been several works that verified the improvement in the outage capacity via this classical i.i.d. shadowing model [6–8, 54, 55].

Unfortunately, the spatial correlations consisting of the shadowing correlation and multi-path fading correlation exist widely in realistic wireless communication systems; therefore, the effects of these correlations should not be ignored if we try to make the results more authentic. More specifically, in [25], as well as in the above chapter, we devised novel upper and lower bounds for the distribution of the sum rate by employing a MU-MIMO CAS with spatial correlation. As a result, we found that there is a significant effect on the system sum rate corresponding to low values of the CDF due to the shadowing correlation and a diminishing effect on the average sum rate due to the Rayleigh fading correlation. Regrettably, the effects of these correlations on the DAS were not mentioned owing to the different propagation paths of each user–port pair, which makes the theoretical analysis of the MU-MIMO DAS mathematically difficult.

In [56], we investigated and compared the characteristics of the MU-MIMO system sum rate in a CAS and DAS under the effects of distance-dependent path loss, spatially correlated shadowing, i.i.d. Rayleigh fading, and inter-cell interference. A statistical analysis revealed that a DAS can improve the system sum rate performance compared to a CAS for the case under consideration. However, this improvement would become smaller with an increase in the shadowing correlation. Although the results in our study [56] could be applied in wireless communication systems with a realistic attenuation environment, the presented model loses generality because the fading correlation that

usually occurred when the antennas are distributed within a small interval space was not considered.

Furthermore, in the previous studies, we employed a typical peer-to-peer (P2P) system model to analyze the performances of the CAS and DAS and modeled the shadowing correlation on both the transmit and receiver sides by the same well-known exponential decay function [41]. However, the shadowing correlation on the transmit side, i.e., the BS side in the downlink transmission, should be modeled using another type of function owing to the large difference in antenna heights on the transmit and receiver sides that makes an extension of the conclusions obtained from the P2P systems to cellular wireless communication systems not straightforward.

To the best of the author's knowledge, there have been few previous studies on the comparison of the channel capacity for a cellular CAS and DAS with spatial correlation and inter-cell interference. On the basis of this fact, we have attempted to bridge this gap by analytically investigating these sum rates by employing a more detailed spatial correlation model for the DAS with MU-MIMO transmissions.

3.2 System Model and Sum Rate Expression

We consider the multi-cell CAS and DAS downlink system model in Fig. 4.1 in which the radius of each hexagonal cell is R_{cell} . The system consists of J cells (indexed with $j \in \{0, \dots, J-1\}$), and there are N_t (indexed with $n \in \{1, \dots, N_t\}$) centralized or distributed antennas in cell j . We define the generalized CAS as the one that has centralized antennas spaced at a small distance, typically tens of wavelengths, and the generalized DAS as the one in which all of distributed antennas are spaced at an inter-antenna distance of $0.5R_{\text{cell}}$, typically tens of hundreds of meters [38]. Optical fibers are employed to transfer information and signaling between the antennas and the BS within each cell, and all signals from the same cell are jointly processed in the corresponding BS.

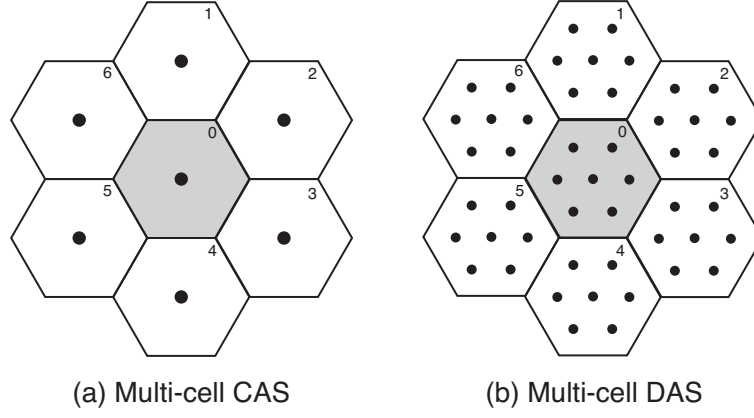


Figure 3.1: Illustration of a homogeneous cellular CAS and DAS consisting of J macro-hexagon omni-cells with identical radii R_{cell} . N_t antennas are centrally or dispersedly sited in each cell. The shaded and blank areas represent the interested and interference cells, respectively. $J = 7$, $N_t = 7$.

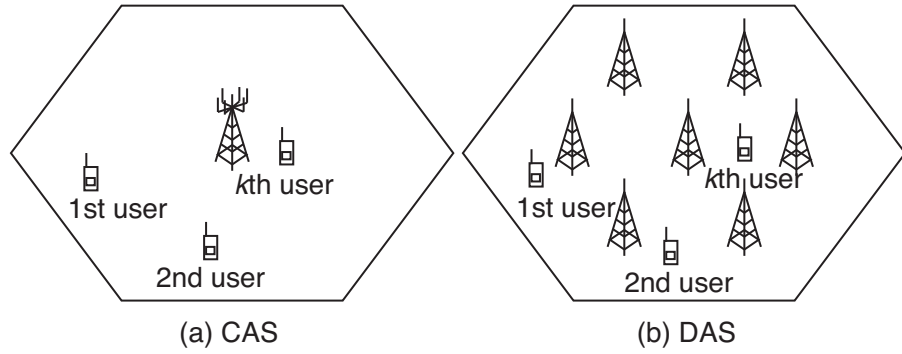


Figure 3.2: Illustration of interested cell 0 with centralized and distributed antenna schemes. K users, each equipped with one antenna, communicate with the BS and are uniformly distributed in the interested cell.

In interested cell 0, as shown in Fig. 3.2, there are K (indexed with $k \in \{1, \dots, K\}$) users, each equipped with one antenna, that communicate with the BS using a carrier frequency f . In general, we assume that the users within interested cell 0 are randomly deployed in

the corresponding coverage area with an independent uniform distribution. Note that we should use the same configuration on the user side to accurately compare the differences in the sum rate in both the CAS and DAS because the primary difference between a CAS and a DAS is that the BS locates antennas centrally or dispersely. Therefore, in our model, we assumed that each user is equipped with one antenna to reduce the system complexity.

Hereafter, we use ZF precoding to cancel the inter-user interference. ZF precoding is the well-known linear precoding scheme that can be achieved, in which the BS transmits signals in a way to isolate the users' transmissions and thus decouples the multi-user channel into multiple independent sub-channels, i.e., it nulls multi-user interference signals [57]. This scheme has been studied extensively in multi-user systems. In this study, we combine the ZF precoding technique with our system and then concentrate on investigating the system sum rate in interested cell 0 considering the effects of inter-cell interference and spatial correlation.

Through the composite fading channels (i.e., with both Rayleigh fading and shadowing), the received signal vector of K users in interested cell 0 is expressed as

$$\mathbf{y} = \tilde{\mathbf{H}}\mathbf{W}\mathbf{P}^{\frac{1}{2}}\mathbf{x} + \mathbf{n}, \quad (3.1)$$

where the $K \times N_t$ combined matrix $\tilde{\mathbf{H}}$ consists of each channel attenuation matrix $\mathbf{H}_j \in \mathbb{C}^{K \times N_t} \forall j$, $\mathbf{H}_j = (\mathbf{h}_{j1}^T, \dots, \mathbf{h}_{jK}^T)^T$, and $\mathbf{h}_{jk} \in \mathbb{C}^{1 \times N_t} \forall j, k$. $\mathbf{W} = (\mathbf{w}_1, \dots, \mathbf{w}_K)$ is the precoding weight matrix with the element $\mathbf{w}_k \in \mathbb{C}^{N_t \times 1}$. $\mathbf{P} = \text{diag}\{p_1, \dots, p_K\}$ is the transmit power scaling factor matrix. $\mathbf{x} \in \mathbb{C}^{K \times 1}$ is the vector of the transmitted symbols that are drawn from a zero-mean Gaussian codebook with a unit average power, i.e., the entries of $\mathbf{x}$ are modeled as $\mathcal{CN}(0, 1)$ RVs. The block length of the codebook is sufficiently long so that it encounters all possible channel realizations for ergodicity [37, 58]. The

complex noise term $\mathbf{n} \in \mathbb{C}^{K \times 1}$ has zero mean with $\mathbb{E}\{\mathbf{n}\mathbf{n}^H\} = N_0 \mathbf{I}_K$, where N_0 is the noise power.

Using the combined channel method that was proposed in [59] and applied in [56], the matrix $\tilde{\mathbf{H}}$ can be considered as a combination of interference channel matrices expressed as

$$\tilde{\mathbf{H}} = \mathbf{Q}^{-\frac{1}{2}} \mathbf{H}_0, \quad (3.2)$$

where $\mathbf{Q} = \text{diag}\{q_k\}_{k=1}^K$, and q_k takes the following form:

$$q_k = 1 + \sum_{j=1}^{J-1} \frac{\rho_j}{N_t} \mathbf{h}_{jk} \mathbf{h}_{jk}^H, \quad (3.3)$$

where $\rho_j = P_{t,j}/N_0$ is the transmit signal-to-noise ratio (SNR) in cell j , and $P_{t,j}$ is the total transmit power from cell j . In fact, the main idea of the combined channel method described in [59] is changing the expression form of the mutual information (eq. 4 in [59]) by invoking the fact that $\det(\mathbf{I} + \mathbf{A}\mathbf{B}) = \det(\mathbf{I} + \mathbf{B}\mathbf{A})$ for the square matrices $\mathbf{A}$ and $\mathbf{B}$. On the basis of the results in [59], the authors suggest that we can directly consider the combined channel matrix as a product of the matrix $\mathbf{Q}^{-\frac{1}{2}}$ that includes the information about the interference signals and the matrix $\mathbf{H}_0$ that includes the information about the desired signals. Note that we assumed the inter-cell interference at each receive antenna consists of mutually independent complex Gaussian RVs to obtain (3.3) [5, 60], and the transmit power in each interference cell is uniformly allocated. In fact, a more generalized method to analyze the characteristics of the system sum rate for multi-cell CASs and DASs with the case under consideration is by solving the sum rate maximization (SRM) problem via an efficient algorithm, e.g., the one proposed in [19]. In this study, the inter-cell interference model can be simplified to (3.3) with the assumption that we mentioned before because we mainly focus on the behavior of the sum rate changes in interested cell 0 [56].

Assuming that the channel-state information at the transmitter (CSIT) side is available, the average transmit power in interested cell $P_{t,0}$ is distributed uniformly among all users. According to [25], the instantaneous received SNR at the k th user with the ZF precoding weight matrix $\mathbf{W} = \tilde{\mathbf{H}}^\dagger$ [16] is equal to

$$\gamma_k = \frac{\rho_0}{K \left[\left(\tilde{\mathbf{H}} \tilde{\mathbf{H}}^H \right)^{-1} \right]_{kk}}. \quad (3.4)$$

By taking the expectation over the instantaneous received SNR with respect to the combined channel matrix $\tilde{\mathbf{H}}$, we have the ergodic system sum rate that is essentially the sum of the throughputs contributed from all users:

$$C = \mathbb{E} \left\{ \sum_{k=1}^K \log_2 (1 + \gamma_k) \right\}, \quad (3.5)$$

where C is taken over all channel realizations. The main challenge is investigating the characteristics of the CAS and DAS by (3.5) to obtain the sum rate C that depends on different user patterns. Certainly, the effects of spatially correlated shadowing, correlated fading, and inter-cell interference are considered.

3.3 Channel Model

The channel attenuation matrix $\mathbf{H}_j$ is not only affected by Rayleigh fading but also by shadowing and path loss. For our scenario, $\mathbf{H}_j$ can be generated from the following expression:

$$\mathbf{H}_j = \mathbf{D}_j^{\frac{1}{2} \circ} \circ \mathbf{S}_{\text{cor},j}^{\frac{1}{2} \circ} \circ \mathbf{R}_{\text{cor},j}, \quad (3.6)$$

where the entries of the matrix $\mathbf{D}_j \in \mathbb{R}^{K \times N_t}$ represent the path loss. Thus, $\mathbf{D}_j = (\mathbf{d}_{j1}^T, \dots, \mathbf{d}_{jK}^T)^T$, where the vector $\mathbf{d}_{jk} \in \mathbb{R}^{1 \times N_t}$ has N_t elements, and the n th element can be expressed as $[\mathbf{d}_{jk}]_n = d_{jkn}^{-\zeta}$, where d_{jkn} denotes the distance between the k th user and the n th antenna at the BS site in cell j , and ζ is the path loss exponent.

