

Large-time behavior of solutions for
the equations of a viscous gas

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1. Introduction

We consider one-dimensional flow of a compressible fluid. In the Lagrange mass coordinate (t, x) , the motion of the fluid is described by the following equations.

$$(1.1) \quad \begin{aligned} v_t - u_x &= 0, & u_t + p_x &= (\mu u_x / v)_x, \\ (e + u^2/2)_t + (pu)_x &= (\kappa \theta_x / v + \mu u u_x / v)_x. \end{aligned}$$

Here $v > 0$ is the specific volume, u the velocity, $\theta > 0$ the absolute temperature, e the internal energy, μ the coefficient of viscosity and κ the coefficient of heat-conductivity. Let us denote the entropy by s . It is known that among five thermodynamic variables v, θ, p, e and s , only two of them are independent. In fact they may all be considered as smooth functions of $(v, \theta), (v, s), (v, e)$ or (p, s) . We write $p = p(v, \theta) = \hat{p}(v, s)$ and $e = e(v, \theta)$ and assume that

$$(1.2)_1 \quad \partial p(v, \theta) / \partial v < 0, \quad \partial p(v, \theta) / \partial \theta > 0, \quad \partial e(v, \theta) / \partial \theta > 0,$$

$$(1.2)_2 \quad \partial^2 \hat{p}(v, s) / \partial v^2 > 0.$$

Notice that these conditions are satisfied for the case of an ideal polytropic gas:

$$(1.3) \quad p = R\theta/v = \hat{R}v^{-\gamma}e^{(\gamma-1)s/R}, \quad e = R\theta/(\gamma-1) + \text{constant},$$

where $R > 0$ is the gas constant, $\gamma > 1$ is the adiabatic exponent and $\hat{R}$ is a positive constant. We also assume that μ and κ are smooth functions of two independent thermodynamic variables and satisfy one of the following two conditions.

$$(1.4)_1 \quad \mu > 0, \quad \kappa > 0 \quad (\text{viscous heat-conductive fluid}),$$

$$(1.4)_2 \quad \mu \equiv 0, \quad \kappa > 0 \quad (\text{inviscid heat-conductive fluid}).$$

We shall study the large-time behavior of solutions to the initial value problem for (1.1). Our main result is as follows: If the initial data are close to a given constant state, then a unique smooth solution of (1.1) exists for all time $t \geq 0$ and approaches the superposition of the nonlinear and linear diffusion waves constructed in terms of the self-similar solutions of the Burgers equation and the linear heat equation as $t \rightarrow \infty$.

We remark that the same asymptotic result has been obtained in [5] for a wide class of systems including (1.1).

Notations

We introduce several function spaces. Let $p \in [1, \infty]$, $\beta \in \mathbb{R}$ and $s \geq 0$. L^p denotes the usual Lebesgue space on $\mathbb{R}$, with the norm $|\cdot|_p$. L^p_β denotes the space of functions $f = f(x)$ such that $(1+|x|)^\beta f \in L^p$, with the norm $|\cdot|_{p,\beta}$. H^s denotes the space of functions $f = f(x)$ such that $\partial_x^\ell f \in L^2$ for $0 \leq \ell \leq s$, with the norm $\|\cdot\|_s$. Note that $H^0 = L^2$.

and $\|\cdot\|_0 = |\cdot|_2$. $C^0([0,\infty); H^S)$ is the space of continuous functions on $[0,\infty)$ with values in H^S .

2. Preliminaries

We first choose v and θ as independent thermodynamic variables and write $p = p(v, \theta)$, $e = e(v, \theta)$ and $s = s(v, \theta)$. The thermodynamic law $de = \theta ds - pdv$ gives

$$(2.1) \quad e_v = -(p - \theta p_\theta), \quad s_v = p_\theta, \quad s_\theta = e_\theta / \theta,$$

where we used abbreviations such as $e_v = \partial e(v, \theta) / \partial v$. When v and s are regarded as independent variables, we write $\theta = \hat{\theta}(v, s)$, $p = \hat{p}(v, s)$ and $e = \hat{e}(v, s)$. Using (2.1), we obtain

