

結び目解消操作

神戸大学理学部 中西康剛 (Yasutaka Nakanishi)

この報告では、以前に作成した最小交叉数が 9 以下の素な結び目の結び目解消数の表について、訂正及び修正をまずします。その後、結び目解消操作及び一般化された結び目解消操作について参考論文のリストを挙げることにします。結び目解消数 $u(K)$ の評価に用いたのは、次の定理です。

定理 1 ([Ms]) 結び目 K について、

$$0 \leq |\sigma(K)|/2 \leq g^*(K) \leq u(K)$$

が成り立つ。ただし、 $\sigma(K)$ は K の signature を、 $g^*(K)$ は K の 4 次元種数を表す。

定理 2 ([N1]) 結び目 K について、

$$0 \leq m(K) \leq u(K)$$

が成り立つ。ただし、 $m(K)$ は K の Alexander 行列の最小次数を表す。

定理 3 ([L1], [KaM]) 2 橋結び目 K については、 $u(K) = 1$ か $u(K) \geq 2$ であるかは判定できる。

これらと [N2] で掲げた次の予想に基づいて表が作られています。

予想 結び目の最小交叉の射影図には、結び目解消操作を施せば、結び目解消数が少なくなるような交叉が必ず存在する。

なお、以前の表で、 $u(9_{29}) = 1$ としましたが、以上のどれかに引っかかるわけではなく、1 または 2 に訂正します。

表において、 N_n は Alexander-Briggs notation で、 N は最小交叉数を、 n は交叉数 N の中での順番を表す。 σ' は signature の絶対値の半値を、 g^* は 4 次元種数を、 m は Alexander 行列の最小次数を表す。また、未確定の数値については、A で 1 または 2 を、X で 2 または 3 を表す。

Table of Unknotting Number

k	σ'	g^*	u	m	k	σ'	g^*	u	m	k	σ'	g^*	u	m	k	σ'	g^*	u	m
3_1	1	1	1	1	8_8	0	0	2	1	9_8	1	1	2	1	9_{29}	1	1	A	1
4_1	0	1	1	1	8_9	0	0	1	1	9_9	3	3	3	1	9_{30}	0	1	1	1
5_1	2	2	2	1	8_{10}	1	1	A	1	9_{10}	2	2	X	1	9_{31}	1	A	2	1
5_2	1	1	1	1	8_{11}	1	1	1	1	9_{11}	2	2	2	1	9_{32}	1	A	A	1
6_1	0	0	1	1	8_{12}	0	1	2	1	9_{12}	1	1	1	1	9_{33}	0	1	1	1
6_2	1	1	1	1	8_{13}	0	1	1	1	9_{13}	2	2	X	1	9_{34}	0	1	1	1
6_3	0	1	1	1	8_{14}	1	1	1	1	9_{14}	0	1	1	1	9_{35}	1	1	X	2
7_1	3	3	3	1	8_{15}	2	2	2	1	9_{15}	1	1	2	1	9_{36}	2	2	2	1
7_2	1	1	1	1	8_{16}	1	A	A	1	9_{16}	3	3	3	1	9_{37}	0	1	2	2
7_3	2	2	2	1	8_{17}	0	1	1	1	9_{17}	1	A	A	1	9_{38}	2	2	X	A
7_4	1	1	2	1	8_{18}	0	A	2	2	9_{18}	2	2	2	1	9_{39}	1	1	1	1
7_5	2	2	2	1	8_{19}	3	3	3	1	9_{19}	0	1	1	1	9_{40}	1	A	2	2
7_6	1	1	1	1	8_{20}	0	0	1	1	9_{20}	2	2	2	1	9_{41}	0	0	2	2
7_7	0	1	1	1	8_{21}	1	1	1	1	9_{21}	1	1	1	1	9_{42}	1	1	1	1
8_1	0	1	1	1	9_1	4	4	4	1	9_{22}	1	1	1	1	9_{43}	2	2	2	1
8_2	2	2	2	1	9_2	1	1	1	1	9_{23}	2	2	2	1	9_{44}	0	1	1	1
8_3	0	1	2	1	9_3	3	3	3	1	9_{24}	0	1	1	1	9_{45}	1	1	1	1
8_4	1	1	2	1	9_4	2	2	2	1	9_{25}	1	1	A	1	9_{46}	0	0	2	2
8_5	2	2	2	1	9_5	1	1	2	1	9_{26}	1	1	1	1	9_{47}	1	A	2	2
8_6	1	1	2	1	9_6	3	3	3	1	9_{27}	0	0	1	1	9_{48}	1	A	2	2
8_7	1	1	1	1	9_7	2	2	2	1	9_{28}	1	1	1	1	9_{49}	2	2	X	2