3.3.1 Spatially Correlated Shadowing

The entries of the matrix $\mathbf{S}_{\text{cor},j} \in \mathbb{R}^{K \times N_t}$ represent the spatially correlated shadowing; thus, $\mathbf{S}_{\text{cor},j} = (\mathbf{s}_{j1}^T, \dots, \mathbf{s}_{jK}^T)^T$ with the identical auto-correlation matrix $\boldsymbol{\Theta}_{\text{S},r} \in \mathbb{R}^{K \times K}$ among every row, where the vector $\mathbf{s}_{jk} \in \mathbb{R}^{1 \times N_t}$ has N_t elements, and the n th element can be expressed as $[\mathbf{s}_{jk}]_n = 10^{0.1s_{jkn}}$, where $s_{jkn} \in \mathcal{N}(\mu_{s_{jkn}}, \sigma_{s_{jkn}}^2)$ is a Gaussian RV. In this study, we consider a homogeneous¹ cellular system; therefore, the shadowing RVs for each propagation follow an identical distribution, i.e., $s_{jkn} \in \mathcal{N}(\mu, \sigma^2) \forall j, k, n$. From the previous experiments reported in [41, 61], it was found that the shadowing auto-correlation (SAC), i.e., the correlation between the RVs s_{jkn} and $s_{jk'n}$, can be modeled via an exponential decay function; therefore, the (l, m) th element of matrix $\boldsymbol{\Theta}_{\text{S},r}$ can be obtained from

$$[\boldsymbol{\Theta}_{\text{S},r}]_{lm} = \exp\left(-\frac{L_{lm}}{d_{\text{cor},r}} \ln 2\right), \quad (3.7)$$

where L_{lm} denotes distance between the l th user and the m th user, and $d_{\text{cor},r}$ is the shadowing correlation distance on the user's side. The terminology "shadowing correlation distance" was used in [41], depends on the environment, and corresponds to the distance at which the correlation drops to 0.5.

The spatial correlation between the RVs s_{jkn} and $s_{j'kn'}$ can be modeled using a suitable shadowing cross-correlation (SCC) model², which should incorporate two key variables, i.e., the angle-of-arrival (AOA) difference that represents the angle between the two paths from different BSs to the user and the relativity of the two path lengths [62, 63]. When the AOA difference is small, the two path profiles share many common elements and are expected to have high correlation. On the other hand, the longer path length

¹The terminology "homogeneous" usually means describing a material or system that has the same properties at every point in space; in other words, uniform without irregularities.

²The terminology "shadowing cross-correlation" means that the shadowing correlation occurs at a given time instant when the links between a user and two different BSs are considered.

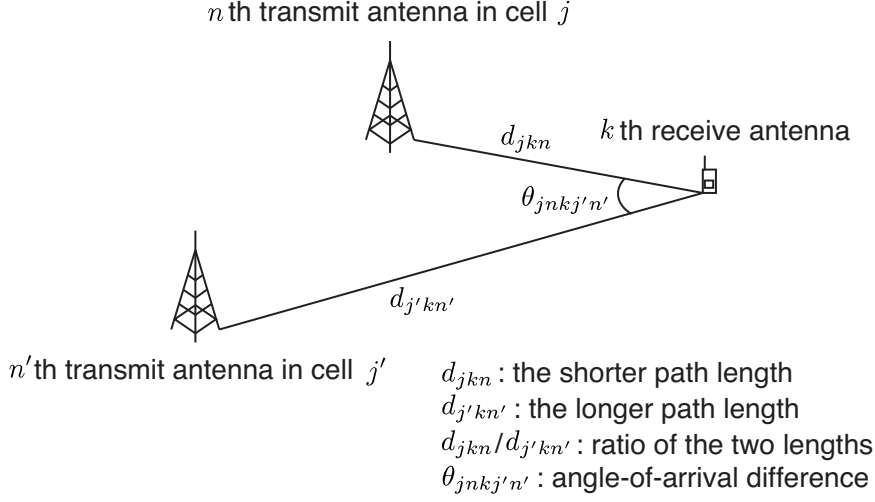


Figure 3.3: An explanation of the shadowing cross-correlation model.

incorporates more elements that are not common to the shorter path as it increases; therefore, the correlation decreases.

The verified SCC model in [62, 64] is taken into account in this study. In Fig. 3.3, the SCC coefficient for two antennas at the BS sited in cell j and the BS sited in cell j' with distances d_{jkn} and $d_{j'kn'}$ ($d_{jkn} < d_{j'kn'}$) from the user k and the AOA difference of $\theta_{jnkj'n'}$ in this model can be expressed as

$$\epsilon_{jnkj'n'} = \begin{cases} \sqrt{\frac{d_{jkn}}{d_{j'kn'}}} & 0 \leq \theta_{jnkj'n'} < \theta_{\text{cor},t} \\ \left(\frac{\theta_{\text{cor},t}}{\theta_{jnkj'n'}}\right)^\kappa \sqrt{\frac{d_{jkn}}{d_{j'kn'}}} & \theta_{\text{cor},t} \leq \theta_{jnkj'n'} \leq \pi, \end{cases} \quad (3.8)$$

where κ is referred to as a parameter determined in practice by the size and height of the terrain and the height of the BS [64]. $\theta_{\text{cor},t}$ corresponds to the threshold angle depending upon the shadowing correlation distance $d_{\text{cor},t}$ on the BS side and can be defined as

$$\theta_{\text{cor},t} = 2 \tan^{-1} \left(\frac{d_{\text{cor},t}}{2d_{jkn}} \right). \quad (3.9)$$

On the basis of these descriptions of the shadowing auto-correlation and cross-correlation,

we first define the following matrix to generate the matrices $\mathbf{S}_{\text{cor},j}$:

$$\begin{aligned}\mathbf{S}_{\text{cor}} &= \left(\text{vec} \left\{ \mathbf{S}_{\text{cor},j}^T \right\}_{j=0}^{J-1} \right)^T \\ &= (\mathbf{S}_{\text{cor},0}, \dots, \mathbf{S}_{\text{cor},J-1})\end{aligned}\quad (3.10)$$

that consists of $\mathbf{S}_{\text{cor},j}$ in decibels. Then, according to [65], the matrix $\mathbf{S}_{\text{cor}}$ can be obtained from

$$\mathbf{S}_{\text{cor}} = \mathbf{\Theta}_{\text{S},\text{r}}^{\frac{1}{2}} \cdot \text{diag} \{ \mathbf{s}_{\text{i.i.d.},k} \}_{k=1}^K \cdot \text{vec} \left\{ \mathbf{\Theta}_{\text{S},\text{t},k}^{\frac{1}{2}} \right\}_{k=1}^K, \quad (3.11)$$

where $\mathbf{s}_{\text{i.i.d.},k} \in \mathbb{R}^{1 \times JN_t} \forall k$ has JN_t mutually independent Gaussian RVs whose mean is μ , and whose variance is σ^2 . The matrix $\mathbf{\Theta}_{\text{S},\text{t},k} \in \mathbb{R}^{JN_t \times JN_t} \forall k$ represents a joint SCC matrix for user k . The (a,b) th element of $\mathbf{\Theta}_{\text{S},\text{t},k}$ with $a = jN_t + n$ and $b = j'N_t + n'$ is the cross-correlation coefficient between the n th antenna in cell j and the n' th antenna in cell j' that can be calculated by (3.8).

3.3.2 Correlated Rayleigh Fading

The entries of the matrix $\mathbf{R}_{\text{cor},j} \in \mathbb{C}^{K \times N_t}$ represent the correlated Rayleigh fading. On the basis of the same methods for generating the shadowing attenuation matrices described in 3.3.1, $\mathbf{R}_{\text{cor},j}$ can be generated by defining the matrix $\mathbf{R}_{\text{cor}} = (\mathbf{R}_{\text{cor},0}, \dots, \mathbf{R}_{\text{cor},J-1})$, and $\mathbf{R}_{\text{cor}}$ can be obtained from the following expression:

$$\mathbf{R}_{\text{cor}} = \mathbf{\Theta}_{\text{R},\text{r}}^{\frac{1}{2}} \cdot \text{diag} \{ \mathbf{r}_{\text{i.i.d.},k} \}_{k=1}^K \cdot \text{vec} \left\{ \mathbf{\Theta}_{\text{R},\text{t},k}^{\frac{1}{2}} \right\}_{k=1}^K, \quad (3.12)$$

where the entries of $\mathbf{r}_{\text{i.i.d.},k} \in \mathbb{C}^{1 \times JN_t} \forall k$ are modeled as i.i.d. $\mathcal{CN}(0,1)$ RVs. $\mathbf{\Theta}_{\text{R},\text{r}}$ and $\mathbf{\Theta}_{\text{R},\text{t},k}$ are Rayleigh fading correlation matrices on the user and BS sides. From previous studies, the fading correlation can be modeled via two well-known correlation models: exponential correlation with

$$\mathbf{\Theta}_{\text{R},\text{r}} = \mathbf{I}_K \quad (3.13)$$

and

$$[\Theta_{\text{R},\text{t},k}]_{ab} = \begin{cases} \beta_j^{|n-n'|} & j = j' \\ 0 & j \neq j', \end{cases} \quad (3.14)$$

where $a = jN_{\text{t}} + n$, $b = j'N_{\text{t}} + n'$, and $\beta_j \in [0, 1) \forall j$ [52]; and a recent model proposed in [39]. The advantage of using the latter model is that the effects of the essential elements that affect the fading correlation, e.g., the AOA and inter-antenna distance, can be clearly illustrated. However, more time should be used for the simulation owing to the complexity. Therefore, we employ the first exponential correlation model to generate the correlated fading considering that our main goal is to analyze the shadowing attenuation. It must be noted that the fading correlation matrix on the user side $\Theta_{\text{R},\text{r}}$ can be considered as an identity matrix $\mathbf{I}_K$ because K users are geographically separated in general.

3.4 Simulation Parameters and Results

3.4.1 Simulation Parameters

In this simulation, a homogeneous cellular area consists of seven macro-hexagon omniscells with identical radii $R_{\text{cell}} = 100$ m. The seven users are assumed to be uniformly distributed in interested cell 0. The parameter κ in the SCC model (3.8) is set to 0.3, and the BS antenna height is set to 15 m [64]. The carrier frequency $f = 2 \times 10^9$ Hz. On the basis of previous experimental reports in [64] and [41], the shadowing correlation distance on the transmit and receiver sides can be set to 20 m at a carrier frequency of 2×10^9 Hz. In fact, some results in [56] show that the effect of the shadowing correlation is mainly dominated by the ratio of the correlation distance to the cell radius, i.e., the normalized shadowing correlation distance. Therefore, we denote ξ as the normalized shadowing correlation distance and use subscripts t and r to identify the state of the transmit and receiver sides, respectively. It is worth mentioning that, according to (3.7),

the correlation coefficient in the SAC model is a function of $L_{lm}/d_{\text{cor},r}$; therefore, we can reasonably consider that $\xi_r = d_{\text{cor},r}/R_{\text{cell}}$ is the dominant parameter. This is because $d_{\text{cor},r}$ should also be normalized by the cell radius to ensure that the solution of (3.7) remains the same if the distance between two positions, e.g., L_{lm} , is normalized by the cell radius R_{cell} and then used in the considered system. Moreover, once the locations of the BS and user are fixed, the correlation coefficients then depend only on ξ_r . These are the main reasons that the authors in [56] used ξ_r to analyze the performance of the sum rate in their system. The same analysis and conclusion for $\xi_t = d_{\text{cor},t}/R_{\text{cell}}$ in our SCC model is also applicable because the correlation coefficient in (3.8) is a function of the threshold angle $\theta_{\text{cor},t}$, which depends on the ratio of $d_{\text{cor},t}$ to d_{jkn} . Some results about the effects of ξ_t and ξ_r will be shown in next subsection. The detailed simulation parameters are listed in Table 4.1.

3.4.2 Simulation Results

In this subsection, we compare the characteristics of MU-MIMO in a CAS and DAS via the calculation of the CDF and expectation of the system sum rate via a Monte Carlo numerical computation method. The CDF is necessary because we can investigate the average and the sum rate corresponding to arbitrary values of the CDF. The results illustrate the effects of spatial correlation and inter-cell interference on the channel average sum rate and its distribution. In particular, the effects of the shadowing correlation on the BS and user sides are also demonstrated.

Figures 3.4 and 3.5 show comparisons of the CDF and average of the system sum rate in a CAS and DAS under the effects of spatial correlation and inter-cell interference employing the i.i.d. and spatially correlated shadowing models, respectively. To easily analyze the characteristics of the system sum rate with interference, we assume that all

Table 3.1: Simulation parameters.

