

SPECTRAL STRUCTURE OF THE EARTH TIDES AND RELATED PHENOMENA — GRAVIMETRIC RECORD —

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Abstract

The spectral structure of the earth tides and some related phenomena have been obtained from the record observed with an Askania gravimeter at Kyoto by the Fourier transform method.

The harmonic constants for 8 principal constituents of the diurnal and semi-diurnal tides are determined and compared with those by the least squares method, showing no significant differences. Spectral features for longer period ranges up to 30 days are also investigated, and several strong peaks with periods of 3.7, 4.8, 5.6, 7.1, 11.1, 11.9, 13.7, 14.7, 16.2, 18.5 and 29.6 days are found in the spectrum.

1. Introduction

Since Darwin established a method of harmonic analysis in 1883, it has been developed by a number of investigators. The methods devised by Doodson, Pertzev, Lecolazet and Lennon, have been widely used in tidal analysis, and the results obtained made important contributions to the study on elastic properties of the earth. The fundamental principle of the usual methods is to isolate tidal constituents with particular angular velocities to be obtained, by an application of linear combinations, eliminating a drift.

It is well known that tidal phenomena consist of three species; semidiurnal, diurnal and long-period. The usual methods of tidal analysis have successfully been applied to determine harmonic constants for the constituents, but would not be valid for getting the whole features of the phenomena. To investigate the spectral structure of the earth tides and related phenomena would be an important and interesting problem in the field of tidal studies. This can be attained by a spectral analysis with an application of the Fourier transform, and it is expected that unknown phenomena, if they were existing, might be detected from the spectrum. The Fourier transform method has the features

that it can be applied to an arbitrary length of observational records, without the previous knowledge for the angular velocities of tidal constituents and also particular techniques for eliminating a drift, while the usual methods have several limitations to these points. The recent remarkable progress of electronic computers made it easy to introduce this method into the analysis of observational data amounting to great volumes.

Harrison and others [1963] first applied the power spectrum and Fourier transform techniques to the earth tidal data of 30 to 40 days observed with LaCoste Romberg tidal gravimeters. A few articles of similar studies have since been published (Jobert [1963], Slichter et al. [1964]). They have determined the gravimetric factor and phase lag for several tidal constituents, by comparing the spectra of an observational record with those of the corresponding theoretical curve.

In the present study, the Fourier transform is applied to a continuous record of one year duration observed with a gravimeter at Kyoto. Spectral structures of semidiurnal and diurnal tides are investigated, and harmonic constants for the principal tidal constituents are determined by a normalization for the length of the record. The obtained results are then compared with those determined by the least squares method which was applied to the same data. The spectral analysis covers also longer period ranges up to 30 days.

2. Method of analysis

A gravimetric record generally consists of the earth tides, effects of oceanic tides, disturbances due to meteorological changes, the instrumental drift and others. In the Fourier transform technique, the record is not necessarily treated as a tidal function but as a simple time series involving some signals and noise. It is expected that this technique will provide directly the whole spectral structure of tidal phenomena, yielding specific amplitudes and phases for continuous frequencies over the ranges to be investigated.

Let an arbitrary time function be $f(t)$, then its Fourier transform is given by

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt \\ &= a(\omega) - ib(\omega) \end{aligned}$$

where

$$\begin{aligned} a(\omega) &= \int_{-\infty}^{+\infty} f(t) \cos \omega t \, dt, \\ b(\omega) &= \int_{-\infty}^{+\infty} f(t) \sin \omega t \, dt. \end{aligned}$$

If a function of frequency $F(\omega)$ is given, $f(t)$ is expressed by the formula,

$$\begin{aligned} f(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega \\ &= \frac{1}{\pi} \int_0^{+\infty} |F(\omega)| \cos[\omega t + \varphi(\omega)] d\omega. \end{aligned}$$

The amplitude and phase spectra, $|F(\omega)|$ and $\varphi(\omega)$, should be obtained by

$$\begin{aligned} |F(\omega)| &= \sqrt{[a(\omega)]^2 + [b(\omega)]^2}, \\ \varphi(\omega) &= \tan^{-1}[-b(\omega)/a(\omega)]. \end{aligned}$$

