

Effect of Quasi-holes on the Structure of Odd-mass Nuclei

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It is well known that the Tamm-Dancoff approximation (TDA) is less adequate than the random phase approximation (RPA) for the treatment of the quadrupole and octupole collective vibration in even-even nuclei since the latter includes the ground state correlation.¹⁾ Although in previous theories RPA has been employed for the description of the even-even core, essentially only the TDA scheme has been used for the odd nucleus since quasi-hole, or the backward scattering effect for an unpaired nucleon, was not taken into account. In the present paper, it is shown that a description including this effect explains some important features of odd nuclei not predicted by previous theories.^{2,3)}

We introduce the Hamiltonian and the coupled equations of motion of quasi-particles and quasi-holes. The Hamiltonian is

$$H = H_{\text{BCS}} + (H_{22} + H_{40} + H_{31} + \text{c.c.}) \quad (1)$$

where the operators have been ordered in normal form, i.e., all the creation operators α_{jm}^+ are placed to the left of the destruction operators α_{j-m} . The indices n_1 and n_2 of $H_{n_1 n_2}$ refer to the numbers of α_{jm}^+ and α_{j-m} , respectively. The equation of motion becomes⁴⁾

$$[H, \alpha_{jm}^+] = E_j \alpha_{jm}^+ + \sum_{j'} K_{jj'} (\lambda \mu j' m' | jm) \alpha_{j'm'}^+ \{Q_{\lambda\mu}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \\ + \sum_{j'} M_{jj'} (\lambda \mu j' m' | jm) (-)^{j'-m'} \alpha_{j'-m'} \{Q_{\lambda\mu}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \quad (2a)$$

$$[H, (-)^{j-m} \alpha_{j-m}] = -E_j (-)^{j-m} \alpha_{j-m} - \sum_{j'} K_{jj'} (\lambda \mu j' m' | jm) (-)^{j'-m'} \alpha_{j'-m'} \{Q_{\lambda\mu}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \\ + \sum_{j'} M_{jj'} (\lambda \mu j' m' | jm) \alpha_{j'm'}^+ \{Q_{\lambda\mu}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \quad (2b)$$

where⁵⁾ $K_{jj'} \equiv -S_{\lambda}^{'-1/2} (2\lambda + 1/2j + 1)^{1/2} \langle j' \| q_{\lambda} \| j \rangle v_{jj'}$

$$M_{jj'} \equiv -S_{\lambda}^{'-1/2} (2\lambda + 1/2j + 1)^{1/2} \langle j' \| q_{\lambda} \| j \rangle u_{jj'}$$

Here E_j is energy of a quasi-particle with spin j and $Q_{\lambda\mu}^+$ ($Q_{\lambda-\mu}$) represents the creation (destruction) operator of a phonon of multipole order λ , μ for the even-even core. It is assumed that $Q_{\lambda\mu}^+$ and $Q_{\lambda-\mu}$ commute with α_{jm}^+ and α_{j-m} , also that $Q_{\lambda\mu}|0\rangle = 0$. The physical meaning of the second assumption is: the ground state $|0\rangle$ of the even-even core in an odd nucleus is the same as the ground state of its even-even neighbor. $S_{\lambda}^{'-1/2}$, the strength of the phonon-quasi-particle and phonon-quasi-hole interactions, is calculated in the scheme of RPA (hereafter denoted $S_{\lambda}^{'-1/2}(\text{e-e})$).²⁾ In the present cal-

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culation, the BCS equations were solved for two major shells employing level energies which are almost the same as those found from Mottelson, Nilsson, and Prior's orbits ($\delta=0$).⁶⁾ The strengths of the pairing and quadrupole forces were determined to fit the energy gaps obtained from the even-odd mass difference and the energies of the first 2^+ states in neighboring even-even nuclei, respectively. For one quasi-particle and hole (in the two major shells) with zero, one, and two phonons, the equations of motion are linearized by employing the following relations⁷⁾

$$\begin{aligned}
 \langle jm | [H, \alpha_{jm}^+] | N, JM \rangle &\simeq (E_j^0 - N\hbar\omega_\lambda) \langle jm | \alpha_{jm}^+ | N, JM \rangle \\
 \langle jm | \{Q_{2\mu}^+ + (-)^{-\mu} Q_{2-\mu}\} \alpha_{j'm'}^+ | N, JM \rangle \\
 &= \sum_{N', J' M'} \langle jm | \alpha_{j'm'}^+ | N', J' M' \rangle \langle N', J' M' | \{Q_{2\mu}^+ + (-)^{-\mu} Q_{2-\mu}\} | N, JM \rangle
 \end{aligned}$$

Here $|jm\rangle$, E_j^0 , and $|N, JM\rangle$ represent a state with spin j in odd nucleus, its energy eigenvalue, and the N -phonon state with spin J , respectively. In the second equation the last term on the right hand side is calculated in the same manner as for an even-even nucleus.

