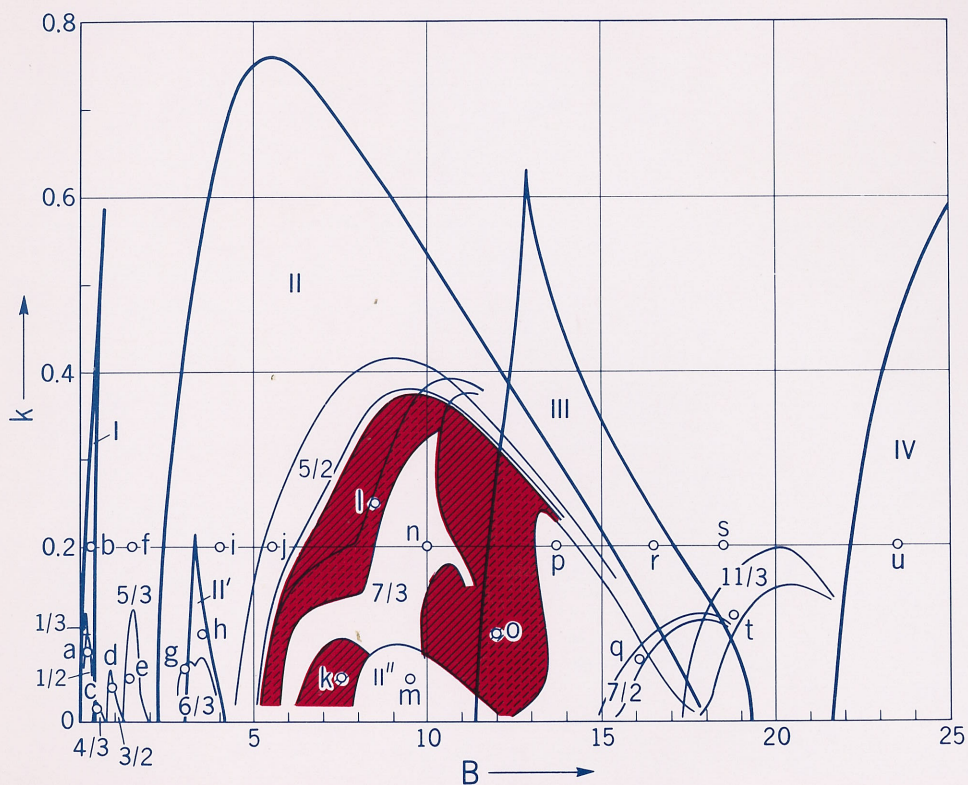


The Road to Chaos



by
Yoshisuke Ueda

About the book

For the first time Yoshisuke Ueda's early research in nonlinear dynamics of forced oscillators is presented here together with his subsequent studies of Duffing's equation. The early work has been retranslated into English, and reset with the original figures enlarged to suit their high quality of illustration, which sets the international standard in this field.

The abridged Ph.D. dissertation treats five different second-order nonlinear oscillators:

Chapter 1 compares averaging analysis and analog and digital simulations of the cubic Duffing equation with experimental observations of a series resonant circuit;

Chapter 2 considers a parametrically forced Duffing oscillator and a parallel resonant circuit;

Chapters 3 through 5 examine entrainment and hysteresis in the forced van der Pol, van der Pol/Duffing, and Rayleigh/Duffing mixed type equations.

Pioneering articles first published in 1970 and 1973 illustrate the tangled invariant manifold structure of what has since become known as chaos, in both its transient and steady state manifestations. The fundamental nature of steady state chaos is described in terms of unstable periodic motions, and is related to Birkhoff's theory of recurrence and central motions.

Further selections include studies of the power spectrum of chaotic attractors of Duffing's equation; an early example of a chaotic attractor explosion, or interior crisis; an expanded and up-to-date version of the 1980 picture book survey of Duffing's equation; and a lively personal reminiscence.

About the author

Yoshisuke Ueda is a professor in the Department of Electrical Engineering of Kyoto University. He received his Ph.D. from Kyoto University; as a graduate student in Chihiro Hayashi's laboratory he first encountered chaos in analog simulations of a nonlinear oscillator in 1961, and became the pioneer of chaos studies in Japan. Professor Ueda's research interests have included geometric phase space analysis of nonlinear oscillators through numerical simulation, behavior of nonlinear delay-differential equations, experimental and theoretical study of synchronous machines, and the stability of electrical power systems.

