

OBSERVATION OF THE ATMOSPHERIC TIDE EFFECTS ON THE EARTH'S DEFORMATIONS

By

Izuo OZAWA

(Received October 23, 1967)

Abstract

In this paper the observations of the earth's tidal strains and tilt have been performed at Osakayama Observatory, and the effects depending on the atmospheric tide in the earth's strains and tilt are calculated using the load-deformation coefficients of the earth reported by I. M. Longman.

According to this calculation and these observations, our estimations seem to be accurate that the atmospheric tide effects in the earth tide amount to about 20% as large as the direct effects due to the astronomical tide.

1. Introduction

All of the lithosphere, hydrosphere and atmosphere of the earth are deformed periodically by the tide generating force due to the attractions and thermal radiations of the celestial bodies. And their oscillations themselves influence one another. Especially, the effects in the crustal deformations depending on the oceanic tides are remarkable at the coast. We have got many studies concerning the oceanic tide effects (Takahashi [1930], Hagiwara *et al.* [1948], Nishimura [1950], Ozawa [1957] and so on).

The pressure amplitudes of the atmospheric tide are not so large as those of the oceanic tide at the coast, but the amplitude pattern of the semi-diurnal component of the atmospheric tide has a quite regular pattern of the spheroidal type. Moreover, their pressure oscillation acts immediately on the ground everywhere. So, the atmospheric tide effect occupies the important part of the crustal tidal deformation at the places distant from the ocean.

Many studies (Takeuchi *et al.* [1965], Longman [1963] etc.) have been performed to calculate theoretically the loading strains together one of the free oscillations on the model earth like the Gutenberg's.

We are able to apply effectively these theoretical calculations for that of the atmospheric tide effects. We have many difficulties in the calculations of the loading strains depending on the oceanic tide, because of their complex patterns especially for their nearing ones. On the contrary, we have no difficulty in the

application to the calculation of the atmospheric tide effects, since the pattern is very simple and has no discontinuity.

In this paper, the present author has performed the precise observations of the tidal strains at Osakayama, and calculates the atmospheric tide effect on the crustal strains and tilt using the load-deformation coefficients for determining the deformation of the earth calculated by I. M. Longman. Then, these observed values and these calculated ones of the effects are compared with each other.

2. Fundamental theory

The pattern of the solar semi-diurnal component S_2 of the atmospheric tide is quite simple form of the spheroidal type, and its amplitude is about sixteen times as large as that of the lunar semi-diurnal component M_2 (Chapman *et al.* [1956]). B. Haurwitz *et al.* [1957] have obtained the empirical expression for the traveling tide w_2 of S_2 component as follow,

$$w_2(\theta, \phi) = 1.23\{\tilde{P}_2^2(\theta, \phi) - 0.182\tilde{P}_4^2(\theta, \phi)\}\sin(2T + 2\phi + 158^\circ)\text{mb}, \dots\dots\dots(1)$$

where θ is the colatitude, ϕ is the east longitude, $\tilde{P}_n^m$ is Legendre Schmidt's normalized associated function of degree n and order m , T is the Greenwich Standard Time.

Let u_r' , u_θ' and u_ϕ' be the radial, colatitudinal and longitudinal components of the displacement depending on the atmospheric tidal pressure, we may write these components as follows,

$$\begin{aligned} u_r' &= \sum_n y_{1,n}(r) \left(\frac{r}{a}\right)^n (w_n(\theta, \phi), \\ u_\theta' &= \sum_n y_{3,n}(r) \left(\frac{r}{a}\right)^n \frac{\partial w_n(\theta, \phi)}{\partial \theta}, \\ u_\phi' &= \sum_n y_{3,n}(r) \left(\frac{r}{a}\right)^n \frac{\partial w_n(\theta, \phi)}{\sin \theta \partial \phi}, \end{aligned} \dots\dots\dots(2)$$

and at the earth's surface

$$y_{1,n}(a) = \frac{h_n'}{g}, \quad y_{3,n}(a) = \frac{l_n'}{g}, \dots\dots\dots(3)$$

and the perturbation of the potential ψ' by the deformation is shown as

$$\psi' = \sum_n (1 + k_n' - h_n')/g \cdot \left(\frac{a}{r}\right)^n w_n, \dots\dots\dots(3')$$

where r is the distance from the earth's center to an observing point, a is the mean radius of the earth, g is the mean value of the gravity acceleration at the observing point, and h_n' , l_n' and k_n' are the Green's functions for the determining deformation of the earth under surface mass loads i.e. load-deformation coefficients, respectively.