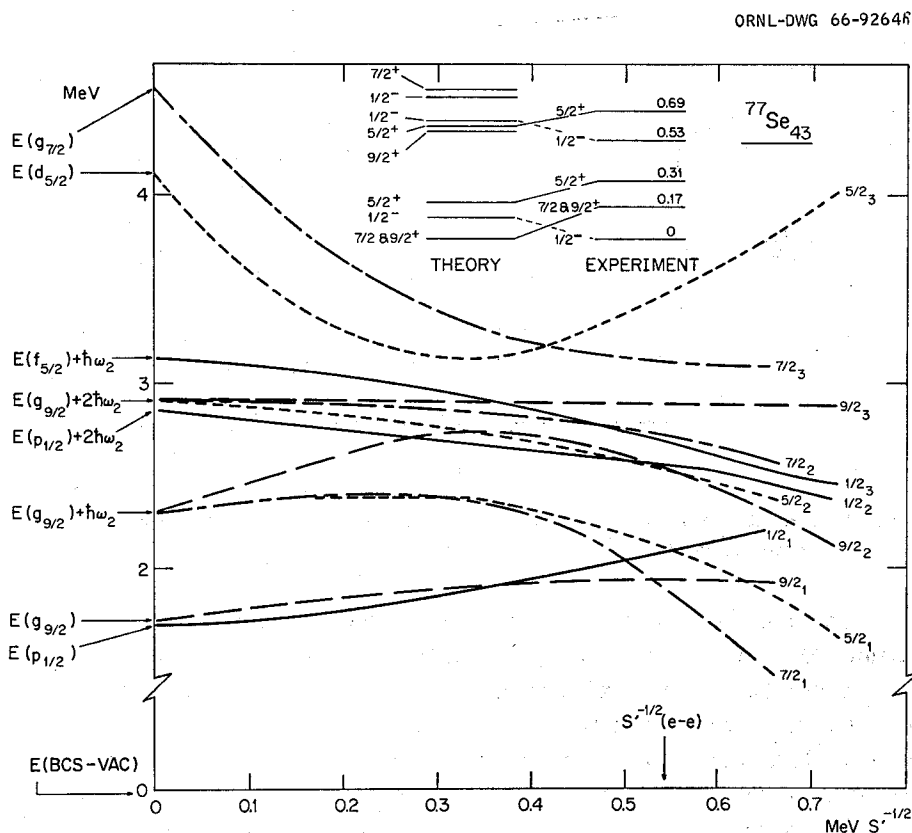


Fig. 1. Dependence on $S_2^{-1/2}$ of the level ordering for Se^{77} calculated taking account of the quasi-hole effect. The value of $S_2^{-1/2}(e-e)$ is estimated from the 2^+ level energies of Se^{76} and Se^{78} .

Figure 1 shows the predicted level ordering in $^{77}\text{Se}_{43}$ as a function of the strength $S'^{-1/2}$. Figure 2 shows the level ordering when the quasi-hole effect is omitted.⁸⁾ The striking features evident from comparison of the two figures are:

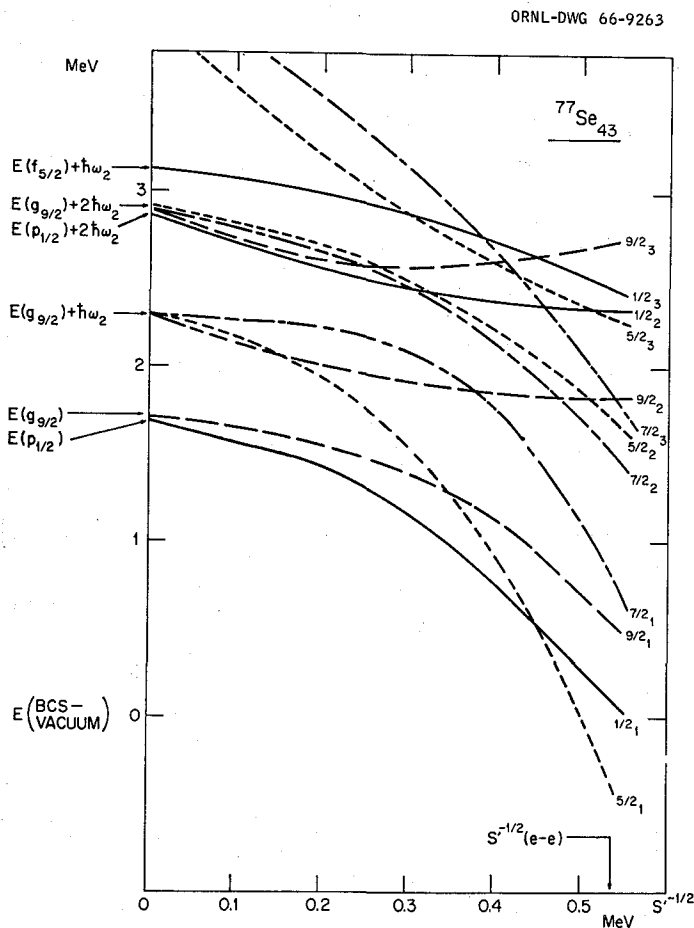


Fig. 2

Fig. 2. Dependence on $S_2'^{-1/2}$ of the level ordering for ^{77}Se calculated neglecting the quasi-hole effect.

