

LOCALIZATION IN TWO DIMENSIONS: EXPERIMENTS IN SILICON INVERSION LAYERS

S. Kawaji

Department of Physics, Gakushuin University
Mejiro, Toshima-ku, Tokyo, 171 Japan

1. Weak Localization and Negative Magnetoresistance in Silicon Inversion Layers

In 1979, Abrahams, Anderson, Licciardello and Ramakrishnan¹⁾ have developed a scaling theory of the Anderson localization. The theory has shown that the localization in 2D random systems has particular features that electrons are always localized at absolute zero even if the impurity scattering is very weak and the conductance decreases with sample size L , logarithmically in high conductance region and exponentially in low conductance region. Anderson, Abrahams and Ramakrishnan²⁾ have shown based on Thouless's Argument³⁾ that the sample size L can be replaced by $\sqrt{D\tau_c}$ at finite temperatures, where D is the diffusion coefficient and τ_c is the inelastic scattering time. Thus, the $\log L$ dependence of conductance can be replaced by $\log T$ dependence due to the temperature dependence of the inelastic scattering time $\tau_c \propto T^{-p}$. Their theory has explained $\log T$ dependence of conductance in metallic films observed by Doran and Osheroff⁴⁾. Bishop, Tsui and Dynes⁵⁾ have explained their observations of $\log T$ dependence of conductance in Si-MOSFETs by the scaling theory of Anderson localization.

On the basis of the scaling theory of the Anderson localization, Hikami, Larkin and Nagaoka⁶⁾ and Altshuler, Kmel'nizkii, Larkin and Lee⁷⁾, have developed the theory of negative magnetoresistance in the weakly localized regime (WLR) ($2\pi e_F \tau / \hbar \gg 1$) of 2D systems. In WLR the quantum interference between states with e^{ikr} and e^{-ikr} causes spacially localized states which produce the correction term δ' to the classical Drude conductivity σ_0 . The quantum correction is sensitive to perturbations which destroy time reversal symmetry. Therefore, an external magnetic field applied perpendicularly to the 2D systems suppresses the quantum correction to the conductivity due to the weak localization and produces negative magnetoresistance. The theory has explained recent experiments in Si-MOSFETs by Kawaguchi and Kawaji^{8,9,10)}, Uren, Davies, Kaveh and Pepper¹¹⁾, Wheeler¹²⁾ and Bishop, Dynes and Tsui¹³⁾.

The quantum correction to the conductivity in WLR of 2D systems has logarithmic dependence on temperatures, electric fields as well as on the magnetic fields. However, in these dependences, the negative magnetoresistance is probably most useful to investigate microscopic parameters in the localization theory such as the inelastic scattering time.

The quantum correction to the conductivity due to the localization depends also on the magnetic scattering and the spin-orbit interaction⁶⁾ though they are not important in Si-MOS inversion layers. In n-channel Si-MOS inversion layers, the intervalley scattering plays a dominant role in the quantum correction due to the localization¹⁴⁾.

In addition to the localization theory, Altshuler, Arnov and Lee¹⁵⁾ have pointed out that the mutual coulomb interactions of electrons in 2D systems produce another correction to the Drude conductivity which depends also on $\log T$. Fukuyama¹⁶⁾ has pointed out that the correction due to the mutual interactions causes also a magnetoresistance which depends on $\log H$ and reduces the negative magnetoresistance due to the localization theory.

In view of the theoretical progress in the quantum correction to the Drude conductivity due to the localization effect and the interaction effect described above, detailed analysis of negative magnetoresistance experiments in Si-MOS inversion layers affords us fruitful informations on electron

localization and interactions in 2D electron systems. In this situation, the controllability of the electron number density in a single sample by direct electrical means makes the 2D systems in Si-MOSFETs the best for studying those fundamental properties of the 2D electron systems.

Following Hikami, Larkin and Nagaoka⁶⁾, the change conductivity with magnetic field B applied perpendicularly to the 2D system having impurity Coulomb scattering only is written as

$$\begin{aligned}\Delta\sigma_B &= \sigma(B) - \sigma(0) \\ &= -\frac{n_v \alpha e^2}{2\pi^2 \hbar} \left[\psi\left(\frac{1}{2} + \frac{1}{a\tau}\right) - \psi\left(\frac{1}{2} + \frac{1}{a\tau_e}\right) - \ln \frac{\tau_e}{\tau} \right]\end{aligned}\quad (1)$$

where n_v is the valley degeneracy, α is a constant prefactor, τ is the relaxation time due to normal impurity scattering, τ_e is the inelastic scattering time and $a = 4DeB/\hbar$ where D is the diffusion coefficient.