As the expression (1) consists of the term of $\tilde{P}_2^2$ and that of $\tilde{P}_4^2$, the displacement components can be expressed with the linear combinations of two terms concerning to $\tilde{P}_2^2$ and $\tilde{P}_4^2$. Hence, we obtain the strain elements or tilt components from the formulas (1), (2), and (3) and the relations between the strain elements or tilt components and the displacements as follows,

$$\begin{aligned}
 e_{\theta\theta}' &= \left[\frac{r^2 A_2}{a^3 g} \left\{ 2l_2' \cos 2\theta + \frac{1}{2} h_2' (1 - \cos 2\theta) \right\} + \frac{r^4 A_4}{a^5 g} \left\{ 2l_4' \left(\cos 4\theta - \frac{1}{7} \cos 2\theta \right) \right. \right. \\
 &\quad \left. \left. + \frac{h_4'}{56} (3 + 4 \cos 2\theta - 7 \cos 4\theta) \right\} \right] \cos 2(T + \phi - \epsilon), \\
 e_{\phi\phi}' &= \left[\frac{r^2 A_2}{a^3 g} \left\{ l_2' (\cos 2\theta - 3) + \frac{1}{2} h_2' (1 - \cos 2\theta) \right\} + \frac{r^4 A_4}{a^5 g} \left\{ \frac{l_4'}{14} (7 \cos 4\theta - 16 \cos 2\theta - 15) \right. \right. \\
 &\quad \left. \left. + \frac{h_4'}{56} (3 + 4 \cos 2\theta - 7 \cos 4\theta) \right\} \right] \cos 2(T + \phi - \epsilon), \\
 e_{\theta\phi}' &= \left[\frac{r^2 A_2}{a^3 g} (-4l_2' \cos \theta) + \frac{r^4 A_4}{a^5 g} l_4' \frac{1}{2} (7 \cos 3\theta - \cos \theta) \right] \sin 2(T + \phi - \epsilon), \\
 \gamma_s' &= \left[\frac{r^2 A_2}{a^3 g} (1 + k_2' - h_2') \sin 2\theta + \frac{r^4 A_4}{a^5 g} (1 + k_4' - h_4') \left(\sin 3\theta \cos \theta - \frac{9}{14} \sin 2\theta \right) \right] \\
 &\quad \times \cos 2(T + \phi - \epsilon), \\
 \gamma_w' &= \left[\frac{r^2 A_2}{a^3 g} (1 + k_2' - h_2') (1 - \cos 2\theta) + \frac{r^4 A_4}{a^5 g} (1 + k_4' - h_4') \left(\cos 2\theta + \frac{1}{6} \right) \sin \theta \right] \\
 &\quad \times \sin 2(T + \phi - \epsilon), \\
 A_2 &= 1.065 \text{ mb}, \\
 A_4 &= -0.876 \text{ mb}, \\
 \epsilon &= 146^\circ.
 \end{aligned}
 \tag{4}$$

According to I. M. Longman [1963], $h_2' = -1.007$, $k_2' = -0.310$, $l_2' = 0.030$, $h_4' = -1.059$, $k_4' = -0.133$ and $l_4' = 0.062$. Inserting these values and formula (1) into the formulas (4), we have following numerical values, for the linear strains

$$\begin{aligned}
 e_{\theta\theta}' &= 0.0876 \times 10^{-8} \cos(2t - 112.0^\circ), \\
 e_{\phi\phi}' &= 0.1044 \times 10^{-8} \cos(2t - 112.0^\circ), \\
 e_{\theta\phi}' &= 0.0202 \times 10^{-8} \sin(2t - 112.0^\circ), \\
 e_{S38^\circ W}' &= 0.0984 \times 10^{-8} \cos(2t - 117.9^\circ), \\
 e_{S52^\circ E}' &= 0.0944 \times 10^{-8} \cos(2t - 123.6^\circ),
 \end{aligned}$$

