

THE VARIATION OF THE SEA-LEVEL AND THE BAROMETRIC PRESSURE WITH CHANDLER'S PERIOD

BY

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Introduction :

The author suggested fourteen years ago that the mean sea-level and the barometric pressure seemed to change with Chandler's period which appeared in the latitude variation. 1) The more detailed systematic study was completed soon after, but the paper prepared at that time has been left unpublished. Hence, on this occasion the paper is rearranged and presented here.

The statistical data utilized in this study were monthly mean values of the latitude at the Mizusawa Observatory, the sea-level at four tidal stations, and the barometric pressure at five meteorological stations. The data of the precipitation at Sapporo were also taken up as supplementary. (Table 1)

Table 1 Statistical data.

Place	Lat. N.	Long. E.	Data
Sapporo	43°03	141°35	Bar. Pres. (1889-1948), Precipitation (1889-1947)
Oshro	42.21	140.86	Sea-level (1906-1948)
Mizusawa	39.13	141.13	Latitude (1900-1947), Bar. Pres. (1906-1947)
Wajima	37.60	136.90	Sea-level (1900-1947)
Kanazawa	36.56	136.65	Bar. Pres. (1886-1948)
Yokohama	35.43	139.65	Bar. Pres. (1897-1948)
Aburatsubo	35.15	139.62	Sea-level (1900-1944)
Hosojima	32.42	131.67	Sea-level (1900-1947)
Miyazaki	31.92	131.43	Bar. Pres. (1886-1948)

All of the data were previously corrected with the annual variation by subtracting respectively the mean values of the same-named months during the whole interval of years. Then smoothed values taken from the means of running five ones were adopted as the final observed monthly values. Some false periods might have been entered in our running means due to the so-called Slutsky-Yule's effect.

2) But these might be taken out of consideration so far as the present subject concerned because these were estimated as 9.7 mth and 4 mth.

Harmonic analysis of the Buys-Ballot's table :

The Buys-Ballot's table with a presumed period of 14 mth was formed from the final observed monthly values for each time-interval of $14 \text{ mth} \times 8 = 9.33 \text{ yr}$, beginning at the March of every even year, and each set of the mean values $x_0, x_1, \dots, x_{13}$ was calculated in the corresponding table. Thus the series of such sets at the successive epochs were obtained with respects to the latitude, the sea-

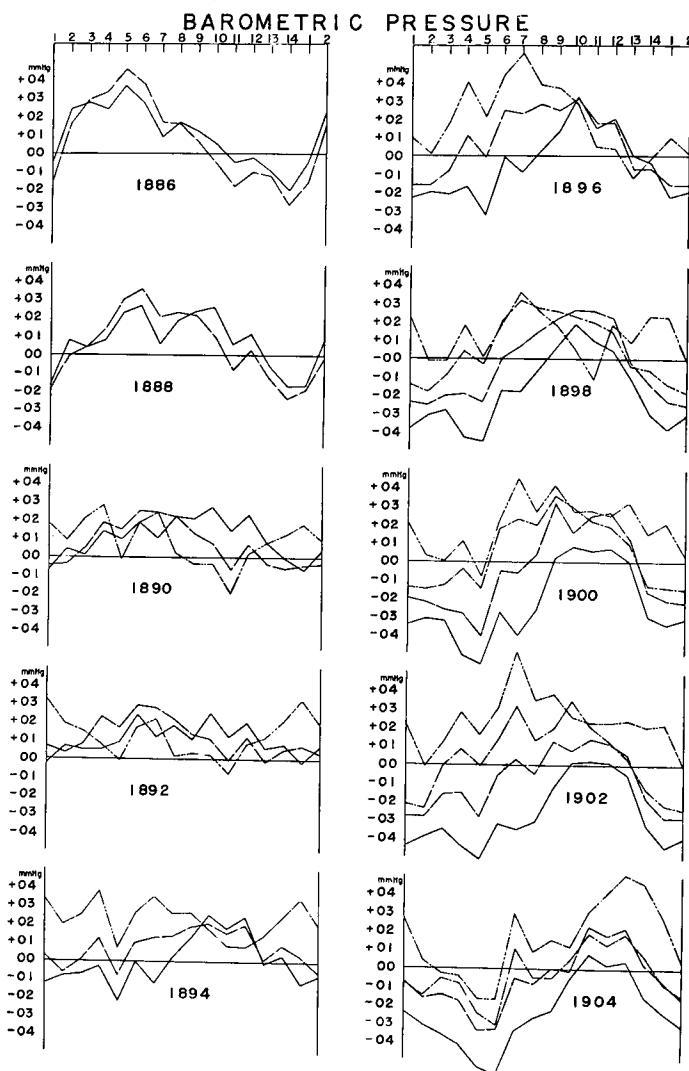


Fig. 1 (a) Change of the barometric pressure corrected for the annual variation.

level and so forth at the respective places. That all the curves of x_i in the series of the latitude were almost perfectly simple harmonic was nothing but a matter which we had anticipated. But as for these of the sea-level and the barometric pressure too, they might be approximated by sine-curves, especially when the amplitude was found large. Fig. 1 shows as an illustration these curves for the barometric pressure.

