

学位申請論文

Studies on Tax and Social Security Reforms in a
Dynamic General Equilibrium Framework

(租税・社会保障政策の動学一般均衡分析)

C H E N M I N G Y A N

Studies on Tax and Social Security Reforms in a Dynamic General Equilibrium Framework

A Dissertation Submitted to
the Graduate School of Economics,
Kyoto University

by

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December 2025

Abstract

This dissertation examines how tax and social security policy reforms affect household behavior and macroeconomic outcomes. Dynamic general equilibrium overlapping-generations models with incomplete markets and heterogeneous households are used in the analysis. This dissertation consists of three studies, the third of which is co-authored with Shinichi Nishiyama.

Chapter 1 extends a large-scale Bewley-type overlapping generations model with heterogeneous households, calibrated to the 2022 Japanese economy, to analyze the effects of pension reform proposals on individual households, the aggregate economy, and social welfare. By incorporating institutional features such as the reduction of pension benefits for working retirees, as well as both the intensive and extensive margins of labor supply, the model better captures households' optimizing behavior in response to pension reforms. The results suggest that cutting pension benefits by 15% and raising the eligibility age by five years would both increase private wealth, thus capital stock, labor supply, and newborn households' welfare in the long run, provided that pension and consumption tax rates are adjusted to balance the government budget. Removing pension adjustments for working retirees would also promote labor supply, however decrease capital stock and newborn households' welfare slightly. It would also lead to a small increase in households' pension contribution burdens, which suggests that the Old-Age Pension System for Working Retirees has played a useful role in containing expenditure pressures of the pension sector, although it reduces labor supply among older households.

Chapter 2 extends a large-scale Bewley type overlapping generations model by adding sex and marital status to examine how tax and pension policies in Japan affect female labor supply and the macroeconomy. The model faithfully replicates Japan's institutional framework, including detailed implementations of the Category 3 insured person system and the (special) exemption for spouses under the income taxation. Three pension reforms are evaluated: abolishing the Category 3 insured person system, eliminating the (special) exemption for spouses, and the introduction of the family quotient system in income taxation. The results suggest that the Category 3 insured person system discourages women's labor supply, and removing it may lead to improvements in the overall economy. Eliminating the (special) exemption for spouses would increase female labor supply but worsen overall economic outcomes. The introduction of the family quotient system would negatively effect women's labor supply while worsen overall economic outcomes.

Chapter 3 extends the Barro–Becker model of endogenous fertility and intergenerational transfers by incorporating human capital investment in children and life-cycle savings decisions. Furthermore, this study transforms the model to a Bewley-type heterogeneous agent OLG-dynasty model by incorporating

idiosyncratic shocks to labor income, human capital, and wealth. Calibrating the model to the U.S. economy, this study quantitatively analyzes the potential impacts of child-related policies—such as child allowances, education subsidies, and estate tax cuts—on heterogeneous households’ decision-making, economic growth, and income and wealth distribution. Policy experiments in this study suggest that child allowances would increase the fertility rate in the short run but decrease human capital investment and wealth accumulation in the long run. Therefore, the long run effects of child allowances on the fertility rate would likely be small. The effects of child-related policies on income and asset inequality and intergenerational mobility is generally limited. One reason for this is that optimal decisions by altruistic households involve intergenerational risk sharing.

Acknowledgment

I would like to express my deepest gratitude to my main supervisor, Professor Shinichi Nishiyama, for his continuous guidance, invaluable advice, and encouragement throughout my graduate studies. I would also like to express my sincere appreciation to my supervisors, Professor Akihisa Shibata and Professor Shuhei Takahashi, for providing thoughtful comments and guidance throughout my academic development.

I am grateful to Professor Kohei Iwasaki and Professor Kazushige Matsuda for their valuable comments and suggestions. Parts of this dissertation benefited from presentations at the 2025 Kyoto Spring Workshop on Macroeconomics, the Spring 2025 Midwest Macroeconomics Meeting, the 2025 World Congress of the Econometric Society, and the 2025 meetings of the Japanese Economic Association. I sincerely thank the participants at these conferences for their constructive feedback.

I also acknowledge with gratitude that part of this dissertation was supported by JST SPRING (Grant Number JPMJSP2110). I am also grateful for the supportive environment provided by my current workplace, Nagasaki University, which enabled me to focus on completing this dissertation during the final stage of my doctoral program. Finally, I would like to thank my family and friends for their support and understanding throughout my studies.

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Chapter 1

Analyzing Japan's Public Pension Reform in an OLG Economy with Heterogeneous Households

1.1 Introduction

Low birth rates and aging populations are becoming a major problem for many countries. Of particular concern is the decline in the working-age population, which directly affects public pension systems. Many countries have adopted pay-as-you-go public pension systems, in which the contributions collected from the working-age population are used to pay pension benefits to the elderly generation. However, due to declining birth rates, the number of households in the younger generation is decreasing, and the total amount of contributions that can be collected is decreasing. At the same time, the total amount of pension benefits paid is increasing due to growing numbers of older-generation households. This has led to an imbalance in pension finances, and if no corrective measures are taken, there is a risk that it will lead to the eventual collapse of the public pension system.