In the analysis of experiments^{8,9,10)}, we consider that the constant prefactor α includes effects of the intervalley scattering¹⁴⁾ and the mutual Coulomb interactions¹⁶⁾ which are not taken into account by Hikami et al⁶⁾. Therefore, in our analysis, τ_e and α have been determined to reproduce most well experimental magnetoconductivity by eq. (1). Wheeler¹²⁾ and Bishop, Dynes and Tsui¹³⁾ have employed $n_v \alpha = 1$ neglecting the mutual interactions and assuming the strong intervalley scattering limit. However, the best fit of eq.(1) to experiments has been obtained in our analysis by adjusting both $n_v \alpha$ and τ_e .

Figure 1 shows changes in the conductivity with magnetic field in n-channel (001) Si-MOSFETs with the peak electron mobilities at 4.2 K of $13000 \text{ cm}^2/\text{V}\cdot\text{s}$.⁸⁾ The experimental points (crosses) are fitted to an approximation of eq.(1) expressed by solid lines.

Temperature dependence and electron concentration dependence of inelastic scattering time are shown in Fig. 2 and Fig. 3, respectively. The vertical broken line in Fig. 3 indicates the electron concentration N_s for $2\pi e_F \tau / \hbar = \pi$ or $k_F \Lambda = 1$ where k_F and Λ are the Fermi wave number and the mean free path of electrons, respectively. The open and solid squares in Fig. 3 are deduced from magnetoconductivity experiments by use of eq.(1) for $n_v = 1$. Open and solid circles are similarly deduced from the same data for the same N_s for $n_v = 2$.

Following Landau argument, τ_e due to the electron-electron process in the pure metal is written as

$$\tau_e(\text{el-el, pure}) \approx \hbar \frac{\epsilon_F}{(k_B T)^2} \quad (2)$$

Equation (2) gives $\tau_e \approx 5.6 \times 10^{-10} \text{ s}$ for n-channel (001) Si MOS inversion layers ($n_v = 2$, $m^* = 0.19 m_0$) at $N_s = 1 \times 10^{12} \text{ cm}^{-2}$ at $T = 1 \text{ K}$.

Recently Abrahams, Anderson, Lee and Ramakrishnan¹⁷⁾ have calculated τ_e due to the electron-electron process in the dirty 2D system. Their results is written as

$$\tau_e(\text{el-el, dirty}) = -\frac{\tau \epsilon_F}{k_B T \ln(T/T_1)} \quad (3)$$

where τ is the normal elastic impurity scattering time, $k_B T_1 = \hbar D \epsilon^2 / e^4$ and $\epsilon = \hbar D q_s^2$ where q_s is the inverse screening length.

Calculation of the inelastic scattering time due to the electron-phonon process which is applicable to 2D systems in semiconductor inversion layers has not yet been performed. Shinba, Nakamura, Fukuchi and Sakata¹⁸⁾ have calculated the energy relaxation time from the 2D electron system to the phonon system in Si(001) and (111) n-channel inversion layers. Their results show that

$$\tau_{\text{relax}}(\text{el-ph}) \propto \frac{\epsilon_F^{1/2}}{T^2} \quad (\text{for } T_e \sim T) \quad \text{and} \quad \propto \frac{\epsilon_F^{1/2}}{T^3} \quad (\text{for } T_e \gg T) \quad (4)$$

at temperature T below 7 K where T_e is the electron temperature. The inelastic scattering time due

to the electron-phonon process $\tau_{\epsilon}(\text{el-ph})$ has probably the same dependences on ϵ_F and T to those in $\tau_{\text{relax}}(\text{el-ph})$.

Results of N_S -dependence of τ_{ϵ} in Fig. 3 show that $\tau_{\epsilon} \propto N_S$ which is in accordance with the theoretical results of the electron-electron process in eq(2) and eq(3). However, the exponent p of the temperature dependence of $\tau_{\epsilon} \propto T^{-p}$ in experiments lies in the range of $1 < p < 2$; $p = 1.75$ at $V_G = 200 \text{ V}$ ($N_S = 5.8 \times 10^{12} \text{ cm}^{-2}$) and $p = 1.3$ at $V_G = 20 \text{ V}$ ($N_S = 6.8 \times 10^{11} \text{ cm}^{-2}$). The tendency in the N_S -dependence of p is not unreasonable when one considers that the system approaches to the pure metal limit at high N_S and to the dirty limit at low N_S . The inelastic scattering time calculated by eq(2) for the pure metal limit is larger than experiments but not so far from them; $\tau_{\epsilon}(\text{theor})/\tau_{\epsilon}(\text{exp}) \approx 6 \sim 15$ for $V_G = 200 \text{ V}$ in the temperature range in Fig. 4. This result suggests that some other scattering processes exist to reduce the τ_{ϵ} and P . However, the inelastic scattering time calculated by eq(3) for $V_G = 20 \text{ V}$ is one order of magnitude smaller than the experiments whose temperature dependence is close to the results of eq(3) derived for the dirty 2D system.

