

GEPNER TYPE STABILITY CONDITION AND KUZNETSOV EQUIVALENCE

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ABSTRACT. This is a proceeding article of my talk ‘Gepner type stability condition and Kuznetsov equivalence’ at the Kinoshita Algebraic Geometry Symposium 2013.

1. INTRODUCTION

1.1. Conjectural Gepner type stability conditions. Historically, it has been observed that there is a curious relationship among two kinds of algebraic varieties: cubic fourfolds and K3 surfaces (cf. [BD85], [Voi86], [Has00], [Kuz10]). Our purpose is to apply the above classical observation to a modern problem in graded matrix factorizations.

Definition 1.1. *For a homogeneous polynomial*

$$W \in A := \mathbb{C}[x_1, x_2, \dots, x_n]$$

of degree d , a graded matrix factorization consists of data

$$P^0 \xrightarrow{p^0} P^1 \xrightarrow{p^1} P^0(d)$$

where P^i are graded free A -modules of finite rank, p^i are homomorphisms of graded A -modules, $P^i \mapsto P^i(1)$ is the shift of the grading, satisfying $p^1 \circ p^0 = p^0 \circ p^1 = \cdot W$.

The homotopy category $\text{HMF}(W)$ of graded matrix factorizations of W has a structure of a triangulated category. In general, there is the notion of stability conditions on triangulated categories by Bridgeland:

Definition 1.2. ([Bri07]) *A stability condition σ on a triangulated category $\mathcal{D}$ consists of a pair $(Z, \{\mathcal{P}(\phi)\}_{\phi \in \mathbb{R}})$*

$$Z: K(\mathcal{D}) \rightarrow \mathbb{C}, \quad \mathcal{P}(\phi) \subset \mathcal{D}$$

where Z is a group homomorphism (called central charge) and $\mathcal{P}(\phi)$ is a full subcategory (called σ -semistable objects with phase ϕ) satisfying the following conditions:

- *For $0 \neq E \in \mathcal{P}(\phi)$, we have $Z(E) \in \mathbb{R}_{>0} \exp(\sqrt{-1}\pi\phi)$.*
- *For all $\phi \in \mathbb{R}$, we have $\mathcal{P}(\phi + 1) = \mathcal{P}(\phi)[1]$.*
- *For $\phi_1 > \phi_2$ and $E_i \in \mathcal{P}(\phi_i)$, we have $\text{Hom}(E_1, E_2) = 0$.*

- For each $0 \neq E \in \mathcal{D}$, there is a collection of distinguished triangles

$$E_{i-1} \rightarrow E_i \rightarrow F_i \rightarrow E_{i-1}[1], \quad E_N = E, \quad E_0 = 0$$

with $F_i \in \mathcal{P}(\phi_i)$ and $\phi_1 > \phi_2 > \cdots > \phi_N$.

Let τ be the autoequivalence of $\text{HMF}(W)$ sending $P^\bullet$ to $P^\bullet(1)$. We are interested in constructing a specific type of a Bridgeland stability condition on $\text{HMF}(W)$, which has a symmetric property with respect to τ . It is formulated in the following conjecture [Wal05], [KST07], [Todc]:

Conjecture 1.3. *There is a Bridgeland stability condition*

$$\sigma_G = (Z_G, \{\mathcal{P}_G(\phi)\}_{\phi \in \mathbb{R}})$$

on $\text{HMF}(W)$, where the central charge Z_G is given by

$$Z_G \left(\bigoplus_{i=1}^N A(m_i) \right) \Leftrightarrow \bigoplus_{i=1}^N A(n_i) = \sum_{i=1}^N \left(e^{\frac{2\pi m_i \sqrt{-1}}{d}} - e^{\frac{2\pi n_i \sqrt{-1}}{d}} \right).$$

and the set of semistable objects satisfy $\tau \mathcal{P}_G(\phi) = \mathcal{P}_G(\phi + 2/d)$.

An expected stability condition in the above conjecture was called *Gepner type* in [Todb], as we will explain below.