Parameters	Values
Radius of hexagon cell	$R_{\text{cell}} = 100 \text{ m}$
Carrier frequency	$f = 2 \times 10^9 \text{ Hz}$
BS antenna height	15 m
Number of interference cells	$J = 7$
Number of user samples	1000
Number of transmit antennas	$N_t = 7$
Number of users	$K = 7$
Path loss exponent	$\zeta = 4$
Shadowing model	spatially correlated
Shadowing mean value	$\mu = 0 \text{ dB}$ [66]
Shadowing standard deviation	$\sigma = 8.0 \text{ dB}$ [66]
Correlation distance on user side	$d_{\text{cor},r} = 20 \text{ m}$ [41]
Correlation distance on BS side	$d_{\text{cor},t} = 20 \text{ m}$ [64]
SCC parameter	$\kappa = 0.3$ [64]
Number of shadowing samples	200
Rayleigh fading model	spatially correlated
Fading correlation for CAS	$\beta_j = 0.1 \forall j$
Fading correlation for DAS	i.i.d.
Fading correlation for MU	i.i.d.
Number of fading samples	1000

of the BSs sited at interference cells have the same total transmit power $P_{t,j}$ for $j \in \{1, \dots, J-1\}$ and set their normalized transmit SNR equal to 0 and 10 dB, respectively.

From these figures, we find that the CDF and average of the system sum rate in a DAS are improved significantly compared to those in a CAS, even under the effects of spatially correlated shadowing and inter-cell interference. This improvement can be considered as the contributions from reducing the access distance and shadowing correlation due to the separation of the antennas and users that occurred. However, the gap between the CAS and the DAS becomes smaller with increasing interference power. In fact, on the basis of the result we found before, the DAS can enhance the desired signals by reducing the access distance and shadowing correlation, which also means it enhances the interference signals for the same reasons. Consequentially, the improvement in the system sum rate

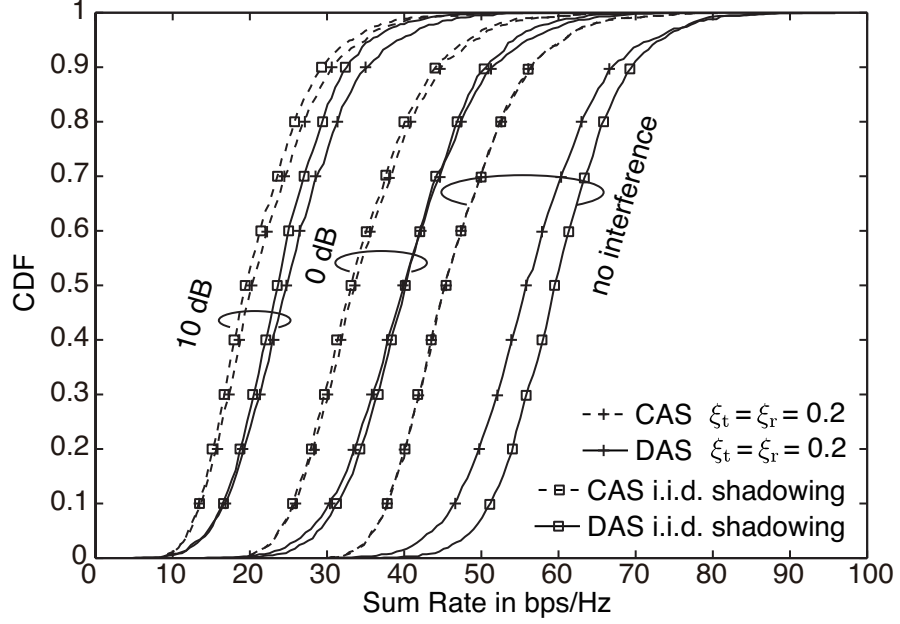


Figure 3.4: Comparison of the CDF of the system sum rate in a CAS and DAS under the effects of spatial correlation and inter-cell interference for SNR = 20 dB.

for the DAS decreases gradually with increasing inter-cell interference.

Moreover, these figures notably show that the sum rate decreases slowly with increasing interference power compared to the i.i.d. shadowing model in the case of correlated shadowing. The reason can be elucidated via analyzing (3.2) and (3.3) and noting the diminishing effects of the shadowing correlation on the inter-cell interference matrix $\mathbf{Q}$, which increases the eigenvalues of the matrix $\mathbf{Q}^{-\frac{1}{2}}$ that directly affect the speed of sum rate changes.

For illustrating the effect of shadowing correlation, in Fig. 3.6, we demonstrate some relationships between the average system sum rate and the normalized shadowing correlation distance (ξ_t and ξ_r) in a CAS and DAS without inter-cell interference. Note that, in this simulation, ξ_t and ξ_r can range from 0 to ∞ by changing shadowing correlation distance $d_{\text{cor},t}$ and $d_{\text{cor},r}$, respectively, and the shadowing RVs become independent

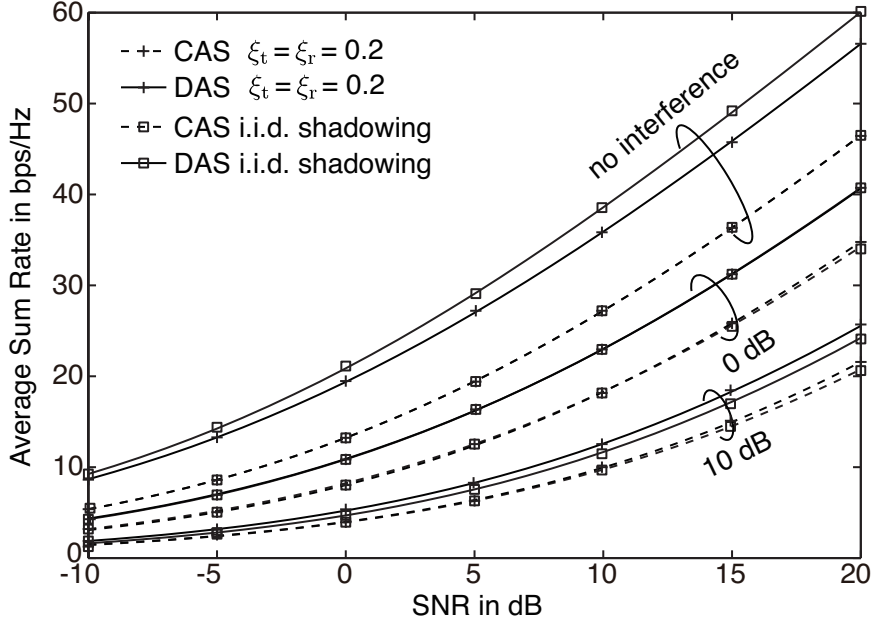


Figure 3.5: Comparison of the average system sum rate in a CAS and DAS under the effects of spatial correlation and inter-cell interference.

when equal to 0; on the other hand, the shadowing RVs become completely correlated when equal to ∞ . In our correlated shadowing model, we assume $\xi_t, \xi_r \in (0, 1]$, i.e., $d_{\text{cor},t}, d_{\text{cor},r} \in (0, R_{\text{cell}}]$, to be practically sufficient. It is worth mentioning that the value of the shadowing correlation distance depends on the parameters of a realistic environment. Therefore, the ranges of $d_{\text{cor},t}$ and $d_{\text{cor},r}$ in a realistic environment should be obtained by a large number of experiments. However, we consider that both ranges of $d_{\text{cor},t}$ and $d_{\text{cor},r}$ follow our assumption to generally analyze the effect of the shadowing correlation distance on the sum rate changes.

Figure 3.6 shows that the average system sum rate in a DAS considerably decreases because of the shadowing correlation. In contrast, there appears to be no change in the CAS. Actually, in [25], we devised some novel upper and lower bounds for the distribution of the system sum rate in an MU-MIMO CAS and concluded that the shadowing

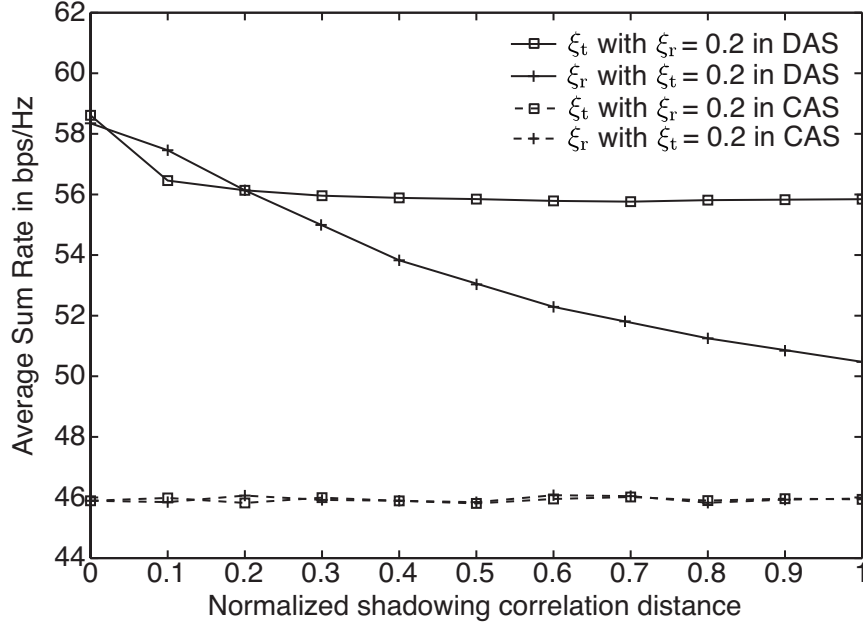


Figure 3.6: The effects of SAC and SCC on the average system sum rate in a CAS and DAS without inter-cell interference: SNR = 20 dB.

correlation equally changes the distribution by increasing the sum rate corresponding to high values of the CDF and decreasing the sum rate corresponding to low values of the CDF; however, the shadowing correlation does not have any effect on the average sum rate. The lower two curves in Fig.3.6 also validate this conclusion. Furthermore, there are two different types of sum rate changes in a DAS. The reason is that the shadowing correlation distance on the BS side $d_{\text{cor},t}$ only affects the threshold angle $\theta_{\text{cor},t}$ in our SCC model that can be formulated by (3.8) and (3.9). Consequentially, the change in the system sum rate would tend to a fixed value at a particular point depending upon the threshold angle. On the other hand, the parameter $d_{\text{cor},r}$ directly affects the SAC coefficients; therefore, the average sum rate would decrease with increasing ξ_r .

3.5 Summary

In this study, we investigated the characteristics of the MU-MIMO system sum rate in a CAS and DAS under the effects of distance-dependent path loss, spatially correlated shadowing, correlated Rayleigh fading, and inter-cell interference via a Monte Carlo numerical computation method. To generate the channel attenuation matrices, we first introduced two different types of functions to model the SAC and SCC and a typical exponential decay function to model the Rayleigh fading correlation. Thus, we successfully obtained the CDF of the system sum rate and its average. Computer simulation results indicated that a DAS offers higher system sum rate performance than a comparable CAS in the case under consideration. However, this improvement decreased with increasing interference power. Furthermore, compared to the i.i.d. shadowing model, the decrease in the system sum rate due to the increase in the interference power became slow under the effect of shadowing correlation.

Chapter 4

Exploration on Clustering Scheme for Large-scale MU-MIMO DASs

In Chapter 2, we theoretically analyzed the effects of composite fading channel with correlation on the performance of sum rate in MU-MIMO CASs, and then an important conclusion for CASs has been verified that is the spatial correlation significantly affects the system sum rate corresponding to low values of the CDF due to the shadowing correlation and a diminishing effect on the average sum rate due to the Rayleigh fading correlation. However, the average sum rate does not seem affected by the shadowing correlation. In Chapter 3, we clarified that the MU-MIMO DASs can significantly improve the performance of sum rate including the average value under the effects of correlated composite fading and inter-cell interference. As the final step to achieve a large-scale DAS with MU-MIMO transmission for the improvement of system sum rate, in this chapter, we explore and propose a simple scheme to dynamically format multiple clusters at a reduced computational cost.

The introduction of this chapter is described in Sect. 4.1. In Sect. 4.2, we define the system model and describe the mathematical background for the channel attenuation model in which the spatial correlation is considered. In the same section, we also derive several expressions that can be used to calculate the sum rate and formulate the problems.

In Sect. 4.3, a simple dynamic CS targeting to maximize the system sum rate is presented. In Sect. 4.4, we compare the characteristics of the proposed and other dynamic CSs using a simulation method. In Sect. 4.5, we analyze the complexity of the described CSs. Concluding remarks are presented in Sect. 4.6.

Notation: We use upper- and lower-case boldface to denote matrices and vectors, respectively. The $n \times n$ identity matrix is denoted by $\mathbf{I}_n$, and the (i, j) th element of a matrix is denoted by $[\mathbf{A}]_{ij}$. The expectation value operator is written as $\mathbb{E}\{\cdot\}$. We use calligraphic font $\mathcal{A}$ to denote an integer collection and $|\mathcal{A}|$ to indicate the size of this collection. The symbols $(\cdot)^\dagger$ and $(\cdot)^H$ represent the pseudo-inverse and Hermitian transpose of a matrix, respectively, and “ $\circ$ ” denotes the Hadamard product, i.e., $[\mathbf{A} \circ \mathbf{B}]_{ij} = [\mathbf{A}]_{ij}[\mathbf{B}]_{ij}$. To distinguish the square root of a matrix $\mathbf{A}^{\frac{1}{2}}$, we use $\mathbf{A}^{\frac{1}{2}\circ}$ to denote the positive square root of $\mathbf{A}$ with respect to the Hadamard product such that $\mathbf{A}^{\frac{1}{2}\circ} \circ \mathbf{A}^{\frac{1}{2}\circ} = \mathbf{A}$. The symbol $\sim \mathcal{CN}(M, \Sigma)$ denotes a complex Gaussian random variable with mean M and covariance Σ , and $\text{diag}\{\cdot\}$ and $\text{vec}\{\cdot\}$ represent a diagonal matrix and a column vector, respectively.