The constant prefactor α shows an interesting behaviour against the electron concentration N_S as shown in Fig. 4. Fukuyama^{19, 20)} has accounted for the behaviour at higher N_S region by the intervalley impurity scattering effect on the prefactor of the logarithmic correction to the Drude conductivity.

Fukuyama²⁰⁾ has shown that the intervalley scattering plays an important role in many valley systems and we have

$$\alpha = 1 + F\left(\frac{1}{2} - n_V\right)/n_V \quad \text{for } \tau' \gg \tau_{\epsilon} \quad (5a)$$

$$\alpha = \frac{1}{n_V} \left[1 - \frac{F}{2n_V}\right] \quad \text{for } \tau' \ll \tau_{\epsilon} \quad (5b)$$

where F is the average of the matrix element of scattering in RPA screened Coulomb interactions between two states on the Fermi surface introduced by Altshuler et al⁷⁾ and τ' is intervalley impurity scattering time. The quantity F is a function of $2k_F/q_s$ where q_s is inverse screening length and $F \rightarrow 1$ as $2k_F/q_s \rightarrow 0$ and $F \rightarrow 0$ as $2k_F/q_s \rightarrow \infty$. In the strong screening limit, α is unity when $\tau' \gg \tau_{\epsilon}$ and α is $1/n_V$ when $\tau' \ll \tau_{\epsilon}$. The inelastic scattering time τ_{ϵ} is proportional to N_S as shown in Fig. 3. Therefore, when $F = 0$, the prefactor α is expected to cross over from $\alpha \approx 1$ for $\tau' \gg \tau_{\epsilon}$ to smaller value $\alpha \sim 0.5$ for $\tau' \ll \tau_{\epsilon}$ with increasing N_S . Actually, following Fukuyama's calculation, $F \approx 0.8$ at $N_S = 6 \times 10^{12} \text{ cm}^{-2}$, then we have from eq(5b) $\alpha \approx 0.4$. This is close to experimental value of α at $N_S = 6 \times 10^{12} \text{ cm}^{-2}$ in Fig. 4.

Fukuyama¹⁹⁾ has pointed out that the sharp drop of α with decreasing N_S at low N_S region ($N_S < 5 \times 10^{11} \text{ cm}^{-2}$) in Fig. 4 is consistent with the removal of the valley degeneracy, i.e. $n_V = 2 \rightarrow 1$ at low N_S . Bloss, Sham and Vinter²¹⁾ have shown that intravalley exchange and correlation induce a first order phase transition and $n_V = 2 \rightarrow 1$ occurs roughly below $N_S = 3 \times 10^{11} \text{ cm}^{-2}$. Experimental magnetoconductivity data have been analyzed assuming that $n_V = 1$ and results are shown by squares in Figs. 3 and 4. They appear more reasonable than the values of τ_{ϵ} , P and α which are obtained by taking $n_V = 2$ at the same electron concentration N_S .

Kawaguchi and Kawaji^{22,23)} have applied the negative magnetoresistance to measurements of the temperature of electrons in silicon MOS inversion layers at high electric fields to evaluate the deformation potential constants in the electron-phonon interaction.

In stationary state at high source-drain field E_{SD} , the rate of energy gain per electron from field, $\sigma_0 E_{SD}^2 / N_S$, is equal to the rate of energy loss to the lattice system, $-\langle d\epsilon/dt \rangle$. The energy loss per electron from the electron system to the lattice by electron-phonon scattering has been calculated by Shinba et al²⁴⁾ by using the deformation potential constants of bulk silicon, $\Xi_u = 12 \text{ eV}$, $D = \Xi_d/\Xi_u = -0.67$. Calculated values of $-\langle d\epsilon/dt \rangle$ (solid lines) and experimental data of $\sigma_0 E_{SD}^2 / N_S$

(solid and open circles) are shown in Fig. 5 as a function of $(T_e - T_L)$. Good agreement between theoretical results and experiments shows that the deformation potential constants in the inversion layer are almost same to the bulk values.

2. Localization in Strong Magnetic Fields and the Quantum Hall Effect in Silicon MOS Inversion Layers

In n-channel MOS inversion layers in Si(001) surfaces at sufficiently low temperatures and in sufficiently strong magnetic fields, it is possible to realize an extreme-quantum-limit condition ($k_B T < \Gamma < \hbar\omega_c$), where Γ the broadening of the Landau levels and ω_c the cyclotron frequency²⁵⁾. In such a system, there exist gap regions in the density of states of electrons between the boundaries of each Landau level. When random potentials are incorporated in the system, localized states are expected to exist at higher and lower edges of each Landau level. If the strength of the random potential is weaker than the mutual Coulomb interaction of electrons, the strong magnetic field is expected to enhance correlated states of electrons, i.e., the Wigner solid or the CDW state.