Since available records have a finite duration, the spectrum that one can actually obtain is not exactly $F(\omega)$, but the Fourier transform of a truncated function $f(t)$ which is zero outside the finite interval $|t| \leq T$, that is defined by the form of convolution $\int_{-\infty}^{+\infty} F(\omega) \frac{\sin T(\omega - x)}{\pi(\omega - x)} dx$. In the present case, however, the duration of observation $2T$ is long enough for the angular frequencies considered, or $\omega T \gg 1$, so that the convolution may well be approximated by $F(\omega)$. For practical calculation of $a(\omega)$ and $b(\omega)$, numerical integrations were performed on an IBM 7090 computer over the above time interval, following the trapezoid or Simpson's rule, after a linear drift of data has been removed. The difference between results from the two rules was too small to give serious effects on final values, in the present case. No data window or frequency window has been applied to get the amplitude and phase spectra, because it is one of the present purposes to determine the absolute amplitudes for specific frequencies. It is to be remarked here that, in order to estimate the absolute amplitudes of tidal constituents, $|F(\omega)|$ was multiplied by the coefficient $1/T$ for its normalization, while some investigators (Harrison et al. [1963], Jobert [1963], Slichter et al. [1964]) used to take a method of comparing the observed amplitude spectrum with the theoretical one, which can be computed from the corresponding tidal curve based on the perfectly rigid earth.

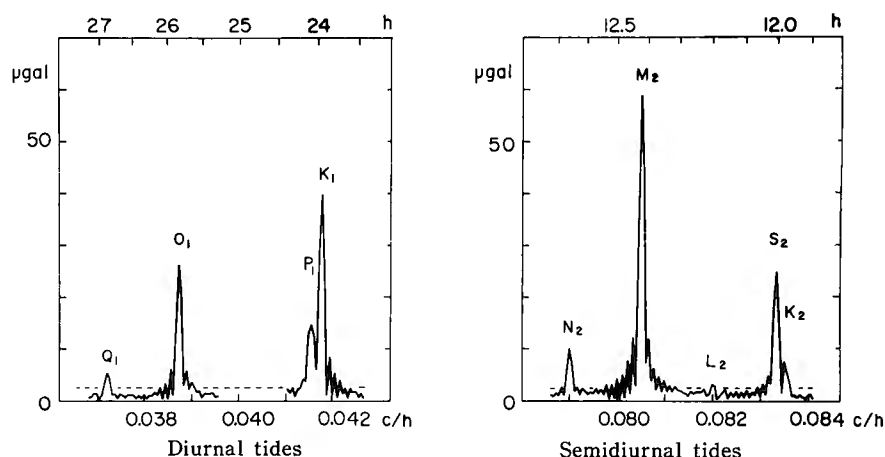
3. Data

Analyzed here is the earth tidal record which has been obtained by an Askania gravimeter at the Geophysical Institute of Kyoto University (35°02'N, 135°47'E, h=60 m) during the period from August 1, 00h00m, 1959 to July 31, 23h00m, 1960 (UT) (Nakagawa [1962]). The number of digitized readings at one hour interval amounts to 8,784, so that this should give a frequency resolution of 0.0002 c/h in the spectral analysis. A trace amplitude of 1 mm on the record corresponds to 2.566 μgal in gravity variation. The reading error, being less than 0.5 mm, is therefore estimated to be within 1.3 μgal .