$$(2.2) \quad \begin{aligned} \hat{\theta}_v &= -\theta p_\theta / e_\theta, & \hat{p}_v &= p_v - \theta p_\theta^2 / e_\theta, & \hat{e}_v &= -p, \\ \hat{\theta}_s &= \theta / e_\theta, & \hat{p}_s &= \theta p_\theta / e_\theta, & \hat{e}_s &= \theta, \end{aligned}$$

where $\hat{\theta}_v = \partial \hat{\theta}(v, s) / \partial v$, etc. In particular, we have $\hat{p}_v < 0$ by (1.2)₁. Similarly, choosing v and e as independent variables and writing $\theta = \tilde{\theta}(v, e)$, $p = \tilde{p}(v, e)$ and $s = \tilde{s}(v, e)$, we obtain

$$(2.3) \quad \begin{aligned} \tilde{\theta}_v &= (p - \theta p_\theta) / e_\theta, & \tilde{p}_v &= (p_v - \theta p_\theta^2 / e_\theta) + p p_\theta / e_\theta, & \tilde{s}_v &= p / \theta, \\ \tilde{\theta}_e &= 1 / e_\theta, & \tilde{p}_e &= p_\theta / e_\theta, & \tilde{s}_e &= 1 / \theta, \end{aligned}$$

where $\tilde{\theta}_v = \partial \tilde{\theta}(v, e) / \partial v$, etc. In particular, we have $\tilde{p}_v - p \tilde{p}_e = \hat{p}_v$.

3. Vector form of the system

Put $E = e + u^2/2$. Then (1.1) is regarded as a system for (v, u, E)

and is rewritten in the vector form

$$(3.1) \quad w_t + f(w)_x = (G(w)w_x)_x,$$

where $w = (v, u, E)^T$, $f(w) = (-u, p, pu)^T$, and $G(w)$ is the matrix given by (3.3) below. We denote by $A(w)$ the Jacobian of $f(w)$ with respect to w . Then (3.1) is equivalent to

$$(3.1)' \quad w_t + A(w)w_x = (G(w)w_x)_x.$$

$A(w)$ and $G(w)$ are given explicitly as follows.

$$(3.2) \quad A(w) = \begin{pmatrix} 0 & -1 & 0 \\ \tilde{p}_v & -u\tilde{p}_e & \tilde{p}_e \\ u\tilde{p}_v & p - u^2\tilde{p}_e & u\tilde{p}_e \end{pmatrix},$$

$$(3.3) \quad G(w) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mu/v & 0 \\ \kappa\tilde{\theta}_v/v & \mu u/v - \kappa u\tilde{\theta}_e/v & \kappa\tilde{\theta}_e/v \end{pmatrix}.$$

By straightforward calculations, using (2.2) and (2.3), we know that the eigenvalues of $A(w)$ are given by

$$(3.4) \quad \lambda_1(w) = -(-\hat{p}_v)^{1/2}, \quad \lambda_2(w) = 0, \quad \lambda_3(w) = (-\hat{p}_v)^{1/2}.$$

These are all real and distinct since $\hat{p}_v < 0$ by (1.2)₁. This means that the inviscid system $w_t + f(w)_x = 0$ is strictly hyperbolic. The corresponding right and left eigenvectors, $r_j(w)$ and $\ell_j(w)$, are

$$(3.5) \quad \begin{aligned} r_j(w) &= a_j(1, -\lambda_j, -u\lambda_j - p)^T, \quad j=1,3, \\ r_2(w) &= a_2(\tilde{p}_e, 0, -\tilde{p}_v)^T, \end{aligned}$$

$$(3.6) \quad \begin{aligned} \ell_j(w) &= b_j(-\tilde{p}_v, -\lambda_j + u\tilde{p}_e, -\tilde{p}_e), \quad j=1,3, \\ \ell_2(w) &= b_2(p, -u, 1), \end{aligned}$$

where $a_j b_j \neq 0$, $j=1,2,3$. We choose a_j and b_j such that $2a_j b_j = 1/(-\hat{p}_v)$, $j=1,3$, and $a_2 b_2 = 1/(-\hat{p}_v)$. In this case we have

$$(3.7) \quad \langle \ell_j(w), r_k(w) \rangle = \delta_{jk}, \quad j,k=1,2,3,$$

where $\langle \cdot, \cdot \rangle$ denotes the standard inner product of $\mathbb{R}^3$. When $(1.2)_2$ is assumed, we determine a_j such that $a_j = -2\lambda_j/\hat{p}_{vv}$, $j=1,3$, and $a_2 = \theta/(-\hat{p}_v)$, where $\hat{p}_{vv} = \partial^2 \hat{p}(v,s)/\partial v^2$. Then we have