References on Unknotting Operation

- [Be] D. Bennequin, *Entrelacement et equations de Pfaff*, Asterisque **107-108** (1983), 87 – 161.
- [Bl] S. A. Bleiler, *A note on unknotting number*, Math. Proc. Cambridge Phils. Soc. **96** (1984), 469 – 471.
- [BW] M. Boileau and C. Weber, *Le problème de J. Milnor sur le nombre gordien des nœuds algébriques, Nœuds, tresses et singularités*, Monographie No. 31 de L'Enseign Math., 31, Univ. de Genève, 1983, pp. 49 – 98.
- [CL] T. D. Cochran and W. B. R. Lickorish, *Unknotting information from 4-manifolds*, Trans. Amer. Math. Soc. **297** (1986), 125 – 142.
- [FMS] S. Fukuhara, Y. Matsumoto, and O. Saeki, *An estimation for the unknotting numbers of torus knots*, Topology and its Appl. **38** (1991), 293 – 299.
- [KaM] T. Kanenobu and H. Murakami, *Two-bridge knots with unknotting number one*, Proc. Amer. Math. Soc. **98** (1986), 499 – 502.
- [Kw] A. Kawauchi (ed.), *Musubimeriron ("Knot Theory" in Japanese)*, Springer-Tokyo, Tokyo, 1990.
- [Ki1] S. Kinoshita, *On Wendt's theorem of knots*, Osaka Math. J. **9** (1957), 61 – 66.
- [Ki2] S. Kinoshita, *On Wendt's theorem of knots, II*, Osaka Math. J. **10** (1958), 259 – 261.
- [Kb1] T. Kobayashi, *Minimal genus Seifert surfaces for unknotting number 1 knots*, Kobe J. Math. **6** (1989), 53 – 62.
- [Kb2] T. Kobayashi, *Generalized unknotting operations and tangle decompositions*, Proc. Amer. Math. Soc. **105** (1989), 471 – 478.

- [Kb3] T. Kobayashi, *Fibered links and unknotting operations*, Osaka J. Math. **26** (1989), 699 – 742.
- [Kh 1] P. Kohn, *Two-bridge links with unlinking number one*, Proc. Amer. Math. Soc. **113** (1991), 1135 – 1147.
- [Kh2] P. Kohn, *Unknotting two component link*, preprint.
- [Kn] H. Kondo, *Knots of unknotting number 1 and their Alexander polynomials*, Osaka J. Math. **16** (1979), 551 – 559.
- [KrM] F. B. Kronheimer and T. S. Mrowka, *Gauge theory for embedded surfaces, I*, preprint.
- [L1] W. B. R. Lickorish, *The unknotting number of a classical knot*, Contemp. Math. **44** (1985), 117 – 121.
- [Me] W. W. Menasco, *A proof of the Bennequin-Milnor unknotting conjecture*, preprint.
- [Ml] J. W. Milnor, *Singular Points of Complex Hypersurfaces*, Ann. of Math. Studies, **61**, Princeton Univ. Press, Princeton, 1968.
- [Ms] K. Murasugi, *On a certain numerical invariant of link types*, Trans. Amer. Math. Soc. **117** (1965), 387 – 422.
- [Mk1] H. Murakami, *Algebraic unknotting operation*, Q & A in General Topology **8** (1990), 283 – 292.
- [N1] Y. Nakanishi, *A note on unknotting number*, Math. Seminar Notes Kobe Univ. **9** (1981), 99 – 108.
- [N2] Y. Nakanishi, *Unknotting numbers and knot diagrams with the minimum crossings*, Math. Seminar Notes Kobe Univ. **11** (1983), 257 – 258.
- [R] K. Reidemeister, *Knotentheorie*, Ergebnisse der Mathematik, **1**, Springer-Verlag,

Berlin, 1932.

- [Sa] T. Sakai, *A remark on the Alexander polynomials of knots*, Math. Seminar Notes Kobe Univ. **5** (1977), 451 – 456.
- [Sc1] M. Scharlemann, *Unknotting number one knots are prime*, Invent. Math. **82** (1985), 37 – 56.
- [Sc2] M. Scharlemann, *Unknotting number, genus, and companion tori*, Math. Ann. **280** (1988), 191 – 205.
- [Sh1] T. Shibuya, *Some relations among various numerical invariants for links*, Osaka J. Math. **11** (1974), 313 – 322.
- [Tr] A. G. Tristram, *Some cobordism invariants for links*, Math. Proc. Camb. Phil. Soc. **66** (1969), 251 – 264.
- [Wn] H. Wendt, *Die Gordische Auflösung von Knoten*, Math. Z. **42** (1937), 680 – 696.
- [We] S. H. Weintraub, *Inefficiently embedded surfaces in 4-manifolds*, Algebraic Topology Aarhus 1978, Lecture Notes in Math., 763, Springer-Verlag, 1979.
- [Ym1] M. Yamamoto, *Lower bounds for the unknotting numbers of certain torus knots*, Proc. Amer. Math. Soc. **86** (1982), 519 – 524.
- [Ym2] M. Yamamoto, *Lower bounds for the unknotting numbers of certain iterated torus knots*, Tokyo J. Math. **9** (1986), 81 – 86.