Then assuming a set of x_i 's to be expressed by

$$x_i = C_0 + \sum_{p=1}^h a_p \cos \frac{2\pi}{14} pi + \sum_{p=1}^h b_p \sin \frac{2\pi}{14} pi + \delta_i = C_0 + \sum_{p=1}^h C_p \sin \left(\frac{2\pi}{14} pi + \sigma_p \right) + \delta_i, \quad (1)$$

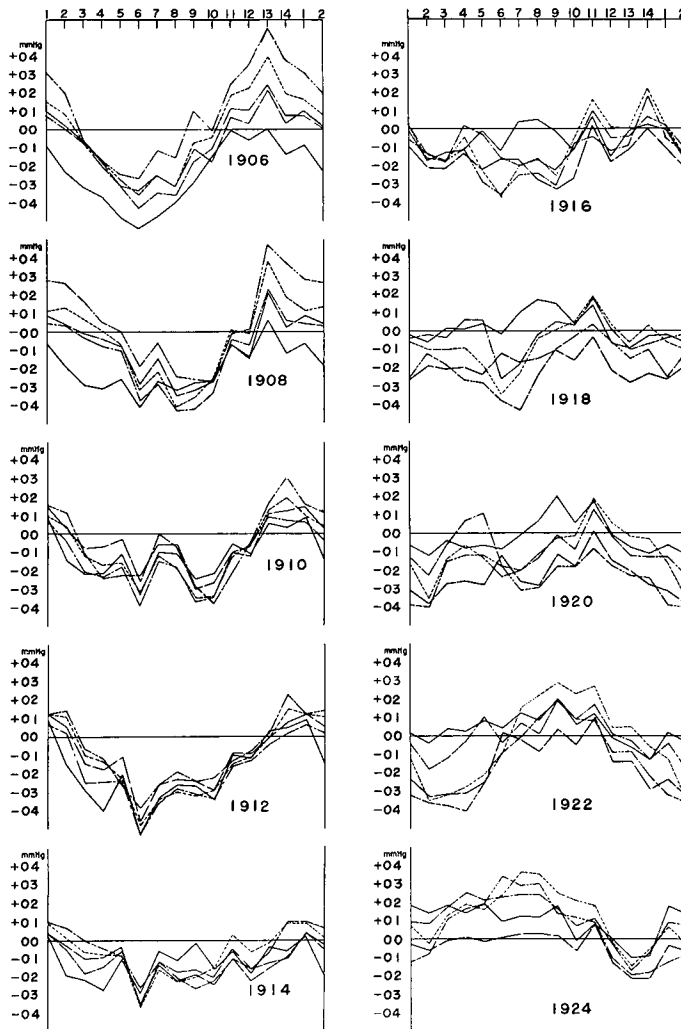


Fig. 1 (b) Continued.

$$(i=0, 1, \dots, 13)$$

where δ_i meant a purely contingent quantity, we required the harmonic constants for every set. In Figs. 2 a, b, c and 3 a, b, c are shown C_0 , C_1 , C_2 , a_1 of the sea-level as well as the barometric pressure and as for the precipitation at Sapporo those are included in the latter Figs.

Significance tests of the harmonic constants:

It seemed appropriate to examine at first whether the observed harmonic constants were merely fictitious ones caused by the sampling errors or not. If we assume that the individual monthly value x_i is composed of periodic terms and a

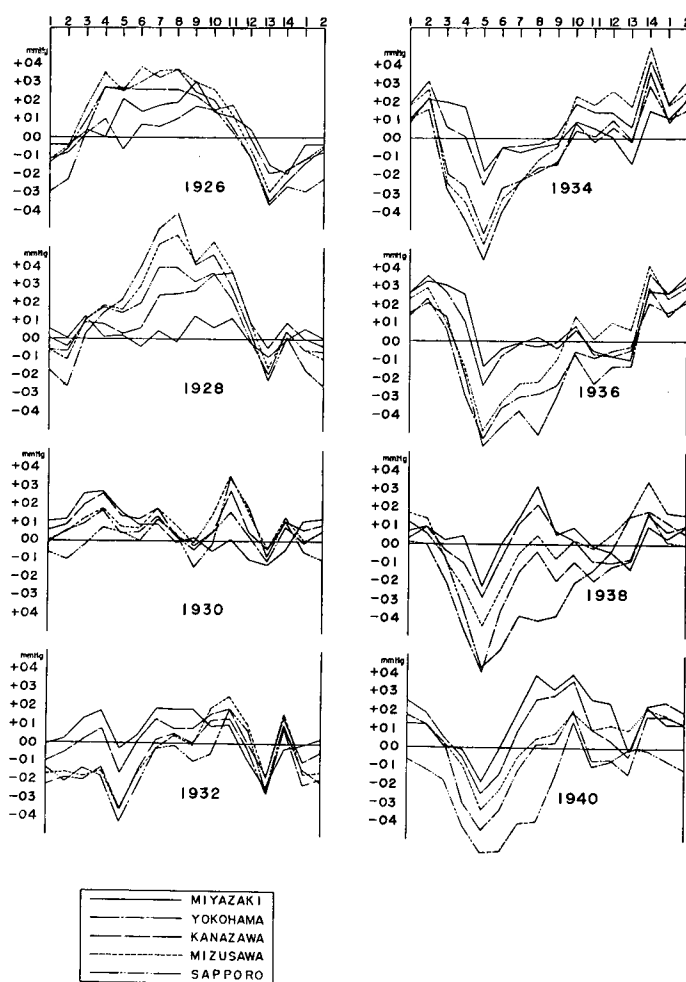


Fig. 1 (c) Continued.

purely random term as expressed by (1), then the F -test provides some statistical criterion on the significant difference between a pair of independent unbiased estimates of the variance, the one from the harmonic constants and the other from the random quantity. For a null hypothesis where $a_p = \bar{a}_p$ and $b_p = \bar{b}_p$ (the bar corresponds to the presumed value) should hold, is used the expression of

$$F = \frac{N-2h-1}{4} \left\{ (a_p - \bar{a}_p)^2 + (b_p - \bar{b}_p)^2 \right\} / \left\{ s^2 - \frac{1}{2} \sum_{q=1}^h (a_q^2 + b_q^2) \right\} \quad (D.F.: 2, N-2h-1), \quad (2)$$

while for another null hypothesis that $a_{p+q} = \bar{a}_{p+q}$, $b_{p+q} = \bar{b}_{p+q}$ ($q=1, 2, \dots$), it is to be replaced with the following

$$F' = \frac{N-2h-1}{4k} \sum_{q=1}^k \left\{ (a_{p+q} - \bar{a}_{p+q})^2 + (b_{p+q} - \bar{b}_{p+q})^2 \right\} / \left\{ s^2 - \frac{1}{2} \sum_{r=1}^h (a_r^2 + b_r^2) \right\} \quad (D.F.: 2k, N-2h-1) \quad (3)$$

