

Optical Model Parameters of Several s-d Shell Nuclei for 28 MeV Alpha-Particle Scattering

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Optical model parameters of Mg, Si, S, and Ar for alphaparticle elastic scattering ($E_\alpha \cong 28$ MeV) are calculated by the DWBA method using an automatic search program. For S and Ar, the values of real potential obtained are about 90 and 75 MeV, respectively. They are deeper than those of Mg and Si which are about 56 and 52 MeV, respectively.

I. INTRODUCTION

The parameter search of the optical model potential has been extensively made for many nuclei and in various incident energies of protons, deuterons, tritons, helions and alpha-particles.¹⁾ For protons and deuterons some average set of optical model parameters have been obtained which is applicable over considerable energy range and mass number. It seems, however, data concerning alpha-particles are relatively little compared with other particles. In this report, the elastic scattering of alpha-particles from several s-d shell nuclei are newly analysed by a computer program, EARCH, which was developed by Wada²⁾ at Institute of Physical and Chemical Research, and can search optimum set of the optical model parameters by automatic search of χ^2 minimum.

The experimental data had been taken at the Kyoto University cyclotron³⁾ and were preliminarily analysed by manual change of limited number of parameters.⁴⁾ A part of the same data also had been analysed by Satchler.⁵⁾

II. THEORY AND METHOD OF ANALYSIS

The optical model potential used in the present analysis is

$$V(r) = -V_R \{1 + \exp[(r - R_R)/a_R]\}^{-1} - iW_I \{1 + \exp[(r - R_I)/a_I]\}^{-1} + V_C(r), \quad (1)$$

where

$$V_C(r) = (zz'e^2/2R_C)(3 - r^2/R_C^2), \quad r \leq R_C$$

$$= zz'e^2/r, \quad r \geq R_C$$

$$R_i = r_i A^{1/3}.$$

The automatic search program, SEARCH, finds a set of parameters of Eq. (1) which gives the best fit to the data by minimizing the quantity

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$$\chi^2 = (1/N) \sum_{i=1}^N \{[\sigma_{th}(\theta_i) - \sigma_{exp}(\theta_i)] / \Delta\sigma_{exp}(\theta_i)\}^2, \quad (2)$$

where σ_{th} and σ_{exp} are the theoretical and experimental differential cross sections at angle θ_i , respectively, and $\Delta\sigma_{exp}$ is the experimental error of the $\sigma_{exp}(\theta_i)$.

A parameter set which minimizes the value of χ^2 depends upon a starting set of initial values of the parameters. Thus the search finds many χ^2 minima which are corresponding to different sets of initial parameters.

After several trials, a method of obtaining the best set of parameters is adopted in this report to find out the smallest value of the χ^2 :

1) At first we fix the value of $r_R(=r_I)$ and $a_R(=a_I)$ and W_I . The starting value of W_I is relatively insensitive and converges to a same value over considerable range of energies of the starting value of W_I . Then we change only the starting value of V_R over wide energy range (10~200 MeV) in step of 15 MeV or less, and search final converged values of V_R and associated χ^2 minimum. A proper final value of V_R which has the smallest value of the minimum χ^2 thus obtained is determined to be the most proper starting value.

2) With this value of V_R fixed and the unchanged value of $r_R(=r_I)$, $a_R(=a_I)$, only the starting value of W_I is changed over energy range of 2~20 MeV in step of 2 MeV. Usually only a final value of W_I was obtained with the different starting value of W_I . This final value is adopted for the most suitable starting value of W_I .

3) With the fixed value of $r_R(=r_I)$ and $a_R(=a_I)$, the best set of values of V_R and W_I are searched automatically of which starting values are given in 1) and 2).

4) With the final values of V_R and W_I obtained after the procedure 3), only two parameters a_R and a_I are changed automatically to search the best fit.

5) With the fixed final values of V_R , W_I , a_R , and a_I , only two parameters r_R and r_I are changed next.

6) Finally all parameters, V_R , W_I ; r_I , r_R ; a_R , and a_I are automatically changed at the same time to find the minimum χ^2 fit with the starting values obtained by the procedure 3), 4), and 5), respectively.

7) The value of r_C is always fixed in 1.40 fm. The values of $\Delta\sigma_{exp}(\theta_i)$ are assumed always 5% of $\sigma_{exp}(\theta_i)$. (Almost the same final parameter set was obtained with experimental errors. See later discussion.)

As for the initial fixed values of W_I , $r_R(=r_I)$, and $a_R(=a_I)$, values obtained in the preliminary process reported in Ref. 4 was used.

