

Long-Range spectral statistics of systems with infinitely many components

— Investigation of classically integrable systems based on the Berry-Robnik theory —

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(2005 年 1 月 30 日受理)

ベリーとロブニックの理論とは、古典力学系の位相空間が軌道の繋がらない独立な領域から成るときに、半古典極限における量子系のエネルギー準位が、これらの領域から供給される準位成分の統計的に独立な重ね合わせで与えられると主張するものである [1]。すると、無数の不変トラスを持つ可積分量子系のエネルギー準位列は無数個の準位成分によって表される。この仮定はパンディの公式 [2] を用いることによってスペクトル硬度、準位数分散、2 準位相関関数などの統計量を用いて表現することができる。本稿では無数個の準位数成分を持つ量子系の準位数分散を導出し、ポアソン統計からのずれを議論している。

By extending our previous argument[3] to the long-range spectral statistics of systems with infinitely many components, the level number variance[4] of classically integrable quantum systems is investigated. This observable is obtained by applying the theory proposed by Berry and Robnik[1] and the mathematical framework of Pandey[2] to systems with infinitely many components.

The Berry-Robnik theory relates the formation of the eigenenergy levels to the phase-space geometry by assuming that the sequence of the energy spectrum is given by the superposition of statistically independent sub-spectra, which are contributed respectively from eigenfunctions localized onto the invariant regions in phase space. Such independent sub-spectra is a consequence of the condensation of energy eigenfunctions on disjoint regions in the classical phase space and of the lack of mutual overlap between their eigenfunctions, and thus can be expected only in the semi-classical limit where the Planck constant tends to zero, $\hbar \rightarrow 0$. This mechanism is sometimes referred to as *the principle of uniform semi-classical condensation of eigenstates*[5].

This picture by Berry and Robnik is expressed by the statistical observable of eigenenergy levels as follows: Consider a system whose classical phase space is decomposed into N -disjoint regions which provide independent spectral components. When the entire level sequence is a product of the statistically independent superposition of N sub-sequences, the number variance

$\Sigma^2(L)$ (i.e., the average variance of the number of levels in an interval containing on average L levels) can be described by using those of sub-sequences, Σ_i^2 as

$$\Sigma^2(L; N) = \sum_{i=1}^N \Sigma_i^2(\rho_i L). \quad (1)$$

This formula is first proposed by Pandey[2], where $\rho_i (i = 1, 2, 3, \dots, N)$ denotes the statistical weights of individual spectral components. In the Berry-Robnik theory, the statistical weights of individual components are equivalent to the phase volume(Liouville measure).

In terms of the cluster function $Y_{2,i}(r)$ of a sub-sequence[6], $\Sigma_i^2(\rho_i L)$ is described as

$$\Sigma_i^2(\rho_i L) = \rho_i L - 2 \int_0^{\rho_i L} (\rho_i L - r) Y_{2,i}(r) dr, \quad (2)$$

where $Y_{2,i}(r)$ satisfies: $\int_{-\infty}^{+\infty} Y_{2,i}(r) dr = 1$.

In addition to equations (1) and (2), we assume the following two conditions for the statistical weights(Liouville measures):

- Assumption (i): The statistical weights of individual components uniformly vanish in the limit of infinitely many regions: $\max_i \rho_i \rightarrow 0$ as $N \rightarrow +\infty$.
- Assumption (ii): The weighted mean of the cumulative cluster function $c_i(L; \rho_i) = 2 \int_0^{\rho_i L} Y_{2,i}(r) dr$ uniformly converges as $\lim_{N \rightarrow +\infty} \sum_{i=1}^N \rho_i c_i(r; \rho_i) = \bar{c}(r)$.

Under assumptions (i) and (ii), the equation (1) leads to the following formula in the limit of $N \rightarrow +\infty$,

$$\Sigma_c^2(L) = L - \int_0^L \bar{c}(r) dr, \quad (3)$$

When the mean level spacings of individual components are sparse enough, one may expect $\bar{c}(r) = 0$, and the limiting number variance (3) of the whole energy sequence reduce to that of the Poisson statistics: $\Sigma_{\bar{c}=0}^2 = L$. In general, one may expect $\bar{c}(L) \neq 0$ which corresponds to a strong clustering of levels of individual components leading to the singular cluster function $Y_{2,i}(L)$ and the non-zero cumulative function $c_i(L; \rho_i)$. Thus, deviations from Poisson statistics are possible even when the energy levels of individual components are statistically independent.

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