4. Results and discussion

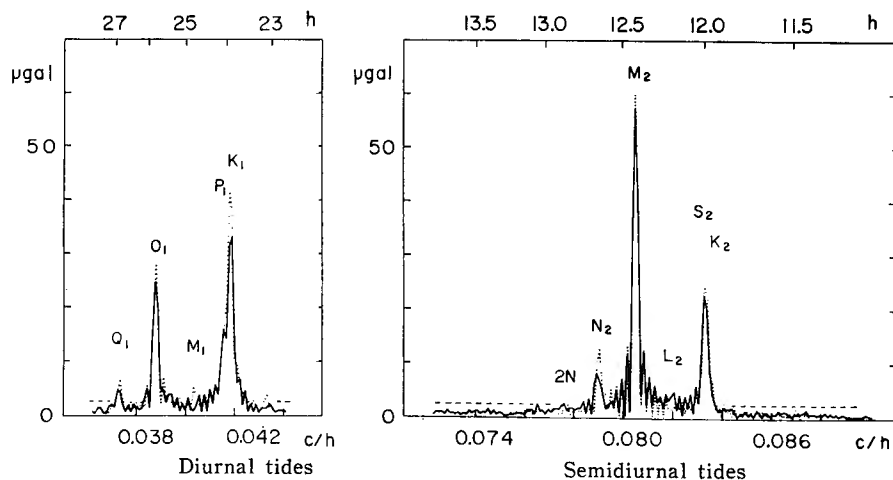
(1) Spectral structure of the earth tides

The Fourier spectrum around diurnal and semidiurnal tidal frequencies obtained from the one year's record is shown in Fig. 1. It can be seen that prominent spectral peaks occur for the principal tidal constituents, M_2 (principal lunar semidiurnal), S_2 (principal solar semidiurnal), N_2 (larger lunar elliptic semidiurnal), K_2 (luni-solar declinational semidiurnal), L_2 (smaller lunar elliptic



Period of analysis : August 1, 00:00, 1959 - July 31, 23:00, 1960 (UT)

Fig. 1.



Period of analysis : August 1, 00:00, 1959 - January 31, 23:00, 1960 (UT)
 — February 1, 00:00, 1960 - July 31, 23:00, 1960 (UT)

Fig. 2.

semidiurnal), K_1 (luni-solar declinational diurnal), O_1 (lunar declinational diurnal), Q_1 (lunar elliptic diurnal) and P_1 (solar declinational diurnal), indicating an excellent agreement with theoretically predicted features. Although each constituent should be represented by a line spectrum, a finite duration of the observation adds it a narrow side lobe. It is to be noted that the two semidiurnal constituents S_2 and K_2 whose frequencies differ only by 0.00023 c/h (0.0328 mean solar days) have been resolved each other by this method. Some irregular noise at the wings of strong spectral peaks may be due to the truncation of the record without applying any smoothing window. This could be a source of contamination, if tidal lines to be investigated are not widely spaced as in the above case. The broken line shown in Fig. 1 indicates an upper limit of reading errors.

Fig. 2 shows the same kind of spectrum computed from the former and latter half year data of the present gravimetric record, with dotted and solid lines, respectively. The K_2 -constituent can no longer be separated from S_2 , and it is also somewhat difficult to identify a sharp peak of P_1 . It is noticed that the amplitudes of all the principal constituents for the former half year are larger than those for the latter, and that the ratio is not always the same for all the constituents. A similar tendency has also been recognized in the gravimetric factor calculated by Lecolazet's method using the same data (Nakagawa [1962]). The fluctuation of a recording sensitivity, instrumental noise and digitizing error are unlikely to be sources of the discrepancy up to 10%. This could be caused by meteorological disturbances or any other fluctuations of gravity field, but it should be investigated in more detail from various standpoints.

(2) *Amplitude and phase of the principal tidal constituents*

Since it has been able to get the spectral features of the diurnal and semi-diurnal tides, the absolute amplitude and phase of 8 principal constituents are discussed in this section. They were estimated by applying the Fourier transform to the full year record for their theoretical angular frequencies. The results are tabulated in Table 1, together with the corresponding values determined by the least squares method (Nakagawa and Satô [1966]) and theoretical values predicted for the perfectly rigid earth. Table 2 gives the gravimetric factor G and phase lag κ computed from the above results. It is understood that the obtained values from the Fourier method are reasonable results. The present analysis shows a mean level of background noise with about $0.7 \mu\text{gal}$, as shown in Figs. 1 and 2, which may be considered as the uncertainty of each tidal amplitude. The uncertainty yields errors of 0.014 in G and 0.75° in κ for the