1.2. Motivation for Conjecture 1.3. Below we explain the motivation of the above conjecture. Given a triangulated category $\mathcal{D}$, the set of Bridgeland stability conditions on $\mathcal{D}$ is known to form a complex manifold. If $\mathcal{D} = D^b \text{Coh}(X)$ for a Calabi-Yau manifold X , then the space of stability conditions $\text{Stab}(X)$ is expected to be related to the stringy Kähler moduli space $\mathcal{M}_K$ of X , that is the moduli space of complex structures of a manifold $X^\vee$ mirror to X . More precisely, the space $\mathcal{M}_K$ is conjectured to be embedded into the double quotient stack

$$[\text{Auteq}(X) \backslash \text{Stab}(X) / \mathbb{C}]$$

via solutions of Picard-Fuchs equations which the period integrals on $X^\vee$ satisfy. For instance if $X = (W = 0)$ is a quintic 3-fold in $\mathbb{P}^4$, then $\mathcal{M}_K$ is given by the quotient stack

$$\mathcal{M}_K = [\{\psi \in \mathbb{C} : \psi^5 \neq 1\} / \mu_5].$$

The point $\psi = \infty$ is called *large volume limit*, the point $\psi^5 = 1$ is called *conifold point* and the point $\psi = 0$ is called *Gepner point*, as described in Figure 1. We refer to [Toda] for the conjectural description of the embedding map.

Near the large volume limit, Bridgeland stability conditions are expected to be approximations of the classical Gieseker stability condition on $\text{Coh}(X)$. On the other hand, the stability condition at the Gepner

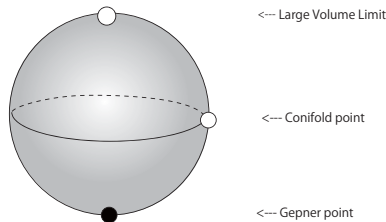


FIGURE 1. Stringy Kähler moduli space of a quintic 3-fold

point is expected to be a stability condition in Conjecture 1.3 (which is presumably unique in some sense) via Orlov equivalence [Orl09]

$$\mathrm{HMF}(W) \xrightarrow{\sim} D^b \mathrm{Coh}(X).$$

The name ‘Gepner type’ in [Todc] comes from the above expectation.

Another motivation comes from Donaldson-Thomas (DT) theory in [Tho00]. If there exists a desired stability condition σ_G in Conjecture 1.3, where W is a defining equation of a quintic 3-fold, then it should define the DT type invariant

$$\mathrm{DT}_G(\gamma) \in \mathbb{Q}, \quad \gamma \in \mathrm{HH}_0(W)$$

which counts σ_G -semistable graded matrix factorizations $P^\bullet$ satisfying $\mathrm{ch}(P^\bullet) = \gamma$. Here $\mathrm{HH}_0(W)$ is the Hochschild homology of $\mathrm{HMF}(W)$, and $\mathrm{ch}(\ast)$ is the Chern character map for graded matrix factorizations [PV12]. The Gepner type property in Conjecture 1.3 yields $\mathrm{DT}_G(\gamma) = \mathrm{DT}_G(\tau_\ast \gamma)$ which, together with the wall-crossing arguments [JS12], [KS], imply a non-trivial constraint among classical DT invariants on a quintic 3-fold. We expect that such a constraint is useful in computing DT invariants, and proving automorphic properties of the generating series of DT invariants predicted in string theory.

1.3. Main result. It has turned out that proving Conjecture 1.3 is a hard problem. A crucial issue is that there is no natural heart of a t-structure on $\mathrm{HMF}(W)$ which is intrinsic with respect to graded matrix factorizations. So far Conjecture 1.3 is known in the following cases: $n = 1$ [Tak], $d < n = 3$ [KST07], $n \leq d \leq 4$ [Todc], and some other weighted cases [KST07], [Todc]. The case $n = d = 5$ is the quintic 3-fold case, and we are not able to prove Conjecture 1.3 in this case at this moment. The strategy in [Todc] was to apply Orlov’s result [Orl09] which relates $\mathrm{HMF}(W)$ with $D^b \mathrm{Coh}(X)$ for $X = (W = 0) \subset \mathbb{P}^{n-1}$, and construct desired stability conditions in the geometric side.

