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N-gonal multilayer symmetric revolute linkage deployed from bundle to surface of revolution

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Deployable structures composed of hinge joints (revolute joints) with inclined axis have a potential to realize a rich variety of deformation with a small number of members, which enables it to be lightweight with low-risk of failure. In this paper, a systematic method is proposed to obtain new hinge-jointed structures which have N -dihedral symmetry and can be folded into a straightly bundled shape. The proposed structure, which we call N -MLSRL, is composed of some layers. Each layer is an assemblage of $2N$ bars and $2N$ inclined hinges, which can be deformed from a regular $2N$ -gonal frame into an entirely straight rod shape, which is a generalization of a 4-bar Bennett linkage. For the case N is less than or equal to 3, the N -MLSRL has a single degree of freedom. For an application to design of deployable structures, a method is introduced to generate the structure expanded to a predefined target surface of revolution. For designing the realistic detail considering the finite sizes of the joints, a technique called hinge offsets is used. The proposed method is applied to a horn-shaped structure, a ball-shaped structure and a dome-shaped structure as its numerical examples. The validity of the proposed method is confirmed by a physical model of a dome-shaped structure including hinge offsets.

1 INTRODUCTION

Deployable frames, which allow large deformation without any strain in each member, has a wide range of application in various fields of engineering. In the field of mechanics, deployable frames are applied to many in-

dustrial usage such as aerospace antennas, lifters or grabbers of robots. For architectural applications, deployable frames can be used for retractable roofs, building environmental equipments such as automatic sunshade devices, and temporary structural frames such as deployable shelter and outdoor-tent for temporal use. The classifications of such deployable frames are proposed by several researchers, (for details, see Fenci and Currie [1]). For example, various spatial mechanisms which are composed of several scissor-like elements (SLEs) have been studied by many researchers, e.g., Gantes [2], Temmerman [3] and Tsuda et al. [4] for specific architectural applications. While the scissor mechanism has the advantage with its simple design methodology and large potential for various fields of application, this mechanism tends to include a large number of members and joints, which sometimes causes a high risk of failure.

As another example of deployable frame, a spatial linkage which includes inclined hinges can be considered. In this frame, members are connected with revolute joints (hinge joints). In contrast to SLEs, directions of axes of inclined hinges are not limited to be parallel, which leads to a 3D deformation of the whole frame. A closed-loop linkage with revolute joints has been studied as one of the basic components of the hinge-jointed movable structure, and has been applied to many industrial usage. In this linkage, multiple rigid bodies (members) are connected by the same number of hinge joints to form a ring. If the hinge axes are all parallel to each other and perpendicular to the plane of the ring, the linkage is a 2D mechanism in which the displacements

of all members are parallel to the plane. The majority of engineering applications of hinge-connected closed-loop linkages are 2D. On the other hand, if the hinge axes are inclined (not parallel to each other), each member exhibits 3D displacements. Due to the difficulty in dealing with its non-linear geometric compatibility conditions, closed-loop linkages with 3D hinge joints have been mainly limited to some academic study and had few industrial applications. As some examples of application, Qi et al. [5] proposed a large ring-shaped structure consisting of plane-symmetric Bricard linkages [6]. Other than assembling closed-loop linkages, an optimization-based method to generate hinge-jointed deployable frame is also proposed by Ohsaki et al. [7].

According to the Chebychev-Grübler-Kutzbach mobility criterion [8], closed-loop linkages with inclined hinges generally have mobility if the number of members is greater than or equal to 7. While in this case the degree-of-freedom (DOF) of the linkage is generally larger than 1, recently it has been shown numerically that the linkage can deform one-dimensionally if a particular set of angles of hinge axes is selected (see Schönke et al. [9] and Kaji et al. [10] for detail). On the other hand, even if the number of members is 6 or less, the linkage still has mobility if some special conditions are satisfied, which is called *overconstrained mechanism* and has a long history of studies. The minimum number of members of overconstrained mechanism is 4; thus, overconstrained linkages with revolute joints have four, five and six members, which are called $4R$, $5R$ and $6R$ linkages, respectively. These linkages have been studied by various researchers, e.g., Bennett [11] ($4R$), Myard [12] and Goldberg [13] ($5R$), and Sarrus [14], Bricard [6], Goldberg [13] and various other types [15, 16, 17] for $6R$ linkages. Most of the $5R$ and $6R$ linkages can be regarded as combinations or derivatives of two typical $4R$ Bennett linkage and $6R$ Bricard linkage [18]. As an application of the linkage to architectural and other large deployable structures, network or tiling patterns of these linkages are proposed by, e.g. Chen [18] and Chen and You [19]. Chen [18] used Bennett linkage and threefold-symmetric Bricard linkage [13, 20] to propose foldable network structures. These two linkages have a common property; they are both foldable from the shapes of regular $2N$ -sided polygons, i.e., a square ($N = 2$) and a regular hexagon ($N = 3$), into completely folded bundles. Gan and Pellegrino [21] proposed a scheme for numerical analysis to simulate deformation paths of these linkages. In these researches, compatibility equations of these linkages are derived using approaches mainly following Denavit and Hartenberg [22]. For the cases of $N \geq 4$, a similar mechanism is also considered by Crawford et al. [23], although in these cases

appropriate constraints must be added to have a single DOF.

In this paper, using the $2N$ -gonal closed-loop linkages, we propose a systematic method to obtain new hinge-jointed structures which have N -dihedral (D_N) symmetry, i.e., N rotational and N reflectional symmetries, and can be folded into a straightly bundled shape, for application to architectural engineering and various other fields. First, a general method is introduced to describe the motion of these $2N$ -gonal linkages introducing some dimension variables with respect to a single parameter along the deformation path. Next, a new deployable structure is proposed by stacking these linkages vertically and connecting them by additional revolute joints with horizontal axes. We derive a systematic procedure to express the deformation process of this new deployable structure by a single parameter. It is shown that the shape of the new deployable structure can be controlled by defining a surface of revolution on which all joints of the structure are located when it is fully opened. This structure has a single DOF if $N = 3$. When $N \geq 4$, DOF of the proposed structure is greater than 1 but it can be used as a base model whose DOF can be reduced effectively by various ways, e.g., controlling rotation angles of all hinges synchronously by rotary actuators, although these methods are not discussed in detail in this paper. A method of introducing hinge offsets for dealing with real (finite) sizes of the members and revolute joints, which is inevitable for designing the realistic details for actual fabrication, is also discussed in this paper. The proposed method is applied to generation of a horn-shaped structure and a ball-shaped structure as numerical examples, and validated by designing and fabricating a physical model of a dome-shaped structure, while the design methodology we propose can be applied to generate other variety of deployable structures with N -dihedral symmetry.

2 N -GONAL SYMMETRIC REVOLUTE LINKAGE

2.1 Definition

In a cartesian coordinate system xyz , a regular $2N$ -gonal frame inscribed in a circle $x^2 + y^2 = 1$ is considered as shown in Fig. 1. The frame consists of $2N$ members which have the same length $L = 2 \sin \frac{\pi}{2N}$ and the adjacent members are connected by inclined hinge joints (revolute joints). These joints, if we ignore the size of their hinges, can be regarded as nodes between members. The first point of them is placed on x axis, which we call P^1 , and the other points are named $Q^1, P^2, Q^2, \dots, P^N, Q^N$ in the order along the circle as shown in Fig. 1. At each node P^k and Q^k ($k = 1, \dots, N$) whose position vec-

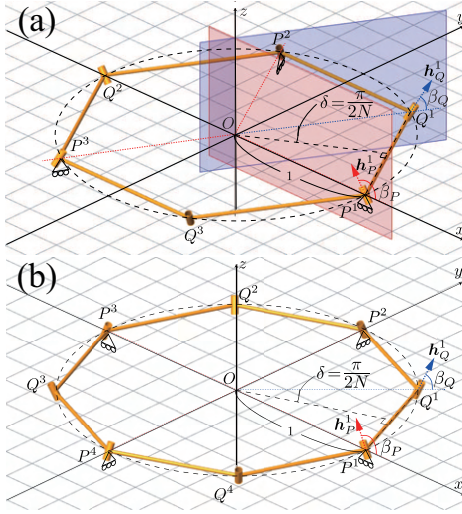


Fig. 1: N -SRL; (a) Case $N = 3$, (b) Case $N = 4$

tors are denoted by $\mathbf{p}^k \in \mathbb{R}^3$ and $\mathbf{q}^k \in \mathbb{R}^3$, respectively, two adjacent members are connected by inclined hinge joints. In this study, we simply call them as P -hinge and Q -hinge, respectively. Let $\mathbf{h}_P^k \in \mathbb{R}^3$ ($k = 1, \dots, N$) denote the unit direction vectors of the hinge axes at P^k , where $\mathbf{h}_P^k$ is set parallel to the plane containing P^k and z axis, and the angle between two vectors $\mathbf{h}_P^k$ and $\mathbf{p}^k$ is equal to β_P as shown in Fig. 1. Similarly, $\mathbf{h}_Q^k \in \mathbb{R}^3$ ($k = 1, \dots, N$), the direction vectors of the hinge axes at Q^k , are set to be parallel to the plane containing Q^k and z axis with the angle between two vectors $\mathbf{h}_Q^k$ and $\mathbf{q}^k$ equal to β_Q . In this study, β_P and β_Q are defined by $\beta_P = \pi - \beta_0$ and $\beta_Q = \beta_0$, respectively, where β_0 is a constant angle determined by the following equation [23] (see Appendix for detail) using $\delta = \pi/(2N)$ as shown in Fig. 1:

$$\begin{aligned} \beta_0 &= \frac{\pi}{2} - \arctan\left(\frac{\sin \delta}{\cos^2 \delta}\right) \\ &= \arctan\left\{\frac{1}{(\tan \delta)\sqrt{1 + (\tan \delta)^2}}\right\} \end{aligned} \quad (1)$$

