

Tangential Representations at Fixed Points

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1. BASIC PROBLEMS

Let G be a finite group throughout this paper. We mean by a (real) G -module a real G -representation (space) of finite dimension. Let $\mathcal{S}(G)$ denote the set of all subgroups of G and let $\mathcal{P}(G)$ denote the subset of $\mathcal{S}(G)$ consisting of all subgroups of prime power order. Unless otherwise stated, M will stand for a (smooth) G -manifold. S. Cappell-J. Shaneson referred the next problem to a basic problem on Algebraic and Differential Topology.

Problem (Basic Problem A). Let $x, y \in M^G$. How similar is a neighborhood of x to that of y as G -spaces?

If $x \in M^G$, then we can regard the tangent space $T_x(M)$ at x in M as a G -module. Thus the problem above is equivalent to ask

Problem (Basic Problem B). How similar is $T_x(M)$ to $T_y(M)$ as G -modules?

A specific case of the problem was posed by P. A. Smith.

Problem (Smith Problem). If Σ is a homotopy sphere with exactly two fixed points x and y , then is $T_x(\Sigma)$ isomorphic to $T_y(\Sigma)$ as G -modules?

We would like to study this problem in a slightly generalized form. Now let $\mathfrak{A}(2)$ denote the family of all (smooth) G -actions on manifolds with exactly 2 fixed points and let $\mathfrak{X} \subset \mathfrak{A}(2)$. We say that G -modules V and W are $\mathfrak{X}$ -related, and write $V \sim_{\mathfrak{X}} W$, if there exists a smooth G -action on $M \in \mathfrak{X}$ such that $M^G = \{a, b\}$, $T_a(M) \cong_G V$ and $T_b(M) \cong_G W$. Let $\text{RO}(G)$ denote the real representation ring of G . We define the $\mathfrak{X}$ -relation set $\text{RO}(G, \mathfrak{X})$ of G by

$$\text{RO}(G, \mathfrak{X}) = \{[V] - [W] \in \text{RO}(G) \mid V \sim_{\mathfrak{X}} W\}$$

Problem (Basic Problem C). Describe $\mathrm{RO}(G, \mathfrak{X})$ in terms of Algebra (or Representation Theory)

We say that a G -action on a disk D has a *linear boundary action* if the boundary ∂D is G -diffeomorphic to the unit sphere $S(V)$ for some G -module V . A G -action on a homotopy sphere Σ is called a *G -semilinear sphere* if Σ^H is a homotopy sphere for each $H \leq G$. G -modules V and W are called *$\mathcal{P}$ -matched* if $\mathrm{res}_P^G V \cong_P \mathrm{res}_P^G W$ for all $P \in \mathcal{P}(G)$.

We will discuss Basic Problem C for the following subfamilies of $\mathfrak{A}(2)$.

$$\mathfrak{E} = \{G\text{-actions on Euclidean spaces} \in \mathfrak{A}(2)\}$$

$$\mathfrak{D} = \{G\text{-actions on disks} \in \mathfrak{A}(2)\}$$

$$\mathfrak{D}_{\partial\text{-lin}} = \{G\text{-actions on disks with linear boundary action} \in \mathfrak{A}(2)\}$$

$$\mathfrak{S} = \{G\text{-actions on homotopy spheres} \in \mathfrak{A}(2)\}$$

$$\mathfrak{S}_{\text{s-free}} = \{\text{semi free actions} \in \mathfrak{S}\}$$

$$\mathfrak{S}_{CS} = \{\Sigma \in \mathfrak{S} \text{ such that } |\Sigma^H| = 2 \text{ or } \Sigma^H \text{ is connected } (\forall H \leq G)\}$$

$$\mathfrak{S}_{\text{s-lin}} = \{G\text{-semilinear spheres} \in \mathfrak{A}(2)\}$$

$$\mathfrak{pS} = \{\Sigma \in \mathfrak{S} \ (\Sigma^G = \{x, y\}) \text{ such that } T_x(\Sigma) \text{ and } T_y(\Sigma) \text{ are } \mathcal{P}\text{-matched}\}$$

With this notation, the Smith Problem is equivalent to ask whether $\mathrm{RO}(G, \mathfrak{S}) = 0$ or not.

Here we may remark the following.