and for the tilts

$$\begin{aligned}
 \gamma_s' &= 0.000813'' \cos(2t - 292.0^\circ), \\
 \gamma_w' &= 0.000880'' \sin(2t - 292.0^\circ), \\
 \gamma_{N38^\circ E}' &= 0.000839'' \cos(2t - 152.2^\circ).
 \end{aligned}
 \tag{5}$$

On the other hand, we have the S_2 -components of the direct effects of the

earth tide using the result, $h_2=0.612$, $k_2=0.302$ and $l_2=0.083$, shown by H. Takeuchi *et al.* [1965] and others as follows, for strains

$$\begin{aligned}e_{\theta\theta} &= 0.630 \times 10^{-8} \cos 2t, \\e_{\phi\phi} &= 0.242 \times 10^{-8} \cos 2t, \\e_{\theta\phi} &= 0.168 \times 10^{-8} \sin(2t - 180^\circ), \\e_{S38^\circ W} &= 0.490 \times 10^{-8} \cos(2t - 9.6^\circ), \\e_{S52^\circ E} &= 0.398 \times 10^{-8} \cos(2t - 348.1^\circ),\end{aligned}$$

and for tilts

$$\begin{aligned}\eta_s &= 0.00237'' \cos 2t, \\\eta_w &= 0.00413'' \sin 2t, \\\eta_{N38^\circ E} &= 0.00314'' \cos(2t - 233.9^\circ).\end{aligned}\tag{6}$$

Comparing the results (5) and (6), our attentions are drawn to that the atmospheric tide effects are near to twenty percents as large as the direct effects due to the astronomical tide.

Considering the effects of the ground deformations caused by the oceanic tide, we find the amplitude ratio between the linear strain e'' by a point mass loading and that of the tilt η'' on the semi-infinite elastic body whose Lamé's elastic constants λ and μ is as follow,

$$\frac{e''}{\eta''} = \frac{\mu}{\lambda + \mu} \frac{\cos 2\alpha}{\sin \alpha}, \tag{7}$$

where α is the angle between the azimuth of the loading point and that of the observing direction. Averaging all the azimuths, we may make roughly estimation that the ratio e''/η'' is from 1/3 to 1/4. The present author [1967] has made estimation that the tidal tilt at Osakayama due to the oceanic tide is nearly equal to the direct effect. From these estimations, the atmospheric tide effects in the strain are nearly equal to the oceanic tide effects. Moreover, the oceanic tide effects in the areal and the vertical strains are nil on the ground surface, but that by the atmospheric tide Σ' does not cancel as follow,

$$\Sigma' = 0.1920 \times 10^{-8} \cos(2t - 112.0^\circ).$$

This value nearly amounts to twenty percents as large as the direct effect in the tidal strain.

The vertical component of tidal strain by the atmospheric tide is

$$\begin{aligned}e_{rr'} &= \frac{-2\lambda}{a(\lambda + 2\mu)} \{ (h_2' - 3l_2')w_2 + (h_4' - 10l_4')w_4 \} \\&\approx - \left(\frac{1}{3} \sim \frac{1}{2} \right) (e_{\theta\theta}' + e_{\phi\phi}') \\&\approx (0.064 \sim 0.096) \times 10^{-8} \cos(2t - 292.0^\circ).\end{aligned}\tag{9}$$

3. Observations

The observations concerning this paper have been performed at Osakayama Observatory, locating at $135^{\circ}51.5'$ of the east longitude and $34^{\circ}59.6'$ of the north latitude. The observing instruments are set on the rock bed of the clay slate stratum, at places from scores meters to one hundred meters under the ground. The amplitude of the annual variation of the room temperature is about 0.15°C . The one ends of the two tunnels for observing rooms are perfectly buried with a thick artificial hill. So, the air flow through the tunnels is cut perfectly.