$$(3.8) \quad \langle \nabla \lambda_j(w), r_j(w) \rangle = 1, \quad j=1,3,$$

$$(3.9) \quad \langle \nabla s(w), r_2(w) \rangle = 1.$$

Here the gradient ∇ is with respect to w , and $s = s(w)$ is the entropy. Since $\langle \nabla \lambda_j(w), r_j(w) \rangle \neq 0$, $j=1,3$, the first and the third characteristic fields are genuinely nonlinear in the sense of Lax [7]. While the second field is linearly degenerate ([7]) because we have $\langle \nabla \lambda_2(w), r_2(w) \rangle = 0$ by $\lambda_2(w) = 0$.

4. Global existence and decay of solution

We consider (3.1) with the initial condition

$$(4.1) \quad w(0,x) = w_0(x),$$

where $w_0 = (v_0, u_0, E_0)^T$ with $E_0 = e_0 + u_0^2/2$. We seek a solution of (3.1), (4.1) in a neighborhood of a constant state $\bar{w} = (\bar{v}, \bar{u}, \bar{E})^T$, where $\bar{v} > 0$, $\bar{u} \in \mathbb{R}$, and $\bar{E} = \bar{e} + \bar{u}^2/2$ with $\bar{\theta} = \bar{\theta}(\bar{v}, \bar{e}) > 0$. We have the following

global existence result.

Theorem 4.1. ([6], see also [4]) *Assume (1.2)₁, and (1.4)₁ or (1.4)₂. If $w_0(x) - \bar{w}$ is small in H^s , $s \geq 2$, then the initial value problem (3.1), (4.1) has a unique global solution $w(t, x)$ in an appropriate function space. In particular, we have $w - \bar{w} \in C^0([0, \infty); H^s)$ and $\|w(t) - \bar{w}\|_s \leq C \|w_0 - \bar{w}\|_s$ for $t \in [0, \infty)$, where C is a constant. Moreover, the solution $w(t, x)$ converges to the constant state $\bar{w}$ uniformly in $x \in \mathbb{R}$ as $t \rightarrow \infty$.*

This result is proved by an energy method which makes use of the following properties: The system (3.1) has an entropy function and is transformed into a symmetric system of hyperbolic-parabolic type which satisfies the stability condition. We refer the reader to [4], [5] for the details. See also [1], [2].

Next we study a decay rate of the difference $w(t, x) - \bar{w}$ for $t \rightarrow \infty$. The linearized system of (3.1) around the constant state $\bar{w}$ is

$$(4.2) \quad w_t' + A(\bar{w})w_x' = G(\bar{w})w_{xx}'.$$

Denote by e^{tR} the semigroup of (4.2). We have

$$(4.3) \quad |\partial_x^\ell(e^{tR}f)|_2 \leq Ce^{-ct} |\partial_x^\ell f|_2 + C(1+t)^{-(1/2+\ell-k)/2} |\partial_x^k f|_1,$$

where $0 \leq k \leq \ell$, C and c are positive constants, and $f = f(x)$ is a function such that the norms on the right hand side of (4.3) are finite (see [9]). Making use of (4.3), we obtain the following

Theorem 4.2. ([4]) *Assume (1.2)₁, and (1.4)₁ or (1.4)₂. If $w_0(x) - \bar{w}$ is small in $H^s \cap L^1$, $s \geq 3$, then the solution $w(t, x)$ of (3.1) constructed in Theorem 4.1 satisfies*

$$(4.4) \quad |\partial_x^\ell(w(t) - \bar{w})|_2 \leq C N_s (1+t)^{-(1/2+\ell)/2}, \quad t \in [0, \infty),$$

where $\ell \geq 0$, $3\ell \leq s-2$, C is a constant and $N_s = \|w_0 - \bar{w}\|_s + |w_0 - \bar{w}|_1$.