4.1 Introduction

4.1.1 Review of Previous Works

In this study, we focus on solving the clustering problem in large-scale MU-MIMO DASs with multiple carriers. In fact, clustering using orthogonal carriers in large-scale MU-MIMO DASs can be considered arbitrary resource allocation among users, BSs, and carriers. Initially, a static CS, which depends on the locations of users and BSs, has been proven to improve the efficiency of the frequency spectrum [9, 10]. In this scheme, the geographical area is divided into a fixed cluster structure, with each cluster having a unique, fixed carrier. The limitation of this CS, however, is a lack of diversity with respect to changing channel conditions, because the clusters are static.

To improve the above-described CS, an advanced location-based adaptive (LOCA) CS was considered in [11]. In this scheme, the carrier associated with its cluster may be changed for every channel realization, and can be dynamically allocated to the appropriate cluster. Although this scheme can achieve a frequency diversity gain, its limitation is also a lack of space diversity because of the fixed cluster structure.

On the other hand, an optimal allocation among users, BSs, and carriers for dynamic clustering is obtained by exhaustively searching all possible cluster formations¹ for each channel realization and selecting the one that yields the maximum capacity. However, such an method is not practically feasible because the computational cost increases exponentially as the number of users and BSs increases.

To reduce the computational cost, several antenna selection methods [67, 68] have been studied over the past decade. In particular, norm-based method [67] remain a popular method because of their simplicity. The main drawback, however, is that they may lead to a much lower capacity when some elements of the channel matrix are close to being linearly dependent. In addition, these methods cannot be directly used in our study, mainly because they only consider point-to-point MIMO system with a single carrier (SC).

To format clusters, a clustering approach [69] for multi-cell cooperative processing network with SC have been studied. Although this approach can dynamically select and group BSs to format clusters with a less capacity loss, the users served by which BSs have been already assigned before running their algorithm, and the number of cooperating BSs in each cluster, i.e., the cluster sizes are also fixed. These facts restrict its use and make an extension of the algorithm to our multi-carrier based systems not straightforward.

In recent years, several methods that concentrated on carrier allocation have been studied, such as that in [23, 24]. Further, a highest-complexity method, i.e., exhaustive

¹The terminology “cluster formation” is defined as a formatted cluster in which the cluster size, allocated users and BSs, and carrier assigned to the cluster are decided.

search over BSs and carriers, for dynamic clustering have been used in a recent study [21]. Additionally, an iterative algorithm has been proposed to maximize the capacity [24]. In these studies, although the carriers can be selected and assigned to users or BSs so that multiple clusters can be formatted, the system is still considered to use multiple point-to-point transmissions, because the MU-MIMO transmission has not been performed in each cluster. Moreover, due to the combinations of users and BSs in [21] or the combinations of BSs and carriers in [24] are not considered, the concept of dynamic clustering over the dimensions of transmitter, receiver, and carrier is not involved in these studies.

4.1.2 Our Study

Taking a general survey over the past decade, in particular, the past few years, we found that there have been few studies on clustering for large-scale MU-MIMO DASs with multi-carrier. In order to bridge this gap and contribute to our future works, in the present study, we explore and propose a simple scheme to dynamically format multiple clusters at a reduced computational cost for the considered system. In contrast to the previous studies, in each cluster, MU-MIMO transmission is performed using a carrier selected from among several alternative carriers. Moreover, the proposal can further achieve dynamic allocation for all users (receivers), BSs (transmitters), and carriers at the same time so as to maximize the system sum rate. Our method starts with several empty sets over all carriers and then adds one user and one BS per step to the set corresponding to the carrier used. In each step, the user and BS with the highest contribution to the sum rate from among all clusters is added to the appropriate set. We expand this method and use it to solve the existing problems.

Furthermore, to the best of the author's knowledge, there have been also few studies on clustering with spatially correlated channel attenuation, such as [23], and only a small number of studies considering simplified correlation models, such as that in [24]. The

shadowing in these kinds of studies, for instance, was considered to be either independent or completely correlated in the same cluster, and either independent or correlated with a constant value between the different clusters. As these methods are simple, they cannot accurately simulate realistic environments, because the correlation distance, a previously unmentioned parameter, plays an important role in channel models [4]. We have attempted to make a connection by introducing suitable correlated models described in Chapter 3.

Motivated by the previous discussion, to establish a large-scale MU-MIMO DAS, in the present study, we first focus on a single cell DAS with spatial correlation and then propose a CS using multiple orthogonal carriers to achieve MU-MIMO transmissions in each cluster. We also investigate the performance of proposal by observing the CDF and the expectation value of the system sum rate. Here, we refer to the calculation method for sum rate proposed in [25] with the assumption that power is uniformly allocated among all of the active users, so as to reduce the computational cost. Furthermore, we compare the characteristics of the proposal to those of other CSs, such as exhaustive search, traditional LOCA CS [11], and improved norm-based CS, which are described below.

4.2 System Model and Problem Formulation

4.2.1 System Model

We consider a single cell MU-MIMO DAS using multiple orthogonal carriers. The system consists of $|\mathcal{B}|$ ($\mathcal{B} := \{1, \dots, B\}$ indexed with b) distributed BSs, each equipped with a single antenna connected to a sub-processor (SP), and $|\mathcal{U}|$ ($\mathcal{U} := \{1, \dots, U\}$ indexed with u) users are distributed in a coverage area with length R_{len} and width R_{wid} . We define the generalized DAS as the DAS in which all BSs are symmetrically distributed in the area [38], and the shortest distance between two adjacent BSs is defined as R_{ant} . Optical

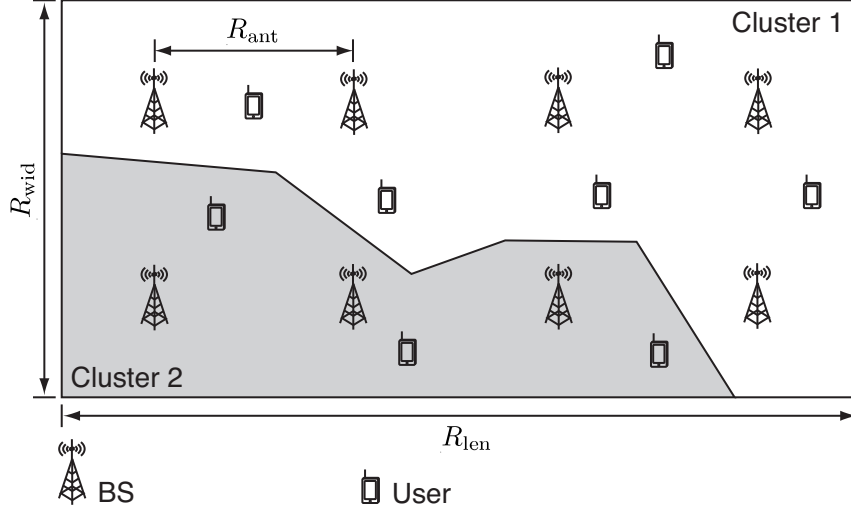


Figure 4.1: Illustration of a single cell MU-MIMO DAS using multiple orthogonal carriers in which $|\mathcal{B}| = 8$ cooperating BSs, each equipped with a single antenna, simultaneously serve $|\mathcal{U}| = 8$ users. $|\mathcal{J}| = 2$ clusters are represented by the shaded or blank areas.

fibers are used to transfer information and for signaling between the BSs and the SP. All signals are jointly processed in this SP.

A downlink scenario in which $|\mathcal{J}|$ ($\mathcal{J} := \{1, \dots, J\}$ indexed with j) mutually orthogonal carriers can be allocated to $|\mathcal{J}|$ clusters is considered. In cluster j , $|\mathcal{B}_j| \leq |\mathcal{B}|$ ($\mathcal{B}_j \subset \mathcal{B}$) BSs cooperate under a linear precoding framework and simultaneously communicate with $|\mathcal{U}_j| \leq |\mathcal{U}|$ ($\mathcal{U}_j \subset \mathcal{U}$) users using carrier j . To simplify our system model and achieve a total number of spatial degrees of freedom (DOF) of $|\mathcal{B}_j|$ in cluster j using a low-complexity MIMO linear precoding scheme, such as ZF precoding, we assume that each user is equipped with one antenna and that $|\mathcal{U}_j| = |\mathcal{B}_j| \forall j$ [37].

Note that, to further reduce the system complexity, we assume that each user or BS uses at most one carrier; however, we allow more than one user or BS to share a carrier, i.e., it follows that for each carrier j , if $b \in \mathcal{B}_j$ and $u \in \mathcal{U}_j$, $b \notin \mathcal{B}_{j'}$ and $u \notin \mathcal{U}_{j'}$ for all $j \neq j'$. Furthermore, to achieve a higher spatial multiplexing gain [70] over all clusters,

we ensure that all BSs are assigned during each clustering operation. An illustration of the system is presented in Fig. 4.1.

4.2.2 Channel Model

In the present study, composite fading channels (i.e., with both Rayleigh fading and shadowing) are considered. The channel matrix from $|\mathcal{B}|$ BSs to $|\mathcal{U}|$ users on the j th carrier can be generated by

$$\mathbf{H}(\mathcal{U}, \mathcal{B}, j) = \mathbf{D}^{\frac{1}{2}\circ}(\mathcal{U}, \mathcal{B}, j) \circ \mathbf{S}_{\text{cor}}^{\frac{1}{2}\circ}(\mathcal{U}, \mathcal{B}, j) \circ \mathbf{R}_{\text{cor}}(\mathcal{U}, \mathcal{B}, j), \quad (4.1)$$

where the entries of the matrix $\mathbf{D}(\mathcal{U}, \mathcal{B}, j) \in \mathbb{R}^{|\mathcal{U}| \times |\mathcal{B}|}$ represent the path loss. Thus, $\mathbf{D}(\mathcal{U}, \mathcal{B}, j) = (\mathbf{d}_1, \dots, \mathbf{d}_B)$ for all j , where the vector $\mathbf{d}_b \in \mathbb{R}^{|\mathcal{U}| \times 1}$ has $|\mathcal{U}|$ elements, and the u th element can be expressed as $[\mathbf{d}_b]_u = d_{ub}^{-\zeta}$, where d_{ub} denotes the distance between the u th user and the b th BS, and ζ is the path loss exponent.

The entries of the matrix $\mathbf{S}_{\text{cor}}(\mathcal{U}, \mathcal{B}, j) \in \mathbb{R}^{|\mathcal{U}| \times |\mathcal{B}|}$ represent the spatially correlated shadowing. Thus, $\mathbf{S}_{\text{cor}}(\mathcal{U}, \mathcal{B}, j) = (\mathbf{s}_{1j}, \dots, \mathbf{s}_{Bj})$ with the spatial autocorrelation matrix $\mathbf{\Theta}_{\text{S},r} \in \mathbb{R}^{|\mathcal{U}| \times |\mathcal{U}|}$ among every row, where the vector $\mathbf{s}_{bj} \in \mathbb{R}^{|\mathcal{U}| \times 1}$ has $|\mathcal{U}|$ elements, and the u th element can be expressed as $[\mathbf{s}_{bj}]_u = 10^{0.1s_{ubj}}$, where $s_{ubj} \in \mathcal{N}(\mu_{ubj}, \sigma_{ubj}^2)$ is a Gaussian random variable (RV). The relationship between $\mathbf{S}_{\text{cor}}(\mathcal{U}, \mathcal{B}, j)$ and $\mathbf{\Theta}_{\text{S},r}$ is expressed below.

Previous experiments [41,61] revealed that the shadowing autocorrelation (SAC), i.e., the correlation between the RVs s_{ubj} and $s_{u'bj}$, can be modeled via an exponential decay function. Therefore, the (u, u') th element of matrix $\mathbf{\Theta}_{\text{S},r}$ can be obtained from

$$[\mathbf{\Theta}_{\text{S},r}]_{uu'} = \exp\left(-\frac{L_{uu'}}{d_{\text{cor},r}} \ln 2\right), \quad (4.2)$$

where $L_{uu'}$ denotes the distance between the u th and the u' th users, and $d_{\text{cor},r}$ is the shadowing correlation distance on the user's side. The “shadowing correlation distance”

[41] depends on the environment and corresponds to the distance at which the correlation drops to 0.5.