Japan is one of the countries facing the challenge. According to the Ministry of Health, Labour and Welfare, Japan's total fertility rate in 2023 was 1.20, the lowest ever recorded. Data from the Statistics Bureau of Japan shows that in the same year, the proportion of people aged 65 and older reached a historic high of 29.1% of the total population. In response to these pressing demographic shifts and to enhance the sustainability of the pension system, the Japanese government gradually increased public pension contribution rates from 2004 to 2017 and is currently in the process of raising the pension eligibility age from 60 to 65. Further reforms are under consideration, including potentially extending the eligibility age beyond 65 and expanding the coverage of employees' pensions to include more contributing households.

The present study extends a Bewley-type incomplete market overlapping generations model with heterogeneous households by incorporating Japan's detailed tax and public pension system, together with both the intensive and extensive margins of labor supply. It analyzes the potential impact of Japan's public pension reform proposals on individual households, the aggregate economy, and social welfare by

computing both the steady-state equilibrium and the transition path.

One of the main contributions of the present study is that it incorporates Japan's two-tier public pension system, which consists of the National Pension System, the Employees' Pension Insurance System, and the Old-Age Pension System for Working Retirees into the model. In the latter system, pension benefits for individuals aged 65 and older are reduced according to an earnings test when they continue working, which can influence their optimization problem. The second contribution is that the model incorporates both the intensive and extensive margins of labor supply. Households choose whether and how much to work based on their lifetime utility, which generates a more realistic pattern of labor force participation and, through the pension system modeled to reflect actual institutional rules, affects the future pension benefits households receive. These features allow the model to better capture the potential effects of public pension reforms on households' optimizing behavior, particularly in labor supply decisions.

Three pension reform policies are analyzed in this study: a 15% proportional reduction in public pension benefits, a five-year increase in the pension eligibility age, and the removal of pension adjustments and suspensions in the Old-Age Pension System for Working Retirees. While increasing pension contributions is often considered a possible reform option, Japan has already raised the contribution rate significantly from 13.58% in 2004 to 18.30% in a phased manner. Given this substantial increase, this study argues that further raising the contribution rate would put too much pressure on the working generation and should be avoided. Therefore, this study excludes further examination of this policy option. In the model economy, pension contribution rates are adjusted to balance the pension budget, and the consumption tax rate is adjusted to balance the rest of the government budget, with the aim of maintaining fiscal balance under each policy reform.

The main findings of the present study are as follows. A 15% reduction in public pension benefits would increase capital stock by 6.1%, labor supply by 0.7%, and total output by 2.9% in the long run. Households' pension contribution burden would decline by up to 15.6%, and future households would be better off on average by 5.0% in the consumption equivalence measure. Raising the pension eligibility age by five years would lead to a larger increase in labor supply of 1.9% but a smaller impact on capital accumulation, which would lead to a 3.1% increase in capital stock. Households' pension contribution burden would decline by up to 15.2%, and future households would be better off on average by 2.8% in the consumption equivalence measure. The third policy change, the removal of pension adjustments and suspensions in the Old-Age Pension System for Working Retirees, would lead to a relatively smaller overall impact. Labor supply rises by 0.4%, private consumption rises by 0.4%, while household welfare declines by 0.1% in the long run.

These results suggest that reducing pension benefits by 15% is the most effective pension policy reform for long-run economic growth and fiscal sustainability, while raising the eligibility age generates a

larger increase in labor supply. However, it is politically challenging as it permanently reduces pension benefits for all households. Raising the eligibility age is more acceptable to households as it extends the contribution-paid period, resulting in higher future pension benefits. However, considering the health condition and overall well-being of elderly households, raising the eligibility age is unlikely to serve as a sustainable solution. As a more balanced approach, rather than raising the pension eligibility age, policies should focus on creating an environment where elderly households that are willing and able to work can do so. Moreover, when balancing pension finances, further increases in pension contribution rates to prevent benefit cuts may not always be necessary. Removing pension adjustments and suspensions in the Old-Age Pension System for Working Retirees has a relatively smaller impact than the other two policy reforms, but leads to a clear increase in labor supply. This suggests that pension adjustments and suspensions create distortions by discouraging elderly households from supplying more labor. However, removing the adjustments would increase households' pension contribution burden and reduce their welfare.

A large number of previous studies have quantitatively analyzed the potential impact of pension reform proposals. Some of these studies use a large-scale but deterministic overlapping generations model with representative households. Auerbach and Kotlikoff (1987) examine the effects of cuts in social security benefits and increases in the retirement age in the US economy. Their study is widely regarded as the pioneering work in applying a large-scale OLG model to analyze the effects of policy reforms. Fehr (2000) accounts for rising life expectancy and declining fertility in Germany to examine the impacts of pension reforms, such as an increase in the retirement age and a gradual decline in pension level. de la Croix et al. (2013) analyze the impact of eliminating early retirement benefits on elderly workers' labor supply and pension costs in the European nations. Vogel et al. (2017) construct an open-economy model of three European nations: France, Germany and Italy, and they evaluate the effectiveness of several policy adjustments, such as raising the retirement age, in mitigating the economic and welfare impacts of population aging. Heer et al. (2020) identify the threshold dependency ratios of the US and 14 European countries at which tax revenues can no longer sustain the planned level of transfers to retirees.