5. Approximation by uniformly parabolic system

We first note that the matrix $A(w)$ has the spectral resolution $A(w) = \sum \lambda_j(w) P_j(w)$, where $P_j(w) = r_j(w) \ell_j(w)$ and the summation is taken over all $j=1,2,3$. We then define the matrix $D(w)$ by

$$(5.1) \quad D(w) = \sum_{j=1}^3 \kappa_j(w) P_j(w),$$

where $\kappa_j(w) = \langle \ell_j(w), G(w) r_j(w) \rangle$ with $G(w)$ given by (3.3). By straightforward calculations we have

$$(5.2) \quad \begin{aligned} \kappa_j(w) &= (-\mu \hat{p}_v + \kappa p_s^2 / \theta) / (-2v \hat{p}_v), \quad j=1,3, \\ \kappa_2(w) &= (-\kappa p_v) / (-v e_\theta \hat{p}_v). \end{aligned}$$

Note that these coefficients are all positive by (1.2)₁, and (1.4)₁ or (1.4)₂.

Now we consider the system

$$(5.3) \quad z_t + f(z)_x = D(\bar{w}) z_{xx},$$

with the initial condition $z(0, x) = w_0(x)$. The system (5.3) is semilinear and uniformly parabolic, and hence has a unique global solution $z(t, x)$, provided that $w_0(x) - \bar{w}$ is small in H^s , $s \geq 1$. The linearized system of (5.3) around the constant state $\bar{w}$ is

$$(5.4) \quad z_t' + A(\bar{w}) z_x' = D(\bar{w}) z_{xx}'.$$

Denote by e^{tS} the semigroup of (5.4). We easily obtain the estimate

$$(5.5) \quad |\partial_x^\ell(e^{tS}f)|_2 \leq Ce^{-ct}|\partial_x^\ell f|_2 + C(1+t)^{-(1/2+\ell-k)/2}|\partial_x^k f|_1,$$

where $0 \leq k \leq \ell$, C and c are positive constants. Making use of (5.5), we know that if $w_0(x) - \bar{w}$ is small in $H^s \cap L^1$, $s \geq 1$, then the solution $z(t, x)$ of (5.3) satisfies

$$(5.6) \quad |\partial_x^\ell(z(t) - \bar{w})|_2 \leq CN_s(1+t)^{-(1/2+\ell)/2}, \quad t \in [0, \infty),$$

where $0 \leq \ell \leq s$ and C is a constant.

Furthermore, we can show that for $t \rightarrow \infty$, the solution $w(t, x)$ of (3.1) is well approximated by the solution $z(t, x)$ of (5.3). More precisely, we have the following

Theorem 5.1. ([5]) *Assume (1.2)₁, and (1.4)₁ or (1.4)₂. If $w_0(x) - \bar{w}$ is small in $H^s \cap L^1$, $s \geq 5$, then we have*

$$(5.7) \quad |\partial_x^\ell(w(t) - z(t))|_2 \leq CN_s(1+t)^{-(3/2+\ell)/2+\alpha}, \quad t \in [0, \infty),$$

where $\ell \geq 0$, $3\ell \leq s-5$, C is a constant, and $\alpha > 0$ is a small fixed constant.

This approximation result is based on the following better decay estimate for the difference between the semigroups e^{tR} and e^{tS} .

$$(5.8) \quad |\partial_x^\ell(e^{tR} - e^{tS})f|_2 \leq Ce^{-ct}|\partial_x^\ell f|_2 + C(1+t)^{-(3/2+\ell-k)/2}|\partial_x^k f|_1,$$

where $0 \leq k \leq \ell$, C and c are positive constants.

6. Diffusion waves

Following Liu [8], we shall construct the diffusion waves. First we determine the coefficients $\delta_j(\bar{w})$, $j=1,2,3$, by

$$(6.1) \quad \int_{-\infty}^{\infty} (w_0(x) - \bar{w}) dx = \sum_{j=1}^3 \delta_j(\bar{w}) r_j(\bar{w}).$$

Put $\delta(\bar{w}) = (\delta_1(\bar{w}), \delta_2(\bar{w}), \delta_3(\bar{w}))$ and assume that $\delta(\bar{w}) \neq 0$. Next we introduce the Riemann invariant. A function of w which are constant in the direction of $r_j(w)$ is called j -Riemann invariant. For each j , we have two independent j -Riemann invariants given below.

$$(6.2) \quad \begin{aligned} & s \text{ and } u + \int^v \lambda_j(v, s) dv \quad \text{for } j=1, 3, \\ & p \text{ and } u \quad \text{for } j=2. \end{aligned}$$

Here the eigenvalue $\lambda_j(w)$ is regarded as a function of v and s .