The first experiment of Shubnikow-de Haas effect in Si-MOSFETs by Fowler, Fang, Howard and Stiles²⁶⁾ has shown the existence of gap regions in the gate voltages where the transverse conductivity σ_{xx} vanishes.

Kawaji and Wakabayashi have performed careful experiments on the finite gap regions in the gate voltages for the vanishing σ_{xx} with various source-drain fields and investigated the magnetic field dependence and the Landau level index dependence of the concentration of immobile electrons²⁷⁾. The magnetic field dependence of the width of the gate voltage for the vanishing conductivity region ($\sigma_{xx} < 10^{-9}$ mho) has shown a result that the sum of the concentration of immobile or localized electrons associated with the higher edge of the $(N-1)$ th Landau level and the lower edge of the N -th Landau level where N is the Landau level index, is approximately given by $[2\pi\ell^2(2N+1)]^{-1}$ as shown in Fig. 6 where ℓ is the radius of the ground Landau orbit. This result means that the electron wave functions become extended or pinned correlated states become free when the whole area of the inversion layer is covered by cyclotron orbits with radius of $(2N+1)^{1/2} \ell$.

There exist several experimental and theoretical investigations which suggest possibilities of the Anderson localization due to random potentials and the Wigner crystallization or the CDW states in 2D systems in strong magnetic fields²⁸⁾. Recent numerical investigations of the Anderson localization by Ando^{28,29,30)} have given the concentration of localized electrons which is in good agreement with our experiments.

Ando, Matsumoto and Uemura³¹⁾ have developed a theory of Hall effect in 2D systems in strong magnetic fields at absolute zero. Their results on the effect of impurities are summarized as follows:

- (1) The Hall conductivity is not affected by the presence of impurities when each Landau level is completely filled and the Hall conductivity is given by

$$\sigma_{xy} = - \frac{Ne^2}{h} \quad (6)$$

when the Fermi level lies in the energy gap between the $(N-1)$ th and N th Landau levels.

- (2) In the case of impurity bands are separated from each Landau level, the Hall conductivity is given by eq.(6) when the Fermi level lies in any spectral gaps lying between the $(N-1)$ th and N th main Landau level, i.e. when the Fermi level lies in gaps between two impurity bands or between an impurity band and the main Landau level.

Igarashi, Wakabayashi and Kawaji^{32,33)} have confirmed experimentally Ando et al's results (1) by Hall voltage measurements for wide samples in 9.8 T at 1.6 K.

In 1980, Kawaji and Wakabayashi³⁴⁾ have reported precise behaviours of the transverse and Hall conductivity and their temperature dependence observed in 15 T at temperatures between 1.5 K and 12 K.

Figure 7 shows an example of the results at 1.5 K which clearly demonstrates the quantized steps in the Hall conductivity for $2e^2/h$ in the spin gap between the $(0+)$ level and $(0++)$ level and $4e^2/h$ in the Landau gap between the $(0+)$ level and the $(1+)$ level.

The quantized Hall conductivity in Fig. 7 demonstrates the characteristic feature of the electron-hole symmetry; i.e., near the spin gap, the Hall conductivity becomes $2e^2/h$ when the Fermi level lies in the localized states in the upper edge of the $(0+)$ level even if the $(0+)$ level is not completely filled or when the holes in the $(0+)$ level are localized as well as the case where the electrons in the $(0++)$ level are localized or the Fermi level lies in the localized states in the lower edge of the $(0++)$ level.

The quantized Hall steps and the electron-hole symmetry in the Anderson localization in Landau levels in 2D systems have been expected by Ando et al's theory of the Hall effect in 2D systems for long. One of the important conclusion reached by Ando et al is their results (2) which is obtained for the case where the impurity bands are separated from the main Landau level. The single-site approximation which Ando et al employed has lead the impurity bands separated from the main Landau level. However, as is well known from the observations of σ_{xx} , localized states exist actually near the edges of each Landau level instead of the impurity bands. Therefore, it is easy to expect the quantized Hall plateaus and the electron-hole symmetry in Landau levels in actual 2D systems.

von Klitzing, Dorda and Pepper³⁵⁾ have independently observed the quantized Hall steps in the Hall resistance measurements. Although their observations lacked the electron-hole symmetry, their results demonstrated that the Hall resistance for the gap between the $(0+)$ level and the $(1+)$ level agrees with the recommended value of $h/4e^2$ in high precision of 3 ppm. On the basis of this fact, they proposed a new method of the determination of the fine structure constant $\alpha = e^2/\hbar c$ by combination of the quantized Hall resistance and the light velocity c . Their proposal has attracted many metrologists and physicists in broad areas to the phenomenon of the quantized Hall effect or the quantum Hall effect^{28,36,37)}.