For the sake of simple notation, we call the hinge-jointed frame in Fig. 1 as a N -gonal symmetric revolute linkage (N -SRL). If $N = 2$ or 3, the frame is well known as a Bennett linkage [11] or threefold-symmetric Bricard linkage [6, 20], respectively, and also known to be classified as *over-constrained mechanism* which means that each mechanism has a single DOF and deforms from regular planar polygonal shape (hereinafter called a *planarly deployed state*) to a state in which all members are bundled into a straight rod shape (named as a *completely folded*

state in this paper) as shown in Fig. 2. During the deformation, it is known that these members and hinge axes retain N rotational symmetry and N reflection symmetry. For the case $N = 3$, Chen et al. [20] showed a threefold-symmetric Bricard linkage that has a bifurcation point at its completely folded state where its DOF increases from 1 to 2. Then, they also showed that a snap-through phenomenon to another compatible path may occur when approaching this bifurcation point if the stiffness of members or hinges is weak. These properties are also true for the N -MLSRL proposed in this study, and care should be taken when designing the N -MLSRL and folding it in practice.

For the case $N \geq 4$, N -SRL is not a stable mechanism by itself since it has more than one DOF. However, Crawford et al. [23] showed that a stable mechanism path from a planarly deployed state to a completely folded state can be provided even if $N \geq 4$ when additional supports or constraints are introduced.

Deformation of N -SRL is studied in detail by Crawford et al. [23]. They expressed an attitude of one member of N -SRL and the directions of hinge axes at its ends by three angles; roll θ_r , pitch θ_p and yaw θ_y , and derived a relationship between these angles (See Eqns. (32a) and (32b) in Appendix for detail). They also expressed the directions of hinge axes β_P and β_Q at member's ends using the three angles as Eqn. (34).

There exist two sets of $(\theta_r, \theta_p, \theta_y)$ satisfying these equations as shown in Eqn. (33). In the next section, a unified approach is proposed to express these two sets by a single parameter.

2.2 Parameter representing deformed state of N -SRL

Here, we introduce a parameter $t \in \mathbb{R}$ defined between -1 and 1 and parameterize the three angles; roll θ_r , pitch θ_p and yaw θ_y related by Eqns. (32a) and (32b) in Appendix, by t as follows:

$$\hat{\theta}_p(t) = \frac{\pi}{2} |t| \quad (-1 \leq t \leq 1) \quad (2)$$

$$\hat{\theta}_r(t) = \begin{cases} \theta_r^+(\theta_p(t)) & (0 \leq t \leq 1) \\ \theta_r^-(\theta_p(t)) & (-1 \leq t \leq 0) \end{cases} \quad (3)$$

$$\hat{\theta}_y(t) = \begin{cases} \theta_y(\theta_p(t), \theta_r^+(\theta_p(t))) & (0 \leq t \leq 1) \\ \theta_y(\theta_p(t), \theta_r^-(\theta_p(t))) & (-1 \leq t \leq 0) \end{cases} \quad (4)$$

where the notation $\hat{(\cdot)}$ represents that these angles are parameterized by t . We call t as a *state parameter*. When t is 0, the N -SRL is in a planarly deployed state with the regular polygonal shape. Furthermore, the N -SRL

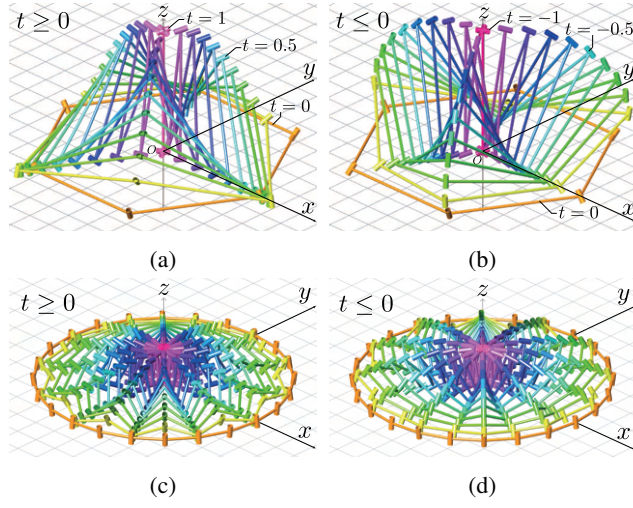


Fig. 2: Deformation path of N -SRL; (a) Positive path with $N = 3$, (b) Negative path with $N = 3$, (c) Positive path with $N = 12$, (d) Negative path with $N = 12$

has two directions of movements to the bundled states as shown in Fig. 2. The first direction is that nodes P^k move outward of the circle. We name it as a *positive path* represented by $t \geq 0$. This deformation continues until it is folded in a bundle at $t = 1$. The other direction that nodes P^k move inward is named a *negative path* denoted by $t \leq 0$. This path also leads to a bundle at $t = -1$. We remark that any deformed state of N -SRL is uniquely and continuously associated with the parameter t between -1 and 1 . As can be seen in Figs. 2, these frames and hinge axes retain N rotational and N reflection symmetries during deformation [23]. Hereinafter, $\theta_p, \theta_r, \theta_y$ are always considered as functions of t , and the notation $(\dot{})$ is omitted.

The parameter t physically means the pitch θ_p divided by $\pi/2$, representing the attitude of each member during deformation. Here, since N -SRLs have two different paths to the same pitch θ_p , we adopt the absolute value symbol in Eqn. (2) so that t corresponding to each of the two paths can be defined with a positive or negative sign.

2.3 Dimension variables of N -SRL

Next, consider new dimension variables r, s and v representing the state of the N -SRL as shown in Fig. 3. The pitch θ_p and yaw θ_y are also shown in the figure. From geometrical relationships in Fig. 3, the following

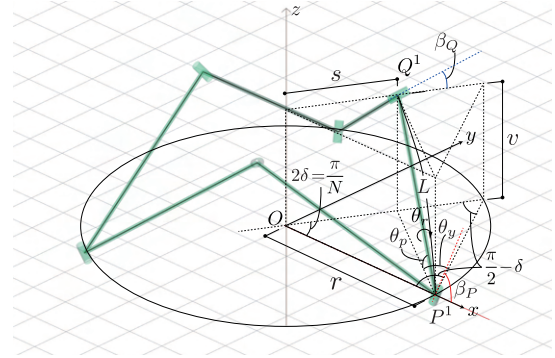


Fig. 3: Dimension variables of N -SRL ($N = 3$)

three equations hold.

$$v = L \sin \theta_p \quad (5)$$

$$s^2 = r^2 + (L \cos \theta_p)^2 - 2rL \cos \theta_p \sin(\theta_y + \delta) \quad (6)$$

$$r^2 = s^2 + (L \cos \theta_p)^2 + 2sL \cos \theta_p \sin(\theta_y - \delta) \quad (7)$$

From Eqns. (6) and (7), the following equation is obtained for general $\theta_p \neq \frac{\pi}{2}$.

$$s \sin(\theta_y - \delta) = r \sin(\theta_y + \delta) - L \cos \theta_p \quad (8)$$

Substituting s of Eqn. (8) to Eqn. (7), the following quadratic equation is derived.

$$\begin{aligned} & (\sin^2 \Delta_2 - \sin^2 \Delta_1) r^2 \\ & + 2L \cos \theta_p \sin \Delta_1 (1 - \sin^2 \Delta_2) r \\ & - L^2 \cos^2 \theta_p (1 - \sin^2 \Delta_2) = 0 \end{aligned} \quad (9)$$

where $\Delta_1 = \delta + \theta_y$ and $\Delta_2 = \delta - \theta_y$. Solving Eqn. (9) for r , the following equation is finally obtained.

$$r = L \cos \theta_p \times \frac{\pm |\cos \Delta_2 \sin \Delta_2 \cos \Delta_1| - \cos^2 \Delta_2 \sin \Delta_1}{\sin^2 \Delta_2 - \sin^2 \Delta_1} \quad (10)$$

Note that, in the right-hand side of Eqn. (10), the sign before the absolute value is determined so that r and s have valid positive values.