Other studies use a large-scale Bewley-type OLG model with heterogeneous households. De Nardi et al. (1999) evaluate the impacts on the macroeconomy and welfare of fiscal reforms, including an increase in the retirement age. Nishiyama and Smetters (2007) examine the effects of partial social security privatization on efficiency over the transition path, using a notion of lump-sum redistribution authority. Nishiyama (2015) incorporates the population projection by the US Social Security Administration and analyzes the effects of an aging population. Díaz-Giménez and Díaz-Saavedra (2009) examine the impact of delaying both the first and normal retirement ages by three years on pension costs and welfare in the Spanish economy. İmrohoroglu and Kitao (2009) explore the role of the intertemporal elasticity of substitution (IES) in shaping the quantitative results of social security reform. İmrohoroglu and Kitao

(2012) incorporate health and medical expenditure risks into their model and analyze the behavior of claiming social security benefit before the normal retirement age. Kitao (2014) examines the effects of various policy options on the macroeconomy and welfare under demographic transitions.

Notable recent studies applying large-scale Bewley-type OLG models to Japan include Kitao (2015, 2017) and Hagiwara (2025). Kitao (2015) incorporates Japan's two-tier public pension system and the integrated government budget to the model, and analyzes several pension reform scenarios that include reducing benefits by 20% and raising the retirement age to 70, to evaluate how these policies can mitigate the contraction of a shrinking economy caused by demographic aging. Kitao (2017) follows the modeling structure of Kitao (2015), analyzes a 20% reduction in the replacement rate and a three-year increase in the retirement age, focusing on how the timing of implementation affects macroeconomic outcomes and social welfare. Hagiwara (2025) analyzes the impact of revising the Old-Age Pension System for Working Retirees on elderly employment, the macroeconomy, pension finance, and welfare.

While these studies have advanced the quantitative analysis of Japan's pension system, each of them omits important institutional features. Kitao (2015, 2017) do not incorporate the Old-Age Pension System for Working Retirees, and therefore cannot capture the reduction in pension benefits that working retirees face under the earnings test. As a result, the model may overestimate the labor supply of households beyond pensionable age. Hagiwara (2025) focuses on the earnings test, but abstracts from the calculation of average historical earnings for pension benefit determination. The present study models pension benefits as a function of both the contribution-paid period and the household's average historical earnings, providing a more realistic representation of the current Japanese system.

The rest of this study is structured as follows. Section 1.2 introduces the large-scale heterogeneous-agent OLG model employed in this study and defines the recursive competitive equilibrium of the model economy. Section 1.3 describes the model calibration and outlines the baseline economy. Section 1.4 presents the results of three policy experiments and analyzes their effects on the model economy. Section 1.5 concludes the study. Appendix 1.A considers a small open economy version of the model.

1.2 The Model

The model used in this study is a standard Bewley-type incomplete market overlapping-generations (OLG) model (e.g., Nishiyama and Smetters, 2014), extended with the Japanese tax system and public pension system. The model economy consists of a large number of heterogeneous and OLG households, a representative firm operating in a perfectly competitive market, and a government with commitment technology. The economy is on a balanced-growth path with a productivity growth rate μ and a population growth rate ν .

1.2.1 The Households

Households are heterogeneous with respect to their wealth, $a \in A = [0, a_{max}]$, average historical earnings, $b \in B = [0, b_{max}]$, working ability, $e \in E = [0, e_{max}]$, and age, $i = 1, \dots, I$. Households enter the economy and begin working at $i = 1$, which corresponds to age 20 in the real world. Every year, households make a stochastic labor supply decision: the likelihood of working is higher when the expected lifetime utility from working exceeds that from non-participation. Households start receiving old-age pension benefits at age $I_1 = 46$ (corresponds to a real age of 65), and may live up to age $I = 80$ (corresponds to a real age of 99).

This model incorporates the Old-Age Pension System for Working Retirees, which allows households aged 65 and older to continue working while receiving pension benefits. The Japanese public pension system consists of two components: the National Pension (NP) and the Employees' Pension Insurance (EPI). The NP is a universal public pension insurance program, with contributions typically starting at age 20 and covering up to 480 months. The EPI, on the other hand, is designed for eligible employees and requires enrollment for those who meet its criteria. In the Japanese economy, enrollment in the EPI entitles individuals to receive benefits from both the NP and the EPI in retirement. Households not enrolled in the EPI must join the NP and pay a fixed NP contribution. Exemptions or deferrals are available for low-income households.