The used instruments are H-59-B type extensometers, devised by I. Ozawa [1965] and a water-tube tiltmeter, devised by T. Eto [1965]. The observations have been performed continuously by the photographic curve methods. The influences of the atmospheric pressure changes and its gradient changes for the extensometers are negligible, because the sensitivities for the ground tilts are less than 0.01 times as large as those of the linear strains. The influence for tiltmeter is equal to the gradient of the atmospheric pressure in the observing room along the observing direction, and is going to be written in the paragraph of the discussions. The standards of these instruments are shown in the following table.

Instrument	Direction of Observation	Span of Observation	Sensitivity
H-59-B type Extensometer	S38°W	20.0 m	$0.070-0.198 \times 10^{-8}/\text{mm}$
H-59-B type Extensometer	S52°E	10.0 m	$0.220 \times 10^{-8}/\text{mm}$
Float type water-tube tiltmeter	N38°E	60.0 m	$0.00346''/\text{mm}$

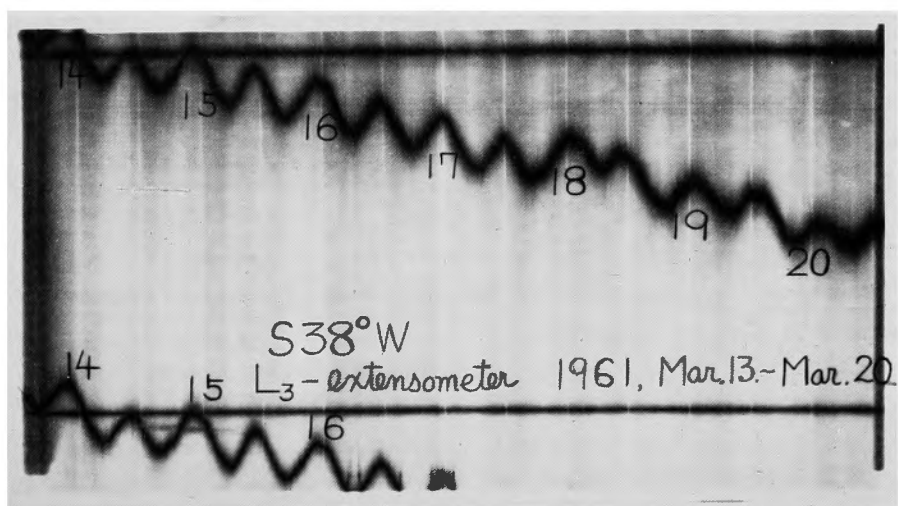


Photo. 1. The recording curve of the extensometer in the direction of S38°W at Osakayama.

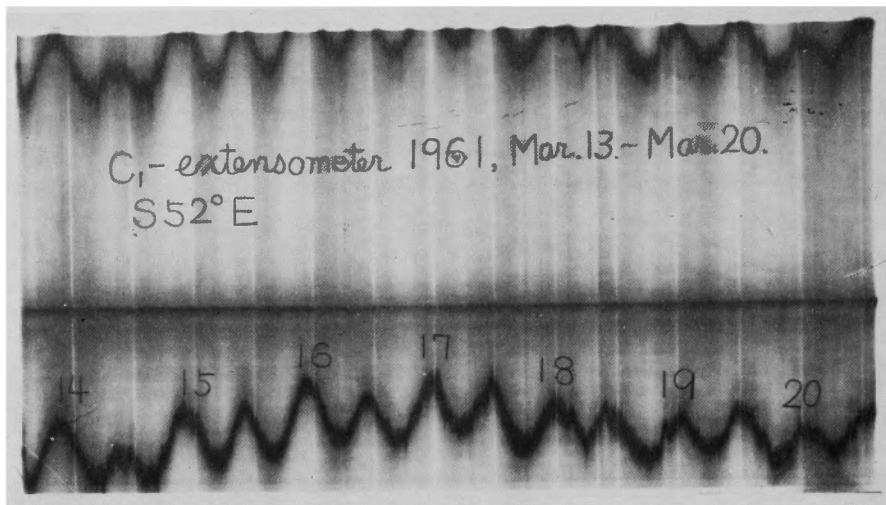


Photo. 2. The recording curve of the extensometer in the direction of S52°E at Osakayama.

