

Reidemeister Torsion of a Homology Lens Space

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In my talk, I announced a formula giving the Reidemeister torsion of a homology lens space obtained by Dehn surgery on a knot in the three sphere associated to the universal abelian covering. And we applied it to get a generalization of Fukuhara's result in [2] that is a generalization of the classification of lens spaces.

After that, I found that essentially same result is already obtained by Turaev [4] in a more general setting. Here I will give the precise statement of result. For a proof of Theorem 1, the reader can refer the paper of Turaev [4] or a self-contained treatment in [3].

Statement of Results

Let k be a knot in the three sphere S^3 . For coprime integers $p > 0$ and q , let $K = L(p, q; k)$ denote a 3-manifold obtained from S^3 by Dehn surgery on k with coefficient p/q . Then $H_1(K, \mathbb{Z})$ is isomorphic to the cyclic group of order p generated by a meridian loop of k . Let $\tilde{K}$ denote the universal abelian covering of K with the covering transformation group Π and let T denote a generator of Π corresponding to the meridian loop of k . We assume that K is triangulated. Then the integral cellular chain group $C_q(\tilde{K}; \mathbb{Z})$ can be considered as a $\mathbb{Z}\Pi$ -free module with the standard basis

determined by K which is well defined up to sign, and up to multiplication, by elements of Π .

Suppose a homomorphism h from Π to the field of complex numbers F is given that takes T into a p -th root of unity $\tau (\neq 1)$. Using h , we can form the chain complex

$$C_* = F \otimes_{\Pi} C_*(\tilde{K}; Z)$$

over F . Then C_q is a finite dimensional vector space over F with the standard basis determined by the basis for $C_q(\tilde{K}; Z)$ above (thus determined by K). For each q , let v_q denote the volume in C_q determined by this basis. Then

Theorem 1. Suppose that the Alexander polynomial of k is $A(t)$. Then C_* is acyclic if and only if $A(\tau) \neq 0$. Therefore, if $A(\tau) \neq 0$, the Reidemeister torsion $\Delta_h(\tilde{K})$ is defined as $\pm h(\Pi) v_0 v_1^{-1} v_2 v_3^{-1}$ and is equal to

$$\pm h(\Pi) A(\tau) (\tau^r - 1)^{-1} (\tau - 1)^{-1}$$

where r is determined by the congruence $qr \equiv 1 \pmod{p}$.

The following generalizes Theorem 2 in Fukuhara [2], which he proved by using EA-matrix, an invariant for closed orientable 3-manifold defined by Fukuhara and Kanno [1].

Theorem 2. Let k and k' be knots in S^3 with trivial Alexander polynomials. Then $L(p, q; k)$ is homeomorphic to $L(p, q; k')$ only if $\pm qq' \equiv 1 \pmod{p}$ or $\pm q \equiv q' \pmod{p}$.

The classification of lens spaces is the case that k and k'

are trivial and Fukuhara's result is the case that k has the trivial Alexander polynomial and k' is trivial.

References

- [1] Fukuhara, S. and Kanno, J.: Extended Alexander matrices of 3-manifolds. Preprint. Tsuda College. Tokyo, Japan(1983).
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