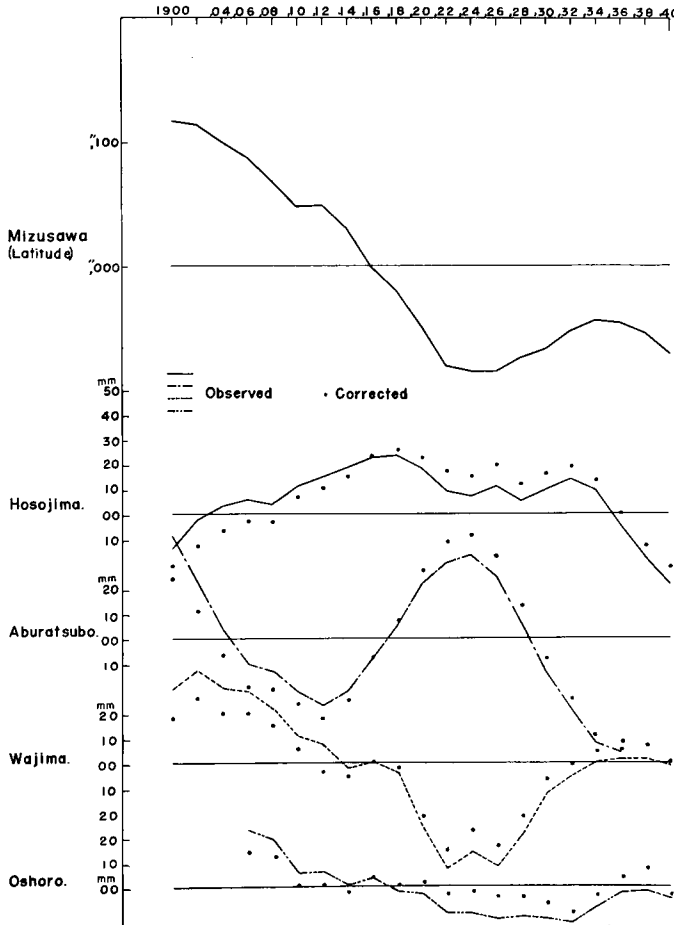


Fig. 2a C_0 (line: observed, dot: corrected) of the sea-level and the latitude.

where $s^2 = \sqrt{\frac{\sum (x - C_0)^2}{N}}$ or the observed variance of the monthly value itself. By means of these formulae any assigned values for the presumed constants $\bar{a}_p$ and $\bar{b}_p$ can be tested.

The following null-hypotheses were tested successively for every set of the harmonic constants of various quantities at the respective epochs. These were:

- i) $\bar{a}_1 = \bar{b}_1 = 0$ or $C_1 = 0$.
- ii) $\bar{a}_1 = a_{10}$, $\bar{b}_1 = b_{10}$, where a_{10} (b_{10}) signifies the mean of a_1 's (b_1 's) at all the stations, tidal or meteorological depending on x_i assigned.
- iii) $\bar{a}_1 = (a_1^2 + b_1^2)^{1/2} (A_1/C_1)$, $\bar{b}_1 = (a_1^2 + b_1^2)^{1/2} (B_1/C_1)$, where A_1 , B_1 , $C_1 = (A_1^2 + B_1^2)^{1/2}$ were the corresponding Fourier constants of the latitude variation.
- iv) $\bar{a}_2 = \bar{b}_2 = 0$ or $\bar{C}_2 = 0$.

The results of the respective significance tests for both sea-level and barometric pressure are summarized in Table 2. It is noted that though every successive four

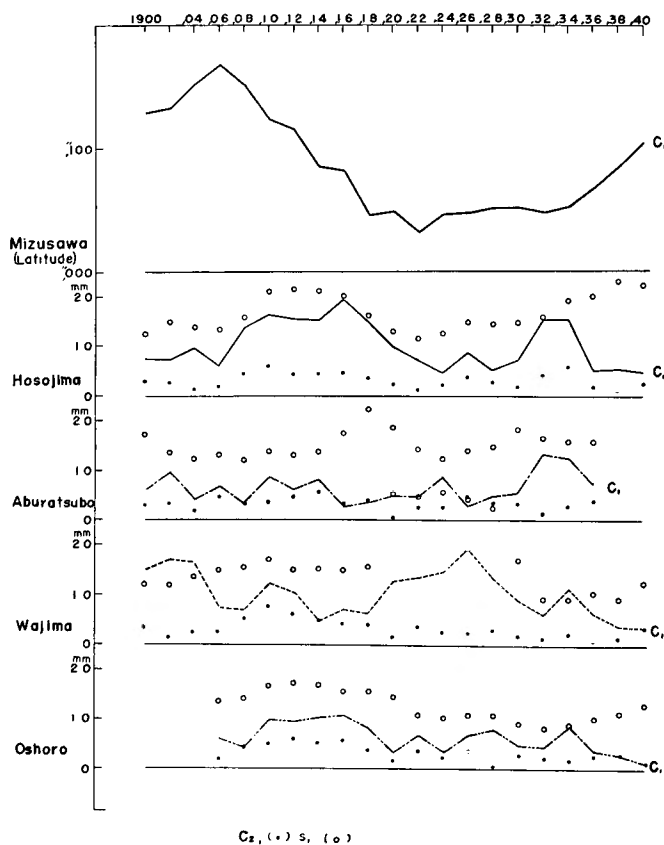


Fig. 2b C_1 (line), C_2 (dot) and s (open circle) of the sea-level and the latitude.