Theorem (G. E. Bredon [2]). *Let $G = C_n$ with $n = p^a$ and $\Sigma \in \mathfrak{S}$ with $\dim \Sigma = 2k$ and $x, y \in \Sigma^G$. Then $T_x(\Sigma) - T_y(\Sigma)$ is divisible by p^h in $\mathrm{RO}(G)$, where $h = \left\lfloor \frac{pk - n}{pn - n} \right\rfloor$.*

By T. Petrie (e.g. [24]), the theorem above implies that if $\dim \Sigma \gg n$ then $T_x(\Sigma) \cong_G T_y(\Sigma)$. Thus, in the case $G = C_n$ with $n = 2^a \geq 8$, the set $\mathrm{RO}(G, \mathfrak{S})$ is not additively closed.

2. PRELIMINARY

Let $\mathcal{H}$ be a set of subgroups of G . G -modules V and W are called $\mathcal{H}$ -matched if $\text{res}_H^G V \cong_H \text{res}_H^G W$ for all $H \in \mathcal{H}$. A G -module V is called $\mathcal{H}$ -free if $V^H = 0$ holds for any $H \in \mathcal{H}$. For $M \subset \text{RO}(G)$, and $\mathcal{H}, \mathcal{K} \subset \mathcal{S}(G)$, we define

$$M_{\mathcal{H}} = \{V - W \in M \mid V \text{ and } W \text{ are } \mathcal{H}\text{-matched}\}$$

$$M^{\mathcal{K}} = \{V - W \in M \mid V, W \text{ are } \mathcal{K}\text{-free}\}$$

$$M_{\mathcal{H}}^{\mathcal{K}} = M_{\mathcal{H}} \cap M^{\mathcal{K}}.$$

By Definition, we have $\text{RO}(G, \mathfrak{p}\mathfrak{S}) = \text{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}$.

In some other papers, V and W are called *Smith equivalent* if $V \sim_{\mathfrak{S}} W$; V and W are called *s-Smith equivalent* if $V \sim_{\mathfrak{S}_{s\text{-lin}}} W$; V and W are called *primary Smith equivalent* if $V \sim_{\mathfrak{p}\mathfrak{S}} W$. The set $\text{Sm}(G) = \text{RO}(G, \mathfrak{S})$ was usually called the *Smith set* and the set $\text{RO}(G, \mathfrak{p}\mathfrak{S})$ *primary Smith set*. By definition, $\text{Sm}(G)_{\mathcal{P}(G)} = \text{RO}(G, \mathfrak{p}\mathfrak{S})$.

A finite group G is called a *mod $\mathcal{P}$ cyclic group* if there exists a normal subgroup P of G such that P is of prime power order and G/P is cyclic. G is called a *mod $\mathcal{P}$ hyperelementary group* if there exists a normal series $P \trianglelefteq H \trianglelefteq G$ such that P and G/H are of prime power order and H/P is cyclic. G is called an *Oliver group* if G is not a mod $\mathcal{P}$ hyperelementary group. Thus G is an Oliver group if and only if G admits a G -action on a disk without fixed points.

Let p be a prime. Let $G^{\{p\}}$ denote the smallest normal subgroup H of G such that G/H has the order of a p -power. We refer $G^{\{p\}}$ to the *Dress subgroup of type p* . Let G^{nil} denote the smallest normal subgroup H of G with nilpotent G/H . It follows that

$$G^{\text{nil}} = \bigcap_q G^{\{q\}}.$$

Let us adopt the following notation.

$$\mathcal{PC}(G) = \{\text{mod-}\mathcal{P} \text{ cyclic subgroups of } G\}$$

$$\mathcal{L}(G) = \{L \in \mathcal{S}(G) \mid L \supset G^{\{p\}} \text{ for some prime } p\}$$

$$\mathcal{M}(G) = \mathcal{S}(G) \setminus \mathcal{L}(G)$$

3. CLASSICAL RESULTS (UNTIL 1996)

There are various affirmative answers to the Smith Problem. It is easy to see that if $V \sim_{\mathfrak{S}} W$ then $\text{res}_P^G V \cong_P \text{res}_P^G W$ for all $P \in \mathcal{P}(G)$ with $|P| \nmid 4$. By Atiyah-Bott and Milnor, $V \sim_{\mathfrak{S}_{\text{s-free}}} W$ implies $V \cong_G W$. Sanchez showed that $V \sim_{\mathfrak{S}} W$ implies $\text{Res}_P^G V \cong_P \text{Res}_P^G W$ for any P of odd-prime-power order.