References on Generalized Unknotting Operation

- [A1] H. Aida, *Unknotting operations of polygonal type*, Tokyo J. Math. **15** (1992), 111 – 121.
- [A2] H. Aida, *The oriented Δ_{ij} -moves on links*, Kobe J. Math. (to appear).
- [F] R. H. Fox, *Congruence classes of knots*, Osaka Math. J. **10** (1958), 37 – 41.

- [HNT] J. Hoste, Y. Nakanishi, and K. Taniyama, *Unknotting operations involving trivial tangle*, Osaka J. Math. **27** (1990), 555 – 566.
- [Ku] L. Kauffman, *On Knots*, Ann. Math. Study 115, Princeton Univ. Press, Princeton, 1987.
- [KB] L. Kauffman and T. F. Banchoff, *Immersions and mod 2 quadratic forms*, Amer. Math. Monthly **84** (1977), 168 – 185.
- [Kr] R. Kirby, *Problems in low dimensional Topology*, Alg. Geom. Topology (Stanford 1976) II Proc. Symp. Pure Math. 32, Amer. Math. Soc., pp. 273 – 312.
- [L2] W. B. R. Lickorish, *Unknotting by adding a twisted band*, Bull. London Math. Soc. **18** (1986), 613 – 615.
- [My] K. Miyazaki, *A solution of Mathieu Conjecture*, Proc. of “Knot Theory and its Application”, Osaka, 1991.
- [MY] K. Miyazaki and A. Yasuhara, *Unknotting operations from a 4-dimensional viewpoint*, in preparation.
- [Mn] J. M. Montesinos, *A note on moves and irregular covering of S^4* , Combinatorial methods in topology and algebraic geometry, Contemp. Math., 44, Amer. Math. Soc., 1985, pp. 345 – 349.
- [Mt] K. Motegi, *Knotting trivial knots and resulting knot types*, preprint.
- [Mk2] H. Murakami, *Some metrics on classical knots*, Math. Ann. **270** (1985), 35 – 45.
- [Mk3] H. Murakami, *Delta-unknotting number and the Conway polynomials*, preprint.
- [MN] H. Murakami and Y. Nakanishi, *On a certain move generating link-homology*, Math. Ann. **284** (1989), 75 – 89.
- [MS] H. Murakami and S. Sakai, *Sharp-unknotting number and Alexander module*,

preprint.

- [N3] Y. Nakanishi, *On Fox's congruence classes of knots, II*, Osaka J. Math. **27** (1990), 207 – 215.
- [N4] Y. Nakanishi, *Replacements in the Conway Third Identity*, Tokyo J. Math. **14** (1991), 197 – 203.
- [N5] Y. Nakanishi, *From a view of localized link theory*, Knots 90, Walter de Gruyter, Berlin and New York, 1992, pp. 173 – 183.
- [NS] Y. Nakanishi and S. Suzuki, *On Fox's congruence of knots*, Osaka J. Math. **24** (1987), 217 – 225.
- [Ok] M. Okada, *Delta-un knotting operations and the second coefficients of the Conway polynomial*, J. Math. Soc. Japan **42** (1990), 713 – 717.
- [Oh1] Y. Ohya, *Un knotting operations of rotation type*, preprint.
- [Oh2] Y. Ohya, *On un knotting operations obtained by rotation and twist*, Proc. of “Knot Theory and its Applications”, Osaka, 1991.
- [Oh3] Y. Ohya, *Un knotting operations and twisting*, in preparation.
- [P1] J. H. Przytycki, *t_k -moves on links*, Proc. U.C.S.C. meetings on “Braids”, Comtemp. Math. **78** (1988), 615 – 656.
- [P2] J. H. Przytycki, *Elementary conjectures in classical knot theory*, preprint.
- [P3] J. H. Przytycki, *$t_3, \overline{t_4}$ moves conjecture for oriented links with matched diagram*, Math. Proc. Cambridge Phil. Soc. **108** (1990), 55 – 61.
- [Sc3] M. Scharlemann, *Smooth spheres in R^4 with four critical points are standard*, Invent. Math. **79** (1985), 125 – 141.
- [Sh2] T. Shibuya, *Self #-un knotting operations of links*, Memoirs Osaka Inst. Tech. **34**

(1989), 9 – 17.

[Sh3] T. Shibuya, *Some metric functions of links*, preprint.

[Sh4] T. Shibuya, *Two self $\#$ -equivalences of links in solid tori*, preprint.

[Te] M. Teragaito, *Composite knots trivialized by twists*, preprint.

[U] Y. Uchida, *On delta-un knotting operation*, preprint.

[Ys] A. Yasuhara, *On slice knots in the complex projective plane*, *Revista Math.* (to appear).