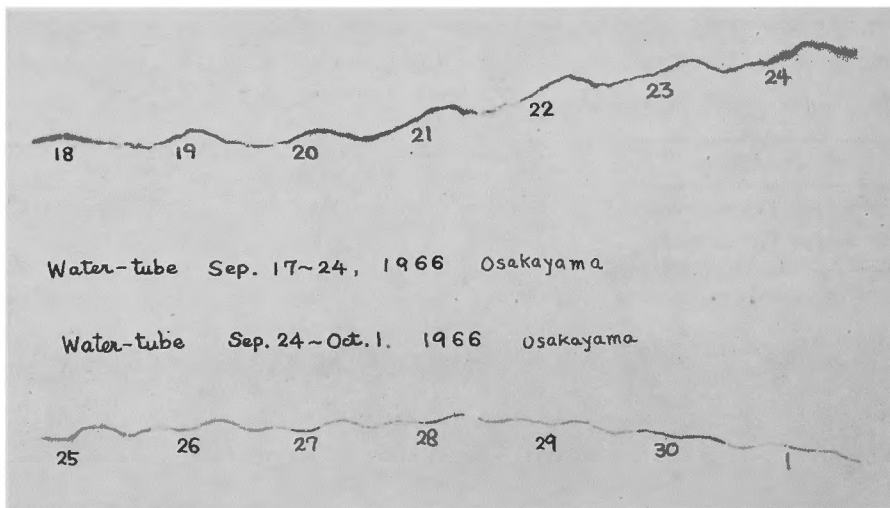


Photo. 3. The recording curve of the water-tube tiltmeter in the direction of N38°E at Osakayama.

We have the observations of the extensometers and the tiltmeter with no discontinuity for from six months to ten months for the tide analyses, respectively. The tide analyses are carried in the following processes.

- (1) The recording curves are read every thirty or sixty minutes.
- (2) The long period's changes are eliminated by means of the running mean's method of 25.0 or 24.5 hours' spans respectively.
- (3) The tidal constants are calculated by the reform method (Ozawa [1963]) of the Darwin's one.

Table 1.

Observing direction	Epoch of analysis	Nos. of analysis	Tidal component			
			S_1		S_2	
			Amplitude	Phase	Amplitude	Phase
S38°W extension	1960. 8. 25	9	0.243×10^{-8}	165.6°	0.132×10^{-8}	339.0°
S52°E extension	1960. 9. 30	9	1.066×10^{-8}	204.8°	0.755×10^{-8}	310.4°
N38°E tilt	1966. 8. 28	7	0.00595''	321.2°	0.00148''	321.2°

Observing direction	Tidal component							
	M_2		O_1		K_1		K_2	
	Amplitude	Phase	Amplitude	Phase	Amplitude	Phase	Amplitude	Phase
S38°W extension	0.457×10^{-8}	13.4°	0.131×10^{-8}	335.5°	0.245×10^{-8}	328.0°	0.056×10^{-8}	355.2°
S52°E extension	1.537×10^{-8}	353.5°	0.775×10^{-8}	8.5°	1.181×10^{-8}	27.2°	0.170×10^{-8}	351.2°
N38°E tilt	0.00616''	290.7°	0.00389''	330.9°	0.00616''	342.9°	0.00048''	242.6°

The analysed values are shown in the table (Table 1). We have the amplitude ratios among of tidal components, S_2/M_2 , K_2/M_2 and O_1/K_1 , from Table 1, and show them together with the respective theoretical values in the following.