Using Eqns. (10), (8) and (5), respectively, dimension variables r, s and v are derived from θ_p, θ_r and θ_y . Then, incorporating Eqns. (2)-(4), we finally obtain r, s and v as the functions of t , which are denoted as $r(t), s(t)$

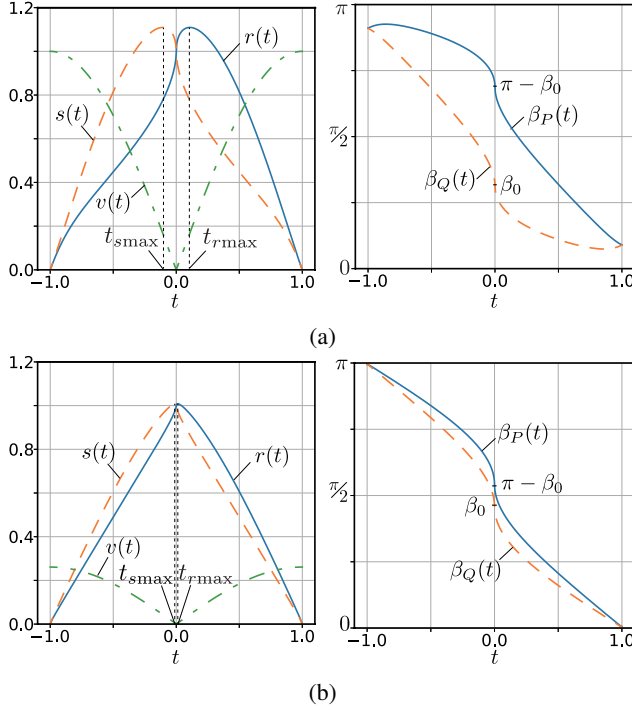


Fig. 4: Dimension variables r, s, v, β_P and β_Q of N -SRL; (a) Case $N = 3$, (b) Case $N = 12$

and $v(t)$, respectively. The shapes of $r(t)$, $s(t)$ and $v(t)$ for $N = 3$ and $N = 12$ are shown as examples in Fig. 4, where $t_{r \max}$ and $t_{s \max}$ are defined as

$$t_{r \max} = \arg \max_{-1 \leq t \leq 1} r(t), \quad t_{s \max} = \arg \max_{-1 \leq t \leq 1} s(t) \quad (11)$$

The values of $t_{r \max}$ and $t_{s \max}$ obtained by Eqn. (11) satisfy $t_{s \max} < 0 < t_{r \max} = -t_{s \max}$.

Similarly, the angles $\beta_P(t)$ and $\beta_Q(t)$ of the hinge axes at P^k and Q^k at the state t are derived by substituting Eqns. (2)-(4) into Eqns. (34a) and (34b) in Appendix. These variables $\beta_P(t)$ and $\beta_Q(t)$ in the cases $N = 3$ and $N = 12$ are shown in Fig. 4, where $\beta_P(0) = \pi - \beta_0$ and $\beta_Q(0) = \beta_0$.

2.4 2D representation of N -SRL

Next, to simplify the following discussion, a 2D representation of the N -SRL is considered. We introduce 2D Rz -plane, where R component is the distance from z axis in the Cartesian coordinate system xyz in Fig. 1. That is, a point $(x_0, y_0, z_0) \in \mathbb{R}^3$ in 3D xyz -space is mapped to a point $(R_0, z_0) \in \mathbb{R}^2$ in 2D Rz -plane where $R_0 = \sqrt{x_0^2 + y_0^2}$. In this manner, nodes $P^1, \dots, P^N$ of N -SRL

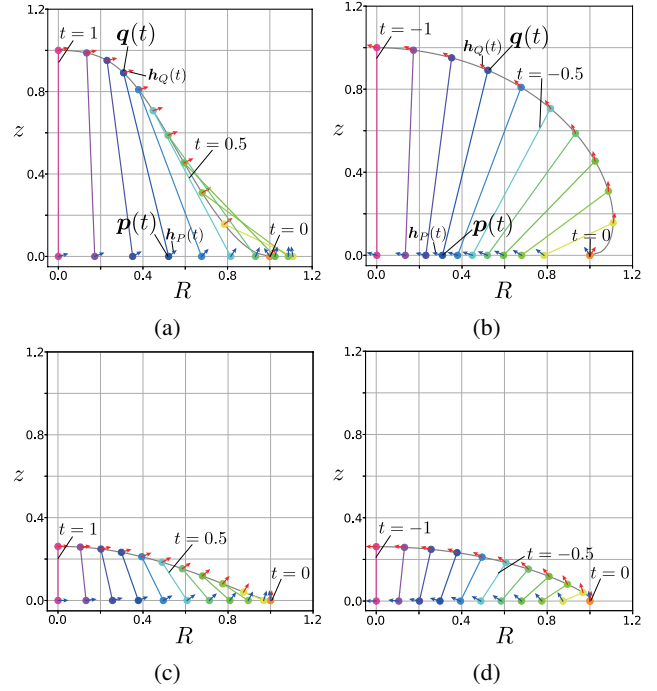


Fig. 5: Trajectories of $\mathbf{p}(t)$ and $\mathbf{q}(t)$ on 2D Rz -plane; (a) Positive path for $N = 3$, (b) Negative path for $N = 3$, (c) Positive path for $N = 12$, (d) Negative path for $N = 12$

at the state t in 3D xyz -space are all depicted by a single point in 2D Rz -plane, which is expressed by a 2D position vector $\mathbf{p}(t) \in \mathbb{R}^2$. Similarly, let a 2D position vector $\mathbf{q}(t) \in \mathbb{R}^2$ in Rz -plane denote nodes $Q^1, \dots, Q^k$ of N -SRL in 3D xyz -space. Using $r(t), s(t), v(t)$ derived in the previous subsection, these vectors can be expressed as $\mathbf{p}(t) = [r(t), 0]^\top$ and $\mathbf{q}(t) = [s(t), v(t)]^\top$ as illustrated in Fig. 5 for $N = 3$ and 12. Note that these frames retain N rotational symmetry and N reflection symmetry. Then hinge axis directions $\mathbf{h}_P^1, \dots, \mathbf{h}_P^N$ of nodes $P^1, \dots, P^N$ (resp. $\mathbf{h}_Q^1, \dots, \mathbf{h}_Q^N$ of nodes $Q^1, \dots, Q^N$) at the state t in 3D xyz -space can also be expressed as a 2D vector $\mathbf{h}_P(t) = [\cos \beta_P(t), \sin \beta_P(t)]^\top$ (resp. $\mathbf{h}_Q(t) = [\cos \beta_Q(t), \sin \beta_Q(t)]^\top$) in 2D Rz -plane. The vectors of hinge axes $\mathbf{h}_P(t)$ and $\mathbf{h}_Q(t)$ for $N = 3$ and 12 at each state are shown in Fig. 5 by blue and red arrows, respectively.

Let $\mathbf{F}(t) \in \mathbb{R}^2$ denote a 2D vector starting at $\mathbf{p}(t)$ and ending at $\mathbf{q}(t)$. Moreover, we define a vector $\mathbf{G}(t) \in \mathbb{R}^2$ obtained by dividing $\mathbf{F}(t)$ by $r(t)$ as follows:

$$\mathbf{G}(t) = \frac{\mathbf{F}(t)}{r(t)} = \frac{\mathbf{q}(t) - \mathbf{p}(t)}{r(t)} = \left[\frac{s(t) - r(t)}{r(t)}, \frac{v(t)}{r(t)} \right]^\top \quad (12)$$

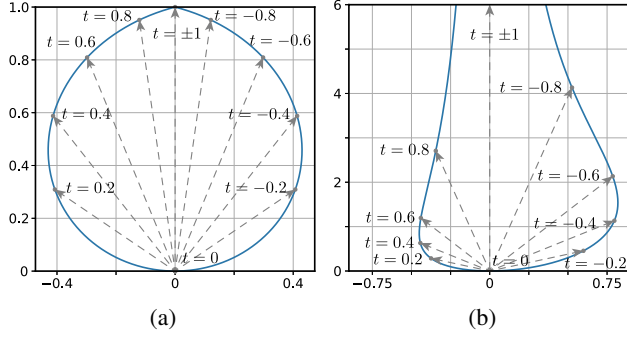


Fig. 6: Trajectories of vectors $F(t)$ and $G(t)$ ($N = 3$); (a) Vector $F(t)$, (b) Vector $G(t)$

Figure 6 shows the trajectory of the vectors $F(t)$ and $G(t)$. These vectors are used for the procedures described in the following section.

3 N-GONAL MULTILAYER SYMMETRIC REVOLUTE LINKAGE (N-MLSRL)

3.1 Shape definition

We obtain a frame by scaling the partially folded N -SRLs defined in the previous section and stacking them vertically as shown in Fig. 7, which we call a *N-gonal multilayer symmetric revolute linkage (N-MLSRL)*. All layers of the N -MLSRL are numbered $1, 2, \dots, m$ from its bottom, where m is the number of layers.

The *initial state* of the N -MLSRL is assumed to be the most deployed state of it, whereas the *completely folded state* of the N -MLSRL is defined as the state where it becomes a bundle as shown in Fig. 8. The initial state of the N -MLSRL satisfies the following rules:

1. Layer i is derived by scaling the N -SRL at the halfway deployed state $t = t_i^0$ relative to the point of origin with scale factor κ_i , rotating $(i-1)\pi/N$ around z axis, and moving parallel to z axis to stack on the layer $i-1$. The nodes of layer i are denoted by $P_i^1, Q_i^1, \dots, P_i^N, Q_i^N$ corresponding to nodes $P^1, Q^1, \dots, P^N, Q^N$ of the N -SRL, respectively, as shown in Fig. 7. Let R_i denote the distance between node P_i^k and z axis at the initial state. Then, the scaling factor κ_i of layer i is associated with $r(t_i^0)$ and R_i by the following equation.

$$\kappa_i = \frac{R_i}{r(t_i^0)}, \quad (i = 1, \dots, m) \quad (13)$$

2. Node P_i^k of layer i is connected to node Q_{i-1}^k locating them at the same position by an *interlayer hinge*,

whose axis is parallel to xy plane and perpendicular to the line in the radial direction.

3. The whole shape of N -MLSRL is determined by parameter t_i^0 and scale factor κ_i of layer i . Designer determines $t_1^0, \dots, t_m^0$ and R_1 , which define the shapes of all layers and the size of the 1st layer. In this study, to simplify movement of the obtained N -MLSRL, t_i^0 is always determined to be smaller than $t_{s \max}$ or larger than $t_{r \max}$, which is described in detail in Sec. 3.2.