To the contrary, there are negative answers to the Smith Problem. T. Petrie showed that if G is an odd-order abelian group containing $C_{pqrs} \times C_{pqrs}$, where p, q, r, s are distinct odd primes, then $\text{RO}(G, \mathfrak{pS}) \neq 0$. In addition, Cappell-Shaneson showed that if $G = C_{4n}$ with $n \geq 2$ then $\text{RO}(G, \mathfrak{S}_{CS}) \neq 0$.

Here we also recall classical results concerned with $\sim_{\mathfrak{e}}$ and $\sim_{\mathfrak{D}}$. By Petrie, if G is an odd-order abelian group, then $\text{RO}(G, \mathfrak{D})^{\mathcal{L}(G)} = \text{RO}(G)_{\mathcal{P}(G)}^{\mathcal{L}(G)}$. R. Oliver showed that if G is not of prime power order, then $\text{RO}(G, \mathfrak{E}) = \text{RO}(G)_{\mathcal{P}(G)}^{\{G\}}$; if G is an Oliver group, then $\text{RO}(G, \mathfrak{D}) = \text{RO}(G)_{\mathcal{P}(G)}^{\{G\}}$.

4. DIMENSION CONDITIONS ON G -MODULES

In order to apply an equivariant surgery theory to a G -manifold M , we require certain properties for M^H , where $H \in \mathcal{S}(G)$. If $V = T_x(M)$ with $x \in M^G$, then $\dim V^H$ is equal to the dimension of the connected component of M^H containing the point x .

Let V be a G -module.

- (1) We say that V satisfies the *strong gap condition* if $\dim V^P > 2 \dim V^H + 2$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$.
- (2) We say that V satisfies the *gap condition* if $\dim V^P > 2 \dim V^H$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$.
- (3) We say that V satisfies the *weak gap condition* if the next dimension condition:
 $(\text{Dim}) \dim V^P \geq 2 \dim V^H$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$
 is satisfied and V satisfies the orientation condition:
 $(\text{Ori}) g : V^H \rightarrow V^H$ preserves orientation for any $g \in N_G(P) \cap N_G(H)$ such that
 $P \in \mathcal{P}(G)$, $P < H \leq G$ and $\dim V^P = 2 \dim V^H$.

A finite group G is called a *gap group* if there exists a G -module V such that V is $\mathcal{L}(G)$ -free and satisfies the gap condition.

5. LAITINEN'S CONJECTURE

E. Laitinen and K. Pawałowski were interested in determining the set $\text{RO}(G, \mathfrak{p}\mathfrak{S})$, namely $\text{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}$.

Conjecture (E. Laitinen). Let G be an Oliver group. Then $\text{RO}(G, \mathfrak{p}\mathfrak{S}) \neq 0$ holds if and only if $\text{RO}(G, \mathfrak{D}) \neq 0$.

For $g \in G$, let (g) denote the conjugacy class $\{aga^{-1} \in G \mid a \in G\}$, and let $(g)^\pm$ denote the *real conjugacy class* $(g) \cup (g^{-1})$. Then a_G stands for the number of all real conjugacy classes $(g)^\pm$ such that $g \in G$ is not of prime power order. If G is an Oliver group, since $\text{RO}(G, \mathfrak{D}) = \text{RO}(G)_{\mathcal{P}(G)}^{\{G\}}$, we obtain $\text{rankRO}(G, \mathfrak{D}) = a_G - 1$.

Theorem (E. Laitinen-K. Pawałowski, K. Pawałowski-R. Solomon, M. Morimoto). *Laitinen's Conjecture has been studied and is affirmative for Oliver gap groups G satisfying one of the following conditions.*

- (1) G is a perfect group [9].
- (2) G is a nonsolvable group:
 - Case $G \not\cong P\Sigma L(2, 27)$: [20].
 - Case $G = P\Sigma L(2, 27)$: $\text{RO}(G, \mathfrak{S}) = \text{RO}(G)_{\mathcal{P}(G)}^{\{G\}} \cong \mathbb{Z}$ [12].
- (3) G has a normal subgroup N such that $G/N \cong C_{pq}$ with distinct odd primes p, q [20].
- (4) G is of odd order [20].