Component	S_2/M_2		O_1/K_1		K_2/M_2	
	Observed	Theoretical	Observed	Theoretical	Observed	Theoretical
S38°W extension	0.291	0.467	0.534	0.710	0.123	0.126
S52°E extension	1.076	0.467	0.655	0.710	0.110	0.126
N38°E tilt	1.28	0.467	0.643	0.710	0.208	0.126

.....(10)

4. Discussions

We have the areal strains from these observations as follows

$$\begin{aligned}
 \Sigma M_2 &= 1.973 \times 10^{-8} \cos(2t - 358.0^\circ), \\
 \Sigma O_1 &= 0.821 \times 10^{-8} \cos(t - 359.7^\circ), \\
 \Sigma K_1 &= 1.322 \times 10^{-8} \cos(t - 14.6^\circ), \\
 \Sigma K_2 &= 0.235 \times 10^{-8} \cos(2t - 341.9^\circ), \\
 \Sigma S_2 &= 0.872 \times 10^{-8} \cos(2t - 314.6^\circ), \quad \dots\dots\dots(11)
 \end{aligned}$$

and the amplitude ratios O_1/M_2 , O_1/K_1 , K_2/M_2 and S_2/M_2 are 0.416 and 0.442, 0.629, 0.119, for their theoretical values 0.417 0.710. 0.126 and 0.467, respectively.

From these results, the phase of M_2 , K_2 , O_1 and K_1 components of areal strain are quite nil, but that of S_2 is big value, -45.4° . This is the reason why

there are no effect but the direct effect in the lunar tide like M_2 , K_2 , O_1 and K_1 . On the contrary, it shows that S_2 component contains some large factor beside the direct effect.

The theoretical value of the S_2 component of the direct areal strain is as follow,

$$0.872 \times 10^{-8} \cos 2t. \quad \dots\dots\dots(12)$$

Subtracting this theoretical S_2 component from that of the observed areal strain, we have following as the other effect

$$0.674 \times 10^{-8} \cos(2t - 247.3). \quad \dots\dots\dots(13)$$

The amplitude of this residual strain (13) is much large, and its phase is almost opposite to that of the calculated atmospheric tide effect. It seems that the reason why the both ends of the span in S52°E component is put near the walls of the gallery, and so the observed strain is influenced by the atmospheric pressure itself and by the effect of thermal stress. Other reasons are that the effect of the higher degree's terms of the pattern of the atmospheric tide are considerable, since at the higher degree, the larger its absolute value of h_n' become, or that the estimation of the direct effect is too large. According to the former of these reasons, we are able to obtain the effect for the observed value of S52°E direction's extension, which is the variation of the section of the gallery due to the atmospheric pressure itself and the thermal stress, subtracting the areal strain depending to the atmospheric tide effect on the uniform the residual strain (13) as follow,

$$0.529 \times 10^{-8} \cos(2t - 259.4^\circ).$$

The phase of this value is nearly equal to that of the atmospheric tide, 292° , and this result almost agree with the proportional coefficients for the linear strain in the other cases of the atmospheric pressure changes.

If the conditions of the gallery and the setting of the base line are improved, we shall be able to depress this influence due to the atmospheric pressure itself and the thermal stress.

Next, we consider the results of the water tube tiltmeter. If an observation is performing in the open-air, it will need the compensation for the gradient of the water-level within the instrument due to the gradient of the atmospheric pressure as follow.

$$0.000452'' \cos(2t - 146.6^\circ), \quad \text{in } N38^\circ E.$$

But, it needs not this compensation, because our observations have been performed in the closed air in the deep gallery. And an effect for this instrument due to the change itself of the atmospheric pressure is negligible.

We are going to calculate the loading tilt due to the oceanic tide. Subtracting the direct effect due to the astronomical tide from the observed value, we have

$$0.00349'' \cos(2t - 28.2^\circ),$$

and also subtracting the atmospheric effect, we get

$$0.00394'' \cos(2t - 18.0^\circ).$$

Similarly, we have the M_2 -component of the oceanic tide effect in the N38°E tilt as

$$0.00617'' \cos(2t - 357.1^\circ).$$

Comparing with S_2 and M_2 components of these effects, we are recognizable that, although the difference between these observed phase lags amount to 89° , the difference between their phases of the oceanic tide effect $S_2^{\circ''} - M_2^{\circ''}$, compensated the atmospheric tide effect for S_2 component, is 20.8° . This value is nearly equal to 25.4° which is the mean value of $S_2^\circ - M_2^\circ$ of the oceanic tide eighteen har-at bors at Kii Peninsula. And the amplitude ratio $S_2^{\circ''}/M_2^{\circ''}$ of the compensated oceanic tide effect, 0.64, is also near to the reasonable value, 0.45.

