

General Heart Construction

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Oct. 28 2009, Kinosaki Symposium

(j.w. with Noriyuki Abe)

Aim

Give an abstract construction

Triangulated category

$\mathcal{C}$

Abelian category

‘heart’

Setting

$(\mathcal{U}, \mathcal{V})$: a pair of subcategories of $\mathcal{C}$

- $\text{Ext}^1(\mathcal{U}, \mathcal{V}) = 0$
- $\forall C \in \text{Ob}(\mathcal{C}), \exists_{\text{dist.}\Delta} U \rightarrow C \rightarrow V[1] \rightarrow U[1]$
($U \in \text{Ob}(\mathcal{U}), V \in \text{Ob}(\mathcal{V})$)

generalizing both cases:

- 1 t -structure ($\Leftrightarrow \mathcal{V} \subseteq \mathcal{V}[1]$)

$$(\mathcal{T}^{\leq 0}, \mathcal{T}^{\geq 0}) \rightarrow \mathcal{T}^{\leq 0} \cap \mathcal{T}^{\geq 0}$$

- 2 cluster tilting ($\Leftrightarrow \mathcal{V} = \mathcal{U}$)
subcategory

$$\mathcal{T} \subseteq \mathcal{C}$$

$$\mathcal{C}/\mathcal{T}$$

(※Any subcategory is assumed to be full, additive, thick and replete.)

Theorem 1 (N-)

$$\mathcal{C} \supseteq \mathcal{H} :=$$

Then

$$\underline{\mathcal{H}} := \mathcal{H}/(\mathcal{U} \cap \mathcal{V}) \text{ is abelian.}$$

Full subcategory

$C \in \text{Ob}(\mathcal{C})$ admitting $\text{dist.}\Delta$'s

$$V \rightarrow W_1 \rightarrow C \rightarrow V[1]$$

$$U[-1] \rightarrow C \rightarrow W_2 \rightarrow U$$

$$(\exists U \in \mathcal{U}, \exists V \in \mathcal{V}, \exists W_1, W_2 \in \mathcal{U} \cap \mathcal{V})$$

generalize

1

&

2

Theorem 2 (Abe, N-)

$\exists H : \mathcal{C} \longrightarrow \underline{\mathcal{H}}$ a cohomological functor

Composition of
some adjoints and
the quotient