Let us focus on the low degree cases of Conjecture 1.3. It is almost trivial to prove it in the $d \leq 2$ cases for any n , so the $d = 3$ case is the non-trivial lowest degree case.

Theorem 1.4. ([Todb]) *Conjecture 1.3 is true in the following cases:*

- $d = 3$ and $n \leq 5$.

- $d = 3$, $n = 6$ and the hypersurface $(W = 0) \subset \mathbb{P}^5$ is a general cubic fourfold containing a plane.

In the next sections, we will give an outline of the proof of the above theorem for $(d, n) = (3, 6)$ case.

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2. ORLOV/KUZNETSOV EQUIVALENCE

2.1. Orlov equivalence. Let W be a homogeneous polynomial with n variables of degree d . We recall Orlov's theorem [Orl09] which relates $\mathrm{HMF}(W)$ with the derived category of coherent sheaves on the hypersurface $X := (W = 0) \subset \mathbb{P}^{n-1}$ by semiorthogonal decompositions (SOD for short). Since we only use the case of $n > d$, we give a statement in this case.

Theorem 2.1. ([Orl09, Theorem 2.5]) *If $n > d$, then there is a fully faithful embedding for each $i \in \mathbb{Z}$*

$$\Phi_i: \mathrm{HMF}(W) \hookrightarrow D^b \mathrm{Coh}(X)$$

and SOD

$$D^b \mathrm{Coh}(X) = \langle \mathcal{O}_X(-i - n + d + 1), \dots, \mathcal{O}_X(-i), \Phi_i \mathrm{HMF}(W) \rangle.$$

In what follows we assume that $d = 3$, $n = 6$ so that $X = (W = 0)$ is a cubic fourfold in $\mathbb{P}^5$. Let $\mathcal{D}_X$ be the semiorthogonal summand of $D^b \mathrm{Coh}(X)$ defined by

$$(1) \quad D^b \mathrm{Coh}(X) = \langle \mathcal{O}_X(-3), \mathcal{O}_X(-2), \mathcal{O}_X(-1), \mathcal{D}_X \rangle.$$

By setting $\Phi = \Phi_1$ in the above notation, Orlov's theorem gives an equivalence

$$(2) \quad \Phi: \mathrm{HMF}(W) \xrightarrow{\sim} \mathcal{D}_X.$$

2.2. Geometry of cubic fourfolds containing a plane. Let $X = (W = 0) \subset \mathbb{P}^5$ be a cubic fourfold which contains a plane P . Let

$$\sigma: \tilde{X} \rightarrow X, \quad \pi: \tilde{X} \rightarrow \mathbb{P}^2$$

be the blow-up at P , the linear projection from P , respectively. The morphism π is a quadric fibration in the projectivization of the rank four vector bundle on $\mathbb{P}^2$, given by

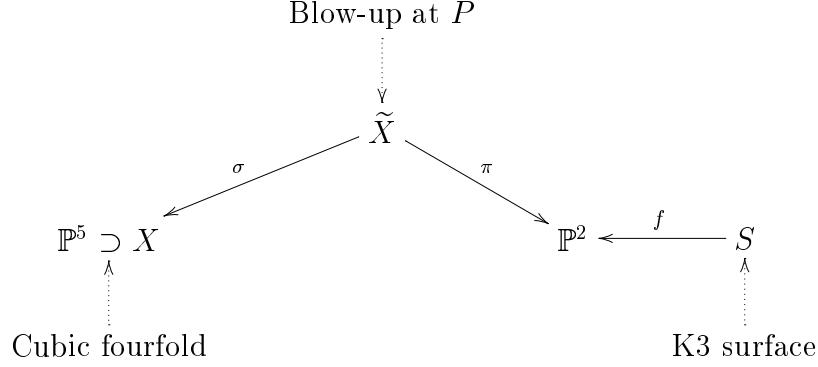
$$E = \mathcal{O}_{\mathbb{P}^2}^{\oplus 3} \oplus \mathcal{O}_{\mathbb{P}^2}(-1).$$