III. RESULTS AND DISCUSSION

The obtained optical model parameters which give optimum fit to the experimental data are listed in Table I. No systematic mass dependence of the parameters cannot be seen. However, it is noticeable that the value of V_R of S and Ar is considerably deeper than that of Mg or Si. In Figs. 1 and 2 are shown experimental points and theoretical curves.

Table I. Optical Model Parameters Which Give Optimum Fit to Experimental Data

Nucleus	E_α (MeV)	V_R (MeV)	W_I (MeV)	r_R (fm)	r_I (fm)	a_R (fm)	a_I (fm)	r_C (fm)	θ_R (mb)	χ^2
Mg	28.4	55.95	10.56	1.72	1.77	0.520	0.450	1.4	1187	39.8
Si	28.3	51.72	10.87	1.69	1.48	0.540	0.679	1.4	1230	6.0
S	27.9	90.44	8.98	1.50	1.85	0.641	0.389	1.4	1273	11.0
Ar	27.6	75.00	13.85	1.60	1.73	0.561	0.431	1.4	1389	11.0

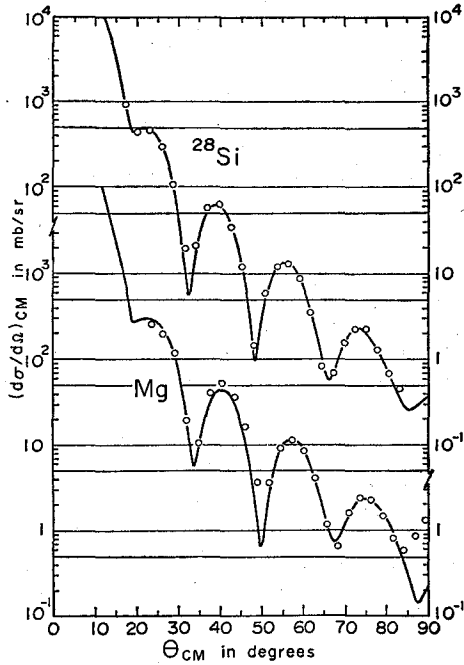


Fig. 1. Experimental (in open circle) and theoretical (in solid curve) cross sections of Mg and Si using parameters in Table I.

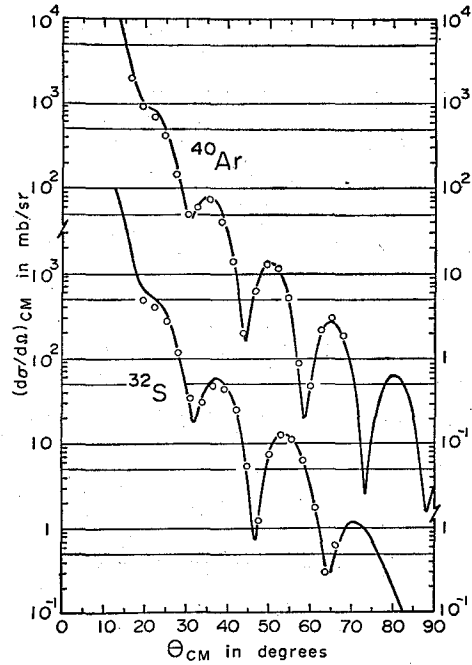


Fig. 2. Experimental (in open circle) and theoretical (in solid curve) cross sections of S and Ar using parameters in Table I.

III.1. Mg

The parameters of four-parameter Woods-Saxon potential given by Satchler⁵⁾ were as follows: $V_R=54.4$ MeV, $W_I=9.8$ MeV, $r_R=r_I=1.70$ fm, and $a_R=a_I=0.529$ fm, using the same data of present analysis. For this set of parameters, one obtains χ^2 of 51.0. An approximate optimum value of V_R is found at 55.5 MeV by the procedure described in II. 1) with the fixed values of $W_I=10.0$ MeV, $r_R=r_I=1.70$, and $a_R=a_I=0.529$. As shown in Table II no other better V_R exists in the energy range of 20 MeV to 200 MeV.

With this value of $V_R=55.5$ MeV, and other fixed parameters, the optimum value of W_I was searched with various starting value in energy range of 2 to 20 MeV in step of 2 MeV. In any case W_I converged to 10.4 MeV. Thus the most proper starting values of V_R and W_I are determined to be 55.5 MeV and 10.4 MeV, respectively. By the procedure described in II. 3)~6), finally one obtains the parameter set listed in Table I.