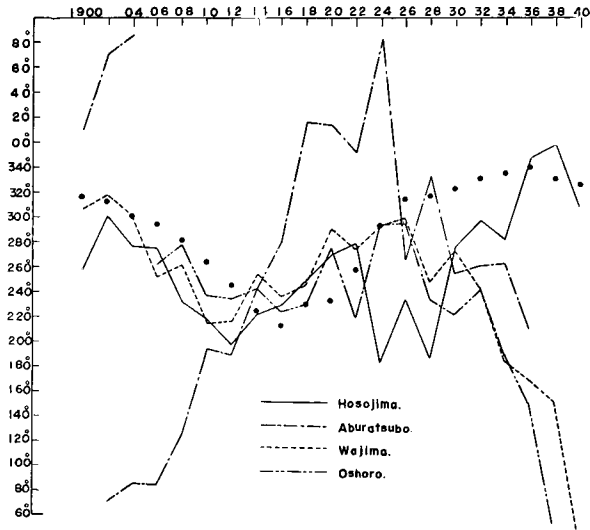
Fig. 2c α_1 of the sea-level (line) and the latitude (dot).

Table 2 Significance tests of the null-hypotheses

Hypothesis	Sta.	(i)			(ii)			(iii)			(iv)			Variance of monthly value (s)
Significance level		*	?	!	*	?	!	*	?	!	*	?	!	
Sea-level	H	6	4	11	0	0	21	8	2	11	0	0	21	mm 24.6-46.4
	A	2	4	13	3	2	14	5	1	13	0	0	19	24.4-45.2
	W	4	4	13	3	3	15	10	3	8	0	0	21	18.7-61.9
	O	1	2	15	1	0	17	8	1	9	0	0	18	17.2-35.3
	All %	13 16.5	14 17.7	52 65.8	7 8.9	5 6.3	67 84.8	31 39.2	7 8.9	41 51.9	0 0	0 0	79 100	
Baro. Pres.	My	9	5	14	1	6	21	5	5	11	0	0	28	mmHg 0.38-0.50
	Y	10	1	12	1	1	21	8	2	11	0	0	23	0.43-0.63
	K	6	7	15	0	3	25	8	1	12	0	0	28	0.41-0.58
	Mz	7	3	8	0	0	18	10	3	5	0	0	18	0.52-0.78
	S	7	3	17	0	3	23	7	6	8	0	0	27	0.56-0.98
	All %	39 31.5	19 15.3	66 53.2	2 1.6	13 10.6	108 87.8	38 38.2	17 16.7	47 46.1	0 0	0 0	124 100	

Note: i) (i), (ii), (iii) and (iv) correspond to the respective null-hypotheses explained in the text.

ii) Marks *, ?, ! correspond to the hypothesis in question which is refuted significantly, irreferably or non-significantly at the levels of 0.01 and 0.05.

iii) As for the variance of the final monthly mean of any set only the extreme values are shown.

sets included more or less common elements, each set for certain station was treated here as if it were independent of each other. The following was concluded from Table 2.

(i) For the first null-hypothesis that the observed periodicity with 14 mth was nothing but apparent and was ascribed to sampling errors, about two-thirds of all the sea-level sets and about a half of the barometric pressure ones were found as nonsignificant at the level of 0.05. While as for about a sixth of the sea-level sets and about a third of the barometric pressure ones, the observed first harmonics were significantly above the randomness, irrespective of the large variance in the monthly datum values.

(ii) With allowance of the reality of Chandler's period, the second hypothesis that the periodic variation was regarded as the same at all the tidal (meteorological)

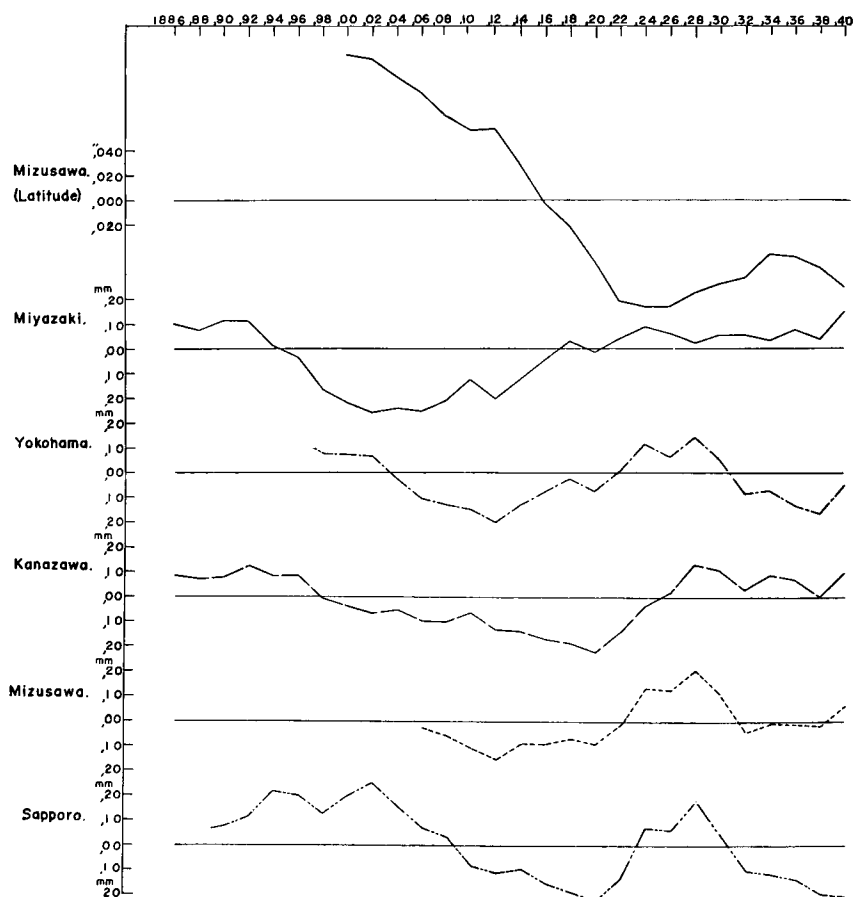


Fig. 3a C_0 of the barometric pressure and the latitude.

stations at every same epoch, might safely be said to be acceptable.