The degeneration locus of π is a sextic $C \subset \mathbb{P}^2$. Let

$$(3) \quad f: S \rightarrow \mathbb{P}^2$$

be the double cover branched along C . The curve C is non-singular for a general cubic fourfold containing a plane. In this case, the associated double cover S is a smooth projective K3 surface. In what follows, we

assume that the cubic fourfold X is general so that C is non-singular. We denote by H a hyperplane in $\mathbb{P}^5$ pulled back to $\tilde{X}$, and h is a hyperplane in $\mathbb{P}^2$ pulled back to $\tilde{X}$ or S . The relevant diagram in this subsection is summarized below:



2.3. Sheaves of Clifford algebras and twisted K3 surfaces. Similarly to the classical construction of Clifford algebras, the morphism π defines the sheaf of Clifford algebras on $\mathbb{P}^2$. It has an even part $\mathcal{B}_0$ and an odd part $\mathcal{B}_1$, which are described as

$$\begin{aligned}
 \mathcal{B}_0 &= \mathcal{O}_{\mathbb{P}^2} \oplus (\wedge^2 E \otimes \mathcal{O}_{\mathbb{P}^2}(-1)) \oplus (\wedge^4 E \otimes \mathcal{O}_{\mathbb{P}^2}(-2)) \\
 \mathcal{B}_1 &= E \oplus (\wedge^3 E \otimes \mathcal{O}_{\mathbb{P}^2}(-1)).
 \end{aligned}$$

We also define $\mathcal{B}_i$ for $i \in \mathbb{Z}$ by the rule $\mathcal{B}_{i+2} = \mathcal{B}_i(1)$. By [Kuz08, Corollary 3.9], every sheaves $\mathcal{B}_i$ are flat over $\mathcal{B}_0$ and we have

$$\mathcal{B}_i \otimes_{\mathcal{B}_0} \mathcal{B}_j \cong \mathcal{B}_{i+j}, \quad \text{for all } i, j \in \mathbb{Z}.$$

In particular, for every i there is an equivalence of abelian categories

$$\otimes_{\mathcal{B}_0} \mathcal{B}_i: \text{Coh}(\mathcal{B}_0) \xrightarrow{\sim} \text{Coh}(\mathcal{B}_0).$$

Here $\text{Coh}(\mathcal{B}_0)$ is the abelian category of coherent right $\mathcal{B}_0$ -modules on $\mathbb{P}^2$.

Let S be the K3 surface obtained as a double cover (3). By [Kuz08, Section 3.5], there exists a sheaf of Azumaya algebras $\mathcal{B}_S$ on S such that $f_* \mathcal{B}_S = \mathcal{B}_0$, and an equivalence

$$f_*: \text{Coh}(\mathcal{B}_S) \xrightarrow{\sim} \text{Coh}(\mathcal{B}_0).$$

The abelian categories $\text{Coh}(\mathcal{B}_0)$, $\text{Coh}(\mathcal{B}_S)$ are also described in terms of twisted sheaves. There exists an element in the Brauer group

$$\alpha \in \text{Br}(S) = H^2(S, \mathcal{O}_S^*), \quad \alpha^2 = \text{id}$$

and an α -twisted vector bundle $\mathcal{U}_0$ of rank two such that $\mathcal{B}_S = \text{End}(\mathcal{U}_0)$ and the functor

$$\text{Coh}(S, \alpha) \ni F \mapsto \mathcal{U}_0^\vee \otimes F \in \text{Coh}(\mathcal{B}_S)$$

is an equivalence. Here $\text{Coh}(S, \alpha)$ is the abelian category of α -twisted coherent sheaves on S (cf. [HS05, Section 1]). Combined with the above equivalences, we obtain the equivalence

$$(4) \quad \Upsilon(-) := f_*(\mathcal{U}_0^\vee \otimes -): D^b \text{Coh}(S, \alpha) \xrightarrow{\sim} D^b \text{Coh}(\mathcal{B}_0).$$