Now, for the genuinely nonlinear field $\lambda_j(w)$, $j=1$ or $j=3$, we define j -diffusion wave $W_j(t, x)$, $W_j = (v_j, u_j, E_j)^T$ with $E_j = e_j + u_j^2/2$, by

$$(6.3)_1 \quad \begin{aligned} s_j(t, x) &= \bar{s}, \quad u_j(t, x) = \bar{u} - \int_{\bar{v}}^{v_j(t, x)} \lambda_j(v, \bar{s}) dv, \\ \lambda_j(v_j(t, x), \bar{s}) - \lambda_j(\bar{v}, \bar{s}) &= Y(t+1, x - \lambda_j(\bar{v}, \bar{s})(t+1); \kappa_j(\bar{w}), \delta_j(\bar{w})). \end{aligned}$$

Here $\bar{s} = \tilde{s}(\bar{v}, \bar{e})$, $\kappa_j(\bar{w})$ and $\delta_j(\bar{w})$ are given by (5.2) and (6.1), respectively, and

$$(6.4)_1 \quad Y(t, x; \kappa, \delta) = \sqrt{\kappa} t^{-1/2} \frac{(e^{\delta/2\kappa} - 1) e^{-\xi^2}}{\sqrt{\pi} + (e^{\delta/2\kappa} - 1) \int_{\xi}^{\infty} e^{-\eta^2} d\eta}, \quad \xi = x/\sqrt{4\kappa t}.$$

The function Y in (6.4)₁ is the self-similar solution of the Burgers equation $y_t + yy_x = \kappa y_{xx}$ and satisfies

$$(6.5) \quad \int_{-\infty}^{\infty} Y(t, x; \kappa, \delta) dx = \delta, \quad t \in (0, \infty).$$

See [3], [8]. Note that $W_j(t, x)$ lies on the curve $R_j(\bar{w})$ defined by $dw/d\tau = r_j(w)$ and $w = \bar{w}$ at $\tau = 0$. Since $\lambda_j(w)$ is monotone along $R_j(\bar{w})$ by (3.8), the relations in (6.3)₁ uniquely determine $v_j(t, x)$ and

$s_j(t, x)$ and therefore all other thermodynamic variables.

For the linearly degenerate field $\lambda_2(w) = 0$, we define 2-diffusion wave $W_2(t, x)$, $W_2 = (v_2, u_2, E_2)^T$ with $E_2 = e_2 + u_2^2/2$, by

$$(6.3)_2 \quad \begin{aligned} p_2(t, x) &= \bar{p}, & u_2(t, x) &= \bar{u}, \\ s_2(t, x) - \bar{s} &= Y(t+1, x; \kappa_2(\bar{w}), \delta_2(\bar{w})), \end{aligned}$$

where $\bar{p} = \tilde{p}(\bar{v}, \bar{e})$, etc., and

$$(6.4)_2 \quad Y(t, x; \kappa, \delta) = \delta(4\pi\kappa t)^{-1/2} e^{-\xi^2}, \quad \xi = x/\sqrt{4\kappa t}.$$

This Y is the self-similar solution of the linear heat equation $y_t = \kappa y_{xx}$ and satisfies (6.5). Notice that $W_2(t, x)$ lies on the curve $R_2(\bar{w})$. The relations in $(6.3)_2$ define $p_2(t, x)$ and $s_2(t, x)$ and therefore all other thermodynamic variables.

Finally, we define $W(t, x)$, the superposition of the diffusion waves, by

$$(6.6) \quad W(t, x) - \bar{w} = \sum_{j=1}^3 (W_j(t, x) - \bar{w}).$$

By straightforward calculations, using $(6.3)_{1,2}$ and $(6.4)_{1,2}$, we have

$$(6.7) \quad W_t + f(W)_x = D(\bar{w})W_{xx} + r_x(t, x) - q(t, x),$$

where $r(t, x)$ and $q(t, x)$ are known functions such that

$$(6.8) \quad \begin{aligned} |\partial_x^\ell r(t, x)| &\leq C |\delta(\bar{w})| e^{-c(t+|x|)}, \\ |\partial_x^\ell q(t, x)| &\leq C |\delta(\bar{w})|^2 (1+t)^{-(4+\ell)/2} \sum_{j=1}^3 e^{-c\xi_j^2}, \end{aligned}$$

where $\ell \geq 0$, $\xi_j = (x - \lambda_j(\bar{w})(t+1))/\sqrt{t+1}$, and C and c are positive constants. For the details, see [5],[8].