In 1985 Ueda was appointed Professor of Power Systems Engineering in the chair formerly held by Chikasa Uenosono. He is an editorial board member of three journals of chaos research, and was recently named to the board of directors of Japan's Institute of Electrical Engineers.

The Science Frontier Express Series

THE ROAD TO CHAOS

Yoshisuke Ueda

**Department of Electrical Engineering
Kyoto University, Japan**

with the assistance of

Ralph H. Abraham

University of California at Santa Cruz

and

H. Bruce Stewart

Brookhaven National Laboratory

Aerial Press, Inc.

P. O. Box 1360, Santa Cruz, CA 95061

Library of Congress Catalog Card Number: 92-73630
ISBN 0-942344-14-6

Copyright © by Aerial Press, Inc.
P. O. Box 1360, Santa Cruz, CA 95061

PREFACE

Aerial Press is proud to present this volume of selected papers by Professor Yoshi Ueda. We actively promoted this project, and persuaded Professor Ueda to help. Bruce Stewart and I named the book, expressing our conviction of the important role of Ueda's work in the creation of chaos theory. So, perhaps a personal word is in order here on the tradition of experimental dynamics in Japan, and its role in my own development.

Let me begin with Ueda's teacher, Chihiro Hayashi (1911-1987). He had studied the works of Lord Rayleigh, Poincaré, Liapunov, Duffing, Van der Pol, and the other classics of dynamics. In an environment of electronic engineering, he evolved his own style in the period 1942-1948, a unique combination of electronic experiments in the style of Van der Pol, classical analysis following Poincaré, and a new level of excellence in graphical presentation. This work appeared in his book in English in 1953 (republished in a much expanded version in 1964 and in 1985), and began to have an influence in Europe and the USA after his visit to MIT in 1955-56. My own discovery of Hayashi's book came relatively late, in the mid-1970s, just as I was beginning the development of the Visual Math Project at the University of California at Santa Cruz, and I must acknowledge the powerful influence of Hayashi's graphics in the visual element of my own work. Many results and illustrations added to Hayashi's second book of 1964 were made by his students: H. Shibayama, M. Kuwahara, Y. Nishikawa, M. Abe and Y. Ueda. In 1984 I went on pilgrimage to Kyoto to meet Hayashi, who treated me very well. At this time Hayashi introduced me to several of his former students, and thus, I met Ueda, whose work was already well-known to our group in Santa Cruz.

At that time I did not know the story, told at last in this book, of the difficult birth of the chaos revolution in Ueda's work in 1961 in Kyoto. But by now it is clear that crucial steps on the road to chaos were taken first in Kyoto, and are still diffusing outward. The exact moment of this bifurcation is now known, as you will see in the Comments by Bruce Stewart at the start of the book, and by Ueda himself at the end. Hayashi was the last master of the periodic paradigm, and Ueda the first master of the chaotic. The following chronology will help in following the steps to chaos described in this book.

- 1942 Hayashi begins his original research program on forced oscillators
- 1953 First edition of Hayashi's book
- 1959 Ueda begins doctoral research with Hayashi on forced oscillations
- 1961 Ueda observes chaos in analog simulation of forced oscillations of the mixed Van der Pol/Duffing system
(Selection 1, Chapter 4, Fig. 4.7; Selection 7, Fig. 1)
- 1963 Ueda observes chaos in analog simulation of the Duffing system
(Selection 7, Chapter 3)
- 1964 Second edition of Hayashi's book with additional data and illustrations by his students
- 1965 Ueda finishes thesis
(Selection 1)
- 1969 Ueda submits his work on chaotic attractors for publication
(Selection 2)
- 1973 Ueda's second paper on chaos published
(Selection 3)
- 1981 Hayashi presents his first paper on chaos in Kiev

It is a great pleasure to introduce this important book, which reports the bifurcation of the graphic experimental tradition of Japan from the periodic epoch of Hayashi into the chaotic era of Ueda, in the first three Selections, and the other outstanding accomplishments of Professor Ueda since those early days. The international community of dynamical systems specialists still have much to learn from this body of work, spanning the past thirty years, and presented here in an accessible single volume for the first time.