Table 1. Results of H and ζ obtained from gravimetric data

Method	Fourier transform		Least squares		Perfectly rigid earth	
	H (μgal)	ζ (degree)	H (μgal)	ζ (degree)	H (μgal)	ζ (degree)
M_2	58.189	181.91	58.073	182.20	50.307	180.00
S_2	25.546	175.27	25.390	174.32	23.465	180.00
N_2	10.084	189.84	10.238	183.90	9.632	180.00
K_2	7.652	174.25	7.228	177.22	6.382	180.00
K_1	46.466	182.14	46.658	182.03	40.930	180.00
O_1	33.494	178.85	33.858	179.88	29.078	180.00
Q_1	6.897	188.91	6.977	182.14	5.567	180.00
P_1	15.142	182.46	15.073	182.06	13.548	180.00

H : Amplitude of the constituent in the case that an inclination of the lunar orbit to the equator equals to its average value of $23^\circ 27' 08''$.

ζ : Time interval from the instant when an imaginary celestial body corresponding to that constituent has transited the meridian of the observation station until the instant when the observed constituent actually reaches the maximum value.

Table 2. Values of G and κ

Method	Fourier transform		Least squares		Comparison	
	G_F	κ_F	G_L	κ_L	G_F/G_L	$\kappa_F - \kappa_L$
M_2	1.157	- 1.91°	1.154	- 2.20°	1.003	+ 0.29°
S_2	1.089	+ 4.73	1.082	+ 5.68	1.006	- 0.95
N_2	1.047	- 9.84	1.063	- 3.90	0.985	- 5.94
K_2	1.199	+ 5.75	1.133	+ 2.78	1.058	+ 2.97
K_1	1.135	- 2.14	1.140	- 2.03	0.996	- 0.11
O_1	1.152	+ 1.15	1.164	+ 0.12	0.990	+ 1.03
Q_1	1.239	- 8.91	1.253	- 2.14	0.989	- 6.77
P_1	1.118	- 2.46	1.113	- 2.06	1.004	- 0.40

G : Gravimetric factor. $G \equiv 1 - 3h/2 + h$ (h and k : Love's numbers).

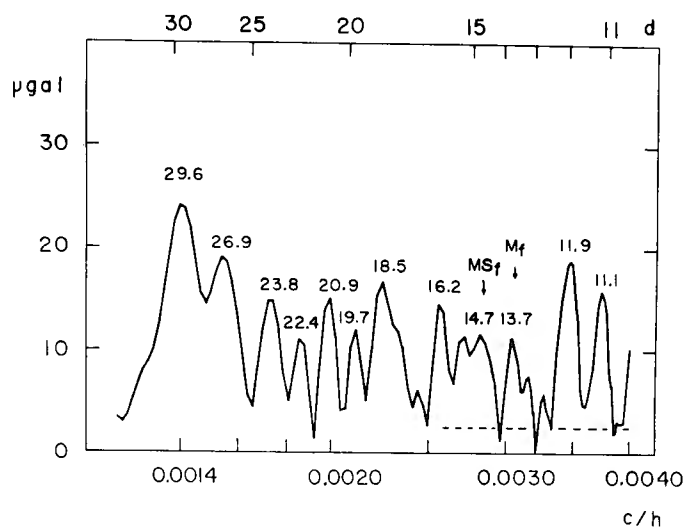
κ : Phase lag. The positive sign of κ shows that the observed tide advances the theoretical one, while the negative sign shows that the former lags behind the latter.

M_2 -constituent. These values are not standard deviations, but should be regarded as mean errors.

As can be seen in Table 2, there are satisfactory agreements between the results by the Fourier transform and least squares techniques, except for the amplitude of K_2 and for the phase lags of N_2 and Q_1 . The discrepancy for the former may be explained by a cross contamination with S_2 , but no definite explanation is found for the latter.