The spatial correlation between the RVs s_{ubj} and $s_{ub'j}$ can be modeled using a suitable shadowing cross-correlation (SCC) model, which should incorporate two key variables, i.e., the angle-of-arrival (AOA) difference that represents the angle between the two paths from different BSs to the user and the relativity of the two path lengths [62, 63]. When the AOA difference is small, the two path profiles share many common elements and are expected to have high correlation. On the other hand, the longer path length incorporates more elements that are not common to the shorter path as it increases; therefore, the correlation decreases.

The verified SCC model in [62, 64] is taken into account in the present study. The SCC coefficient for two BSs with distances d_{ub} and $d_{ub'}$ ($d_{ub} < d_{ub'}$) from the user u and the AOA difference of $\theta_{ubb'}$ in this model can be expressed as

$$\epsilon_{ubb'} = \begin{cases} \sqrt{\frac{d_{ub}}{d_{ub'}}} & 0 \leq \theta_{ubb'} < \theta_{\text{cor},t} \\ \left(\frac{\theta_{\text{cor},t}}{\theta_{ubb'}}\right)^\kappa \sqrt{\frac{d_{ub}}{d_{ub'}}} & \theta_{\text{cor},t} \leq \theta_{ubb'} \leq \pi, \end{cases} \quad (4.3)$$

where κ is a parameter that is determined in practice by the size and height of the terrain and the height of the BS [64]. Moreover, $\theta_{\text{cor},t}$ corresponds to the threshold angle depending on the shadowing correlation distance $d_{\text{cor},t}$ on the BS side and can be defined as

$$\theta_{\text{cor},t} = 2 \tan^{-1} \left(\frac{d_{\text{cor},t}}{2d_{ub}} \right). \quad (4.4)$$

Based on these descriptions of the SAC and SCC, according to the Kronecker model [65], the matrix $\mathbf{S}_{\text{cor}}(\mathcal{U}, \mathcal{B}, j)$ can be obtained from

$$\mathbf{S}_{\text{cor}}(\mathcal{U}, \mathcal{B}, j) = \mathbf{\Theta}_{\text{S},r}^{\frac{1}{2}} \cdot \text{diag} \{ \mathbf{s}_{\text{i.i.d.},uj} \}_{u=1}^U \cdot \text{vec} \left\{ \mathbf{\Theta}_{\text{S},t,u}^{\frac{1}{2}} \right\}_{u=1}^U, \quad (4.5)$$

where $\mathbf{s}_{\text{i.i.d.},uj} \in \mathbb{R}^{1 \times |\mathcal{B}|} \forall u, j$ has $|\mathcal{B}|$ mutually i.i.d. lognormal RVs, the logarithm of which follows a normal distribution with mean μ_{ubj} and variance σ_{ubj}^2 . Moreover, the vectors $\mathbf{s}_{\text{i.i.d.},uj} \forall u$ are also mutually independent. The matrix $\mathbf{\Theta}_{\text{S,t},u} \in \mathbb{R}^{|\mathcal{B}| \times |\mathcal{B}|}$ represents an SCC matrix for user u , and the (b, b') th element of $\mathbf{\Theta}_{\text{S,t},u}$ can be calculated by

$$[\mathbf{\Theta}_{\text{S,t},u}]_{bb'} = \epsilon_{ubb'}, \quad (4.6)$$

where $\epsilon_{ubb'}$ can be obtained from (4.3).

The entries of the matrix $\mathbf{R}_{\text{cor}}(\mathcal{U}, \mathcal{B}, j) \in \mathbb{C}^{|\mathcal{U}| \times |\mathcal{B}|}$ represent the spatially correlated Rayleigh fading that can be obtained from the following expression:

$$\mathbf{R}_{\text{cor}}(\mathcal{U}, \mathcal{B}, j) = \mathbf{\Theta}_{\text{R,r}}^{\frac{1}{2}} \cdot \mathbf{R}_{\text{i.i.d.}}(\mathcal{U}, \mathcal{B}, j) \cdot \mathbf{\Theta}_{\text{R,t}}^{\frac{1}{2}}, \quad (4.7)$$

where the entries of $\mathbf{R}_{\text{i.i.d.}}(\mathcal{U}, \mathcal{B}, j) \in \mathbb{C}^{|\mathcal{U}| \times |\mathcal{B}|} \forall j$ are modeled as i.i.d. $\mathcal{CN}(0, 1)$ RVs. Moreover, $\mathbf{\Theta}_{\text{R,r}} \in \mathbb{R}^{|\mathcal{U}| \times |\mathcal{U}|}$ and $\mathbf{\Theta}_{\text{R,t}} \in \mathbb{R}^{|\mathcal{B}| \times |\mathcal{B}|}$ are Rayleigh fading correlation matrices on the user and BS sides. Based on previous studies, the fading spatial correlation can be modeled via exponential correlation with

$$[\mathbf{\Theta}_{\text{R,r}}]_{uu'} = \beta_{\text{r}}^{|u-u'|} \quad (4.8)$$

and

$$[\mathbf{\Theta}_{\text{R,t}}]_{bb'} = \beta_{\text{t}}^{|b-b'|}, \quad (4.9)$$

where $\beta_{\text{r}}, \beta_{\text{t}} \in [0, 1]$ [52].

Note that, the authors in [71] investigated the correlations of the shadowing and fading between the different frequency bands in urban environments through experimental measurements. The results indicate that there is a high correlation for the shadowing coefficients between all frequency bands. Conversely, the correlation of the fading between different frequency bands is very small.

The reasons can be considered as that, shadowing is mainly caused by blockage from obstacles, behaves in a very similar way for all frequency bands, thus causes very similar attenuation for all frequencies; on the other hand, fading is due to the constructive and destructive interferences between different multipath components. Multipath components at different frequencies have very different path lengths (in terms of wavelengths), and therefore have very different constructive and destructive interferences. Similar results and analysis can also found in [72].

Therefore, based on these considerations, and to reasonably simplify the channel model, in the present study, we assume that the shadowing attenuations between different frequencies, i.e., the matrices $\text{diag}\{\mathbf{s}_{\text{i.i.d.},uj}\}_{u=1}^U \forall j$, are completely correlated. On the other hand, the fading attenuations between different frequencies, i.e., the matrices $\mathbf{R}_{\text{i.i.d.}}(\mathcal{U}, \mathcal{B}, j) \forall j$, are mutually independent.

4.2.3 Sum Rate and Problem Formulation

In these composite fading channels, the received signal in cluster j is expressed as

$$\mathbf{y}_j = \mathbf{H}(\mathcal{U}_j, \mathcal{B}_j, j) \mathbf{W}_j \mathbf{P}_j^{\frac{1}{2}} \mathbf{x}_j + \mathbf{n}, \quad (4.10)$$

where $\mathbf{y}_j \in \mathbb{C}^{|\mathcal{U}_j| \times 1}$ is the received signal vector, and $\mathbf{x}_j \in \mathbb{C}^{|\mathcal{U}_j| \times 1}$ is the vector of the transmitted symbols that is drawn from a zero-mean Gaussian codebook with a unit average power, i.e., the entries of $\mathbf{x}_j$ are modeled as $\mathcal{CN}(0, 1)$ RVs. The block length of the codebook is sufficiently that it encounters all possible channel realizations for ergodicity [58, 73]. The complex noise term $\mathbf{n} \in \mathbb{C}^{|\mathcal{U}_j| \times 1}$ has zero mean with $\mathbb{E}\{\mathbf{n}\mathbf{n}^T\} = N_0 \mathbf{I}_{|\mathcal{U}_j|}$, where N_0 is the noise power. $\mathbf{W}_j = \{\mathbf{w}_{1j}, \dots, \mathbf{w}_{|\mathcal{U}_j|j}\}$ is the precoding weight matrix with element $\mathbf{w}_{uj} \in \mathbb{C}^{|\mathcal{B}_j| \times 1}$. The matrix $\mathbf{P}_j = \text{diag}\{p_{uj}\}_{u=1}^{|\mathcal{U}_j|} \in \mathbb{R}^{|\mathcal{U}_j| \times |\mathcal{U}_j|}$ is the transmit power scaling factor matrix.

Hereafter, we focus on the sum rate in cluster j with ZF precoding. For the simplified series, we assume that carrier $j \in \mathcal{J}$ has a bandwidth that is much smaller than the

coherence bandwidth of the channel, and that the instantaneous CSI for all carriers of all of the user-BS pairs are known to the SP. In fact, only partial CSI can be obtained over a limited rate feedback channel in realistic communication systems. Fortunately, techniques to reduce the feedback rate while maintaining good performance by, for instance, utilizing the frequency domain correlation of precoding vectors, have been proposed in many recent studies [20, 74, 75]. In the present study, because we concentrate mainly on the CS, we assume that channel estimation is prefect.

Hereafter, for the case under consideration, the ZF precoding weight matrix is expressed as $\mathbf{W}_j = \mathbf{H}(\mathcal{U}_j, \mathcal{B}_j, j)^\dagger$ [16]. Therefore, the sum rate with ZF precoding in cluster j can be calculated by

$$C_j = \sum_{u=1}^{|\mathcal{U}_j|} \log_2 \left(1 + \frac{p_{uj}}{N_0} \right) \quad (4.11)$$

with power constraint

$$\sum_{u=1}^{|\mathcal{U}_j|} p_{uj} |\mathbf{w}_{uj}|^2 = P_{t,j}, \quad (4.12)$$

where $P_{t,j}$ is the transmit power in cluster j [16]. Then, the goal of the present study is to find the best assignments for $\mathcal{U}_j$ and $\mathcal{B}_j$ for all j , such that the sum rate C of overall clusters, i.e., $C = \sum_{j=1}^J C_j$, is maximized for given total transmit power $P_t = \sum_{j=1}^J P_{t,j}$ (note that the inter-cluster interference can be ignored because of the orthogonality of the carrier). Mathematically, we can formulate the problem as

$$\begin{aligned} \hat{\mathcal{U}}_j, \hat{\mathcal{B}}_j &= \arg \max_{\substack{\mathcal{U}_j, \mathcal{B}_j \\ j \in \mathcal{J}}} C \\ &= \arg \max_{\substack{\mathcal{U}_j, \mathcal{B}_j \\ j \in \mathcal{J}}} \left(\sum_{j=1}^J \sum_{u=1}^{|\mathcal{U}_j|} \log_2 \left(1 + \frac{p_{uj}}{N_0} \right) \right), \end{aligned} \quad (4.13)$$

where the maximization is subjected to the constraint

$$P_t = \sum_{j=1}^J \sum_{u=1}^{|\mathcal{U}_j|} p_{uj} |\mathbf{w}_{uj}|^2. \quad (4.14)$$

$$C_U = \sum_{j=1}^J \sum_{u=1}^{|\mathcal{U}_j|} \log_2 \left(1 + \frac{\rho}{\sum_{i=1}^J |\mathcal{U}_i|} \left[\left(\mathbf{H}(\mathcal{U}_j, \mathcal{B}_j, j) \mathbf{H}(\mathcal{U}_j, \mathcal{B}_j, j)^H \right)^{-1} \right]_{uu}^{-1} \right) \quad (4.15)$$

It is noteworthy that the sum rate in (4.13) with power constraint (4.14) can be optimally solved using the well-known waterfilling algorithm, because of the assumption of perfect CSI. However, more time should be used for the simulation because of the complexity of calculating the Lagrange multipliers. Therefore, because our primary task is to verify the effectiveness of the dynamic CS and to maintain the simulation time at an acceptable level, we assume that the total transmit power P_t is uniformly allocated among all active users. Thereafter, we can simplify the sum rate C to C_U , which can be expressed as (4.15), where $\rho = P_t/N_0$ denotes the transmit signal-to-noise ratio (SNR) [25]. Then, the optimization problem of (4.13) can be re-expressed as

$$\hat{\mathcal{U}}_j, \hat{\mathcal{B}}_j = \arg \max_{\substack{\mathcal{U}_j, \mathcal{B}_j \\ j \in \mathcal{J}}} C_U. \quad (4.16)$$

Note that, in several cases, not all carriers are used, because of bad channel conditions. Therefore, the sum rate corresponding to unused carriers in (4.15) is set to zero to ensure feasibility.

4.3 Dynamic CSs for Multi-carrier Systems

In this section, we mainly describe the proposed dynamic CS, which attempts to maximize the sum rate in the considered system. The problem of sum rate maximization can be expressed mathematically as (4.16), where the probability distribution of sum rate $C_U(\hat{\mathcal{U}}_j, \hat{\mathcal{B}}_j)$ $j \in \mathcal{J}$ is obtained over all channel realizations. The goal of this study is to investigate the performance of the proposal by observing the CDF and expectation value of this sum rate. We also provide some description of the exhaustive search and the improved norm-based scheme to allow better understanding the proposed scheme.