2.4. Orlov/Kuznetsov equivalence. Let $\mathcal{D}_X$ be the semiorthogonal summand of $D^b \text{Coh}(X)$ given by (1). In [Kuz10], Kuznetsov established an equivalence between $D^b \text{Coh}(\mathcal{B}_0)$ and $\mathcal{D}_X$. A starting point is the fully faithful functor

$$\Psi: D^b \text{Coh}(\mathcal{B}_0) \rightarrow D^b \text{Coh}(\tilde{X})$$

constructed in [Kuz08], defined as a Fourier-Mukai transform

$$\Psi(-) = \pi^*(-) \otimes_{\pi^* \mathcal{B}_0} \mathcal{E}.$$

Here $\mathcal{E}$ is a sheaf of left $\pi^* \mathcal{B}_0$ -modules on $\tilde{X}$ given by the cokernel of the canonical surjection

$$\pi^* \mathcal{B}_0(-2H) \rightarrow \pi^* \mathcal{B}_1(-H) \rightarrow \mathcal{E} \rightarrow 0.$$

As $\mathcal{O}_{\tilde{X}}$ -module, the sheaf $\mathcal{E}$ is locally free of rank four. Kuznetsov [Kuz10] performs a sequence of mutations of SOD of $D^b \text{Coh}(\tilde{X})$, and proves the following result:

Theorem 2.2. ([Kuz10]) *The functor*

$$\Theta: D^b \text{Coh}(\mathcal{B}_0) \rightarrow \mathcal{D}_X$$

given by

$$\begin{aligned} \Theta(F) = \text{Tot}\{ & \mathbf{R} \text{Hom}(\mathcal{O}_{\tilde{X}}(h - H), \Psi(F)) \otimes I_P \rightarrow \mathbf{R} \sigma_* \Psi(F) \\ & \rightarrow \mathbf{R} \text{Hom}(\Psi(F), \mathcal{O}_{\tilde{X}}(-h))^\vee \otimes \mathcal{O}_X(-1)\}. \end{aligned}$$

is an equivalence. Here $I_P \subset \mathcal{O}_X$ is the ideal sheaf of P .

We summarize the equivalences obtained so far in the following corollary:

Corollary 2.3. *There is a sequence of equivalences*

$$D^b \text{Coh}(S, \alpha) \xrightarrow{\Upsilon} D^b \text{Coh}(\mathcal{B}_0) \xrightarrow{\Theta} \mathcal{D}_X \xleftarrow{\Phi} \text{HMF}(W).$$

Here Υ is given in (4), Θ is given in Theorem 2.2 and Φ is given in (2).

3. OUTLINE OF THE PROOF OF THEOREM 1.4 FOR $(d, n) = (3, 6)$

Step 1. *Description of the grade shift functor.*

Our first step is to describe the grade shift functor τ on $\mathrm{HMF}(W)$ in terms of $D^b \mathrm{Coh}(\mathcal{B}_0)$. We define the autoequivalence F_B of $D^b \mathrm{Coh}(\mathcal{B}_0)$ to be

$$F_B := \mathrm{ST}_{\mathcal{B}_1}^{-1} \circ \otimes_{\mathcal{B}_0} \mathcal{B}_{-1}[1].$$

Here $\mathrm{ST}_{\mathcal{B}_1}$ is the Seidel-Thomas twist [ST01] associated to $\mathcal{B}_1$:

$$\mathrm{ST}_{\mathcal{B}_1}(-) = \mathrm{Cone}(\mathbf{R} \mathrm{Hom}(\mathcal{B}_1, -) \otimes \mathcal{B}_1 \rightarrow -).$$

We have the following proposition:

Proposition 3.1. ([Todb, Corollary 3.4]) *The following diagram commutes:*

$$\begin{array}{ccc} D^b \mathrm{Coh}(\mathcal{B}_0) & \xrightarrow{\Phi^{-1} \circ \Theta} & \mathrm{HMF}(W) \\ F_B \downarrow & & \downarrow \tau \\ D^b \mathrm{Coh}(\mathcal{B}_0) & \xrightarrow{\Phi^{-1} \circ \Theta} & \mathrm{HMF}(W). \end{array}$$