In a similar manner as N -SRL, a 2D representation of N -MLSRL in Rz -plane is also introduced as shown in Fig. 9, where nodes P_i^k and Q_i^k ($k = 1, \dots, N$) of layer i are plotted using the 2D position vectors p_i and $q_i \in \mathbb{R}^2$, respectively.

3.2 Deformation path parameter of N-MLSRL

Now we consider the deformation process of N -MLSRL from its initial state.

First, t_i^0 is defined as a state parameter of layer i of N -MLSRL. Layer i consists of N -SRL with scale factor κ_i . Then, we introduce a new parameter u between 0 and 1 which represents the deformation path of N -MLSRL; $u = 0$ at its initial state and $u = 1$ at the completely folded state. Parameter t_i is associated with u as $t_i(u)$ such that the following equations are satisfied for each i .

$$-1 \leq t_i(u) \leq 1 \quad (0 \leq u \leq 1) \quad (14a)$$

$$t_i(0) = t_i^0, \quad |t_i(1)| = 1 \quad (14b)$$

Here, the first and second equations of Eqns. (14b) mean, respectively, that the layer i in its initial state $u = 0$ has the specified shape of N -SRL with its state parameter $t = t_i^0$, and that the layer i is completely folded into a bundle when $u = 1$.

If $t_i(u)$ for layer i is obtained, the deformation of N -MLSRL from its initial state to the completely folded state can be simulated. For simplicity, we only consider the situation that t_i^0 for each i except for the top layer satisfies either $t_i^0 > t_{r \max}$ or $t_i^0 < t_{s \max}$, and that $t_i(u)$ is a monotonically increasing function when $t_i^0 > t_{r \max} > 0$, and a monotonically decreasing function when $t_i^0 < t_{s \max} < 0$. In this situation both $r(t_i(u))$ and $s(t_i(u))$ monotonically decrease as u increases, which means that all nodes approach the z axis monotonically without returning to the opposite direction.

The function $t_1(u)$ for the first layer can be arbitrarily defined under the condition that $t_1(u)$ satisfies Eqns. (14a) and (14b) and monotonicity. Here, as a sim-

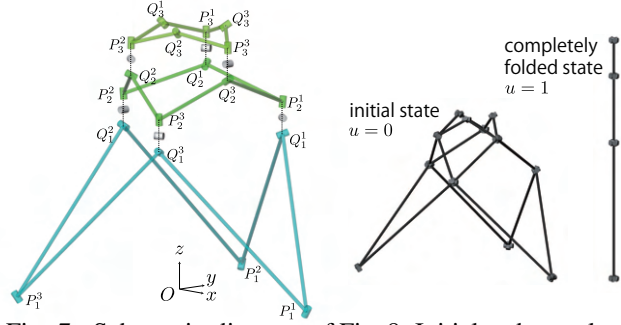


Fig. 7: Schematic diagram of Fig. 8: Initial and completely folded states of N -MLSRL ($N = 3, m = 3$)

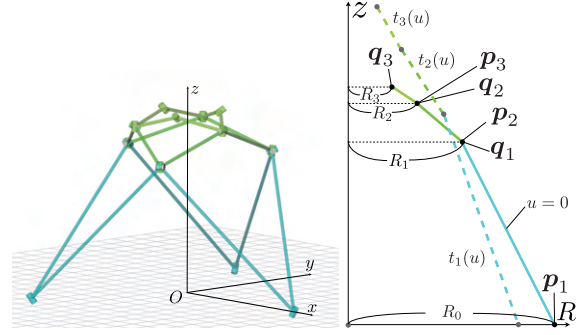


Fig. 9: 2D representation of N -MLSRL

ple example, $t_1(u)$ is defined as the following linear function.

$$t_1(u) = \begin{cases} t_1^0 + u(1 - t_1^0) & \text{if } t_1^0 > t_{r \max} \\ t_1^0 + u(-1 - t_1^0) & \text{if } t_1^0 < t_{s \max} \end{cases} \quad (0 \leq u \leq 1) \quad (15)$$

In this case, the first layer of N -MLSRL deforms monotonically from its initial state $u = 0$ in the direction of t_1 with the same sign as t_1^0 , and finally completely be folded into a bundle at $u = 1$.

For the second and higher layers $i \geq 2$, $t_i(u)$ is determined from the condition that layer i continuously connects with layer $i - 1$. Here, 2D position vectors $\mathbf{p}_i$ and $\mathbf{q}_i$ in Rz -plane can be regarded as vector functions with respect to u . Using the relation $\mathbf{q}_i(u) - \mathbf{p}_i(u) = \kappa_i[s(t_i) - r(t_i), v(t_i)]^\top$, $\mathbf{p}_i(u)$ and $\mathbf{q}_i(u)$ can be expressed as follows:

$$\mathbf{p}_i(u) = \left[\kappa_i r(t_i(u)), \sum_{j=1}^{i-1} \kappa_j v(t_j(u)) \right]^\top \quad (16a)$$

$$\mathbf{q}_i(u) = \left[\kappa_i s(t_i(u)), \sum_{j=1}^i \kappa_j v(t_j(u)) \right]^\top \quad (16b)$$

The connecting condition between layers $i - 1$ and i is expressed as

$$\mathbf{p}_i(u) = \mathbf{q}_{i-1}(u) \quad (17)$$

Substituting Eqns. (16a) and (16b) into Eqn. (17), we obtain the following condition.

$$\kappa_i r(t_i(u)) = \kappa_{i-1} s(t_{i-1}(u)) \quad (18)$$

Using Eqn. (18), $t_i(u)$ can be determined sequentially from $t_{i-1}(u)$. Solving Eqn. (18) for $t_i(u)$, we obtain the following expression:

$$t_i(u) = \begin{cases} T^+(r(t_i(u))) = T^+ \left(\frac{\kappa_{i-1}}{\kappa_i} s(t_{i-1}(u)) \right) & \text{if } t_i^0 > t_{r \max} \\ T^-(r(t_i(u))) = T^- \left(\frac{\kappa_{i-1}}{\kappa_i} s(t_{i-1}(u)) \right) & \text{if } t_i^0 < t_{r \max} \end{cases} \quad (19)$$

where $T^+(r)$ and $T^-(r)$ are the inverses of $r(t)$ satisfying the following inequalities.

$$\left. \begin{aligned} -1 &\leq T^-(r) \leq t_{r \max} \\ t_{r \max} &\leq T^+(r) \leq 1 \end{aligned} \right\} \quad (0 \leq r \leq r_{\max}) \quad (20)$$

Therefore, the condition that $t_i(u)$ is determined from $t_{i-1}(u)$ by Eqn. (19) is summarized as follows.

$$0 \leq \frac{\kappa_{i-1}}{\kappa_i} s(t_{i-1}(u)) \leq r_{\max} \quad (0 \leq u \leq 1) \quad (21)$$

Equation (21) always holds if $\kappa_i > \kappa_{i-1}$ regardless of the value of $t_{i-1}(u)$. However, if $\kappa_i \leq \kappa_{i-1}$, Eqn. (21) may not be satisfied depending on the value of $t_{i-1}(u)$, that is, the connecting condition between layers i and $i - 1$ is not maintained.

We note that $T^+(r)$ and $T^-(r)$ monotonically increase and decrease, respectively, as r decreases. Then, now it can be confirmed that the function $t_i(u)$ is a monotonically increasing or decreasing function when it is determined by either of two equations in Eqns. (19) based on the value of t_i^0 , if t_i^0 for layer i except the top layer is defined not to be between $t_{s \max}$ and $t_{r \max}$. It is difficult to determine the inverses $T^+(r)$ and $T^-(r)$ explicitly. In this study, we estimate these inverses using cubic

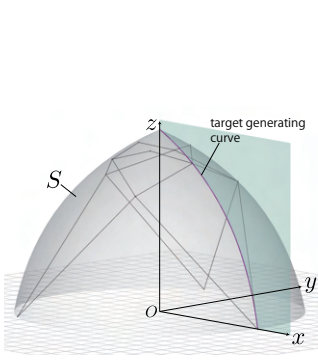


Fig. 10: Fitting N -MLSRL to target surface of revolution S .

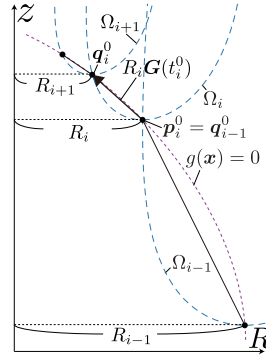


Fig. 11: Fitting procedure on Rz -plane.

interpolation with a sufficient number of discrete samples $[t, r(t)]$ calculated in advance.

4 FITTING N -MLSRL TO TARGET SURFACE OF REVOLUTION

For an application of N -MLSRL to structural design, it is helpful if a designer can determine the deployed shape of N -MLSRL from a surface of revolution which inscribes it. Now we consider an N -MLSRL in which P_i^k and Q_i^k ($k = 1, \dots, N, i = 1, \dots, m$) are located on a predefined surface of revolution S in its initial state. Here S is called *target surface* around the z axis as shown in Fig. 10. As an intersection of S with xz -plane, a corresponding *target generating curve* is defined, and is also depicted as the same shape on Rz -plane expressed as $g(x) = 0$ where $x = (R, z)$. It is assumed that the minimum and maximum of z coordinate of the target generating curve is 0 and z_{max} , respectively, and that the curve $g(x) = 0$ is not parallel to z axis. When all P_i^k and Q_i^k ($k = 1, \dots, N, i = 1, \dots, m$) of the N -MLSRL are on S in its initial state, the following equation holds on the Rz -plane, where p_i and q_i are functions of u , and p_i^0 and q_i^0 denote $p_i(0)$ and $q_i(0)$, respectively:

$$q_i^0 - p_i^0 = \kappa_i F(t_i^0) = \frac{R_i}{r(t_i^0)} F(t_i^0) = R_i G(t_i^0) \quad (22)$$

where $F(t)$ and $G(t)$ are defined by Eqn. (12). Using Eqn. (22), we can determine κ_i and t_i^0 ($i = 1, \dots, m$) by the following procedure.