The exhaustive search first decides all possible sizes for each cluster with constraint $\sum_{j=1}^J |\mathcal{B}_j| = |\mathcal{B}|$. Second, for each cluster size, an exhaustive search of all possible user and BS combinations is performed to obtain all possible cluster formations. Finally, the value that yields the maximum sum rate is selected from all possible cluster formations. This method is not practically feasible because of its high computational cost.

Compared with the recent studies [21, 24] in which the clustering methods are restricted in the dimensions of transmitter and carrier, the proposed CS breaks this restriction and achieves dynamic allocation among all of user, BS, and carrier resources. To describe the selection mechanism, the algorithm starts by setting several empty sets of user and BS for all carriers, then finds one user and one BS from full sets $\mathcal{U}$ and $\mathcal{B}$ with the highest contribution to the system sum rate. Here, the system sum rate can be calculated using (4.15). In next step, adds the selected user and BS to the set corresponding to the carrier used and removes them from sets $\mathcal{U}$ and $\mathcal{B}$. Finally, continues in the same fashion until all of users and BSs are selected. The procedures are summarized in Algorithm 1 to solve the clustering problem in large-scale MU-MIMO DASs.

It is noteworthy that, in several cases, all of the users and BSs communicate using the same carrier, meaning they are assigned to one cluster. This occurrence contradicts our initial motivation. However, noting that all of the CSs mentioned in this paper will use the same formula (4.15) to calculate the sum rate, we can reasonably expect that these cases are not common enough that they influence the effectiveness of the evaluation.

On the other hand, a norm-based method was used in [67] because of its simplicity. However, it cannot be directly used in our study, mainly because it only solves allocation between the users and BSs in point-to-point MIMO systems with SC. In the present study, to extend the norm-based method to MU-MIMO DAS with multi-carrier based clustering, we improve the scheme by selecting one user, one BS, and the carrier corresponding to

Algorithm 1: Proposed scheme

```

1 Input:  $\mathbf{H}(\mathcal{U}, \mathcal{B}, j) \forall j, \rho, |\mathcal{U}|, |\mathcal{B}|, |\mathcal{J}|$ ;
2 Output:  $\hat{\mathcal{U}}_j, \hat{\mathcal{B}}_j, \forall j$ ;
3 Initialization: Set  $\hat{\mathcal{U}}_j = \emptyset, \hat{\mathcal{B}}_j = \emptyset, \forall j$ , and  $C_{U, \max} = 0$ ;
4 Repeat:
5   for  $u = 1$  to  $|\mathcal{U}|$  do
6     if  $u \notin \hat{\mathcal{U}}_j \forall j$  then
7       for  $b = 1$  to  $|\mathcal{B}|$  do
8         if  $b \notin \hat{\mathcal{B}}_j \forall j$  then
9           for  $j = 1$  to  $|\mathcal{J}|$  do
10            Set  $\mathcal{U}_i = \hat{\mathcal{U}}_i, \mathcal{B}_i = \hat{\mathcal{B}}_i, i \in \mathcal{J}$ ;
11            Add  $u$  and  $b$  to sets  $\mathcal{U}_j$  and  $\mathcal{B}_j$ ;
12            Calculate the sum rate using (4.15) and input the result to  $C_U$ ;
13            if  $C_U > C_{U, \max}$  then
14              Let  $C_{U, \max} = C_U$ ;
15              Let  $\hat{u} = u, \hat{b} = b, \hat{j} = j$ ;
16            end
17          end
18        end
19      end
20    end
21  end
22  Add  $\hat{u}$  and  $\hat{b}$  to sets  $\hat{\mathcal{U}}_j$  and  $\hat{\mathcal{B}}_j$ ;
23  Reset  $C_{U, \max}, \hat{u}, \hat{b}, \hat{j}$ ;
24 Until all users and BSs are selected.

```

the u , b , and j of an element in the channel matrix $\mathbf{H}(\mathcal{U}, \mathcal{B}, \mathcal{J})$ with the largest Euclidean norm. Briefly, the improved norm-based CS created using Algorithm 2.

4.4 Simulation Parameters and Results

4.4.1 Simulation Parameters

In the simulation, two homogeneous² areas, i.e., $R_{\text{len}} \times R_{\text{wid}} = 400 \text{ m} \times 200 \text{ m}$ and $R_{\text{len}} \times R_{\text{wid}} = 400 \text{ m} \times 400 \text{ m}$, are considered. As shown in Fig. 4.1 and Fig. 4.2(a), $|\mathcal{B}| = 8$ and $|\mathcal{B}| = 16$ BSs are symmetrically distributed in these areas with a distance of $R_{\text{ant}} = 100 \text{ m}$.

²When describing a material or system, the term “homogeneous” indicates having the same properties at every point in space; in other words, uniform, without irregularities.

Algorithm 2: Improved norm-based scheme

```

1 Input:  $\mathbf{H}(\mathcal{U}, \mathcal{B}, j) \forall j, |\mathcal{U}|, |\mathcal{B}|, |\mathcal{J}|$ ;
2 Output:  $\hat{\mathcal{U}}_j, \hat{\mathcal{B}}_j, \forall j$ ;
3 Initialization: Set  $\hat{\mathcal{U}}_j = \emptyset, \hat{\mathcal{B}}_j = \emptyset, \forall j$ , and  $EN_{\max} = 0$ ;
4 Repeat:
5 for  $u = 1$  to  $|\mathcal{U}|$  do
6   if  $u \notin \hat{\mathcal{U}}_j \forall j$  then
7     for  $b = 1$  to  $|\mathcal{B}|$  do
8       if  $b \notin \hat{\mathcal{B}}_j \forall j$  then
9         for  $j = 1$  to  $|\mathcal{J}|$  do
10          Calculate the Euclidean norm and input the result to  $EN$ , i.e.,
11           $EN = \sqrt{\mathbf{H}(u, b, j)\mathbf{H}(u, b, j)^H}$ ;
12          if  $EN > EN_{\max}$  then
13            Let  $EN_{\max} = EN$ ;
14            Let  $\hat{u} = u, \hat{b} = b, \hat{j} = j$ ;
15          end
16        end
17      end
18    end
19 end
20 Add  $\hat{u}$  and  $\hat{b}$  to sets  $\hat{\mathcal{U}}_j$  and  $\hat{\mathcal{B}}_j$ ;
21 Reset  $EN_{\max}, \hat{u}, \hat{b}, \hat{j}$ ;
22 Until all users and BSs are selected.

```

The parameter κ in the SCC model (4.3) is set to 0.3, and the BS height is set to 15 m [64]. We consider a simplified MU-MIMO DAS with a center frequency of 2.000×10^9 Hz with 1.000×10^6 Hz separation. According to experimental results reported in [76] and using the assumption of two homogeneous areas, the shadowing parameters μ_{ubj} and σ_{ubj} at a frequency band of 2.000×10^9 Hz with a path loss exponent $\zeta = 4$ can be set at $\mu = 0$ dB and $\sigma = 9.6$ dB for all u, b , and j . Furthermore, based on the previously reported experimental results [41], [64], the shadowing spatial correlation distance on the user and BS sides can be set to 20 m in this frequency band. The detailed simulation parameters are listed in Table 4.1. Note that, in the simulation, the fading spatial correlation that occurs on both the user and BS sides can be considered to be independent because, in

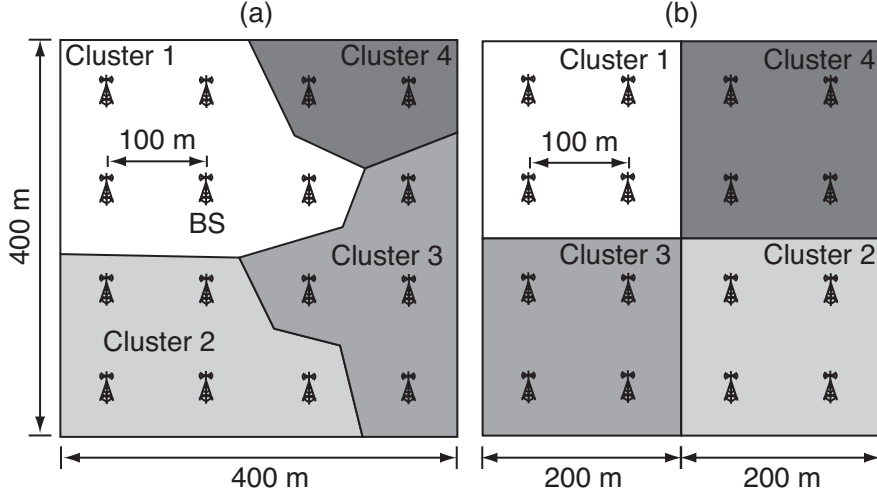


Figure 4.2: (a) Illustration of a MU-MIMO DAS using multiple orthogonal carriers, in which $|\mathcal{U}| = 16$, $|\mathcal{B}| = 16$, and $|\mathcal{J}| = 4$. (b) An example of CS using LOCA method with $|\mathcal{U}| = 16$, $|\mathcal{B}| = 16$, and $|\mathcal{J}| = 4$.

general, the $|\mathcal{U}|$ users and $|\mathcal{B}|$ BSs are geographically separated.

4.4.2 Simulation Results

In this subsection, we compare the performance of four different types of CS (namely, exhaustive search, the proposed CS, the improved norm-based CS, and the traditional LOCA CS) via calculations of the CDF and expectation value of the system sum rate using a Monte Carlo numerical computation method.

The traditional LOCA method is an advanced version of CS (relative to the existing static CS). In the existing static CS, as mentioned in the introduction, carriers are uniquely allocated to users and BSs based on their current location. The most significant difference between these two methods is that in the LOCA method, the carrier associated with its cluster may be changed based on the particular channel realization. Therefore, the SP can maximize the sum rate by dynamically allocating these carriers to appropriate clusters. Two examples of CS using the LOCA method are depicted in Fig. 4.3

Table 4.1: Simulation parameters.

Parameters	Values
Dimensions of area in Fig. 4.1	$R_{\text{len}} = 400$ m $R_{\text{wid}} = 200$ m
Dimensions of area in Fig. 4.2	$R_{\text{len}} = 400$ m $R_{\text{wid}} = 400$ m
The shortest distance between two adjacent BSs	$R_{\text{ant}} = 100$ m
Number of BSs	$ \mathcal{B} = 8, 16$
BS height	15 m
Number of users	$ \mathcal{U} = 8, 16$
Number of user samples	100
Number of carriers	$ \mathcal{J} = 2, 4$
Center frequency	2.000×10^9 Hz
Frequency separation	1.000×10^6 Hz
Path loss exponent	$\zeta = 4$
Shadowing model	spatially i.i.d. and cor.
Shadowing cor. in freq. domain	completely correlated
Shadowing mean value	$\mu = 0$ dB [66]
Shadowing standard deviation	$\sigma = 9.6$ dB [76]
Cor. distance on user side	$d_{\text{cor},r} = 20$ m [41]
Cor. distance on BS side	$d_{\text{cor},t} = 20$ m [64]
SCC parameter	$\kappa = 0.3$ [64]
Number of shadowing samples	200
Fading spatial cor. on user side	i.i.d. ($\beta_r = 0$)
Fading spatial cor. on BS side	i.i.d. ($\beta_t = 0$)
Fading cor. in freq. domain	mutually independent
Number of fading samples	1000

and Fig. 4.2(b), where the coverage areas are divided into two and four square clusters, respectively, with sizes of $200 \text{ m} \times 200 \text{ m}$.

Note that we should use the same distribution on the user side to accurately compare the differences between the CSs. Noting the system configurations, in the simulation, we assumed that users (and the same number of BSs) are uniformly distributed in each square area. This assumption allows convenient comparison of the LOCA with other CSs. In spite of this, in the present study, we emphasize that all users can be uniformly

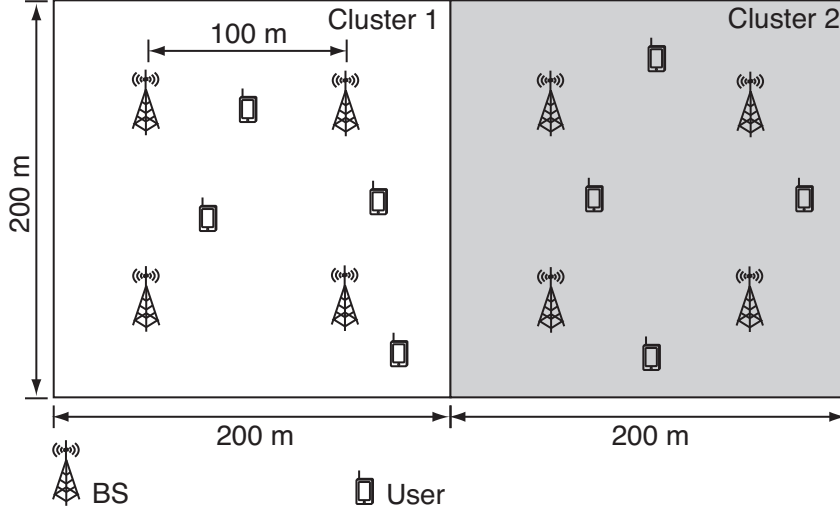


Figure 4.3: Example of CS using LOCA method with $|\mathcal{U}| = 8$, $|\mathcal{B}| = 8$, and $|\mathcal{J}| = 2$.

distributed in the coverage areas, without loss of generality.