(iii) The third hypothesis that the variation of the sea-level (barometric pressure) was in the same phase as that of the latitude variation at Mizusawa was refuted significantly at the level of 0.01 for about four-tenths of the sea-level sets as well as of the barometric pressure ones. Then the sea-level (barometric pressure) variation under consideration was not necessarily ascertained as "parallel" to that of the latitude.

(iv) For the last hypothesis it turned out non-significant without exception. Accordingly the second harmonic with a period of 7 mth lay at random.

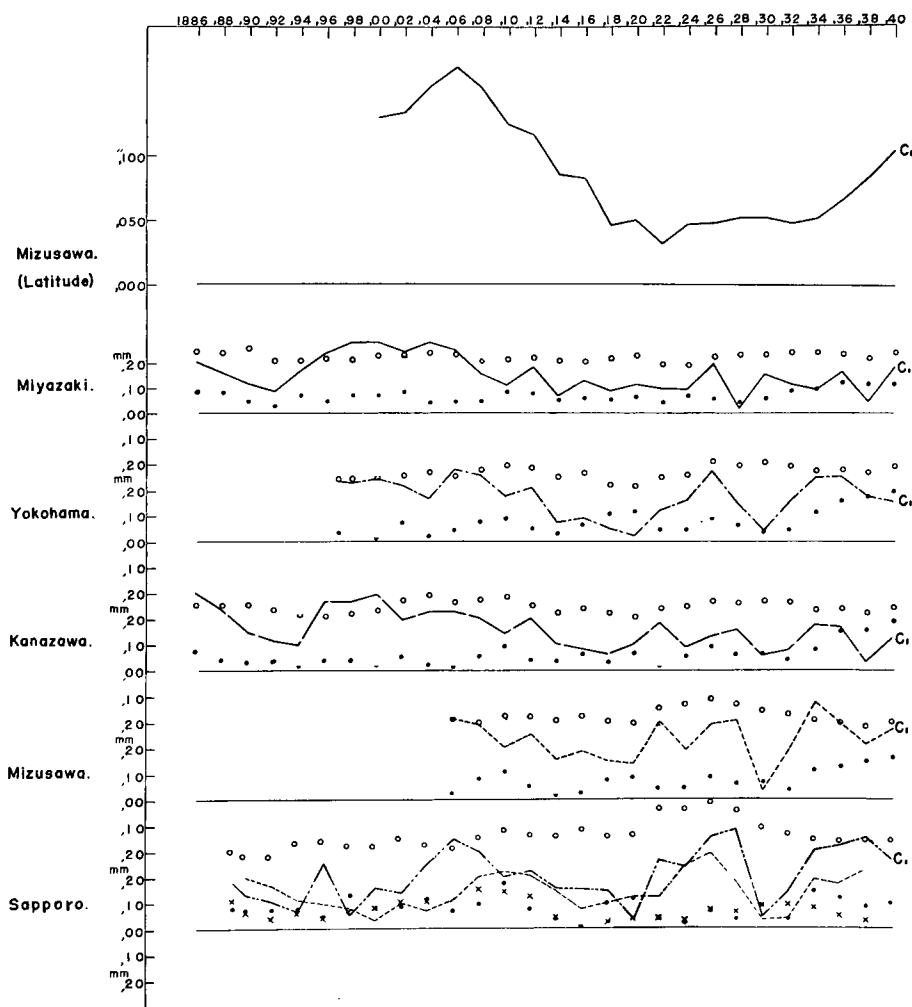


Fig. 3b C_1 (line), C_2 (dot) and s (open circle) of the barometric pressure. Only for Sapporo C_1 (---) and C_2 ($\times$) of the precipitation are shown.

The above-mentioned facts indicate that an assumed like periodicity in the variations of both sea-level and barometric pressure is likely to be real, though random disturbance struck out the phenomenon considerably. The ratio of variance of the monthly value to the numerical value of amplitude was in general less in the barometric pressure than in the sea-level, so that Chandler's periodicity could appear somewhat more distinctly in the barometric pressure than in the sea level.

Secular change in the time-interval of period :

Because the common variation of the sea-level (barometric pressure) might be accepted for all the tidal (meteorological) stations, the mean amplitude and the initial phase were compared with those of the latitude (Figs. 4 and 5). It is difficult to recognize a close relationship between the sea-level (barometric pressure) and the latitude in Figs. 4 and 5, as anticipated already from the test of the third null-hypothesis. But in this connection some caution is necessary because the time-interval of period had been assumed to be constant or exactly 14 mth. If the variation of, say, the sea-level has a true period n_0 different from the assumed one $n = n_0 + \Delta n$, it will appear as if the phase at successive epoch changes secularly together with lessening of the amplitude. Such effect may be readily seen by a simple approximate formula as follows, so far as an allowance is made that the true period and the phase are kept as constant in a time interval between two