STEP 1 Let $q_0^0 \in \mathbb{R}^2$ be the position vector of the intersection of the target generating curve $g(x) = 0$ and R axis on the Rz -plane. Set $i = 1$.

STEP 2 Let the position vector p_i^0 be equal to q_{i-1}^0 and $R_i (> 0)$ denote the R coordinate of p_i^0 . The curve Ω_i is depicted as the trajectory of the point $x(t_i) \in \mathbb{R}^2$ ($-1 \leq t_i \leq 1$) defined as follows, as shown in Fig. 11.

$$\Omega_i : x(t_i) = p_i^0 + R_i G(t_i) \quad (23)$$

STEP 3 Find $t_i^0 (\neq 0)$ which expresses the parameter t_i at the intersection of the curve Ω_i and the target generating curve $g(x) = 0$, which is obtained by solving the equation $g(x(t_i^0)) = 0$ for t_i^0 , as illustrated in Fig. 11. Terminate the calculation if there is no solution t_i^0 , or if the z component of the vector $R_i G(t_i^0)$ is smaller than a predefined *minimum layer height*.

STEP 4 Substitute t_i^0 calculated in STEP 3 to Eqn. (13) to obtain κ_i . The 2D position vector q_i^0 is derived by the following equation.

$$q_i^0 = p_i^0 + R_i G(t_i^0) \quad (24)$$

STEP 5 Let $i \leftarrow i + 1$. Return to STEP 2.

All the position vectors p_i^0 and q_i^0 determined by the above procedure satisfy $g(p_i^0) = g(q_i^0) = 0$. To find t_i^0 in STEP 3, a nonlinear programming problem is solved for minimizing the error of the equation.

5 HINGE OFFSETS CONSIDERING FINITE SIZE OF HINGE JOINTS

5.1 Hinge offsets

The methods described in the previous sections have shown how to generate N -MLSRL whose all nodes in its initial state is deployed onto the target surface of revolution S , where the hinge joints between members are assumed to have infinitesimal size. However, for actual fabrication, the finite sizes of the hinge joints should be considered. In this section, for incorporating finite sizes of all the hinge joints between layers, we use the technique of *hinge offsets* for offsetting the hinge axis as shown in Fig. 12. In Fig. 12, M_i^k is the center point of the interlayer hinge between node Q_i^k in layer i and P_{i+1}^k in layer $i + 1$. Let $m_i(u) \in \mathbb{R}^2$ denote the 2D position vector of M_i^k on Rz -plane. Here, u is a deformation path parameter in the range $0 \leq u \leq 1$, as defined in the previous section. In Fig. 12, $w_{Q,i}$ and $w_{P,i}$ express Q_i - and P_i -hinge offset lengths, respectively, around the interlayer hinge between layers i and $i + 1$. Then, the following equations hold on Rz -plane, as illustrated in Fig. 13 corresponding to $u = 0$

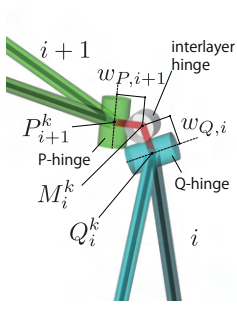


Fig. 12: Schematic diagram of hinge offsets

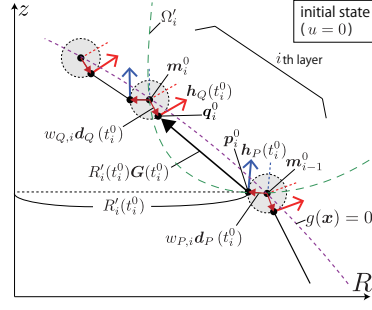


Fig. 13: Fitting procedure considering hinge offsets

where $\mathbf{m}_i^0$ denotes $\mathbf{m}_i(0)$.

$$\mathbf{p}_i(u) = \mathbf{m}_{i-1}(u) + w_{P,i} \mathbf{d}_P(t_i(u)) \quad (25a)$$

$$\mathbf{q}_i(u) = \mathbf{m}_i(u) + w_{Q,i} \mathbf{d}_Q(t_i(u)) \quad (25b)$$

In Eqn. (25), we call $\mathbf{d}_P(t) = [d_{P1}(t), d_{P2}(t)]^\top$ and $\mathbf{d}_Q(t) = [d_{Q1}(t), d_{Q2}(t)]^\top \in \mathbb{R}^2$ as *P-hinge offset vectors*, respectively. They are unit vectors obtained by rotating $\mathbf{h}_P(t)$ and $\mathbf{h}_Q(t)$ by $\pi/2$ and $-\pi/2$, respectively. The shape of layer i between nodes P_i^k and Q_i^k is defined by scaling the N -SRL at the state $t = t_i^0$. Therefore, the following relation holds for the scaling parameter κ_i , where R'_i is the distance between nodes P_i^k of layer i and z axis:

$$\kappa_i = \frac{R'_i}{r(t_i^0)}, \quad (i = 1, 2, \dots, m) \quad (26)$$

In this case, we can derive the connecting condition between adjacent layers $i-1$ and i from Eqn. (25) replacing i in Eqn. (25b) with $i-1$ and eliminating $\mathbf{m}_{i-1}(u)$ as follows.

$$\begin{aligned} \mathbf{p}_i(u) - w_{P,i} \mathbf{d}_Q(t_i(u)) \\ = \mathbf{q}_{i-1}(u) - w_{Q,i-1}(u) \mathbf{d}_Q(t_{i-1}(u)) \end{aligned} \quad (27)$$

The equality of R components of Eqn. (27) can be written as

$$\begin{aligned} \kappa_i r(t_i(u)) - w_{P,i} d_{P1}(t_i(u)) \\ = \kappa_{i-1} s(t_{i-1}(u)) - w_{Q,i-1} d_{Q1}(t_{i-1}(u)) \end{aligned} \quad (28)$$

which can be regarded as the modified Eqn. (18) with the consideration of hinge offsets. Using Eqn. (28), $t_i(u)$ can

be determined from $t_{i-1}(u)$ even if hinge offsets are considered. Equation (28) is a nonlinear equation, which is to be solved numerically. Similar to the case where the hinge offsets are not considered, $t_i(u)$ may not be obtained when the values of $\kappa_i, \kappa_{i-1}, w_{P,i}$ and $w_{Q,i-1}$ are not appropriate for constructing N -MLSRL.

It should be noted that consideration of hinge offsets is not necessary even if the hinges have finite sizes. That is, it is possible to design a realistic detail of hinge joints of N -MLSRL without hinge offsets, for example, by combining a universal joint and a hinge (revolute) joint. In general, however, designing the hinge detail without hinge offsets is complicated and hard to manufacture. Therefore, in the numerical example shown in Sec. 6.4, hinge offsets described above are used to design a realistic and constructable hinge detail.

5.2 Fitting N -MLSRL with hinge offsets to target surface of revolution

Finally, we propose a method to generate N -MLSRLs with hinge offsets which are deployed on a predefined surface of revolution S around z axis. In this case, we consider the situation that all the center points of interlayer hinges $M_i^k (k = 1, \dots, N, i = 1, \dots, m-1)$ in its initial state are on the surface S . This is equivalent to the following condition.

$$g(\mathbf{m}_i^0) = 0, \quad (i = 1, \dots, m-1) \quad (29)$$

Here, STEPs 1-5 described in Sec. 4 are modified as follows, as illustrated in Fig. 13.

STEP 1 Let $\mathbf{m}_0^0 \in \mathbb{R}^2$ be the position vector of the intersection of the target curve $g(\mathbf{x}) = 0$ and R axis on the Rz -plane. Set $i = 1$.

STEP 2 Let the position vector $\mathbf{p}_i(t_i)$ be equal to $\mathbf{m}_{i-1}^0 + w_{P,i} \mathbf{d}_P(t_i)$ and $R'_i(t_i)$ denote the R coordinate of $\mathbf{p}_i(t_i)$. The curve Ω'_i is depicted as the trajectory of a point $\mathbf{x}(t_i) \in \mathbb{R}^2$ ($-1 \leq t_i \leq 1$) defined as follows, where $\mathbf{G}(t)$ is a vector defined by Eqn. (12):

$$\begin{aligned} \Omega'_i : \mathbf{x}(t_i) = \mathbf{m}_{i-1}^0 + w_{P,i} \mathbf{d}_P(t_i) \\ + R'_i(t_i) \mathbf{G}(t_i) - w_{Q,i} \mathbf{d}_Q(t_i) \end{aligned} \quad (30)$$

STEP 3 Find $t_i^0 (\neq 0)$ as the parameter t_i when the curve Ω'_i and the target generating curve $g(\mathbf{x}) = 0$ intersect, which is obtained by solving the equation $g(\mathbf{x}(t_i^0)) = 0$. Terminate the calculation if there is no solution, or if the z component of the vector $R'_i(t_i^0) \mathbf{G}(t_i^0)$ is smaller than the predefined minimum layer height.