i) Levels of quasi-particle near the Fermi level are pushed up because of their interaction (M'_{ij} term in Eq. 2) with the quasi-holes.

ii) Levels having the same spin and parity lie fairly close together in Fig. 1 in contrast with their separated positions in Fig. 2. The close spacing of Fig. 1 is consistent with observation, close to the ground state of odd nuclei with strong vibrational nature, of a number of doublets having the same spin and parity.⁹⁾

iii) The pushing-up effect, due to the quasi-hole, on the so called anomalous-coupling $7/2^+$ and $5/2^+$ levels is weaker than for normal states. This is another reason for the low-lying position of the anomalous-coupling states in addition to our previous explanation.⁶⁾

iv) Though the effect of the quasi-hole on level energies of odd nuclei, particularly on level energies of quasi-particles lying near the Fermi level, is very remarkable, it changes transition probabilities, multipole moments, spectroscopic factors, and so on by less than 10% from values obtained by the usual calculations.

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- (2) S. Yoshida, *Nucl. Phys.*, **38**, 380 (1962).
- (3) L. S. Kisslinger and R. A. Sorensen, *Rev. Mod. Phys.*, **35**, 853 (1963).
- (4) In the RPA scheme, one may also add the following terms:

$$\sum_{j'j_1j_1'} L_{jj':j_1j_1'}(\lambda\mu j' m' | jm) \alpha_{jm}^+ \{ \phi_{j_1j_1'}(-)^{\lambda-\mu} Q_{\lambda-\mu} + \varphi_{j_1j_1'} Q_{\lambda\mu}^+ \}$$

and

$$\sum_{j'j_1j_1'} L_{jj':j_1j_1'}(\lambda\mu j' m' | jm) (-)^{j'-m'} \alpha_{j'-m'} \{ \phi_{j_1j_1'}(-)^{\lambda-\mu} Q_{\lambda-\mu} + \varphi_{j_1j_1'} Q_{\lambda\mu}^+ \}$$

$$L_{jj':j_1j_1'} \equiv -\chi_\lambda (2\lambda+1/2j+1)^{1/2} \langle j_1' \| q_\lambda \| j' \rangle \langle j \| q_\lambda \| j_1 \rangle v_{j_1'j'} u_{jj_1} W(jj_1j'j_1'; \lambda\lambda)$$

to the right hand side of Eqs. 2a and 2b, respectively. They are numerically smaller than the terms including $K_{jj'}$ and $M_{jj'}$. Here, χ_λ represents a strength of the two body interaction of 2^λ -pole. Explicit expressions for $\varphi_{jj'}$ and $\phi_{jj'}$ can be seen in Ref. 2.

- (5) Here $q_\lambda \equiv i^\lambda w_\lambda Y_\lambda$, $w_\lambda \equiv [(m\omega_0/\hbar)^{1/2} r]^\lambda$
 $u_{j,j} \equiv U_j V_j + V_{j'} U_{j'}$, $v_{j,j} \equiv U_j U_j - V_{j'} V_{j'}$

where r is the radial co-ordinate of nucleon, m being its mass. Harmonic oscillator wave functions, with angular frequency ω_0 , are used for single-particle wave functions. U_j and V_j mean the probability of orbit j being empty and occupied, respectively.

- (6) It is indispensable to take accounts of several major shells for an explanation of the anomalous-coupling states on the basis of the BCS method and RPA. H. Ikegami and M. Sano, *Phys. Lett.*, **21**, 323 (1966).
- (7) Half the solutions of the equations of motion, whose energies tend to those of a quasihole with zero, one, and two phonons in the limit of $S_2^{-1/2} = 0$ are unphysical and must be rejected as in the RPA for even-even nuclei.
- (8) The result shown in Fig. 2 is essentially almost the same with those presented in Ref. 6.
- (9) For examples, low-lying levels having the same spin and parity are:
 The ground and 0.265 MeV states; $3/2^-$ in As^{75} .
 The 0.249 and 0.439 MeV states; $5/2^-$ in Sc^{77} .
 The 0.131 and 0.279 MeV states; $5/2^+$ in Br^{77} .
 The ground and 0.261 MeV states; $3/2^-$ in Br^{79} , etc.