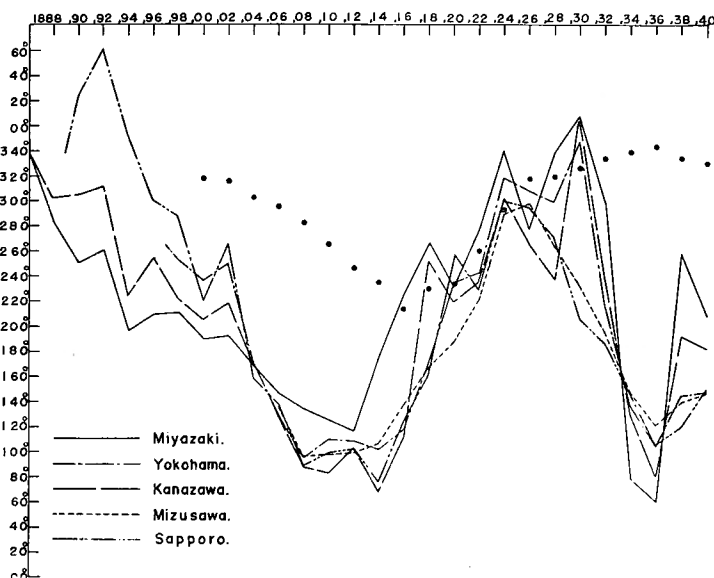


Fig. 3c α_1 of the barometric pressure (line) and the latitude (dot).

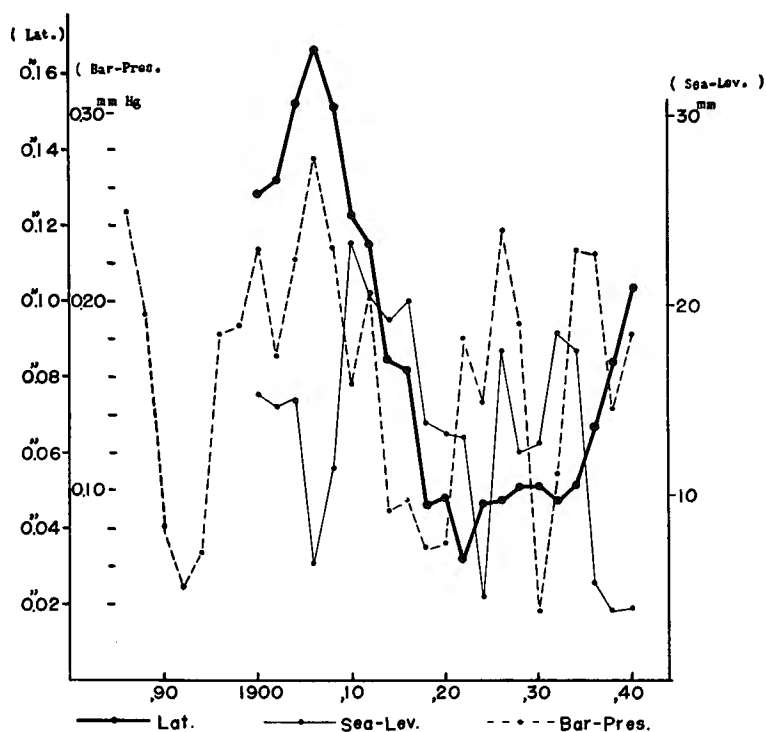


Fig. 4 Change of the mean amplitude.

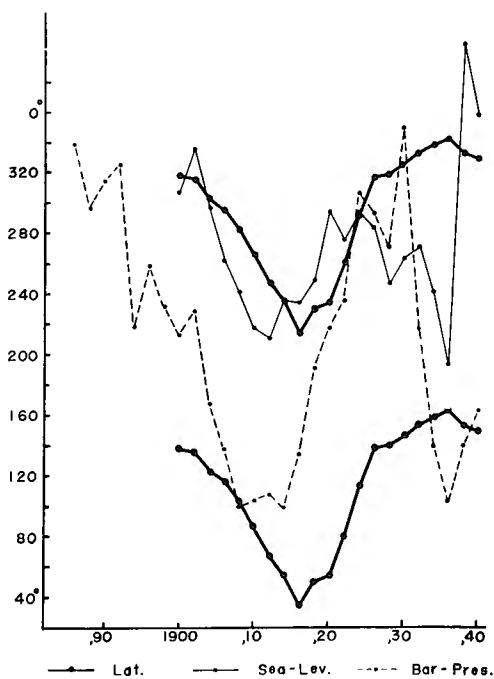


Fig. 5 Change of the mean initial phase.

successive epochs.

$$a' - a = \frac{24}{7} \frac{\Delta n}{n}, \quad (4)$$

where a and a' signified the initial phases at two successive epochs respectively. Mathematically speaking an error of $(a' - a)$ does not exceed $\pm 30^\circ$ if the original variation contains no second harmonic and $|\Delta n/n|$ is less than 0.1. But in reality the resulting error in Δn might be expected to be larger because of deviation from the assumed condition and of the sampling error in the utilized harmonic constants.

The mean of $(a' - a)$'s for all the tidal (meteorological) stations was compared with $(a' - a)$ of the latitude variation. Individual variations from the corresponding mean were $\pm 56^\circ$ (m.e.) for the sea-level and $\pm 42^\circ$ (m.e.) for the barometric pressure, consequently the variance in the mean value itself was estimated as about $\pm 30^\circ$ (m.e.) for the sea-level and about $20^\circ \sim 30^\circ$ (m.e.) for the barometric pressure according to the number of stations taken. In Fig. 6 the secular change of period is shown with regards the latitude, the sea-level and the barometric pressure.