Ralph Abraham
Santa Cruz

TECHNICAL COMMENTS

Since the first appearance of the “Picture book” [16]* in 1980, a wide audience has learned about chaos and nonlinear behavior in Duffing’s equation and other forced oscillators through Yoshisuke Ueda’s research papers. His earlier research is not so widely known, but deserves consideration. Like his contemporary Lorenz, he charted new territory, and those who follow can benefit from another look at the origins of the chaos paradigm.

Chaos was first noticed and recorded by Ueda at the end of 1961 as part of his doctoral thesis research on forced oscillators under the guidance of Chihiro Hayashi. Ueda had determined the approximate regions of entrainment of a forced self-oscillator by harmonic balance methods. As Kyoto University did not yet have a general use digital computer, his predictions could at first only be tested by analog simulation. What Ueda found outside the regions of entrainment did not fit into the then-known universe of nonlinear oscillation theory. By the time the thesis was completed in 1965, chaos had appeared in the Duffing equation as well, but Hayashi saw no reason to look beyond the accepted concept of almost periodic oscillation, so the issue was not joined in the thesis [3], nor in the 1968 monograph condensed from it [5].

By 1969 Ueda was persuaded that a previously unheralded type of long-term or steady oscillation existed, which needed further attention. He submitted the manuscript, published in 1970 [7], giving examples of what are now called a chaotic attractor and a fractal basin boundary.

In 1973, Ueda published a paper [10] setting forth his understanding of chaos, following a close reading of G. D. Birkhoff’s theory of recurrence and central motions.

These first three selections have been revised for this collection, with the purpose of improving the English translation. Care has been taken to avoid any changes which would be out of keeping with Ueda’s understanding at the time the papers were first written. A few phrases and sentences have been added for clarity or emphasis; these additions are often repetitions or minor variations of points made elsewhere in the same work.

Doctoral Thesis This text is condensed, and includes revisions made to Chapter 3 for the 1968 monograph.

*Numbers in brackets refer to the List of Publications in English at the end of this volume.

It should be noted that the term beat oscillation has been substituted in Chapters 4 and 5 wherever the thesis misleadingly used the term almost periodic oscillation to describe chaotic behavior.

Figure 1.2 has a particularly interesting history. A similar figure with $k = 0.2$, computed and drawn by Ueda, will be found on page 129 of Hayashi's 1964 book "Nonlinear Oscillations in Physical Systems." The second unstable region in both figures was a source of misunderstanding between Hayashi and Ueda. For $k = 0.2$, this region is predominated by chaotic attractors, while for $k = 0.4$ chaotic transients are typical; see also Fig. 7 of [28]. By adopting $k = 0.4$ for the thesis, Ueda avoided contentious discussion about whether a new phenomenon had been observed.

Throughout the thesis, harmonic balance methods, despite their limitations, prove useful even in conditions where their justification appears weak. The fact that chaos arises every time harmonic balance fails must have given Ueda some encouragement.

An Announcement The second selection might appear as little more than the observation that not all steady nonlinear oscillations can be explained as almost periodic oscillations. The conclusions are dressed in modesty, but the contents are rewarding: beautiful examples of tangled invariant manifold structures, including a little-known candidate for the first example of a fractal basin boundary in a differential equation. These figures are of course not mere illustrations but hard-won data. The task of constructing invariant manifolds with analog computers and electromechanical pen plotters is not an enviable one, but it would foster a solid intuitive understanding of the significance of homoclinic structure.

Randomly Transitional Phenomenon The third selection presents Ueda's understanding of the new phenomenon now called chaotic attractors. The variety of examples underscores the widespread, generic occurrence of the phenomenon. Homoclinic structures are extensively documented, the Birkhoff-Smith theorem on periodic points is quoted, and the existence of order 2^n subharmonics is inferred via the Levinson index theorem. The term "randomly transitional phenomenon" is proposed to describe the ceaseless shifting among an infinity of unstable subharmonics.