Let $SG(m, n)$ denote the n th small group of order m given by the computer software GAP [5].

Theorem (A. Koto-M. Morimoto-Y. Qi, M. Morimoto, T. Sumi). *Laitinen's Conjecture fails and $\text{RO}(G, \mathfrak{S}) = 0$ for Oliver groups G satisfying one of the following conditions.*

- (1) $G = \text{Aut}(A_6)$ (nongap group, $G/G^{\text{nil}} = C_2 \times C_2$) [14].

- (2) $G = SG(72, 44)$ (gap group, $G/G^{nil} = C_6$) [28].
- (3) $G = SG(288, 1025)$ (gap group, $G/G^{nil} = C_6$) [28].
- (4) $G = SG(432, 734)$ (nongap group, $G/G^{nil} = C_2$) [28].
- (5) $G = SG(576, 8654)$ (nongap group, $G/G^{nil} = C_2 \times C_2$) [28].
- (6) $G = SG(1176, 220)$ (gap group, $G/G^{nil} = C_3$) [7].
- (7) $G = SG(1176, 221)$ (gap group, $G/G^{nil} = C_3$) [7].

6. DETERMINATION OF $RO(G, \mathfrak{p}\mathfrak{S})$

Throughout this section, let G be an Oliver group.

Theorem (K. Pawałowski-R. Solomon [20]). *Let G be an Oliver group.*

- (1) *If G is a gap group, then $RO(G, \mathfrak{S})_{\mathcal{P}(G)}^{\mathcal{L}(G)} = RO(G)_{\mathcal{P}(G)}^{\mathcal{L}(G)}$.*
- (2) *If G is either an Oliver group of odd order or a nonsolvable group $\not\cong \text{Aut}(A_6)$, $P\Sigma L(2, 27)$ and if $a_G \geq 2$, then $RO(G)_{\mathcal{P}(G)}^{\mathcal{L}(G)} \neq 0$.*

Let us define the following subsets of $RO(G)$.

$$RO[\mathcal{H}^{\mathcal{L}}](G) = \{V - W \in RO(G) \mid V, W \text{ are } \mathcal{L}(G)\text{-free and satisfy (Dim)}\}$$

$$RO[\mathcal{W}^{\mathcal{L}}](G) = \{V - W \in RO(G) \mid V, W \text{ are } \mathcal{L}(G)\text{-free and satisfy (Dim), (Ori)}\}$$

where (Dim) and (Ori) stand for the dimension condition and the orientation condition, respectively, appearing in the weak gap condition (see Section 4).

By definition,

$$2 \cdot RO[\mathcal{H}^{\mathcal{L}}](G) \subset RO[\mathcal{W}^{\mathcal{L}}](G) \subset RO[\mathcal{H}^{\mathcal{L}}](G).$$

If G is a gap group, then $RO[\mathcal{W}^{\mathcal{L}}](G) = RO(G)_{\mathcal{P}(G)}^{\mathcal{L}(G)}$.

By the Deleting-Inserting Theorem by M. Morimoto stated in [16, Appendix], we obtain the next basic theorem.

Theorem 6.1. *If G is an Oliver group, then*

$$RO[\mathcal{W}^{\mathcal{L}}](G)_{\mathcal{P}(G)} \subset RO(G, \mathfrak{p}\mathfrak{S}) \cap RO(G, \mathfrak{D}_{\partial\text{-lin}}).$$

Corollary 6.2. *If G is an Oliver group with $RO[\mathcal{H}^{\mathcal{L}}](G)_{\mathcal{P}(G)} \neq 0$, then $RO(G, \mathfrak{p}\mathfrak{S}) \neq 0$.*

X.M. Ju applied the theorem above and obtained the next result.

Theorem (X.M. Ju). Let $X_2 = C_2 \times \cdots \times C_2$ be the n -fold cartesian product of C_2 , where $n \geq 1$. Then $G = S_5 \times X_2$ is a nongap Oliver group,

$$\mathrm{RO}(G, \mathfrak{S}) = \mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = \mathrm{RO}(G)_{\mathcal{P}(G)}^{\{A_5\}}$$

and

$$\mathrm{rank}_{\mathbb{Z}} \mathrm{RO}(G)_{\mathcal{P}(G)}^{\{A_5\}} = 2^n - 1.$$

Lemma 6.3 ([7]). Let G be a finite group not of prime power order, N a normal subgroup of G , N_2 a Sylow 2-subgroup of N .