Table II. Starting Value of V_R and Corresponding Optimum V_R' . The Values of W_I , $r_R=r_I$, and $a_R=a_I$ Are Fixed in 10.0, 1.70, and 0.529, Respectively.

Nucleus	V_R (MeV)		V_R' (MeV)	χ^2
Mg	20,	35	38.00	124.7
	50,	65	55.52	49.6
	80,	95	83.60	138.0
	110,	125	118.55	561.6
	140,	155, 170	158.11	1535.4
	185,	200	201.45	3133.4

The fitness is improved at angles around 75° compared with the result in Ref. 5.

By the same method with errors of experimental value, following parameters were obtained: $V_R=56.20$ MeV, $W_I=9.97$ MeV, $r_R=1.71$ fm, $r_I=1.77$ fm, $a_R=0.524$ fm, and $a_I=0.450$ fm.

III.2. Si

Using the parameters in Ref. 5, one obtains 30.5 for the value of χ^2 . As shown in Table III the most proper starting value of V_R is determined to be 55.0 MeV. The final optimum set of parameters is listed in Table I.

Table III. Starting Value of V_R and Corresponding Optimum V_R' . The Values of W_I , $r_R=r_I$, and $a_R=a_I$ Are Fixed in 11.0, 1.66, and 0.555, Respectively.

Nucleus	V_R (MeV)		V_R' (MeV)	χ^2
Si	20,	35	34.38	89.6
	50,	65	55.00	32.3
	80,	95	81.05	107.1
	110,	125	111.75	327.2
	140,	155	147.26	1240.9
	170,	185, 200	186.17	3118.1

By the same procedure and with errors of experimental value the following set of parameters were obtained which is very close to that of all 5% errors: $V_R=51.35$ MeV, $W_I=10.69$ MeV, $r_R=1.70$ fm, $r_I=1.53$ fm, $a_R=0.521$ fm, and $a_I=0.659$ fm.

III.3. S

The preliminary analysis of sulfur reported in Ref. 4 was made by a manual change of four-parameters and seems to be improper in choosing starting values of V_R and W_I . By present analysis it was found that the starting value of V_R should be more than 80 MeV which is considerably deeper than the value in Ref. 4 (see Table IV) and the starting value of W_I should be 10.3 MeV.

In this case the value of V_R strongly depends on the experimental value at angles larger than 60° . Thus it seems to be important to measure at angles greater than

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Table IV. Starting Value of V_R and Corresponding Optimum V_R' . The Values of W_I' , $r_R=r_I$, and $a_R=a_I$ Are Fixed in 10.0, 1.63, and 0.525, Respectively.

Nucleus	V_R (MeV)		V_R' (MeV)	χ^2
S	10,	20	6.77	3115.3
		35	34.89	255.1
	50,	65	55.92	126.2
		80	80.33	37.2
	95,	110	107.26	82.5
	125,	140, 155	140.36	288.6
		170, 185	176.96	792.2
		200	217.00	1691.6

65° to calculate an optimum set of parameters.

The value of χ^2 is greatly improved compared with the result in Ref. 4 which was 355.4.

III.4. Ar

By the procedure II. 1) and 2), the starting values of V_R and W_I are found to be 74.4 MeV and 15.0 MeV, respectively (see Table V). The set of parameters in preliminary analysis (Ref. 4) gives 70.1 for the value of χ^2 . By present analysis one obtains χ^2 of 11.0.

Table V. Starting Value of V_R and Corresponding Optimum V_R' . The Values of W_I , $r_R=r_I$, and $a_R=a_I$ Are Fixed in 10.0, 1.61, and 0.540, Respectively.

Nucleus	V_R (MeV)		V_R' (MeV)	χ^2
Ar	35,	50	58.21	74.9
	65,	80	74.40	13.1
	95,	110	99.05	107.7
	125,	140	129.14	370.7
	155,	170	161.81	695.6
	185,	200	197.32	1020.8

IV. CONCLUSION

By the four-parameter Woods-Saxon potential, moderately good fits to the elastic scattering from Mg, Si, S, and Ar had been obtained.^{4,5)} In the present analysis the number of parameters is increased to six so that better fits were obtained in all four nuclei. Generally speaking, by increasing number of parameters the fits to the experimental data in angles more than 60° were considerably improved compared with the case of four-parameters. Especially in S and Ar, better fits in backwards angles (> 60°) were obtained with relatively deep values of the real potential V_R .

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The analysis was made by an electronic computer, TOSBAC-3400/M41 at the Institute for Nuclear Study, University of Tokyo.

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