In an Appendix, Ueda presents Birkhoff's theory of recurrence and central motions as the context in which the new steady oscillation phenomenon should be understood. Ueda was nervous about his mastery of Birkhoff's theory, but the only defect in the Appendix is omission of Pugh's results showing that generically the central motions consist solely of the nonwandering set, and thus Birkhoff's construction of higher-order nonwandering sets is unnecessary.

Ueda describes the new phenomenon as structurally stable, which would have been read as a technical error by mathematicians at that time. But surely it is the mathematical definition of structural stability which needs mending, not reality! Indeed, Zeeman has recently proposed a way forward in “Nonlinearity,” Vol. 1, p. 115 (1988).

Attractor Explosion and Catastrophe The fifth selection documents a case in which a chaotic attractor undergoes a severe bifurcation. This event later became widely known as an interior crisis due to the work of James Yorke and colleagues, who overlooked Ueda’s claim to precedence.

Those who wish to reproduce Ueda’s results should bear in mind the concept of structural stability and how it relates to analog and digital simulation. Ueda found this bifurcation using analog simulation, which suggested the simple picture described in Section 21.4 of R. H. Abraham and C. D. Shaw, “Dynamics: the Geometry of Behavior” (Second Edition). Upon fine checking with digital simulation, Ueda confirmed this picture, but also found that near the bifurcation threshold, the chaotic attractor collapses to periodic orbits which are stable in small windows of parameter values. Depending on the particular numerical scheme chosen, it could happen that these periodic windows might obscure the simple chaotic explosion scenario. But since the global bifurcation associated with chaotic attractor explosion lies on a generic codimension one manifold in parameter space, it would be possible to slide a small distance along this manifold to slightly different parameters where the digital simulations would reveal the same simple bifurcation suggested by the analog results.

Attractor Extinction In this fifth selection, Ueda mentions another global bifurcation phenomenon, the “extinction of strange attractors” by a “transition chain”; this refers to his discovery of what is now termed a blue sky catastrophe or boundary crisis, first reported in [15]. That paper is not included in the present volume.

This volume concludes with the expanded 1991 version of the famous “Picture book,” and a reminiscence, translated into English by Mrs. Masako Ohnuki with such skill that I had the wonderful sensation of hearing Ueda’s own voice unhindered by the barrier of language.

Bruce Stewart
Upton, New York

TABLE OF CONTENTS

Preface by Ralph H. Abraham	iii
Technical Comments by H. Bruce Stewart	v
1. Some Problems in the Theory of Nonlinear Oscillations, Doctoral Dissertation, Kyoto University, 1965: Chapters 1 to 3 were published as the Monograph with some revision in Chapter 3 by Nippon Printing and Publishing Co., Osaka, Japan (1968)	1
2. On the behavior of self-oscillatory systems with external force, Transactions of the Institute of Electronics and Communication Engineers of Japan, Vol. 53-A, No. 3, pp. 150-158 (1970) with C. Hayashi, N. Akamatsu and H. Itakura	73
3. Computer simulation of nonlinear ordinary differential equations and non-periodic oscillations, Transactions of the Institute of Electronics and Communication Engineers of Japan, Vol. 56-A, No. 4, pp. 218-225 (1973) with N. Akamatsu and C. Hayashi	97
4. Random phenomena resulting from nonlinearity: in the system described by Duffing's equation, Int. J. Non-Linear Mechanics, Vol. 20, No. 5/6, pp. 481-491 (1985): Translated from the Transactions of the Institute of Electrical Engineers of Japan, Vol. 98-A, No. 3, pp. 167-173 (1978)	119
5. Explosion of strange attractors exhibited by Duffing's equation, Non-linear Dynamics, Annals of the New York Academy of Sciences, Vol. 357, pp. 422-434 (1980)	133
6. Survey of regular and chaotic phenomena in the forced Duffing oscillator, Int. J. Chaos, Solitons and Fractals, Vol. 1, No. 3, pp. 199-231 (1991)	147
7. Strange attractors and the origin of chaos, Int. J. Nonlinear Science Today, Vol. 2, No. 2, pp. 1-16 (1992); will also appear in the Proceedings of the International Symposium "The Impact of Chaos on Science and Society", April 15-17, 1991 organized by the United Nations University and the University of Tokyo	183

List of Publications in English on Nonlinear Dynamics	217
Postscript	221
Curriculum Vitae of Yoshisuke Ueda	223