- (1) If $G/N \cong C_2$ and $V \sim_{\mathfrak{S}} W$, then $V^N = 0 = W^N$ or $\mathrm{res}_N^G V \cong_N \mathrm{res}_N^G W$.
- (2) If $G/N \cong C_p$ with p odd prime, N_2 is normal in N , and $V \sim_{\mathfrak{S}} W$ then $V^N = 0 = W^N$ or $\mathrm{res}_N^G V \cong_N \mathrm{res}_N^G W$.

Lemma 6.4 ([7]). Let G be a finite group not of prime power order and G_2 a Sylow 2-subgroup of G .

- (1) If $G/G^{\{2\}} \cong C_2 \times \cdots \times C_2$, then $\mathrm{RO}(G, \mathfrak{S}) \subset \mathrm{RO}(G)^{\{G^{\{2\}}\}}$.
- (2) If G_2 is normal in G and $G/G^{\{3\}} \cong C_3 \times \cdots \times C_3$, then $\mathrm{RO}(G, \mathfrak{S}) \subset \mathrm{RO}(G)^{\{G^{\{3\}}\}}$.

Theorem 6.5 ([12]). Let G be either $SG(864, 2666)$ or $SG(864, 4666)$. Then G is an Oliver group with $G/G^{\mathrm{nil}} \cong C_3$ and

$$\mathrm{RO}(G, \mathfrak{S}) = \mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = \mathrm{RO}(G)_{\mathcal{P}(G)}^{\{G\}} \cong \mathbb{Z}.$$

Let G be a finite Oliver group of order ≤ 2000 . T. Sumi (2006) tried to see whether $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = 0$ or not. Putting his computation together with our results, we can determine whether $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = 0$ or not for G except ones in the next list:

$G(m, n)$	a_G	gap?	G/G^{nil}
$G(864, 4663)$	3	No	C_8
$G(864, 4672)$	5	Yes	$Q_8 \times C_3$
$G(1152, 155470)$	2	Yes	C_6
$G(1152, 157859)$	2	Yes	C_6

List 1

7. CONJECTURES

We have several conjectures related to the Smith Problem which are not yet proved.

Conjecture (S. E. Cappell-J. L. Shaneson). If $V \sim_{\mathfrak{S}_{CS}} W$ and the actions on V and W are pseudofree, then $V \simeq_G W$ (G -homeomorphic).

Conjecture 7.1. If G is an Oliver group with $\mathrm{RO}(G)_{\mathcal{P}(G)}^{\mathcal{L}(G)} \neq 0$, then $\mathrm{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}^{\mathcal{L}(G)} \neq 0$.

Let c_G denote the number of the conjugacy classes (C) of cyclic subgroup C of G such that the order of C is not of prime power order. Let Γ denote the Galois group $\mathrm{Gal}(\mathbb{Q}(\zeta))$, where $\zeta = \exp\left(\frac{2\pi\sqrt{-1}}{|G|}\right)$

Conjecture 7.2. If G is an Oliver group with $c_G \geq 2$, then $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S})^\Gamma \neq 0$

Conjecture 7.3. If G is an Oliver group, then $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \subset \mathrm{RO}(G, \mathfrak{D}_{\partial\text{-lin}})$.