Table 1: Parameters t_i^0 and κ_i of horn-shaped frame

i	t_i^0	κ_i
1	-0.7557	0.7516
2	-0.5151	0.7350
3	-0.2889	0.9691
4	-0.1461	1.3666
5	-0.0678	1.8270

STEP 4 Substitute t_i^0 calculated in STEP 3 to Eqn. (26) to obtain κ_i . The 2D position vector $\mathbf{m}_i^0$ is derived by the following equation.

$$\mathbf{m}_i^0 = \mathbf{m}_{i-1}^0 + w_{P,i} \mathbf{d}_P(t_i^0) + R_i'(t_i^0) \mathbf{G}(t_i^0) - w_{Q,i} \mathbf{d}_Q(t_i^0) \quad (31)$$

STEP 5 Let $i \leftarrow i + 1$. Return to STEP 2.

According to the procedure described above, we can systematically generate N -MLSRL that has hinge offsets at the interlayer connection and is deployed onto the target surface S at its initial state. In this case, however, the parameters $t_i(u)$ of all layers generally do not converge to $|t_i(1)| = 1$ simultaneously. In addition, the possible maximum value of u will generally be slightly smaller than 1 when it is considered to satisfy the conditions for actual constructability and movability due to interference between each member. A numerical and practical example considering hinge offsets is shown in Sec. 6.4.

It should also be noted that, when hinge offsets are included in N -MLSRL, its number of instability modes increases due to the additional degree of freedom of relative rotation around the z axis between vertically adjacent layers. Therefore, if the lengths of hinge offsets are large, the detail of joints needs to be devised to prevent the relative rotation at the initial state for actual use. In the example shown in Sec. 6.4, this issue is considered in the detailed design of hinge joints.

6 NUMERICAL EXAMPLES

6.1 Horn-shaped frame

The proposed method is first applied to a horn-shaped target surface as shown in Fig. 14. The shape of the target generating curve $g(x) = 0$ is indicated by the dotted line in the corresponding Rz -plane in Fig. 15, where $g(x) = 0$ is determined by NURBS curve interpolating three points (0.2, 0), (0.5, 1.1) and (2.4, 2.25). In

Table 2: Parameters t_i^0 and κ_i of ball-shaped frame

i	t_i^0	κ_i	i	t_i^0	κ_i
1	-0.1242	0.7846	10	0.1385	1.3104
2	-0.1331	0.8888	11	0.1184	1.1447
3	-0.1450	1.0143	12	0.1052	1.0146
4	-0.1616	1.1702	13	0.0951	0.9072
5	-0.1866	1.3736	14	0.0871	0.8167
6	-0.2305	1.6681	15	0.0807	0.7391
7	-0.3970	2.4700	16	0.0756	0.6719
8	0.5600	3.2660	17	0.0713	0.6129
9	0.1746	1.5434			

this model, the number of symmetry N is 3 and hinge offset is not considered. The minimum layer height is 0.05. The units are not considered, because they are not significant for this study.

The shape of the model at its initial state obtained by the proposed method is shown in Fig. 14(a), where the number of layers m is 5. Parameters t_i^0 and κ_i of layer i are listed in Table 1. This frame has a single degree of freedom because $N = 3$ and the parameters t_i^0 are less than $t_{s \max} = -0.1077$ in all layers except the top layer. Figures 14(b) and (c) show the deformed frame corresponding to $u = 0.5$ and $u = 1$, respectively. It can be seen that all members at $u = 1$ are folded into a rod. The corresponding 2D representations are shown by three polylines corresponding to $u = 0$, $u = 0.5$ and $u = 1$ in Fig. 15, where the lines in the figure represent line segments connecting $\mathbf{p}_i(u)$ and $\mathbf{q}_i(u)$ calculated using Eqns. (16a) and (16b) for layer i . The directions of hinge axes on each layer's ends are also depicted along the polylines.

Figure 16 shows the variation of $t_i(u)$ defined by Eqn. (19) from t_i^0 during $0 \leq u \leq 1$. From this figure, it can be seen that every layer of this frame follows a negative deformation path $-1 \leq t_i(u) < 0$ and finally converges to $t_i(1) = -1$.

6.2 Ball-shaped frame

Next example is a ball-shaped target surface as shown in Fig. 17(a). In this example, N is 12, which means this frame has multiple degrees of freedom because $N \geq 4$, and therefore unstable if only a single DOF is constrained in the deformation process. However, it is possible to control stability of the frame by introducing some devices or supports; e.g., some rotary actuators

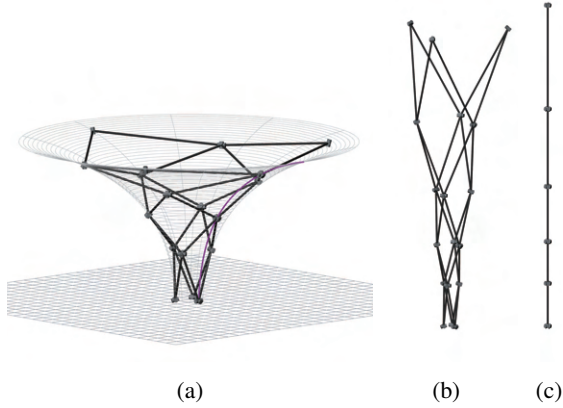


Fig. 14: Horn-shaped frame ($N = 3, m = 5$); (a) Initial state ($u = 0$), (b) Deformed state at $u = 0.5$, (c) Completely folded state ($u = 1.0$)

that mechanically control their hinge rotations. The target generating curve $g(x) = 0$ as shown in Fig. 18 is determined by NURBS curve interpolating three points $(0.666, 0)$, $(1.7982, 0.999)$ and $(0.333, 1.998)$. The minimum layer height is 0.05, and hinge offset is ignored.

Using the proposed method, we obtained a frame shown in Fig. 17(a), which has 17 layers ($m = 17$). The determined parameters t_i^0 and κ_i for each layer are shown in Table 2, where all parameters t_i^0 satisfy either $t_i^0 < t_{s \max} = -0.0106$ or $t_i^0 > t_{r \max} = 0.0106$. Shapes of this frame at $u = 0.5$ and 1 are shown in Fig. 17(b) and (c), respectively. These shapes are also expressed on Rz -plane in Fig. 18. Figures 17(c) and 18 show that this frame is completely folded into a straight rod shape at $u = 1$.

Figure 19 shows the transition of t_i of layer i from t_i^0 shown in Table 2 in the range $0 \leq u \leq 1$. From Fig. 19 and Table 2, it can be confirmed that layers 1 to 7 follow negative paths and converge to $t_i(1) = -1$, while layers 8 to 17 follow positive paths and converge to $t_i(1) = 1$.

6.3 Dome-shaped frame

The proposed method is next applied to a dome-shaped target surface as shown in Fig. 20(a) to generate a foldable dome frame. The number of symmetry N is 3, which means the frame has 1 DOF. The target generating curve $g(x) = 0$ as shown in Fig. 21 is determined by NURBS curve interpolating three points $(1.5, 0)$, $(0.95, 1)$ and $(0, 2.2)$. The minimum layer height is 0.20 and hinge offset is ignored.

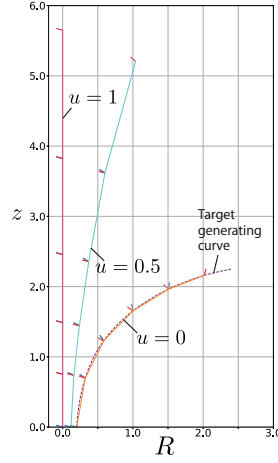


Fig. 15: Representation of horn-shaped frame on Rz -plane

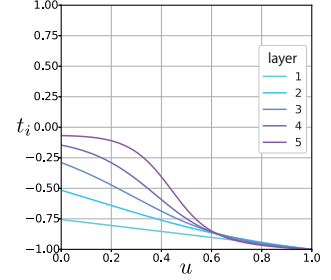


Fig. 16: Parameters t_i of horn-shaped frame

Table 3: Parameters t_i^0 and κ_i of dome-shaped frame with hinge offsets

i	t_i^0	κ_i
1	0.4708	1.7614
2	0.3802	0.8662
3	0.3408	0.4665

Table 4: Parameters t_i^0 and κ_i of dome-shaped frame with hinge offsets

i	t_i^0	κ_i
1	0.4744	1.7400
2	0.3857	0.8155
3	0.3530	0.3996

The obtained frame is shown in Fig. 20(a), which has $m = 3$ layers. Shape parameters are shown in Table 3, where all parameters t_i^0 satisfy $t_i^0 > t_{r \max} = 0.1077$. Figures 20(b) and (c) show the deformed state at $u = 0.5$ and the completely folded state at $u = 1.0$, respectively. The transition of t_i of each layer is shown in Fig. 22. It can be seen that every t_i follows positive path and converges to 1.

6.4 Dome-shaped frame with hinge offsets

Finally, we consider again the dome-shaped target surface S in Sec. 6.3 adding hinge offset to generate actually constructable dome-shaped foldable frame. For this example, the hinge offset lengths of each layer's ends $w_{P,i}$ and $w_{Q,i}$ are all 0.03. Other parameter settings and conditions are the same as those in Sec. 6.3.