For simplicity, the model does not include NP contributions. All working-age households are assumed to be enrolled in the EPI, which means that they pay only EPI contributions and are eligible for benefits from both. Households aged 65 and older who continue working and meet the eligibility requirements for the EPI system are required to remain enrolled and continue paying pension contributions until the age of 70. During this period, their contribution-paid periods and average historical earnings are updated annually. EPI benefits may be adjusted or suspended under the Old-Age Pension System for Working Retirees, depending on the combined amount of employment income and EPI benefits received by each household. Upon reaching age 70 ($I_2 = 51$), households are assumed to cease participation in the EPI system. They are no longer required to pay pension contributions, and their contribution-paid periods and average historical earnings stop being updated. However, their EPI benefits can still be adjusted under the Old-Age Pension System for Working Retirees if they continue working beyond age 70.

The individual state of a household is represented by $\mathbf{s} = (i, e, a, b)$, while the aggregate state of the economy in period t is represented by $\mathbf{S}_t = (x(\mathbf{s}), W_{G,t})$. Here, $x(\mathbf{s})$ is the population density function of households, and $W_{G,t}$ is the government's net worth at the beginning of period t .

The government's policy schedule at the beginning of period t is

$$\Psi_t = \{C_{G,s}, tr_{LS,s}, \tau_{c,s}, \tau_{P,s}(\cdot), \tau_{H,s}(\cdot), tr_{SS,s}(\cdot), \tau_{I,s}(\cdot), q_s, W_{G,s+1}\}_{s=t}^{\infty},$$

where $C_{G,t}$ is the government's general expenditure, $tr_{LS,t}$ is lump-sum transfers to each household, and $\tau_{c,t}$ is a flat tax on consumption. $\tau_{P,t}(\cdot)$ and $\tau_{H,t}(\cdot)$ are functions that calculate contributions to EPI and Health Insurance (HI), respectively. $tr_{SS,t}(\cdot)$ is the function that calculates pension benefits households receive after reaching pension eligibility age. $\tau_{I,t}(\cdot)$ is the function that calculates income tax collected from households. q_t is the uniform transfer received by each households, which in this model is accidental bequests collected and distributed by the government. $W_{G,t+1}$ is the government's net worth at the beginning of period $t + 1$.

1.2.1.1 The Optimization Problem

Let $v(s, \mathbf{S}_t; \Psi_t)$ be the household's value function at the beginning of period t . The optimization problem is then:

$$v(s, \mathbf{S}_t; \Psi_t) = \max_{c, h, a'} \left\{ u(c, h) + \tilde{\beta} \phi_i E[v(s', \mathbf{S}_{t+1}; \Psi_{t+1}) | s] \right\} \quad (1.1)$$

subject to the constraints for the decision variables,

$$c > 0, \quad 0 \leq h < 1, \quad a' \geq 0,$$

and the laws of motion of the individual state variables,

$$s' = (i + 1, e', a', b'), \quad (1.2)$$

$$a' = \frac{1}{1 + \mu} \left[(1 + r_t) a + w_t e h + tr_{SS,t}(i, b) - \tau_{P,t}(i, w_t e h) - \tau_{H,t}(w_t e h, tr_{SS,t}(\cdot)) - \tau_{I,t}(r_t a, w_t e h, tr_{SS,t}(\cdot)) + tr_{LS,t} + q_t - (1 + \tau_{c,t}) c \right], \quad (1.3)$$

$$b' = \mathbf{1}_{\{i \geq I_2\}} b + \mathbf{1}_{\{i < I_2\}} \frac{1}{i} \left[(i - 1) b + \min \left\{ w_t e h - \frac{1}{2} [\tau_{P,t}(\cdot) + \tau_{H,t}(\cdot)], \vartheta_{1max} \right\} \right], \quad (1.4)$$

where c is household consumption, h is household working hours, $u(\cdot)$ is the household's period utility function,

$$u(c, h) = \frac{\left[c^\theta (1 - h)^{1-\theta} \right]^{1-\gamma}}{1 - \gamma}, \quad (1.5)$$

$\tilde{\beta}$ is a growth-adjusted discount factor defined as $\tilde{\beta} = \beta (1 - \mu)^{\theta(1-\gamma)}$, μ is the productivity growth rate, and ϕ_i is the conditional survival probability of a household at the end of age i given that the household is alive at the beginning of age i .

In addition to the intensive margin of labor supply, this model also incorporates the extensive margin. The optimization problem described above applies to employed households. These households choose their working hours, including the option of taking a temporary leave from work in the current period ($h = 0$). In addition to this decision, households also evaluate the lifetime utility they would obtain if they exit employment and spend the current period out of the labor market. By comparing these two lifetime utilities, the share of households that remain in the labor market is determined endogenously for each relevant state. The details of this mechanism are explained in Section 1.3.4 For all possible states s , the state-specific outcome is defined as the weighted average of the two types of households.

In this model, household income is categorized into labor income, interest income, and pension income. Income tax $\tau_{I,t}(\cdot)$ is calculated separately for each category, based on real-world policies. Interest income is taxed separately using a flat tax rate, while pension income is taxed using a combination of progressive deductions and a flat tax rate, and labor income is taxed progressively. The calculations also account for employment income deduction, basic exemption, and the resident tax.