Several high precision measurements of the quantized Hall resistance of Si-MOSFETs have been reported³⁸⁻⁴¹⁾. Yamanouchi et al's³⁹⁾ results are shown in Fig. 8. Their results of the quantized Hall resistance for $h/4e^2$ has lead the inverse of the fine structure constant $\alpha^{-1} = 137.035894$ (± 0.88 ppm) which agrees with the values of α^{-1} so far obtained. Yoshihiro et al⁴²⁾ have measured the Hall resistance of $R_H(i) = h/ie^2$ for $i = 4, 8, 12$ in 9 T at 0.5 K and obtained the result that $iR_H(i)/4R_H(4)$ is unity within the error of ± 0.2 ppm.

Based on a mobility edge model, Kawaji, Wakabayashi and Moriyama⁴³⁾ have tried a phenomenological analysis of the temperature dependence of $\Delta\sigma_{xy}(T) = \sigma_{xy}(T) - (-2e^2/h)$ in the lower half of the $(0++)$ Landau level in Fig. 7. The simple mobility edge model can explain temperature dependence of σ_{xy} only at temperatures lower than 2.5 K. In order to describe the temperature dependence at higher temperatures, we have to introduce phenomenological mobility edges which move to the Landau level edges at higher temperatures, $T > 2.5$ K.

Recently, Ando's⁴⁴⁾ numerical experiments have shown that there is no mobility edge in the $N=0$ Landau level or the localization length diverges at the center of the Landau level. Ono⁴⁵⁾ has obtained analitically similar results.

We have to remind here that the effect of inelastic scattering plays an important role in the weak localization in the absence of strong magnetic fields. As is first discussed by Thouless, the sample size L should be replaced by the inelastic diffusion length $\sqrt{D\tau_c}$ when $\sqrt{D\tau_c} < L$ which is usually satisfied. Therefore, the electrons can not be localized when the localization length ξ is longer than the inelastic diffusion length $\sqrt{D\tau_c}$. Thus an effective mobility edge exists at E_c where $\xi(E_c) = \sqrt{D\tau_c}$. Therefore, it is expected that the temperature dependence of the inelastic scattering time τ_c controls the temperature dependence of the effective mobility edge.

Moriyama and Kawaji⁴⁶⁾ have discussed temperature dependence of the effective mobility edge

derived from the temperature dependence of $d\sigma_{xy}/dN_s$ or $d\sigma_{xy}/dV_G$.

Figure 9 shows temperature dependence of the effective mobility edge E_c/Γ obtained from the temperature dependence of $d\sigma_{xy}/dN_s$ at two different gate voltages. Both curves in Fig. 9 appear to approach to $E_c/\Gamma = 0$ at $T \rightarrow 0$ as has been expected from numerical results by Ando⁴⁴⁾ and analytical results by Ono⁴⁵⁾.

3. Summary and Conclusion

Results of negative magnetoresistance experiments in the weakly localized regime have been well explained by recently developed theories based on Anderson localization in two-dimensional systems and effect of mutual Coulomb interactions on the orbital motion in two-dimensional systems including effects of intervalley impurity scattering. Based on hot-electron effect experiments on negative magnetoresistance we have reached an important conclusion that the deformation potential constants in electron-phonon interaction in inversion layers in the Si-SiO₂ interface are the same to those in bulk silicon.

Most interesting behaviour of two-dimensional systems in random potentials has appeared in strong magnetic fields where the Hall conductivity is quantized to an integer multiple of $-e^2/h$. We have confirmed the electron-hole symmetry in the localization in Landau levels which produce the same integer multiple of $-e^2/h$. Our simple analysis of the Hall conductivity has appeared to show that no mobility edge exists in a Landau level in two-dimensional systems though our experiments have been done at temperatures higher than 1.5 K. Electronic states near the center of a Landau level appear to extend at low temperatures. However, further experiments at lower temperatures should be done before arriving at the final conclusion.

In this review we have not discussed on researches on strongly localized regime in silicon MOS inversion layers. Though we have done some attempts to understand the properties in this regime in the light of the scaling arguments^{47,48)}, we have no theory at finite temperatures in this regime. We hope to find solutions to this problem on a line extended from the researches which have performed on weak magnetic field effect in weakly localized regime and on the strong localization in Landau levels.