The shape of the obtained frame on Rz -plane is shown in Fig. 23 and the determined shape parameters t_i^0 and κ_i are shown in Table 4. As can be seen from

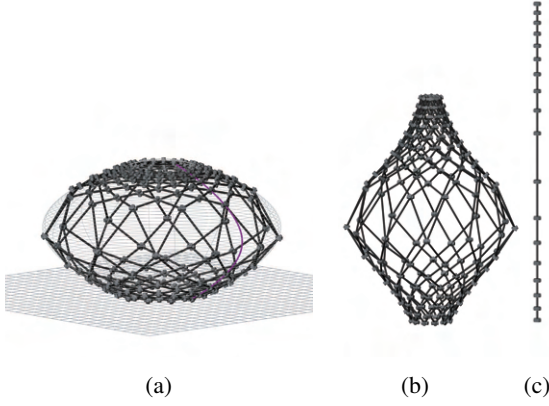


Fig. 17: Ball-shaped frame ($N = 12, m = 17$); (a) Initial state ($u = 0$), (b) Deformed state at $u = 0.5$ (c) Completely folded state ($u = 1.0$)

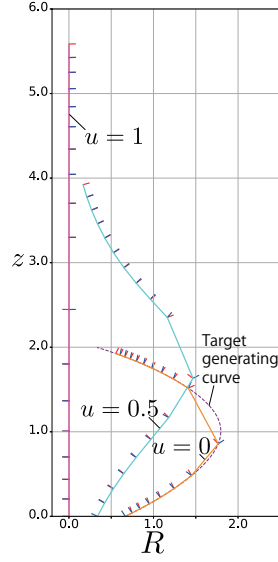


Fig. 18: Representation of ball-shaped frame on Rz -plane

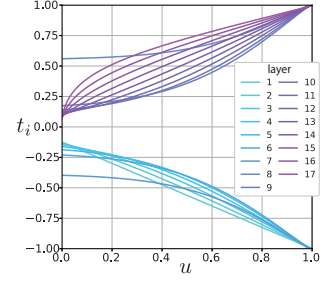


Fig. 19: Parameters t_i of ball-shaped frame

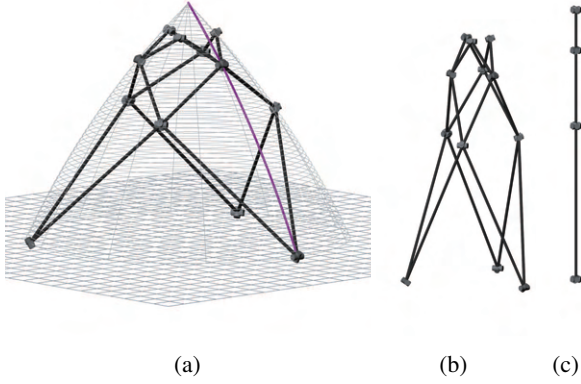


Fig. 20: Dome-shaped frame ($N = 3, m = 3$); (a) Initial state ($u = 0$), (b) Deformed state at $u = 0.5$, (c) Completely folded state (1.0)

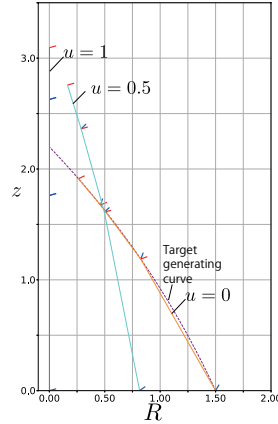


Fig. 21: Representation of dome-shaped frame on Rz -plane

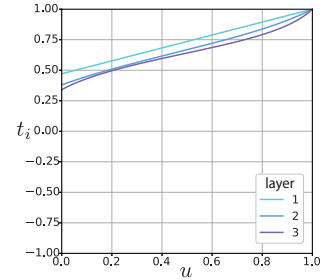


Fig. 22: Parameters t_i of dome-shaped frame

Table 4, the obtained shape parameters are slightly different from those in Table 3 which does not consider hinge offset. Also, in Fig. 23, it can be seen that there are some spaces between vertically adjacent layers which imply existence of hinge offsets, and that the first and second layers are not completely folded even when the third layer is always completely folded into a bundle at $u = 0.976$. Figure 24 shows the transition of t_i for each layer near $u = 1$, which shows that t_3 first reach 1 at $u = 0.976$,

whereas at the same point t_1 and t_2 do not reach 1.

Now we design the detail of the actual structure using the obtained dome-shaped N -MLSRL frame. To avoid member interference in the bundled state, the maximum value of u , which defines the most physically folded state of the structure, is set to 0.936. In this state, the axial lines of members in the obtained N -MLSRL are not parallel to z axis as shown in Fig. 25(a). However, by associating vertically straight shapes for the physical model

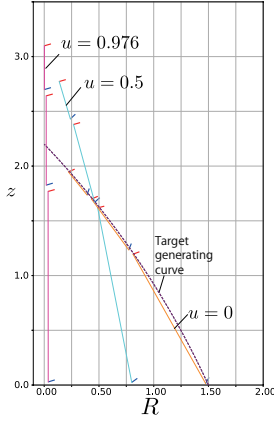


Fig. 23: Representation of dome-shaped frame with hinge offsets on Rz -plane

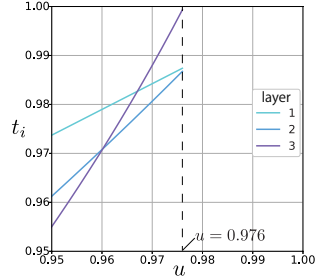


Fig. 24: Parameters t_i of dome-shaped frame with hinge offsets

with the original inclined axial lines in N -MLSRL at this state, i.e., allowing the physical members not to be parallel with the original axial lines as shown in Fig. 25(b), the property of foldability into straight shape is maintained, as shown in Fig. 25(d), where the simulated movement of the designed hinge joint is shown in Fig. 25(c).

Foldability of the proposed dome-shaped frame is verified with a physical model as shown in Fig. 26, where the movement of its hinge joint is the same as the simulated motion shown in Fig. 25(c). Furthermore, in this model, all members in its 1st layer include telescopic joints in the middle. Letting them shrink after folding the whole frame, the frame can be made more compact as shown in Fig. 26(c), which is thanks to the property that the frame can be folded to a straight bundle.

As shown in this physical model, in the actual fabrication, the model cannot be folded to the completely folded state ($u = 1$) due to interference between each member. The final u must be set smaller as the number of layers or the sizes of the members and hinges increase, and the final folded shape departs further from a straightly bundled shape. Users of this method must pay attention to these disadvantage in the application to large deployable structures.

7 CONCLUSION

In this paper, we proposed a method to generate a new deployable structure, N -MLSRL, that has N -dihedral symmetry and can be folded into a straight rod shape.

First, we defined a concept of N -SRL, a regular $2N$ -

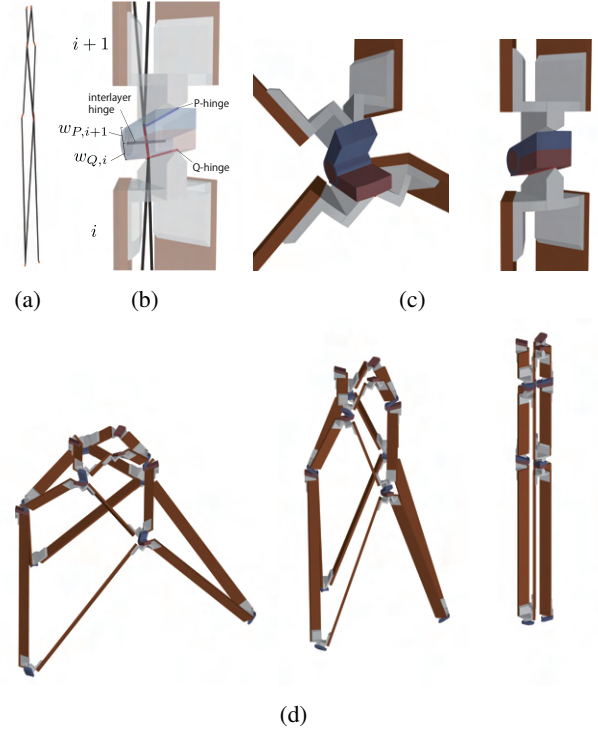


Fig. 25: Hinge detail design using hinge offset; (a) Most physically folded state of N -MLSRL with hinge offsets, (b) Detailed design of hinge joint and connecting members considering hinge offsets, (c) Simulated movement of the designed hinge joint, (d) Simulated deformation process

gonal frame which is composed of $2N$ bars and $2N$ inclined hinge joints and foldable into a bundle. N -SRL can be regarded as an extension of Bennett linkage and threefold-symmetric Bricard linkage to general N -fold symmetry. From the expression of its movement by roll, pitch and yaw which had already been obtained by Crawford et al. [23], we first defined the positive and negative paths of N -SRL using a parameter t and derived its dimensional variables $r(t)$, $s(t)$, $v(t)$. Next, by stacking N -SRLs vertically, the concept of N -MLSRL was defined. The design parameters of the proposed N -MLSRL are N , t_i^0 ($i = 1, \dots, m$) and κ_i ($i = 1, \dots, m$), which represent the number of symmetries of the frame, the shape of layer i at its initial state and its scaling factor, respectively. Note that if either t_i^0 or κ_i is determined then the other is determined automatically to connect with its adjacent layer. Considering the continuously connecting condition between vertically adjacent layers of N -MLSRL, the deformation process of each layer constituting N -MLSRL can be simply described with a single parameter u .