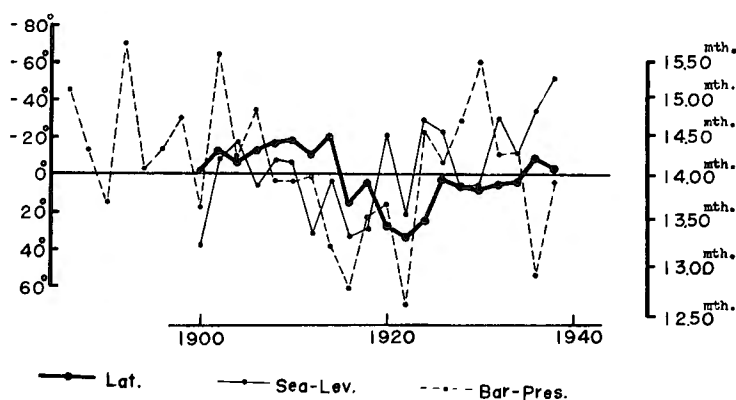


Fig. 6 Change of the periode.

The correlation of $(a' - a)$ between the mean sea-level (barometric pressure) and the latitude at the same epoch was as follows.

	$l-s$	$l-b$	$s-b$
$r_0(\text{observed}) \pm (Pr.(r > r_0) = Pr.(r < r_0) = 0.5)$	$+0.37 \pm 0.13$	$+0.18 \pm 0.14$	$+0.19 \pm 0.14$
$Pr.(r > r_0)$ in a case where a parent population has no correlation or $\rho = 0$.	0.16	0.23	
Significance interval = 0.98	$-0.16 < r < +0.72$	$-0.42 < r < +0.61$	$-0.34 < r < +0.62$
0.90	$-0.01 < r < +0.63$	$-0.21 < r < +0.50$	$-0.18 < r < +0.51$

l : latitude, s : sea-level, b : barometric pressure, ρ : correlation coef. of a parent population.

The probability for getting $r > r_0$ under the null hypothesis of no correlation was found above the significance level of 0.01 for both cases of $l-s$ and $l-b$, so that hypothesis could not be refuted positively. Nevertheless, even if $\rho = +0.71$ ($+0.60$) or $\rho = +0.62$ ($+0.49$) were really true for $l-s$ ($l-b$), our statistical data would inform us that the hypothesis is irreferable or nonsignificant at the usual level as well. As for $s-b$ where $r = +0.19$, nearly the same may be mentioned.

Hence, though we could hardly insist on the close relationship among three curves in Fig. 6, it seemed rather natural that the secular change of the period has a similar general tendency among the sea-level, the barometric pressure and the latitude.

Distributions of the amplitude and the phase:

Some studies were made so as to say of the mean character of Chandler's marine and atmospheric tides regarding all the sets as not time series but merely as a random sample as a whole.

Fig. 7 shows the cumulative distribution of the amplitude of the sea-level* against that of the latitude by adopting a scale factor of 1.0 cm : 0''.100. It is evident that agreement of both distributions is excellent. The same is said as for the corresponding diagram for the barometric pressure with a scale factor of 0.20 mmHg : 0''.100, though the figure is omitted here.

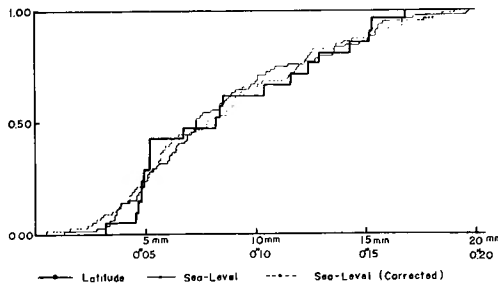


Fig. 7 Cumulative Distribution of the amplitude.

On the other hand the frequency distribution of the mean phase-lag of the sea-level* (barometric pressure) from the latitude is illustrated in Fig. 8. Scattering of the phase-lag is found large as expected from the third F -test. If the marine (atmospheric) tide in question had no connection with the polar motion of the earth, the phase difference between $l-s$ ($l-b$) would take any value in the whole range of $-180^\circ \sim +180^\circ$ with equal probability, accordingly the frequency

* Even if the harmonic constants of the sea-level were corrected with the effect of Chandler's variation of the barometric pressure, the general tendency of the revised values is nearly the same as that of the unrevised as seen Figs. 2a, 7 and 8.

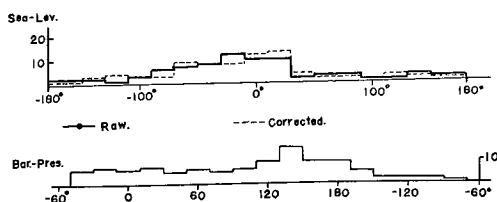


Fig. 8 Frequency distribution of the Phase-lag.

distribution would be of a rectangular type. But such null-hypothesis turned out to be significantly refuted by the χ^2 -test at the significance level of 0.01 for both of $l-s$ and $l-b$.

Hence, as for the phase-lag of the sea-level, the frequency distribution of which appeared nearly symmetric about the mean or 10° , it may well be said that the Chandler's marine tide was practically in the same phase as the Chandler's motion of the earth's axis. While for the barometric pressure, though its frequency distribution of the phase-lag appeared considerably assymmetric about its maximum at about $+1600$, Chandler's atmospheric tide was likely to be lagging behind about $+100^\circ$ in the mean from the earth's polar motion.

Various multiple correlations concerning the barometric pressure :

To see the dependence of the variation of the barometric pressure on that of the latitude more closely, the multiple correlations with regards the amplitudes of both and the phase-lag were analyzed at every station, as shown in Table 3.

The results in Table 3 could be interpreted as follows :

i) At first the scale factor between both amplitudes of b and l may be estimated independently not only from the ratio of the mean $\bar{x}/\bar{z}$ but also from that of the variance S_x/S_z for each station as given below.

