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McKay quiver representations and tiltings

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Aim

(1) Describe : $\mathcal{M}_{C,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathcal{M}_{C',\mathbf{d}}(Q,R)$
tilting theory.
The isomorphism of moduli spaces.

(2) Generalize : Crawley-Boevey's theorem.

Moduli spaces and Representations

Notations

G **: a finite subgroup of** $SL(2,\mathbb{C})$.
$\{\rho_0,\cdots,\rho_n\}$ **: the set of irreducible reps of** G.
(Q,R) **: the McKay quiver of** G.

$$\left.\begin{array}{l}\mathbf{d} := (\dim \rho_i)_{i\in Q_0} : \text{a dimension vector} \\ C : \text{a chamber in the stability space}\end{array}\right\} \overset{GIT}{\rightsquigarrow} \mathcal{M}_{C,\mathbf{d}}(Q,R).$$

$\mathrm{rep}_{C,\mathbf{d}}(Q,R)$ **: the category of -semistable** (Q,R) **reps w.r.t.** $\in C$ **of dimension vector d.**

$(\mathrm{rep}_{C,\mathbf{d}}(Q,R)/\simeq) = \mathcal{M}_{C,\mathbf{d}}(Q,R)$ **as a set.**

Fact

(Kronheimer)
$\mathcal{M}_{C,\mathbf{d}}(Q,R)$ **is the minimal resolution.**

(Nakajima,Lusztig etc.)
Describe : $S : \mathcal{M}_{C,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathcal{M}_{C',\mathbf{d}}(Q,R)$ **as a reflection functor.**

Tilting theory and Weyl groups

Tilting modules

Λ **: a noetherian algebra (e.g.** $\mathbb{C}Q/R$**).**
T **: a right** Λ**-module.**

$$T : \text{tilting} \overset{def}{\Leftrightarrow} \begin{cases} (1)\,\text{proj.dim}_\Lambda T \le 1 \\ (2)\,\mathrm{Ext}^1_\Lambda(T,T)=0 \\ (3)\exists\text{exact sequence} \\ 0\to\Lambda\to T_1\to T_2\to 0 \quad (T_i\in \mathrm{add}\,T)\end{cases}$$

Theorem(Happel, Rickard)

Tilting modules induce the equivalences

$$\mathcal{D}^{\mathrm{b}}(\mathrm{mod}\,\Lambda) \simeq \mathcal{D}^{\mathrm{b}}(\mathrm{mod}\,\mathrm{End}_\Lambda(T)).$$

Theorem (Iyama-Reiten)

$\mathbb{C}Q/R$ **: the preprojective algebra.**
W **: the corresponding Weyl group.**
$I_i := \langle 1-e_i\rangle$ **: an ideal of** $\mathbb{C}Q/R$.
$\mathcal{I} := \{I_{i_1}\cdots I_{i_k} \mid 1\le i_1,\dots,i_k\le n,\ k\in\mathbb{N}\}$ **: a set of ideals of** $\mathbb{C}Q/R$.

(1) $\mathcal{I}\ni\forall T$ **is a tilting module over** $\mathbb{C}Q/R$ **s.t.** $\mathrm{End}_\Lambda(\mathbb{C}Q/R)\cong\mathbb{C}Q/R$.

(2) $\exists$ **a one-to-one correspondence** $W\to\mathcal{I}$ **;**

$$w = s_{i_1}\cdots s_{i_k} \mapsto I_w := I_{i_1}\cdots I_{i_k}$$
(a reduced expression)

Main Theorem

C_G **: fix the chamber defined by** $_0 < 0$ **and** $_i > 0$ $(i\neq 0)$.
(i.e. $\mathcal{M}_{C_G,\mathbf{d}} = G$**-Hilb).**

Main Theorem

There is a category equivalence

$$\mathrm{Hom}_{\mathbb{C}Q/R}(I_w,-) : \mathrm{rep}_{C_G,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathrm{rep}_{C_G^w,\mathbf{d}}(Q,R)$$

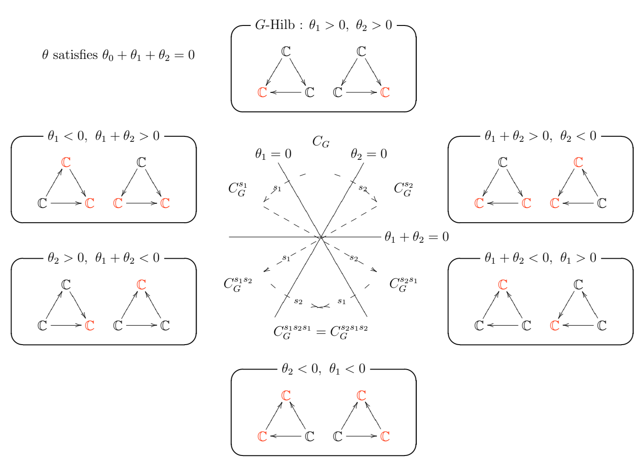
which induces the isomorphism

$$\mathcal{M}_{C_G,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathcal{M}_{C_G^w,\mathbf{d}}(Q,R).$$

Point of proof of Main Theorem

$$\begin{array}{cccc} \mathbb{R}\mathrm{Hom}_{\mathbb{C}Q/R}(I_i,-) : & \mathcal{D}^{\mathrm{b}}(\mathrm{mod}\,\mathbb{C}Q/R) & \xrightarrow{\sim} & \mathcal{D}^{\mathrm{b}}(\mathrm{mod}\,\mathbb{C}Q/R) \\ & \cup & & \cup \\ \mathrm{Hom}_{\mathbb{C}Q/R}(I_i,-) : & \mathrm{rep}_{C,\mathbf{d}}(Q,R) & \xrightarrow{\sim} & \mathrm{rep}_{C^{s_i},\mathbf{d}}(Q,R)\end{array}$$

Example of A_2**-type**



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