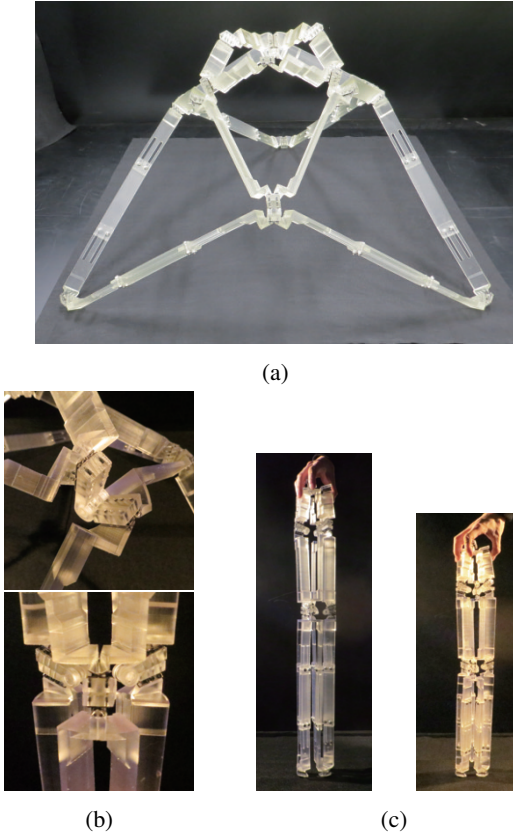


Fig. 26: Physical model of dome-shaped frame with hinge offsets; (a) Initial state, (b) Movement of hinge joints, (c) Most physically folded state. Folded shape can be further shortened with telescopic joints

Then, a methodology to systematically design the initial deployed shape of N -MLSRL was proposed by predefining a surface of revolution S where all nodes of the frame are located. It was shown by numerical examples that various shapes on a surface of revolution can be realized using the proposed procedure. Additionally, the technique of hinge offsets were used to realize the details to avoid interference of the hinge joints which have finite sizes. As mentioned in Sec. 5.1, it is not necessary to use hinge offsets because the assembly of three hinges, namely, P -hinge, Q -hinge and interlayer hinge between vertically adjacent layers can be realized as a combination of a universal joint and a hinge joint. It should also be noted that the use of hinge offsets increases the DOF of relative rotation between vertically adjacent layers. The larger the hinge offset length, the greater the effect, so the designer must pay attention to the relative rotation during the folding process. By taking these points into account, hinge offsets are helpful to design a constructable

N -MLSRL. The effectiveness of hinge offsets was confirmed by the physical model in Sec. 6.4.

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A APPENDIX A: RELATIONS BETWEEN THREE ANGLES EXPRESSING DEFORMATION OF N -SRL

In this section, we summarize the study of Crawford et al. [23] which expressed an attitude of one member of N -SRL by roll θ_r , pitch θ_p and yaw θ_y and derived relations between these angles.

Let (x_e, y_e, z_e) denote the cartesian coordinate fixed to a member between P^1 and Q^1 of N -SRL where x_e is directed toward member axis and z_e is perpendicular to the plane containing N -SRL in the planarly deployed state. Additionally, we assume cartesian coordinate (X_e, Y_e, Z_e) which is fixed in space and coincides with (x_e, y_e, z_e) of this state.

Here, θ_r , θ_p and θ_y are introduced to express the rotation of coordinate axis from (X_e, Y_e, Z_e) to (x_e, y_e, z_e) . The following equations are obtained in any state of N -SRL during its deformation [23].

$$A(\theta_p) \cos^2 \theta_r + B(\theta_p) \cos \theta_r + C(\theta_p) = 0 \quad (32a)$$

$$\theta_y = \arctan \left(-\frac{\sin \theta_r}{\cos \theta_r + \tan^2 \delta \frac{\cos \theta_p}{1 - \sin \theta_p}} \right) \quad (32b)$$

where coefficients in Eqn. (32b) are determined by θ_p as follows:

$$A(\theta_p) = \cos^2 \delta \cos^2 \theta_p$$

$$B(\theta_p) = \sin^2 \delta \{ \cos^2 \delta (2 \sin \theta_p - 1) + \sin^2 \delta \} \cos \theta_p$$

$$C(\theta_p) = -(1 - \sin \theta_p)(1 - 2 \sin^2 \delta \cos^2 \delta + \sin^2 \delta \sin \theta_p)$$

Angle θ_p is 0 or $\pi/2$ when N -SRL is at planarly deployed state or completely folded state, respectively. From Eqn. (32a), an arbitrary pitch θ_p satisfying $0 \leq \theta_p \leq \pi/2$ determines the two solutions $\theta_r = \theta_r^-(\theta_p)$ and $\theta_r^+(\theta_p)$ where $\theta_r^-(\theta_p) \leq \theta_r^+(\theta_p)$. Yaw θ_y can be obtained from θ_p and θ_r by Eqn. (32b), which is expressed by $\theta_y(\theta_p, \theta_r)$. Then there are following two sets of solution $[\theta_p, \theta_r, \theta_y]$ satisfying Eqns. (32a) and (32b) as shown in Fig. 27:

$$[\theta_p, \theta_r, \theta_y] = \begin{cases} [\theta_p, \theta_r^-(\theta_p), \theta_y(\theta_p, \theta_r^-(\theta_p))] \\ [\theta_p, \theta_r^+(\theta_p), \theta_y(\theta_p, \theta_r^+(\theta_p))] \end{cases} \quad (33)$$

Crawford et al. [23] also derived the direction vectors of the hinge axes $[l_{X_e}^P, l_{Y_e}^P, l_{Z_e}^P]^\top$ and $[l_{X_e}^Q, l_{Y_e}^Q, l_{Z_e}^Q]^\top$ at the two ends P^1 and Q^1 of the member with respect

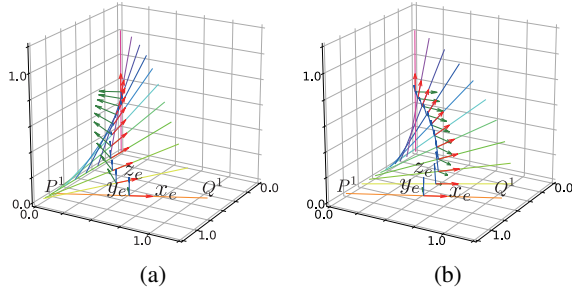


Fig. 27: Two variation paths of an attitude of one member in N -SRL ($N = 3$); (a) Positive path, (b) Negative path

to coordinates (X_e, Y_e, Z_e) using θ_p, θ_r and θ_y . Vector $[l_{X_e}^P, l_{Y_e}^P, l_{Z_e}^P]^\top$ is derived as

$$\begin{bmatrix} \frac{(S\delta C\theta_y C\theta_p - C\delta S\theta_y C\theta_r + C\delta C\theta_y S\theta_p S\theta_r)S\alpha_1 + (S\theta_y S\theta_r + C\theta_y S\theta_p C\theta_r)C\alpha_1}{(S\delta S\theta_y C\theta_p + C\delta C\theta_y C\theta_r + C\delta S\theta_y S\theta_p S\theta_r)S\alpha_1 + (-C\theta_y S\theta_r + S\theta_y S\theta_p C\theta_r)C\alpha_1} \\ \frac{(-S\delta S\theta_p + C\delta C\theta_p S\theta_r)S\alpha_1 + C\theta_p C\theta_r C\alpha_1}{(S\delta S\theta_p + C\delta C\theta_p S\theta_r)S\alpha_1 + C\theta_p C\theta_r C\alpha_1} \end{bmatrix}$$

and $[l_{X_e}^Q, l_{Y_e}^Q, l_{Z_e}^Q]^\top$ is derived as

$$\begin{bmatrix} \frac{(-S\delta C\theta_y C\theta_p - C\delta S\theta_y C\theta_r + C\delta C\theta_y S\theta_p S\theta_r)S\alpha_2 + (S\theta_y S\theta_r + C\theta_y S\theta_p C\theta_r)C\alpha_2}{(-S\delta S\theta_y C\theta_p + C\delta C\theta_y C\theta_r + C\delta S\theta_y S\theta_p S\theta_r)S\alpha_2 + (-C\theta_y S\theta_r + S\theta_y S\theta_p C\theta_r)C\alpha_2} \\ \frac{(S\delta S\theta_p + C\delta C\theta_p S\theta_r)S\alpha_2 + C\theta_p C\theta_r C\alpha_2}{(S\delta S\theta_p + C\delta C\theta_p S\theta_r)S\alpha_2 + C\theta_p C\theta_r C\alpha_2} \end{bmatrix}$$

where Sx and Cx denote $\sin(x)$ and $\cos(x)$, respectively, and α_1 and α_2 are defined as $\alpha_1 = -\alpha_2 = \beta_0 - \pi/2 = -\arctan(\frac{\sin \delta}{\cos^2 \delta})$.

The hinge axes β_P and β_Q can be obtained by the following equations:

$$\beta_P(\theta_p, \theta_r, \theta_y) = \text{atan2}(l_{Z_e}^P, l_{X_e}^P \sin \delta + l_{Y_e}^P \cos \delta) \quad (34a)$$

$$\beta_Q(\theta_p, \theta_r, \theta_y) = \text{atan2}(l_{Z_e}^Q, -l_{X_e}^Q \sin \delta + l_{Y_e}^Q \cos \delta) \quad (34b)$$

where $\text{atan2}(y, x)$ is the function defined as follows.

$$\text{atan2}(y, x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ \pi/2 & \text{if } x = 0 \text{ and } y > 0, \\ -\pi/2 & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0 \end{cases} \quad (35)$$