References

- 1) E. Abrahams, P. W. Anderson, D. C. Licciardello and T. V. Ramakrishnan, Phys. Rev. Letters **42**, 673 (1979)
- 2) P. W. Anderson, E. Abrahams and T. V. Ramakrishnan, Phys. Rev. Letters **43**, 718 (1979)
- 3) D. J. Thouless, Phys. Rev. Letters **39**, 1167 (1977)
- 4) G. J. Dolan and D. D. Osheroff, Phys. Rev. Letters **43**, 721 (1979)
- 5) D. J. Bishop, D. C. Tsui, and R. C. Dynes, Phys. Rev. Letters, **44**, 1153 (1980)
- 6) S. Hikami, A. I. Larkin and Y. Nagaoka, Prog. Theoret. Phys. **63**, 707 (1980)
- 7) B. L. Altshuler, D. E. Kmel'nizkii, A. I. Larkin and P. A. Lee, Phys. Rev. **B22**, 5142 (1980)
- 8) Y. Kawaguchi and S. Kawaji, J. Phys. Soc. Jpn. **48**, 699 (1980)
- 9) Y. Kawaguchi and S. Kawaji, J. Phys. Soc. Jpn. **49**, Suppl. A, 983 (1980)
- 10) Y. Kawaguchi and S. Kawaji, Surf. Sci. **113**, 505 (1982)
- 11) M. J. Uren, R. A. Davis, M. Kaveh and M. Pepper, J. Phys. C. **14**, L 395 (1981)
- 12) R. G. Wheeler, Phys. Rev. **B24**, 4645 (1981)
- 13) D. J. Bishop, R. C. Dynes, and D. C. Tsui, Phys. Rev. **26**, 773 (1982)
- 14) H. Fukuyama, Prog. Theor. Phys. Suppl. **69**, 220 (1981)
- 15) B. L. Altshuler, A. G. Aronov and P. A. Lee, Phys. Rev. Letters **44**, 288 (1980)
- 16) H. Fukuyama, J. Phys. Soc. Jpn. **48**, 2169 (1980)
- 17) E. Abrahams, P. W. Anderson, P. A. Lee, I. V. Ramakrishnan, Phys. Rev. **B24**, 6783 (1981)
- 18) Y. Shinba, K. Nakamura, M. Fukuchi, and M. Sakata, J. Phys. Soc. Jpn. **51**, 157 (1982)
- 19) H. Fukuyama, Surf. Sci. **113**, 489 (1982)
- 20) H. Fukuyama, "Anderson Localization" ed. by Y. Nagaoka and H. Fukuyama, (Springer-Verlag, Berlin, 1982) p.89
- 21) W. L. Bloss, L. J. Sham, and B. Vinter, Surf. Sci. **98**, 250 (1980)
- 22) Y. Kawaguchi and S. Kawaji, Proc. 16th Int. Conf. on Phys. of Semicond., Montpellier, 1982, to be published