	Miy.	Yok.	Kan.	Miz.	Sap.	Mean	Sap. (Prec.)
$\bar{x} : \bar{z}$	1.64	1.90	1.62	2.93	2.63	2.142	8.38
$S_x : S_z$	1.67	1.87	1.55	2.16	2.42	1.934	8.60

Thus the scale factor adopted before in the preceding section was proved as of a right order and yet the ratio of the mean variances between the barometric pressure and the latitude was found also practically the same as this value.

ii) The variance-series consisting of five S_x 's was significantly different from being homogeneous, but that of S_y 's was found to be homogeneous on the usual significance level. Then, by testing the equality of $\bar{y}$'s by the t -test, the mutual

Table 3 Multiple correlations.

	Miyazaki	Yokohama	Kanazawa	Mizusawa	Sapporo	Sapporo (Prec.)
C_1 (Bar. Pres.) $\equiv \bar{x}$	0.142 mmHg	0.165 mmHg	0.141 mmHg	0.228 mmHg	0.228 mmHg	7.18 mm
$(\alpha_1)_l - (\alpha_1)_b \equiv \bar{y}$	+70.°3	+96.°1	+106.°3	+115.°7	+114.°3	+93.°2
C_1 (Latitude) $\equiv \bar{z}$	0."0868	0."0868	0."0868	0."0782	0."0868	0."0860
S_x	0.0697 mmHg	0.0780 mmHg	0.0647 mmHg	0.0826 mmHg	0.1009 mmHg	3.64
S_y	98.°0	82.°7	78.°9	65.°2	72.°7	71.°0
S_z	0."0416	0."0416	0."0416	0."0386	0."0416	0."0423
r_{xy}	+0.386	+0.577 *	+0.441	+0.316	+0.312	+0.376
r_{xz}	+0.666 *	+0.471 ?	+0.616 *	+0.224	+0.110	-0.120
r_{yz}	+0.550 *	+0.567 *	+0.530 ?	+0.580	+0.413	+0.220
$r_{x^{1/2}z}$	+0.030	+0.427	+0.171	+0.243	+0.295	+0.415
$r_{xz \cdot y}$	+0.556 ?	+0.214	+0.502 ?	+0.040	-0.022	+0.224
$r_{yz \cdot x}$	+0.426	+0.410	+0.364	+0.541 ?	+0.402	+0.288
No.	21	21	21	18	21	20

* Significant $Pr_0 < (0.01)$, ? Irrefferable ($0.01 < Pr_0 < 0.05$)

Note: Confidence interval for $\rho=0$ is as follows.

For r_{xy} etc.	Prob.	No.			For $r_{xy \cdot z}$ etc.	Prob.	No.		
		18	20	21			18	20	21
	$P=0.10$	± 0.39	± 0.36	± 0.35		$P=0.10$	± 0.42	± 0.39	± 0.38
	$P=0.05$	± 0.47	± 0.43	± 0.42		$P=0.05$	± 0.48	± 0.46	± 0.44
	$P=0.01$	± 0.57	± 0.55	± 0.54		$P=0.01$	± 0.61	± 0.58	± 0.56

difference of any pair of $\bar{y}$'s was found to be nonsignificant.

iii) With respects to the correlation coefficients r_{xy} , r_{yz} and r_{xz} , some of which differed significantly from 0, it was found to be homogeneous among all the stations by χ^2 -test.

iv) None of the partial correlation coefficients $r_{xy \cdot z}$, $r_{yz \cdot x}$ or $r_{xz \cdot y}$ were not confirmed to be significantly different from $\rho=0$.

v) Among various quantities in Table 3 some systematic dependence on the location of the station could be traced by plotting these against the latitude or the longitude, even though significant differences among the station were hardly proved except in a few cases. Some of the quantities changed continuously, but the others

did discontinuously at nearly the same place (36°N , 140°E).

A trial analysis on the precipitation :

As it seemed certain that the phenomena concerning the barometric pressure are caused by some sort of atmospheric disturbance, it was natural to expect Chandler's period in the other meteorological elements too. A trial analysis on the precipitation at Sapporo brought also similar results as in the case of the barometric pressure as well as the sea-level. For example, the curves from Buys-Ballot's table were approximated by the sine curve (the fact, however, was not found in Wolf's number of the sun-spots!), and the harmonic constants seemed to have some relation with those of the barometric pressure at Sapporo and with the latitude variation as seen in Fig. 3 b. The ratio of the amplitudes between the precipitation and the latitude was $8.35\text{ mm} : 0''.100$ from the means and from the variances respectively (i) in the preceding section), so that the mean 8.5 mm might be adopted. As for the phase-lag the mean was to be about -95° , but individual values were nearly distributed into two groups centered at about -150° and -20° (the distribution was not of rectangular type at the significance level of 0.02), while for the phase difference between the precipitation and barometric pressure at Sapporo, a maximum condensation appeared at about 170° .

Conclusion :

In this paper Chandler's periodicity in the variation of the sea-level and the barometric pressure was statistically investigated in comparison with that of the latitude variation. Though the observed characters of variations were disturbed by sampling errors, such periodicity is likely real from various view points in both of the sea-level and the barometric pressure. It was found that the respective amplitudes were about 1 cm and 0.2 mmHg compared with $0''.1$ for the latitude variation and the respective phase lags from the latitude variation were about $+10^{\circ}$ and 100° in the mean.

Some theoretical discussion is reserved for a separate paper, but it is mentioned that the expected amplitude from the change of the centrifugal force due to Chandler's polar motion turned out to be a third of the observed one as for the sea-level while to be negligible with respect to the barometric pressure. Nevertheless, the excitation effect due to a mechanism similar as that suggested by Rudnick 4) 5) seems to explain the observed results.

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