

Energy-Efficient Threshold Circuits Detecting Global Pattern in 1-Dimensional Arrays

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1 Introduction

Neurons communicate with each other by “firing” (i.e., emitting an electrical signal) for information processing, and a circuit consisting of neurons is often modelled by a combinatorial logic circuit, called a *threshold circuit*. Motivated by a biological fact that a neuron consumes substantially more energy to fire than not to fire [1], Uchizawa, Douglas and Maass proposed a complexity measure, called *energy complexity*, and initiate a study of threshold circuits with small energy complexity [2].

In this paper, we consider a Boolean function, called P_{LR}^n , which Legenstrin and Maass introduced to model a simple task for a pattern recognition on 1-dimensional array [3]. We investigate a relationship between the energy and size of a threshold circuit computing P_{LR}^n .

2 Preliminaries

A *threshold circuit* C is a combinatorial circuit of threshold gates. A threshold circuit C is expressed by a directed acyclic graph; let n be the number of input variables to C , then each node of in-degree 0 in C corresponds to one of the n input variables $x_1, x_2, \dots, x_n$, and the other nodes correspond to threshold gates. We define *size* s

of a threshold circuit C as the number of threshold gates in C . Let $g_1, g_2, \dots, g_s$ be the gates in C . One may assume without loss of generality that $g_1, g_2, \dots, g_s$ are topologically ordered with respects to the underlying graph of C . Let i be an integer such that $1 \leq i \leq s$. For each gate g_i , we denote by $w_{i,1}, w_{i,2}, \dots, w_{i,l_i}$ the weights and by t_i the threshold of the gate g_i , respectively, where the weights and the threshold are real numbers and l_i is the fan-in of the gate g_i . Let $\mathbf{z}_i(\mathbf{x}) = (z_{i,1}(\mathbf{x}), z_{i,2}(\mathbf{x}), \dots, z_{i,l_i}(\mathbf{x})) \in \{0, 1\}^{l_i}$ be an input to g_i for a circuit input $\mathbf{x}$. The output $g_i(\mathbf{z}_i(\mathbf{x}))$ of g_i is defined as follows: $g_i(\mathbf{z}_i(\mathbf{x})) = 1$ if $\sum_{j=1}^{l_i} w_{i,j} z_{i,j}(\mathbf{x}) \geq t_i$; and $g_i(\mathbf{z}_i(\mathbf{x})) = 0$ otherwise. For every input $\mathbf{x} \in \{0, 1\}^n$, the *output* $C(\mathbf{x})$ of C is denoted by $g_s(\mathbf{z}_s(\mathbf{x}))$. Let $f : \{0, 1\}^n \rightarrow \{0, 1\}$ be a Boolean function of n inputs. A threshold circuit C *computes* a Boolean function f if $C(\mathbf{x}) = f(\mathbf{x})$ for every input $\mathbf{x} \in \{0, 1\}^n$. The *energy* e of a threshold circuit C is defined as the maximum number of gates outputting “1” in C , where the maximum is taken over all inputs to C .

For any positive integer n , we define P_{LR}^n as follows: For $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \{0, 1\}^n$ and $\mathbf{y} = (y_1, y_2, \dots, y_n) \in \{0, 1\}^n$, $P_{LR}^n(\mathbf{x}, \mathbf{y}) = 1$ if there exists a pair of indices i and j such that $1 \leq i < j \leq n$, $x_i = y_j = 1$; and $P_{LR}^n(\mathbf{x}, \mathbf{y}) = 0$ otherwise.

3 Our result

We give a construction of energy-efficient threshold circuits computing P_{LR}^n . The following theorem gives an upper bound on the size of threshold circuits computing P_{LR}^n with energy e for any $e \geq 3$.

Theorem 1 *Let n be a positive integer. Then, there is a threshold circuit C computing P_{LR}^n such that C has energy $e \geq 3$ and size $s = O(e \cdot n^{2/(e-1)})$.*

Thus, one can construct an energy-efficient circuit computing P_{LR}^n if it is allowable to use large size.

We also consider the extreme case where a threshold circuit has energy $e = 1$. We prove by construction that a linear number of gates is sufficient.

Theorem 2 *Let n be a positive integer. Then, there is a threshold circuit C computing P_{LR}^n such that C has energy $e = 1$ and size $s = \lceil n/2 \rceil$.*

The following theorem implies that the size of C given in Theorem 2 is optimal.

Theorem 3 *Let n be a positive integer. Let C be any threshold circuit computing P_{LR}^n with energy $e = 1$. Then, the size of C is at least $\lceil n/2 \rceil$.*

References

- [1] P. Lennie. The cost of cortical computation. *Current Biology*, 13:493–497, 2003.
- [2] K. Uchizawa, R. Douglas, and W. Maass. On the computational power of threshold circuits with sparse activity. *Neural Computation*, 18(12):2994–3008, 2006.
- [3] R. A. Legenstein and W. Maass. Neural circuits for pattern recognition with small total wire length. *Theoretical Computer Science*, 287:239–249, 2002.