- 23) Y. Kawaguchi and S. Kawaji, Japan. J. Appl. Phys. 21, L 709 (1982)
- 24) Y. Shinba and K. Nakamura, J. Phys. Soc. Jpn. 50, 114 (1981)
- 25) S. Kawaji, Surf. Sci. 73, 46 (1978)
- 26) A. B. Fowler, F. F. Fang, W. E. Howard, and P. J. Stiles, Phys. Rev. Letters 16, 901 (1966)
- 27) S. Kawaji and J. Wakabayashi, Surf. Sci. 58, 238 (1976)
- 28) T. Ando, "Recent Topics from the Semiconductor Physics" ed. by Y. Toyosawa and H. Kamimura (World Scientific Publication Co., Singapore, 1983) and references cited therein
- 29) T. Ando, Surf. Sci. 113, 182 (1982)
- 30) T. Ando, "Anderson Localization" ed. by Y. Nagaoka and H. Fukuyama (Springer-Verlag, Berlin, 1982), p. 176
- 31) T. Ando, Y. Matsumoto and Y. Uemura, J. Phys. Soc. Jpn. 39, 279 (1975)
- 32) T. Igarashi, J. Wakabayashi, and S. Kawaji, J. Phys. Soc. Jpn. 38, 1549 (1975)
- 33) S. Kawaji, T. Igarashi and J. Wakabayashi, Prog. Theor. Phys. Suppl. 57, 176 (1975)
- 34) S. Kawaji and J. Wakabayashi, "Physics in High Magnetic Fields" ed. by S. Chikazumi and N. Miura (Springer, Berlin, 1981) p. 284
- 35) K. von Klitzing, G. Dorda and M. Pepper, Phys. Rev. Letters 45, 494 (1980)
- 36) M. E. Cage and S. M. Girvin, Comments on Solid State Physics, to be published
- 37) S. M. Girvin and M. E. Cage, Comments on Solid State Physics, to be published
- 38) E. Braun, E. Staben and K. von Klitzing, PTB Mitl. 90, 350 (1980)
- 39) C. Yamanouchi, K. Yoshihiro, J. Kinoshita, K. Inagaki, J. Moriyama, S. Baba, S. Kawaji, K. Murakami, T. Igarashi, T. Endo, M. Koyanagi and N. Nakamura, "Precision Measurements and Fundamental Constants II", ed. by B. N. Taylor and W. D. Phillips (Nat. Bur. Stand. U. S., Spec. Publ. 617, to be published)
- 40) M. E. Cage, R. F. Dziuba, B. F. Fields, C. F. Lavine, and R. J. Wagner, ibid., to be published
- 41) A. Hartland, ibid., to be published
- 42) K. Yoshihiro, J. Kinoshita, K. Inagaki, C. Yamanouchi, J. Moriyama, and S. Kawaji, J. Phys. Soc. Jpn. 51, 5 (1982)
- 43) S. Kawaji, J. Wakabayashi, and J. Moriyama, J. Phys. Soc. Jpn. 50, 3839 (1981)
- 44) T. Ando, J. Phys. Soc. Jpn., to be published
- 45) Y. Ono, J. Phys. Soc. Jpn. 51, 2055 (1982)
- 46) J. Moriyama and S. Kawaji, Solid State Commun. 45 (1983) 511
- 47) S. Kawaji, M. Namiki and N. Hoshi, J. Phys. Soc. Jpn. 49, 1637 (1980)
- 48) N. Hoshi and S. Kawaji, Surf. Sci. 113, 189 (1982)

Figure Captions

- Fig. 1 Magnetoconductivity in an n-channel(100) Si MOSFET with μ (peak at 4.2K) = $13000 \text{ cm}^2/\text{V}\cdot\text{s}$. The experimental points are fitted to the expression (1) with an approximation $\psi(1/2 + 1/\tau a) \approx \ln(1/\tau a)$.⁸⁾
- Fig. 2 Temperature dependence of inelastic scattering time τ_E for $E_F\tau/\hbar = 0.4$ ($V_G = 100 \text{ V}$) and 14.5 (200 V).¹⁰⁾
- Fig. 3 Electron concentration dependence of inelastic scattering time τ_E . $E_F\tau/\hbar = 0.5$ at the vertical broken line.¹⁰⁾ The open and solid squares are deduced from the magnetoconductivity by use of the expression (1) for $n_V = 1$, and circles for $n_V = 2$.
- Fig. 4 Electron concentration dependence of prefactor α at 4.2 K. $E_F\tau/\hbar = 0.5$ at the vertical broken line.¹⁰⁾ A square is deduced from magnetoconductivity by use of the expression (1) for $n_V = 1$, and circles for $n_V = 2$.
- Fig. 5 T_e (electron temperature)- T_L (lattice temperature) versus energy gain per electron from the field ($eE\tau/\hbar$) (solid circles at $T_L = 4.2 \text{ K}$ and open circles at 1.1 K) and energy loss per electron calculated by Shinba and Nakamura (private communication).^{22,23)}
- Fig. 6 Concentration of immobile electrons associated with the higher edge of the N-lth Landau level and the lower edge of the Nth Landau level versus magnetic field for different Landau level index N.²⁷⁾
- Fig. 7 Hall conductivity σ_{xy} and transverse conductivity σ_{xx} versus gate voltage V_G in the $N = 0$ Landau level in an n-channel Si(100) MOSFET measured by Hall current method in 15 T at 1.5 K. Reference 34 contains temperature dependence of the similar data.
- Fig. 8 Results of high precision measurements of Hall resistance R_H and transverse resistance $R_{pp'}$ in the gap between (0+-) and (1++) Landau levels. At gate voltages between 14.4 V and 14.6 V, $R_{pp'}$ passes through a minimum ($\approx 5 \times 10^{-4} \Omega$) and R_H passes through a plateau valued $R_H = 6453.2024 \pm 0.0008 \Omega$. This result was not taken into account in the final result because this measurement was made without oil bath.³⁹⁾
- Fig. 9 Temperature dependence of the effective mobility edge E_C/l .⁴⁶⁾