The above proposition is proved in the following way: the functor τ is described in terms of $\mathcal{D}_X$ under Orlov equivalence Φ as $F_X = \mathrm{ST}_{\mathcal{O}_X} \circ \otimes_{\mathcal{O}_X}(1)$ by [BFK12]. It is enough to prove the commutativity of

$$\begin{array}{ccc} D^b \mathrm{Coh}(\mathcal{B}_0) & \xrightarrow{\Theta} & \mathcal{D}_X \\ F_B \downarrow & & \downarrow F_X \\ D^b \mathrm{Coh}(\mathcal{B}_0) & \xrightarrow{\Theta} & \mathcal{D}_X. \end{array}$$

The above commutativity is proved by proving the commutativity for objects of the form $\Upsilon(\mathcal{O}_x)$ with $x \in S$, and the commutativity of some numerical classes of objects.

Step 2. *Description of the central charge Z_G .*

The next step is to describe the central charge Z_G in terms of α -twisted sheaves on the K3 surface S . Recall that by Corollary 2.3, there is a sequence of equivalences

$$D^b \mathrm{Coh}(S, \alpha) \xrightarrow{\Upsilon} D^b \mathrm{Coh}(\mathcal{B}_0) \xrightarrow{\Theta} \mathcal{D}_X \xleftarrow{\Phi} \mathrm{HMF}(W).$$

Let Z_G be the canonical central charge on $\mathrm{HMF}(W)$ given in Conjecture 1.3. We compute the pull-back of the central charge Z_G on $\mathrm{HMF}(W)$ by the above sequence of equivalences, using the result of Proposition 3.1. The resulting central charge on $D^b \mathrm{Coh}(S, \alpha)$ coincides with an integral over S which appeared in Bridgeland's paper [Bri08]:

Proposition 3.2. ([Todb, Proposition 4.7]) *There is an element $B \in H^{1,1}(S, \mathbb{Q})$ and $c \in \mathbb{C}^*$ such that we have*

$$Z_G \circ \Phi^{-1} \circ \Theta \circ \Upsilon(E) = c \cdot \int_S e^{B - \frac{\sqrt{-3}}{4}h} \text{ch}(E) \sqrt{\text{td}_S}$$

for any $E \in D^b \text{Coh}(S, \alpha)$. Here h is a hyperplane in $\mathbb{P}^2$ pulled back to S .

The Chern character on $D^b(S, \alpha)$ is the *untwisted* Chern character, defined to be the twisted Chern character by Huybrechts-Stellari [HS05], multiplied by the exponential of the minus of the B-field to get back to the untwisted one. Although it takes its value in an algebraic class, it is no longer defined in the integer coefficient.

Step 3. *Construction of a Gepner type stability condition.*

The final step is to construct a corresponding Gepner type stability condition on $D^b \text{Coh}(S, \alpha)$, using the above descriptions of the grade shift functor and the central charge. In this step, we need a further genericity assumption: the Brauer class α is non-trivial. This condition is not satisfied only if X lies in a union of countable many hypersurfaces in the moduli space of cubic fourfolds containing a plane. Let Z'_G be the central charge on $D^b \text{Coh}(S, \alpha)$ defined by

$$Z'_G(E) = - \int_S e^{B - \frac{\sqrt{-3}}{4}h} \text{ch}(E) \sqrt{\text{td}_S}.$$

By the arguments so far, the following result obviously implies Theorem 1.4 as desired:

Theorem 3.3. ([Todb, Theorem 4.13]) *Suppose that $\alpha \neq 1$. Then there is a Bridgeland stability condition $\sigma'_G = (Z'_G, \{\mathcal{P}'(\phi)\}_{\phi \in \mathbb{R}})$ on $D^b \text{Coh}(S, \alpha)$ satisfying*

$$\Upsilon^{-1} \circ F_B \circ \Upsilon \circ \mathcal{P}'(\phi) = \mathcal{P}'(\phi + 2/3).$$

The proof relies on a standard technique on constructions of stability conditions on K3 surfaces [Bri08]. It requires proving the non-existence of ‘bad’ spherical objects, in which we use the $\alpha \neq 1$ assumption.

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