Figures 4.4 and 4.5 show the CDF and expectation values of the sum rates obtained using an exhaustive search, the proposed CS, the improved norm-based CS, and LOCA CS in the considered system with $|\mathcal{U}| = 8$, $|\mathcal{B}| = 8$ and $|\mathcal{J}| = 2$ under the effects of spatially independent and correlated shadowing. The CDF in Fig. 4.4 is plotted against the sum rate, and the expectation value in Fig. 4.5 is a function of the average received SNR.

These figures show that the results obtained using the proposed CS closest to those obtained using an exhaustive search, which demonstrates that the proposal, described in Sect. 4.3, is effective. Furthermore, these figures also show that spatially correlated shadowing has a lower sum rate than spatially independent shadowing because of the decrease in the number of DOFs in shadowing RVs [4].

To further verify the proposed CS, in Fig. 4.6 and Fig. 4.7, we show the CDF and expectation values of the sum rates obtained using the above CSs (excluding the exhaustive search) in the considered system with $|\mathcal{U}| = 16$, $|\mathcal{B}| = 16$ and $|\mathcal{J}| = 4$ under the

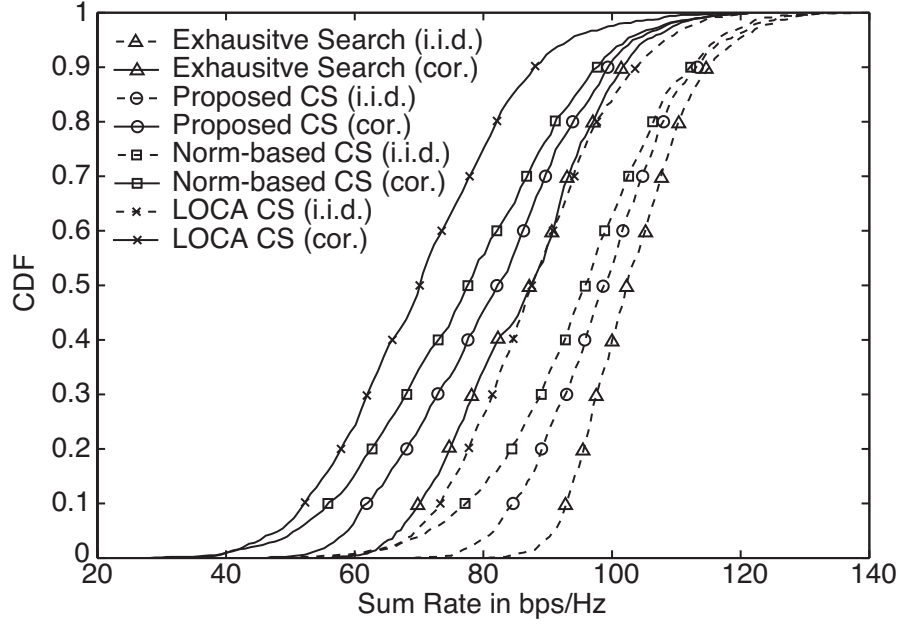


Figure 4.4: CDF of the sum rates obtained using the proposed CS, an exhaustive search, the improved norm-based CS, and LOCA CS under the effects of spatially independent and correlated shadowing for $\text{SNR} = 20$ dB, $|\mathcal{U}| = 8$, $|\mathcal{B}| = 8$ and $|\mathcal{J}| = 2$.

effects of spatially independent and correlated shadowing. Simulation results indicate that the proposed CS provides better performance than the existing schemes, and that the spatially correlated shadowing considerably decreases the sum rates. These results support our previous conclusions.

Note that, to maintain the simulation time at an acceptable level, in the simulations, we considered 16 users and BSs as well as four carriers at most. In fact, there are no special restrictions for the number of users, BSs, and carriers in our study. Furthermore, the results obtained using the exhaustive search are not shown in Fig. 4.6 and Fig. 4.7, primarily because its computational cost is very large, so that the exhaustive search cannot be used with increasing values of $|\mathcal{U}|$, $|\mathcal{B}|$, and $|\mathcal{J}|$ in computer simulation. This fact provides further proof that a dynamic CS with low computational cost and satisfactory performance is necessary. A detailed analysis of the complexity of the mentioned CSs is

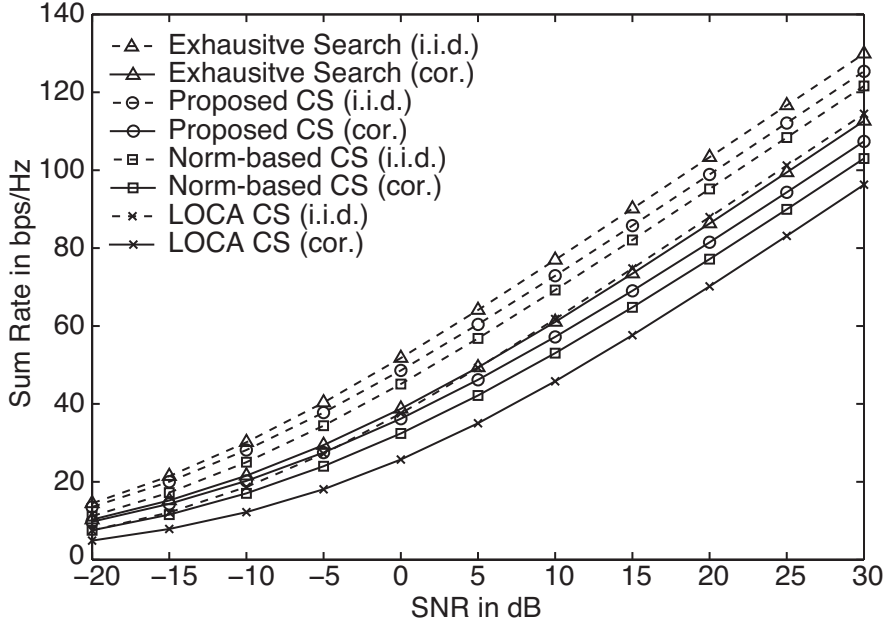


Figure 4.5: Expectation values of the sum rates obtained using the proposed CS, an exhaustive search, the improved norm-based CS, and LOCA CS under the effects of spatially independent and correlated shadowing for $|\mathcal{U}| = 8$, $|\mathcal{B}| = 8$ and $|\mathcal{J}| = 2$.

provided in the following section.

4.5 Complexity Analysis

The optimal search for dynamic CS in the considered system with all possible cluster

sizes $|\mathcal{B}_j|$ involves exhaustive searching over

$$\begin{aligned}
 & \sum_{\mathcal{G}} \binom{|\mathcal{B}|}{|\mathcal{B}_1|} \binom{|\mathcal{B}| - |\mathcal{B}_1|}{|\mathcal{B}_2|} \cdots \binom{|\mathcal{B}| - \cdots - |\mathcal{B}_{J-1}|}{|\mathcal{B}_J|} \times \\
 & \binom{|\mathcal{U}|}{|\mathcal{B}_1|} \binom{|\mathcal{U}| - |\mathcal{B}_1|}{|\mathcal{B}_2|} \cdots \binom{|\mathcal{U}| - \cdots - |\mathcal{B}_{J-1}|}{|\mathcal{B}_J|} = \\
 & \frac{|\mathcal{B}|! |\mathcal{U}|!}{(|\mathcal{U}| - |\mathcal{B}|)!} \sum_{\mathcal{G}} \left(\prod_{j=1}^J (|\mathcal{B}_j|!) \right)^{-2}
 \end{aligned} \tag{4.17}$$

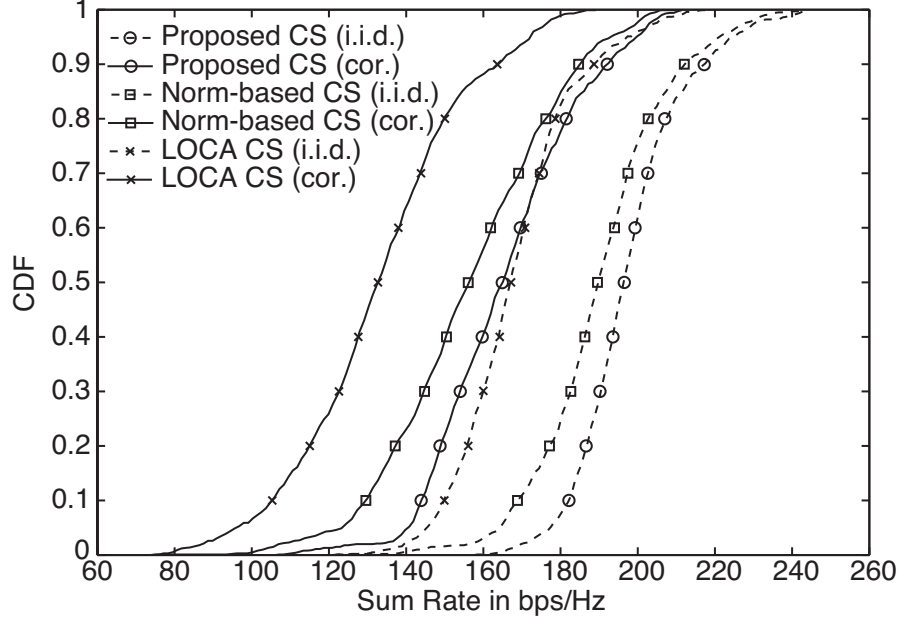


Figure 4.6: CDF of the sum rates obtained using the proposed CS, the improved norm-based CS, and LOCA CS under the effects of spatially independent and correlated shadowing for $\text{SNR} = 20$ dB, $|\mathcal{U}| = 16$, $|\mathcal{B}| = 16$, and $|\mathcal{J}| = 4$.

possible cluster formations for each channel realization, where $\mathcal{G}$ is a finite set of solutions of equation $\sum_{j=1}^J |\mathcal{B}_j| = |\mathcal{B}|$, which can be written as

$$\mathcal{G} := \left\{ (|\mathcal{B}_1|, \dots, |\mathcal{B}_J|) \left| \sum_{j=1}^J |\mathcal{B}_j| - |\mathcal{B}| = 0 \right. \right\}.$$

For each cluster formation, the sum rate in (4.15) must be calculated once. Therefore, the complexity of the exhaustive search grows exponentially with the number of users and BSs, and increases significantly as the number of carriers increases. This phenomenon is evident from Stirling's factorial approximation, according to which

$$X! \approx X^X e^{-X} \sqrt{2\pi X}. \quad (4.18)$$

Both the proposed and the improved norm-based CS must run

$$|\mathcal{J}| \sum_{i=0}^{|\mathcal{B}|-1} (|\mathcal{B}| - i) (|\mathcal{U}| - i) \quad (4.19)$$

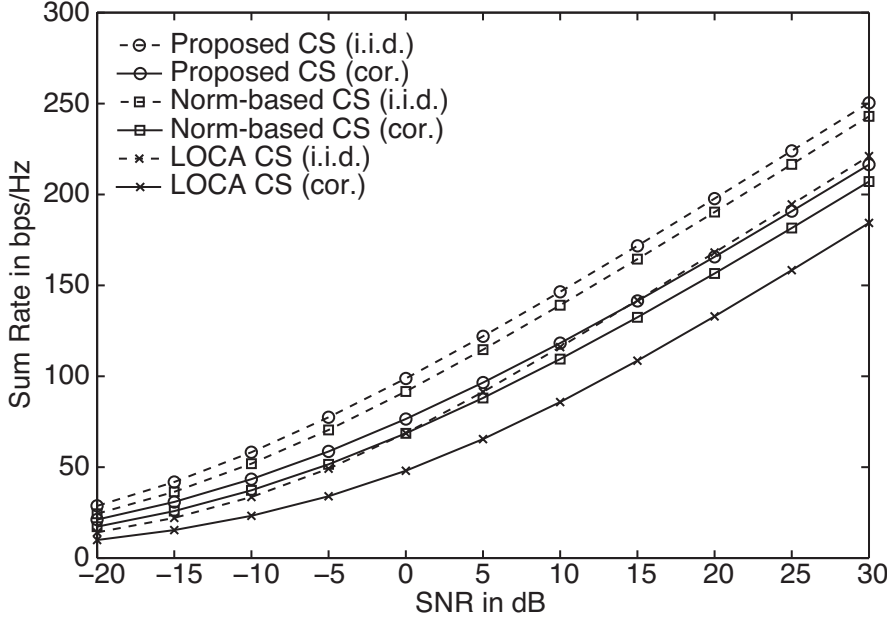


Figure 4.7: Expectation values of the sum rates obtained using the proposed CS, the improved norm-based CS, and LOCA CS under the effects of spatially independent and correlated shadowing for $|\mathcal{U}| = 16$, $|\mathcal{B}| = 16$, and $|\mathcal{J}| = 4$.

loops for each channel realization. In the proposed CS, the calculation of sum rate in (4.15) should be executed during each loop; on the other hand, in the improved norm-based CS, the calculation of the Euclidean norm should be executed instead of the calculation of sum rate. In fact, the calculation of sum rate is more complex than the calculation of the Euclidean norm; therefore, the proposed CS has a higher computational cost than the improved norm-based CS, even though they have the same number of loops.