REFERENCES

- [1] M. F. Atiyah and R. Bott, *A Lefschetz fixed point formula for elliptic complexes: II. Applications*, Ann. of Math. **88** (1968), 451–491.
- [2] G. E. Bredon, *Representations at fixed points of smooth actions of compact groups*, Ann. of Math. (2) **89** (1969), 515–532.
- [3] S. E. Cappell and J. L. Shaneson, *Representations at fixed points*, Group Actions on Manifolds (Boulder, Colo., 1983), Contemp. Math., 36, Amer. Math. Soc., Providence, RI, 1985, pp. 151–158.
- [4] A. L. Edmonds and R. Lee, *Fixed point sets of group actions on Euclidean space*, Topology **14** (1975), 339–345.
- [5] GAP, *Groups, Algorithms and Programming, a System for Computational Discrete Algebra*, Release 4.3, 06 May 2002, URL: <http://www.gap-system.org>.
- [6] X.M. Ju, *The Smith isomorphism question: A review and new results*, RIMS Kokyuroku no. 1569 (2007), Res. Inst. Math. Sci., Kyoto Univ., 43–51.
- [7] A. Koto, M. Morimoto and Y. Qi, *The Smith sets of finite groups with normal Sylow 2-subgroups and small nilquotients*, J. Math. Kyoto Univ. **48** (2008), 219–227.
- [8] E. Laitinen and M. Morimoto, *Finite groups with smooth one fixed point actions on spheres*, Forum Math. **10** (1998), 479–520.
- [9] E. Laitinen and K. Pawłowski, *Smith equivalence of representations for finite perfect groups*, Proc. Amer. Math. Soc. **127** (1999), 297–307.
- [10] J. W. Milnor, *Whitehead torsion*, Bull. Amer. Math. Soc. **72** (1966), 358–426.
- [11] M. Morimoto, *The Burnside ring revisited*, in Current Trends in Transformation Groups, eds. A. Bak et al., pp.129–145, Kluwer Academic Publ., Dordrecht-Boston-London, 2002.
- [12] M. Morimoto, *Construction of smooth actions on spheres*, RIMS Kokyuroku no. 1569 (2007), Res. Inst. Math. Sci., Kyoto Univ., 52–58.
- [13] M. Morimoto, *Fixed-point sets of smooth actions on spheres*, J. K-Theory **1** (2008), 95–128.
- [14] M. Morimoto, *Smith equivalent $\mathrm{Aut}(A_6)$ -representations are isomorphic*, Proc. Amer. Math. Soc. **136** (2008), 3683–3688.
- [15] M. Morimoto and K. Pawłowski, *The equivariant bundle subtraction theorem and its applications*, Fund. Math. **161** (1999), 279–303.

- [16] M. Morimoto and K. Pawalowski, *Smooth actions of Oliver groups on spheres*, Topology **42** (2003), 395–421.
- [17] M. Morimoto, T. Sumi and M. Yanagihara, *Finite groups possessing gap modules*, in: Geometry and Topology, Aarhus 1998, Contemp. Math. 258, Amer. Math. Soc., Providence, 2000, pp. 329–342.
- [18] B. Oliver, *Fixed point sets and tangent bundles of actions on disks and Euclidean spaces*, Topology **35** (1996), 583–615.
- [19] R. Oliver, *Fixed point sets of groups on finite acyclic complexes*, Comment. Math. Helv. **50** (1975), 155–177.
- [20] K. Pawalowski and R. Solomon, *Smith equivalence and finite Oliver groups with Laitinen number 0 or 1*, Algebr. Geom. Topol. **2** (2002), 843–895.
- [21] T. Petrie, *Three theorems in transformation groups*, Algebraic Topology (Aarhus 1978), Lecture Notes in Math., 763, Springer Verlag, Berlin-Heidelberg-New York, 1979, pp. 549–572.
- [22] T. Petrie, *The equivariant J homomorphism and Smith equivalence of representations*, Current Trends in Algebraic Topology (London, Ont., 1981), CMS Conf. Proc. 2, Part 2, Amer. Math. Soc., Providence, RI, 1982, pp. 223–233.
- [23] T. Petrie, *Smith equivalence of representations*, Math. Proc. Cambridge Philos. Soc. **94** (1983), 61–99.
- [24] T. Petrie and J. Randall, *Transformation Groups on Manifolds*, Marcel Dekker, Inc., New York and Basel, 1984.
- [25] T. Petrie and J. Randall, *Spherical isotropy representations*, Publ. Math. IHES **62** (1985), 5–40.
- [26] C. U. Sanchez, *Actions of groups of odd order on compact orientable manifolds*, Proc. Amer. Math. Soc. **54** (1976), 445–448.
- [27] P. A. Smith, *New results and old problems in finite transformation groups*, Bull. Amer. Math. Soc. **66** (1960), 401–415.
- [28] T. Sumi, *Finite groups possessing Smith equivalent, nonisomorphic representations*, RIMS Kokyuroku no. 1569 (2007), Res. Inst. Math. Sci., Kyoto Univ., 170–179.