In equation (1.4), I_2 is used because average historical earnings are updated until age 70 ($i = I_2$). ϑ_{1max} is the upper limit of the standard remuneration used to calculate benefits under the EPI system. In the real economy, social insurance contributions are typically shared between employers and employees. Therefore, in this model, employer-paid insurance contributions are considered part of employees' labor income, effectively making households bear the full cost of these contributions.

In reality, the average historical earnings used to calculate EPI benefits are based on actual salaries, which do not include the employer's share of social insurance contributions. To reflect this, in equation (1.4), labor income $w_t e h$ is adjusted by subtracting half of the total social insurance contributions, ensuring consistency with the real-world calculation of b . The details of the social security system and tax system are explained in Section 1.3.5 and 1.3.6.

1.2.1.2 The Household's Decision Rules

To solve the household's optimization problem for all possible states s , this study first computes the household's decision rules $c(s, \mathbf{S}_t; \Psi_t)$ and $h(s, \mathbf{S}_t; \Psi_t)$ numerically for both types of households described above. For households that choose to exit employment in the current period, $h(\cdot)$ is fixed at 0.

The remaining decision rules a' and b' are then calculated using the obtained values of c and h ,

$$\begin{aligned}
a'(\mathbf{s}, \mathbf{S}_t; \Psi_t) = & \frac{1}{1 + \mu} \{ (1 + r_t) a + w_t eh(\mathbf{s}, \mathbf{S}_t; \Psi_t) + tr_{SS,t}(i, b) \\
& - \tau_{P,t}(i, w_t eh(\mathbf{s}, \mathbf{S}_t; \Psi_t)) - \tau_{H,t}(w_t eh(\mathbf{s}, \mathbf{S}_t; \Psi_t), tr_{SS,t}(\cdot)) \\
& - \tau_{I,t}(r_t a, w_t eh(\mathbf{s}, \mathbf{S}_t; \Psi_t), tr_{SS,t}(\cdot)) + tr_{LS,t} + q_t \\
& - (1 + \tau_{c,t}) c(\mathbf{s}, \mathbf{S}_t; \Psi_t) \},
\end{aligned} \tag{1.6}$$

$$\begin{aligned}
b'(\mathbf{s}, \mathbf{S}_t; \Psi_t) = & \mathbf{1}_{\{i \geq I_2\}} b + \mathbf{1}_{\{i < I_2\}} \frac{1}{i} \left[(i - 1) b + \min \left\{ w_t eh(\mathbf{s}, \mathbf{S}_t; \Psi_t) \right. \right. \\
& \left. \left. - \frac{1}{2} [\tau_{P,t}(i, w_t eh(\mathbf{s}, \mathbf{S}_t; \Psi_t)) + \tau_{H,t}(w_t eh(\mathbf{s}, \mathbf{S}_t; \Psi_t), tr_{SS,t}(\cdot))] , \vartheta_{1max} \right\} \right].
\end{aligned} \tag{1.7}$$

1.2.1.3 The Distribution of Households

Let $x_t(\mathbf{s})$ be the population density function of households at period t , and $X_t(\mathbf{s})$ be the cumulative distribution function. Households enter the economy at age 20 with no assets or earning history, thus $a = b = 0$. However, each household has different working ability e . The total population of 20-year-old households is normalized to unity,

$$\int_{E \times A \times B} dX_t(1, e, a, b) = \int_E dX_t(1, e, 0, 0) = 1. \tag{1.8}$$

Given the population growth rate ν and the conditional survival probability ϕ_i , the law of motion for the growth-adjusted population distribution is

$$x_{t+1}(\mathbf{s}') = \frac{\phi_i}{1 + \nu} \int_{E \times A \times B} \mathbf{1}_{\{a' = a'(\mathbf{s}, \mathbf{S}_t; \Psi_t), b' = b'(\mathbf{s}, \mathbf{S}_t; \Psi_t)\}} \pi(e' | e) dX_t(\mathbf{s}), \tag{1.9}$$

where $\pi(e' | e)$ is the transition probability density function of working ability e from age i to age $i + 1$.

1.2.1.4 The Measure of Welfare

This study employs the veil of ignorance welfare measure to assess the impact of policies on household welfare. This measure represents the welfare effects of a policy as the percentage change in baseline consumption path that would generate the same change in household utility as the policy itself. By translating utility changes into equivalent consumption changes, this measure provides an intuitive way to understand the welfare impact of policies.