Summary

We estimate the effects for the tidal strains and tilt of the earth depending on the atmospherical tide (atmospheric tide effect) using the load-deformation coefficients of the earth calculated by I. M. Longman. According to this study the atmospheric effects amount to near 20% as large as the direct effects due to the astronomical tide generating force.

And we have been performing observations of the tidal strains with two components of H-59-B type's extensometers in the horizontal and orthogonal directions and a water-tube tiltmeter in N38°E direction at Osakayama.

According to our observations, the phase lags of S_2 -components much differ with those of M_2 -components in the linear strains and tilt too. The phases of the M_2 , K_2 , K_1 and O_1 components of the observed areal strain are nil, but S_2 -component that of is 315° . S_2 -component of the residual subtracting theoretical areal strain from the observed one has a considerably large amplitude and an opposite phase to the calculated one. It seems that the residual consists of the atmospheric tide effect on the uniform earth crust and the change of the section of the gallery due to the atmospheric pressure itself and the effect of the thermal stress, and we are able to estimate the change of the section of the gallery due to these effects.

These results show that the atmospheric tide effects are of lot of importance and our estimation for the effects are almost accurate.

Acknowledgements

In conclusion, the author wants to express his hearty thanks to the engineers in our institute, Messrs. K. Morimoto, K. Tanaka and S. Fujii, for their helps by making of the various instruments, and Mrs. M. Shimizu and Miss. C. Kimura for their helps by assistant calculations. And also he expresses his many thanks to Dr. R. Yamamoto for his lending of the literatures of the atmospheric tide.

References

- Chapman, S., 1956; A comparison of the annual solar and lunar atmospheric tides in barometric pressure, as regards their worldwide distribution of amplitude and phase, *J. Atmos. Terres. Phys.*, 8, 1-23.
- Eto, T., 1965; A recording water tube tiltmeter, *Disas. Prev. Res. Inst., Kyoto Univ., Ann.*, 8, 85-88 (in Japanese).
- Hagiwara, T., T. Rikitake, K. Kasahara and J. Yamada, 1949; Observations of the deformation of the earth's surface at Aburatsubo, Miura Peninsula, Part III, *Bull. Earthq. Res. Inst., Tokyo Univ.*, 27, 39-44.
- Haurwitz, B.; Geographical distribution of the solar semidiurnal pressure oscillation, *Metorological Papers, New York Univ.*, 2, No. 5, 1-36.
- Longman, I. M., 1963; A Green function for determining the deformation of the earth under surface mass loads, 2. Computations and numerical results, *J. Geophys. Res.*, 68, 485-496.
- Nishimura, E., 1950; On earth tide, *Trans. Amer. Geophys. Union*, 31, 357-376.
- Ozawa, I., 1957; Study on elastic strain of the ground in earth tides, *Disas. Prev. Res. Inst., Kyoto Univ., Bull.*, 15, 1-36.
- Ozawa, I., 1963; On the mincing of the reading-intervals in the earth tidal analyses, *J. Geod. Soc. Japan*, 9, 14-21 (in Japanese).
- Ozawa, I., 1965; On the extensometer of which magnifier is Zöllner suspension type tiltmeter, and the observations of the earth's strains by means of the instruments, *Anali Geofs., Roma*, 18, 263-278.
- Ozawa, I., 1967; On the tidal observation by means of the recording water-tube tiltmeter, *J. Geod. Soc. Japan*, 12, 151-156 (in Japanese).
- Takahashi, R., 1929; Tilting motion of the earthcrust caused by tidal loading, *Bull. Earthq. Res. Inst., Tokyo Univ.*, 6, 85-108.
- Takeuchi, H. and L. E. Alsop, 1965; Comparison of theoretical and observational expression for elastic strains of the earth, *Bull. Seis. Soc. Amer.*, 55, 153-163.