7. Large-time behavior

We shall show that $W(t, x)$ defined by (6.6) is an asymptotic solution for $t \rightarrow \infty$ of the uniformly parabolic system (5.3). To this end we construct the linear hyperbolic wave $\zeta(t, x)$ as the solution of

$$(7.1) \quad \zeta_t + A(\bar{w})\zeta_x = q(t, x),$$

with the following condition: $\zeta(t, x) \rightarrow 0$ uniformly in $x \in \mathbb{R}$ as $t \rightarrow \infty$. Here $q(t, x)$ is the function in (6.7). By the characteristic method, we have a unique smooth solution $\zeta(t, x)$ satisfying

$$(7.2) \quad |\partial_x^\ell \zeta(t, x)| \leq C |\delta(\bar{w})|^2 \sum_{j=1}^3 \{ (t+1+|x-\lambda_j(\bar{w})(t+1)|^2)^{-(2+\ell)/2} + (t+1+|x-\lambda_j(\bar{w})(t+1)|)^{-(3+\ell)/2} \},$$

where $\ell \geq 0$ and C is a constant. Also, it is shown that for $t \in [0, \infty)$,

$$(7.3) \quad \int_{-\infty}^{\infty} (W(t, x) - \bar{w} + \zeta(t, x)) dx = \sum_{j=1}^3 \delta_j(\bar{w}) r_j(\bar{w}).$$

From (6.1) and (7.3) we know that $w_0(x) - W(t, x) - \zeta(t, x)$ has zero integral for each $t \in [0, \infty)$. By virtue of this property, we have a desired conclusion.

Theorem 7.1. ([5]) Assume (1.2)_{1,2}, and (1.4)₁ or (1.4)₂. Suppose that $w_0(x) - \bar{w}$ is small in $H^s \cap L_\beta^1$, $s \geq 1$ and $\beta \geq 1/2$. Let $z(t, x)$ be the solution of (5.3) and let $W(t, x)$ be the superposition of the diffusion waves defined by (6.6). Then we have

$$(7.4) \quad |\partial_x^\ell (z(t) - W(t))|_2 \leq CM_s (1+t)^{-(1+\ell)/2 + \alpha}, \quad t \in [0, \infty),$$

where $0 \leq \ell \leq s$, C is a constant, $M_s = \|w_0 - \bar{w}\|_s + \|w_0 - \bar{w}\|_{1,1/2}$, and $\alpha > 0$ is a small fixed constant.

In the proof of this theorem, the following estimate for the semigroup e^{tS} plays an essential role: If $f \in L^1_\beta$, $\beta \in [0,1]$, and $f(x)$ has zero integral, then we have

$$(7.5) \quad |\partial_x^\ell(e^{tS}f)|_p \leq Ct^{-(1-1/p+\beta+\ell)/2} \|f\|_{1,\beta}, \quad t \in (0,\infty),$$

where $\ell \geq 0$, $p \in [1,\infty]$ and C is a constant.

We remark that (7.4) is a meaningful asymptotic relation for $t \rightarrow \infty$, because for large t , the L^2 -norm of $\partial_x^\ell(W(t,x) - \bar{w})$ is bounded from below by $c|\delta(\bar{w})|t^{-(1/2+\ell)/2}$ with a positive constant c .

A combination of Theorems 5.1 and 7.1 gives the main result of this paper.

Theorem 7.2. ([5]) *We assume the conditions of Theorem 7.1 with $s \geq 1$ replaced by $s \geq 5$. Then the solution $w(t,x)$ of (3.1) satisfies*

$$(7.6) \quad |\partial_x^\ell(w(t) - W(t))|_2 \leq CM_s(1+t)^{-(1+\ell)/2+\alpha}, \quad t \in [0,\infty),$$

where $\ell \geq 0$, $3\ell \leq s-5$ and C is a constant; M_s and α are the same as those in Theorem 7.1.

This theorem means that the superposition of the diffusion waves defined by (6.6) is also an asymptotic solution for $t \rightarrow \infty$ of the system (3.1).

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