Let $\lambda_{i,1}$ and $\lambda_{1,t}$ be the percentage change in baseline consumption path that would generate the same expected lifetime utility change as the policy itself. $\lambda_{i,1}$ applies to households of each age when the policy is implemented, while $\lambda_{1,t}$ applies to newborn households ($i = 1$) entering the economy

during the transition. Their functional forms are

$$\lambda_{i,1} = \left[\left(\frac{E\bar{v}(\mathbf{s}_i, \mathbf{S}_1; \Psi_1)}{E\bar{v}(\mathbf{s}_i, \mathbf{S}_0; \Psi_0)} \right)^{\frac{1}{\theta(1-\gamma)}} - 1 \right] \times 100, \quad (1.10)$$

$$\lambda_{1,t} = \left[\left(\frac{E\bar{v}(\mathbf{s}_1, \mathbf{S}_t; \Psi_t)}{E\bar{v}(\mathbf{s}_1, \mathbf{S}_0; \Psi_0)} \right)^{\frac{1}{\theta(1-\gamma)}} - 1 \right] \times 100, \quad (1.11)$$

where $\mathbf{s}_1 = (1, e, a, b)$ is the state of a newborn household (age $i = 1$) in the economy, $\bar{v}(\mathbf{s}_1, \mathbf{S}_t; \Psi_t)$ is the lifetime utility of a specific $i = 1$ household at period t after the policy change, weighted over the two employment-status types relevant for the extensive margin. $E\bar{v}(\mathbf{s}_1, \mathbf{S}_t; \Psi_t)$ is the expected lifetime utility of an $i = 1$ household in period t , and it is defined as

$$E\bar{v}(\mathbf{s}_1, \mathbf{S}_t; \Psi_t) = \int_{E \times A \times B} \bar{v}(\mathbf{s}_1, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}_1). \quad (1.12)$$

1.2.1.5 The Supply of Capital and Labor

This study assumes a closed economy, where the capital stock K_t is the sum of total private wealth $W_{P,t}$ and the government's net worth $W_{G,t}$. The total labor supply is the aggregate efficient labor provided by all households. Their functional forms are

$$W_{P,t} = \sum_{i=1}^I \int_{E \times A \times B} a dX_t(\mathbf{s}), \quad K_t = W_{P,t} + W_{G,t}, \quad (1.13)$$

$$L_t = \sum_{i=1}^I \int_{E \times A \times B} e \bar{h}(\mathbf{s}, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}), \quad (1.14)$$

where $\bar{h}(\cdot)$ is the weighted working hours over the two household types.

1.2.2 The Representative Firm

The representative firm employs capital and labor as production factors and operates according to a Cobb-Douglas production function

$$Y_t = A \tilde{K}_t^\alpha \tilde{L}_t^{1-\alpha}, \quad (1.15)$$

where A is the total factor productivity, and $\tilde{K}$ and $\tilde{L}$ are the capital input and labor input in efficiency units chosen by the firm. Letting δ be the depreciation rate, the firm's profit maximizing conditions are

$$\alpha A \left(\frac{\tilde{K}_t}{\tilde{L}_t} \right)^{\alpha-1} - (r_t + \delta) = 0, \quad (1.16)$$

$$(1 - \alpha) A \left(\frac{\tilde{K}_t}{\tilde{L}_t} \right)^{\alpha} - w_t = 0. \quad (1.17)$$

Given the assumption of a closed economy, the factor markets clear such that $\tilde{K}_t = K_t$ and $\tilde{L}_t = L_t$. Gross domestic product, which is here identical to gross national product Y_t , is

$$Y_t = F(K_t, L_t) = (r_t + \delta)(W_{P,t} + W_{G,t}) + w_t L_t. \quad (1.18)$$

1.2.3 The Government

1.2.3.1 Government General Budget

This section defines the government revenue and expenditure in this model. Weighted averages are first taken over households in the two employment-status types, and then aggregated over all states. The Public Pension System contribution revenue $T_{P,t}$ is

$$T_{P,t}(\bar{\tau}_{p,t}) = \sum_{i=1}^I \int_{E \times A \times B} \tau_{P,t}(i, w_t eh(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t); \bar{\tau}_{p,t}) dX_t(\mathbf{s}), \quad (1.19)$$

where $\bar{\tau}_{p,t}$ is the contribution rate of Employees' Pension Insurance. The Public Pension System benefit expenditure $TR_{SS,t}$ is

$$TR_{SS,t}(\psi_t) = \sum_{i=I_1}^I \int_{E \times A \times B} tr_{SS,t}(i, b; \psi_t) dX_t(\mathbf{s}), \quad (1.20)$$

where ψ_t is the pension sector balance factor, adjusted to ensure that the pension sector budget is always balanced.

The Healthcare Insurance contribution revenue $T_{H,t}$ is

$$T_{H,t}(\bar{\tau}_{h,t}) = \sum_{i=1}^I \int_{E \times A \times B} \tau_{H,t}(w_t eh(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t), tr_{SS,t}(\cdot); \bar{\tau}_{h,t}) dX_t(\mathbf{s}), \quad (1.21)$$

where $\bar{\tau}_{h,t}$ is the contribution rate of Health Insurance. The government's income tax revenue $T_{I,t}$ is

$$T_{I,t}(\varphi_t, \bar{\tau}_{k,t}, \bar{\tau}_{ss,t}, \bar{\tau}_{r1,t}, \bar{\tau}_{r2,t}) = \sum_{i=1}^I \int_{E \times A \times B} \tau_{I,t}(r_t a, w_t eh(\mathbf{s}, \mathbf{S}_t; \Psi_t), tr_{SS,t}(\cdot); \varphi_t, \bar{\tau}_{k,t}, \bar{\tau}_{ss,t}, \bar{\tau}_{r1,t}, \bar{\tau}_{r2,t}) dX_t(\mathbf{s}), \quad (1.22)$$