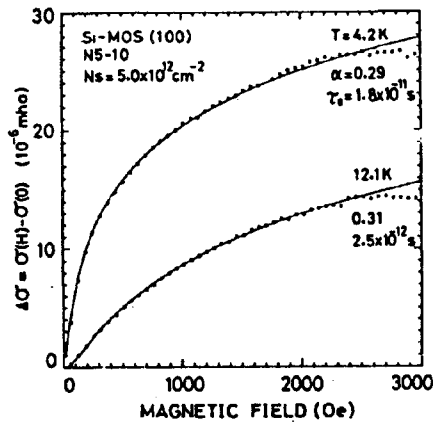


Fig. 1

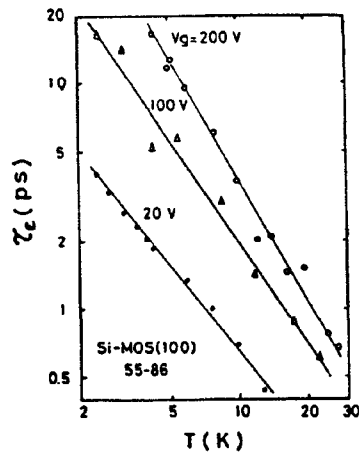


Fig. 2

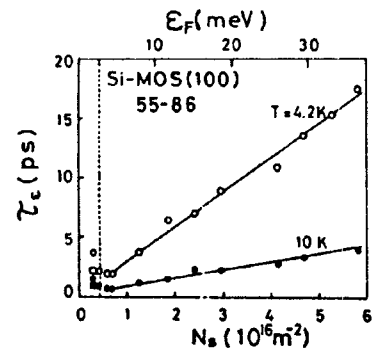


Fig. 3

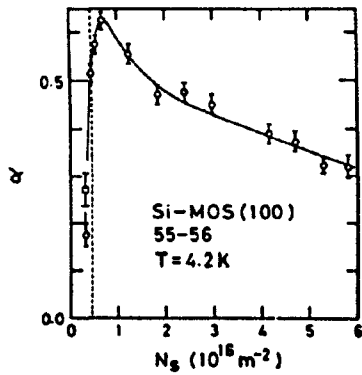


Fig. 4

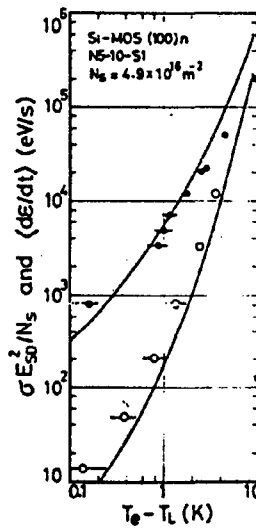


Fig. 5

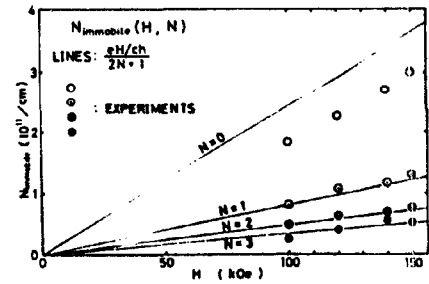


Fig. 6

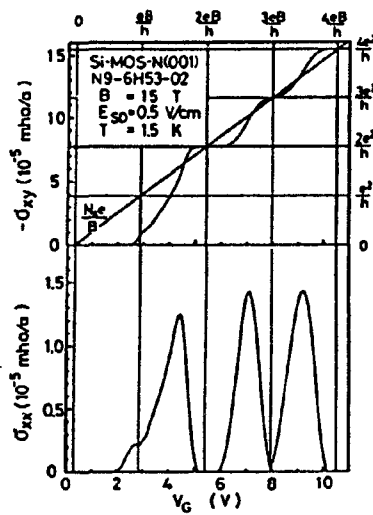


Fig. 7

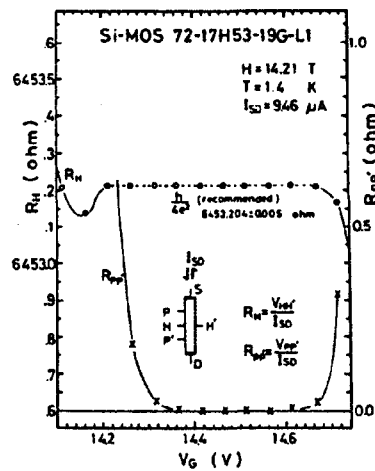


Fig. 8

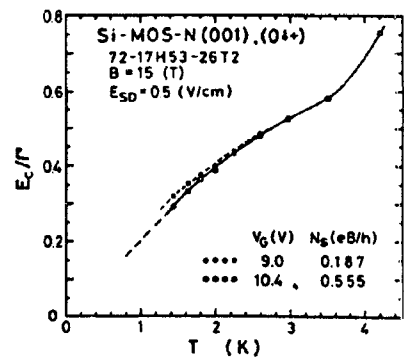


Fig. 9