In the traditional LOCA CS, the SP dynamically allocates $|\mathcal{J}|$ carriers to $|\mathcal{J}|$ clusters, which have been constructed based on the location of users and BSs; therefore, there are $|\mathcal{J}|!$ possible cluster formations, and for each one, the sum rate must be calculated once. Therefore, the complexity of this CS grows exponentially only with the number of carriers $|\mathcal{J}|$. Table 4.2 presents a comparison of the number of running sum rate operations for the exhaustive search, the proposed CS, and LOCA CS. The proposed CS is shown to

Table 4.2: Comparison of computational cost for different CSs.

$(\mathcal{U} , \mathcal{B} , \mathcal{J})$	(8,8,2)	(16,16,2)	(16,16,4)
Exhaustive search	12870	6.0×10^8	1.0×10^{17}
Proposed CS	408	2992	5984
LOCA CS	2	2	24

be much faster than the exhaustive search.

4.6 Summary

To establish a wide service area for MU-MIMO DASs, the use of a dynamic CS is necessary to reduce the enormous amount of computation required for precoding. In the present study, we first explored and proposed a simple method for dynamic clustering in a single cell MU-MIMO DAS and investigated its performance by observing the CDF and expectation value of the system sum rate. We also compared the characteristics of the proposal to other classical CSs (such as the improved norm-based CS and LOCA CS) in terms of sum rate improvement. To make our results more universal, we introduced the spatial correlation in the considered system, in particular, the spatial correlation in shadowing, which should not be ignored in DASs. Computer simulation results indicated that the proposed CS provides better performance than the existing schemes and can achieve a sum rate that is close to that of an exhaustive search but at a lower computational cost. The results also verified that the proposal can be successfully used into the single cell MU-MIMO DASs. Next step, by taking the inter-cell interference into account and introducing the coordinated precoding, we will expand and use the proposal to further solve the clustering problems in multi-cell DASs, and we are willing to believe that the results obtained from the present study can support our future works.

Chapter 5

Concluding Remarks

This thesis has focused on creating a large-scale multi-cell multi-user (MU) multi-input multi-output (MIMO) distributed antenna system (DAS) with using multi-carrier based dynamic clustering and considering the correlated composite fading attenuation which consists of path loss, correlated shadowing and correlated Rayleigh. We theoretically analyzed the effects of spatial correlation on the receiver side by the proposed upper and lower bounds in the traditional MU-MIMO centralized antenna systems (CASs), and then demonstrated the effect of the mentioned correlation on the transmitter side and the effect of inter-cell interference to clarify the improvements of system performance by the MU-MIMO DASs. We also proposed a simple dynamic clustering scheme (CS) using multiple orthogonal carriers to achieve a large-scale DAS with MU-MIMO transmissions.

A detailed analysis of the sum rate distribution in MU-MIMO CASs with zero-forcing (ZF) precoding was presented. Specifically, novel upper and lower cumulative distribution function (CDF) bounds were devised, which can be applied for an arbitrary number of antennas and remain tight across the entire signal-to-noise ratio (SNR) range. In fact, the proposed lower bound becomes exact at high SNRs. More importantly, the proposed bounds are generic because they encompass the composite fading channel model consisting of spatially correlated shadowing and correlated Rayleigh fading, which have practical interest, and can be very easily evaluated.

With the help of these bounds, we gained valuable insights into the effects of the composite fading parameters on the performance of MU-MIMO CASs. For instance, we found that the spatial correlation significantly affects the system sum rate corresponding to low values of the CDF due to the shadowing correlation and a diminishing effect on the average sum rate due to the Rayleigh fading correlation. However, the average sum rate does not seem affected by the shadowing spatial correlation in the CAS. We also derived a relationship between the inter-user distance and sum rate corresponding to 10% of the CDF. A practical conclusion from our results based on the considered system is that the effect of spatially correlated shadowing can be considered to be independent when the inter-user distance is more than five times the correlation distance of shadowing.

In order to verify and clarify the improvements of system performance by the MU-MIMO DAS which is defined as the one that has a base station (BS) with multiple geographically distributed antennas, we investigated the characteristics of the MU-MIMO system sum rate in a CAS and DAS under the effects of distance-dependent path loss, spatially correlated shadowing, correlated Rayleigh fading, and inter-cell interference via a Monte Carlo numerical computation method. To generate the channel attenuations, we first introduced two different types of functions to model the shadowing auto-correlation and shadowing cross-correlation, and a typical exponential decay function to model the Rayleigh fading correlation. Thus, we successfully obtained the CDF of the system sum rate and its average in DASs. Computer simulation results indicated that a DAS with MU-MIMO transmission offers higher channel average sum rate performance than a comparable CAS in the case under consideration. However, this improvement decreased with increasing interference power because the inter-cell interference dominates the sum rate changes in that case. Furthermore, compared to the i.i.d. shadowing model, the decrease in the system sum rate due to the increase in the interference power became slow under the effect of shadowing correlation.

On the basis of the previous results, we clearly understand that DASs with MU-MIMO transmissions can considerably improve the performances of systems mainly because of the decreases of access distance and shadowing correlation both in transmit and receiver sides. Next, in order to create a large-scale multi-cell MU-MIMO DAS, we first explored and proposed a simple dynamic CS for a single cell system and investigated its performance by observing the CDF and expectation value of the system sum rate with considering the effect of the spatial composite fading attenuation. To further verify the effectiveness of the proposal, we also compared the characteristics of the proposal to other classical CSs, such as the improved norm-based CS and location-based adaptive CS in terms of sum rate improvement. Computer simulation results indicated that the proposed CS provides better performance than the existing schemes and can achieve a sum rate that is close to that of an exhaustive search but at a lower computational cost. The results also verified that the proposal can be successfully used into the single cell MU-MIMO DASs.

Certainly, in order to extend the current investigation towards much more complicated real wireless communication networks, further extensive research is required, such as to achieve our big picture that is creating a multiple cell network in where MU-MIMO DASs are considered in each cell. As a future research direction, one target might be to extend the proposed dynamic CS to a multi-cell model with noting that:

- The effect of inter-cell interference must be included in the calculation of system sum rate, so that the coordinated precoding for the optimization of sum rate in multi-cell system should be attracted more attention;
- The large amounts of channel state information due to a large number of users, BSs, and carriers can be only transferred over limited rate channel, so that how to

reduce the feedback rate while maintaining good performance of system is also one of the valuable topics.

If we can, somehow, solve the mentioned issues, then the extension might be possible. We are willing to believe that the results obtained from the present research can support our future works.

Appendix

Calculation about ν th Central Moment of Lognormal Function

We consider the general case of M correlated Gaussian RVs $\mathbf{X} = \{X_m\}_{m=1}^M$, which have an arbitrary correlation matrix $\mathbf{\Theta}$. The corresponding Gaussian RVs follow the joint distribution

$$f_{\mathbf{X}}(\mathbf{X}) = \frac{1}{(2\pi)^{\frac{M}{2}} \sqrt{\det(\mathbf{\Theta})}} \exp\left(-\frac{(\mathbf{X} - \boldsymbol{\mu})^H \mathbf{\Theta}^{-1} (\mathbf{X} - \boldsymbol{\mu})}{2}\right), \quad (1)$$

where $\boldsymbol{\mu} = \{\mu_{X_m}\}_{m=1}^M$ is the vector of the means of the Gaussian RVs. The ν th central moment of a lognormal function, for example, the first term in (2.17), is then written as

$$E_{\nu} = \int_{-\infty}^{\infty} \frac{1}{(2\pi)^{\frac{M}{2}} \sqrt{\det(\mathbf{\Theta})}} \left(\log_2 \prod_{m=1}^M \left(1 + 10^{\frac{X_m}{10}}\right) \right)^{\nu} \times \exp\left(-\frac{(\mathbf{X} - \boldsymbol{\mu})^H \mathbf{\Theta}^{-1} (\mathbf{X} - \boldsymbol{\mu})}{2}\right) d\mathbf{X}. \quad (2)$$

When the decorrelating transformation $\mathbf{X} = \sqrt{2}\mathbf{\Theta}^{\frac{1}{2}}\mathbf{Z} + \boldsymbol{\mu}$ is used, X_m is provided by

$$X_m = \sqrt{2} \sum_{j=1}^M g_{mj} Z_j + \mu_{X_m}, \quad (3)$$

where $g_{mj} = [\mathbf{\Theta}^{\frac{1}{2}}]_{mj}$ is the (m, j) th element of $\mathbf{\Theta}^{\frac{1}{2}}$. Therefore, (2) becomes

$$E_{\nu} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{1}{\pi^{\frac{M}{2}}} \exp(-\mathbf{Z}^H \mathbf{Z}) \left(\log_2 \prod_{m=1}^M \left(1 + 10^{0.1(\sqrt{2} \sum_{j=1}^M g_{mj} Z_j + \mu_{X_m})}\right) \right)^{\nu} dZ_1 \cdots dZ_M. \quad (4)$$

Performing the Gauss-Hermite expansion with respect to Z_1 yields

$$E_\nu \approx \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{1}{\pi^{\frac{M-1}{2}}} \exp \left(- \sum_{i=2}^M |Z_i|^2 \right) \sum_{n_1=1}^N \frac{w_{n_1}}{\sqrt{\pi}} \\ \times \left(\log_2 \prod_{m=1}^M \left(1 + 10^{0.1(\sqrt{2} \sum_{j=2}^M g_{mj} Z_j + \sqrt{2} g_{m1} a_{n_1} + \mu x_m)} \right) \right)^\nu dZ_2 \cdots dZ_M. \quad (5)$$

Proceeding in a similar manner for $Z_2, \dots, Z_M$, we obtain E_ν expressed as

$$E_\nu \approx \sum_{n_1=1}^N \cdots \sum_{n_M=1}^N \left(\prod_{m=1}^M \frac{w_{n_m}}{\sqrt{\pi}} \right) \left(\sum_{m=1}^M \log_2 \left(1 + 10^{0.1(\sqrt{2} \sum_{j=1}^M g_{mj} a_{n_j} + \mu x_m)} \right) \right)^\nu. \quad (6)$$

Note that improved estimates of E_ν can be obtained by increasing the Hermite integration order N .

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Author's Publication List

Journal Papers

1. O. Zhao, and H. Murata, "A Study on Dynamic Clustering for Large-scale Multi-user MIMO Distributed Antenna Systems with Spatial Correlation," *IEICE Trans. Commun.*, vol. E99-B, no. 4, pp. 928–938, April 2016.
2. O. Zhao, and H. Murata, "Sum-rate analysis for centralized and distributed antenna systems with spatial correlation and inter-cell interference," *IEICE Trans. Commun.*, vol. E98-B, no. 3, pp. 449–455, Mar. 2015.
3. O. Zhao, and H. Murata, "Sum rate analysis of MU-MISO systems with ZF beamforming over composite fading channels," *IEICE Trans. Fund.*, vol. E98-A, no. 2, pp. 558–568, Feb. 2015.

International Conference Papers

1. O. Zhao, H. Murata, "Effects of spatial correlation on the sum rate distribution of ZF receivers in MU-MIMO systems," *Proc. IEEE Vehicular Technology Conference (VTC) '14*, pp. 1–4, Seoul, Korea, May 2014.
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1. O. Zhao, X.L. Shi, and H. Murata, "Transmission Performance of In-Vehicle Inter-User Communication by Wireless LAN," *Proc. IEICE General Conference*, B-5-135, pp. 528, Kyushu, March 2016.

2. O. Zhao, X.L. Shi, and H. Murata, "A Simple Method to Simulate Inter-user Links for Collaborative Communications in MIMO Systems," The 38th Symposium on Information Theory and its Applications (SITA2015), 6.2.2, pp. 410–415, Kojima, Okayama, Japan, Nov. 2015.
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Awards

- IEEE VTS Japan Young Researcher's Encouragement Award
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Scholarships and Financial Support

- KDDI Foundation Scholarship for International Students, April 2014–March 2015.
- NEC C&C Foundation Grants for Researchers Attending International Conferences, May 2014.
- The Hara Research Foundation for Researchers Attending International Conferences, Sept. 2013.
- Japan Student Services Organization (JASSO) Monbukagakusho Honors Scholarship for Privately Financed International Students, April 2011–March 2012.