where φ_t is a parameter of labor income tax function, representing the highest progressive tax rate faced by households. $\bar{\tau}_{k,t}$ is the interest income tax rate, $\bar{\tau}_{ss,t}$ is the pension income tax rate, $\bar{\tau}_{r1,t}$ and $\bar{\tau}_{r2,t}$ are the resident tax rates. The consumption tax revenue $T_{C,t}$ is

$$T_{C,t}(\tau_{c,t}) = \tau_{c,t} \sum_{i=1}^I \int_{E \times A \times B} c(\mathbf{s}, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}), \quad (1.23)$$

and the lump-sum transfer spending $TR_{LS,t}$ is

$$TR_{LS,t}(tr_{LS,t}) = \sum_{i=1}^I \int_{E \times A \times B} tr_{LS,t} dX_t(\mathbf{s}). \quad (1.24)$$

The law of motion of the government's net worth $W_{G,t+1}$ is

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} \left[(1+r_t) W_{G,t} + T_{I,t} + T_{C,t} - TR_{LS,t} + T_{P,t} - TR_{SS,t} + T_{H,t} - C_{G,t} \right]. \quad (1.25)$$

1.2.3.2 Accidental Bequests

To simplify the analysis, it is assumed that when a household with assets deceases, those assets are collected by the government and distributed as a lump sum transfer to all households within the same period. Let Q_t be the total accidental bequests collected by the government in period t , $\bar{a}'(\mathbf{s}, \mathbf{S}_t; \Psi_t)$ be the weighted average of end-of-period wealth for households in state $\mathbf{s}$, then

$$Q_t = \sum_{i=1}^I \int_{E \times A \times B} (1 - \phi_i) (1 + \mu) \bar{a}'(\mathbf{s}, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}). \quad (1.26)$$

The accidental bequest q_t each household receives is

$$q_t = \left(\sum_{i=1}^I \int_{E \times A \times B} dX_t(\mathbf{s}) \right)^{-1} Q_t. \quad (1.27)$$

1.2.4 Recursive Competitive Equilibrium

The recursive competitive equilibrium in this economy is defined as follows.

Definition: Recursive Competitive Equilibrium. Let $\mathbf{s} = (i, e, a, b)$ be the individual state of households, $\mathbf{S}_t = (x(s), W_{G,t})$ be the aggregate state of the economy, and Ψ_t be the government policy schedule at the beginning of period t ,

$$\Psi_t = \{C_{G,s}, tr_{LS,s}, \tau_{c,s}, \tau_{P,s}(\cdot), \tau_{H,s}(\cdot), tr_{SS,s}(\cdot), \tau_{I,s}(\cdot), q_s, W_{G,s+1}\}_{s=t}^{\infty}.$$

A time series of the factor prices and government policy variables,

$$\Omega_t = \{r_s, w_s, \varphi_s, \tau_{c,s}, \bar{\tau}_{k,s}, \bar{\tau}_{ss,s}, \bar{\tau}_{r1,t}, \bar{\tau}_{r2,t}, \bar{\tau}_{p,s}, \bar{\tau}_{h,s}, tr_{LS,s}, \psi_s, q_s, C_{G,s}, W_{G,s+1}\}_{s=t}^{\infty},$$

the value functions of households, $\{v(\mathbf{s}, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty}$, the distribution of households, $\{x_s(\mathbf{s})\}_{s=t}^{\infty}$, and the decision rules of households

$$d(\mathbf{s}, \mathbf{S}_s; \Psi_s) = \{c(\mathbf{s}, \mathbf{S}_s; \Psi_s), h(\mathbf{s}, \mathbf{S}_s; \Psi_s), a'(\mathbf{s}, \mathbf{S}_s; \Psi_s), b'(\mathbf{s}, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty},$$

are in a recursive competitive equilibrium if, for all $s = t, \dots, \infty$,

1. each household chooses $d(\mathbf{s}, \mathbf{S}_s; \Psi_s)$ to maximize her lifetime utility, taking factor prices and the government policy as given;
2. each firm chooses its factor inputs $(\tilde{K}_t, \tilde{L}_t)$ and maximizes its own profit;
3. factor markets clear under the factor prices (r_t, w_t) ;
4. the government budget constraint is satisfied;
5. by Walras's Law, the aggregate resource constraint is satisfied.

The economy is in a steady-state equilibrium, thus on the balanced-growth path, if $\mathbf{S}_s = \mathbf{S}_{s+1}$ and $\Psi_s = \Psi_{s+1}$ for all $s = t, \dots, \infty$.