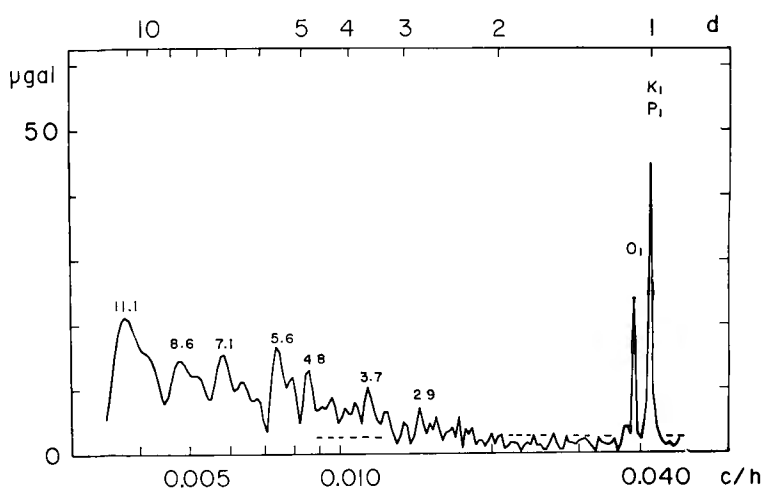
(3) Spectra in long-period ranges

The Fourier analysis has been extended to longer period ranges which are



Period of analysis : August 1, 00:00, 1959 – July 31, 23:00, 1960 (UT)

Fig. 3 (a).



Period of analysis : May 1, 00:00, 1960 – July 31, 23:00, 1960 (UT)

Fig. 3 (b).

eliminated in term of drift in the usual methods of tidal analysis. A continuous observation of one year duration allows us to discuss the spectral structure of gravimetric variation with periods up to one month. The obtained Fourier spectra in the frequency range from 0.0014 to 0.0040 c/h (30 to 10 days) are shown in Fig. 3(a), and those from 0.004 to 0.040 c/h (below 11 days) are given in Fig. 3(b). The latter was computed from 3 months' data. The period in days is indicated on the top of each spectral peak. Spectral peaks corres-

ponding to the fortnightly constituents Mf (lunar declinational, with a period of 13.66 mean solar days) and MSf (luni-solar variation, with a period of 14.77 mean solar days) can be seen in Fig. 3(a), although they do not have large amplitude as predicted theoretically for the present station situated in medium latitudes. The lunar elliptic monthly constituent Mm with a period of 27.55 mean solar days cannot be identified from the spectrum, but there is an apparent peak near the synodical period (29.53 days). The question what the latter means is left unanswered. It is also noted that there are several strong peaks which may be regarded as significant ones in comparison with noise levels but are theoretically unassigned. Table 3 lists the periods and amplitudes of

Table 3. Prominent long-period modes recorded by the Askania gravimeter

Period (day)	Amplitude (μ gal)	Remarks
29.6	25	Mm (27.55 days)
26.9	20	
23.8	15	
22.4	11	
20.9	16	
19.7	12	MSf (14.77 days) Mf (13.66 days)
18.5	17	
16.2	12	
14.7	12	
13.7	12	
11.9	19	
11.1	16	
8.6	15	
7.1	16	
5.6	18	
4.8	13	
3.7	10	
2.9	8	

the prominent peaks. Some of them may be attributable to an instrumental drift, and others might be associated with the change in the ground level due to meteorological or other disturbances. The relation with these phenomena will be investigated in near future by analyzing temperature and barometric records which were obtained concurrently with gravimetric observations. However, a possibility that there could be time variations with the long periods in the gravity field cannot be ruled out. One of the possible methods for looking for the sources would be to compare the present results with observations of the orbital motion of artificial satellites.

5. Conclusion

In the present article, the spectral structure of the earth tides and some related phenomena was obtained from a gravimetric tidal record of one year duration, by the use of the Fourier transform technique. The results showed that spectral peaks for principal constituents of the semidiurnal and diurnal tides are in excellent agreement with theoretical prediction. The amplitude and phase of 8 principal constituents, M_2 , S_2 , N_2 , K_2 , K_1 , O_1 , Q_1 and P_1 , were determined for their proper frequencies, with a certain uncertainty estimated from noise levels. The gravimetric factor and phase lag obtained were then compared with those from the method of least squares, indicating a satisfactory agreement. The present analysis has revealed long-period gravimetric variations with periods from a few days to one month including the fortnightly constituents Mf and MSf .

The sources of the variations are not yet precisely known. It would be necessary to make long term observations simultaneously at many stations to obtain conclusive evidence of the phenomena.

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