1.3 Calibration

This study uses statistical data from the Japanese economy in 2022 to construct the baseline economy for policy experiments. In reality, Japan is still in the process of raising the pension eligibility age from 60 to 65. According to the plan, the pension eligibility age for men will rise to 65 by 2025, and for women by 2030. However, since the analysis includes steady-state equilibrium, the baseline economy assumes that the pension eligibility age has already been raised to 65. Based on the baseline, the policy experiment on raising the eligibility age examines the impact of raising the pension eligibility age from

Table 1.1: Parameters and Values

Parameter		Value	Comment
Maximum Possible Age	I	80	Corresponds to a real-life age of 99
Pension Eligibility Age	I_1	46	
Contribution Free Age	I_2	51	The age at which pension contribution are no longer required
Productivity Growth Rate	μ	0.0070	Average change rate in real GDP per capita in 2003-2022
Population Growth Rate	ν	-0.0011	Average change rate in 2003-2022
Household Survival Rates	ϕ_i		Based on the 23rd Life Tables 2020
Labor Participation Responsiveness	λ	2.0000	Match the age-profile of employment age
Share Parameter of Capital Stock	α	0.4124	SNA data
Discount Factor	β	0.9924	$K/Y = 3.5$ in the baseline
Coefficient of Relative Risk Aversion	γ	3.0000	
Depreciation Rate of Capital Stock	δ	0.0628	SNA data $r = 0.0550$ in the baseline
Share Parameter of Consumption	θ	0.6500	Frisch elasticity 0.5531 for the average worker
Benefit Multiplier of the EPI	κ	0.0055	
Autocorrelation Parameter of Log Income	ρ	0.9500	Income variance by 10-year age groups
Standard Deviation of Log Income Shocks	σ	0.2200	in 2022
Average Earning Ability	$\bar{e}_i$		Estimated by OLS with 2022 wage data
Full Amount of NP Benefit	np	0.0852	¥777,800 in 2022 ^a
Upper Limit of the Standard Remuneration y_P	ϑ_{1max}	1.3471	$12 \times ¥650,000 + 3 \times ¥1,500,000$ $= ¥12,300,000$ in 2022 ^a
Converted ϑ_{1max}	ϑ_{2max}	1.5380	
Annual Adjustment Amount for the Suspension of EPI Benefit	ϑ_{3max}	0.6571	¥6,000,000 in 2024 ^a
Converted Upper Limit of the Standard ^b Remuneration of the HI	ϑ_{4max}	2.7003	$12 \times ¥1,390,000 + ¥5,730,000$ $= ¥22,410,000$ in 2022 ^a
Income Tax Parameters: Tax Rate Limit	φ_t	0.4500	Maximum individual income tax rate
: Curvature	φ_1	1.0623	} Estimated by OLS
: Scale	φ_2	0.6579	
Consumption Tax Rate	$\tau_{c,t}$	0.1000	
Interest Income Tax Rate	$\bar{\tau}_{k,t}$	0.2020	$T_I/Y = 7.2\%$ in the Baseline
Pension Income Tax Rate	$\bar{\tau}_{ss,t}$	0.0500	
Resident Tax Rate 1	$\bar{\tau}_{r1,t}$	0.0500	Apply for interest income
Resident Tax Rate 2	$\bar{\tau}_{r2,t}$	0.1000	Apply for labor and pension income
Converted EPI Contribution Rate	$\bar{\tau}_{p,t}$	0.1603	18.3% in reality
Converted HI Contribution Rate ^b	$\bar{\tau}_{h,t}$	0.0876	10% average contribution rate
Government Consumption	$C_{G,t}$	10.8754	$C_G = T_C + T_I + T_H - TR_{LS}$ in the baseline
Lump-sum Transfers	$tr_{LS,t}$	0.0000	
Pension Sector Balance Factor	ψ_t	0.9878	
Total Factor Productivity	A	0.8153	$w = 1.0$ in the baseline
Accidental Bequests per Households	q_t	0.0285	

Note: ^a One unit of income in the model economy equals to 9,130,702 Japanese Yen.

^b Varies depending on the household's age and labor income.

65 to 70 on the macroeconomy and household welfare. Other settings are based on Japan's current system. The main parameters are summarized in Table 1.1.

1.3.1 Demographics

In Japan, individuals are required to join the public pension system upon turning 20. Accordingly, in this study, the age at which households enter the model ($i = 1$) corresponds to 20 years old in real life. Households are assumed to survive for a maximum of 80 periods ($I_{max} = 80$), which means that all survivors exit the model economy at the end of their 99th year. In the baseline economy, two key ages are considered: I_1 and I_2 . Households begin receiving pension benefits at $I_1 = 46$ (age 65), and stop paying pension contribution at $I_2 = 51$ (age 70).

Considering the impact of COVID-19 on GDP growth, the model sets the productivity growth rate as follows. Using real GDP data (at 2015 constant prices) from the Cabinet Office, Government of Japan's National Accounts, and population data from the Statistics Bureau of Japan, for 2000, 2003, 2019, and 2022, Japan's average annual growth rate of real GDP per capita is calculated over two periods: 2000 to 2019 and 2003 to 2022. The 2000 population figure is the confirmed number from the census, while data for 2003, 2019, and 2022 are Population Estimates as of October 1st. The calculations, based on a compound interest method, yield growth rates of 0.68% for the period 2000 to 2019, and 0.73% for the period 2003 to 2022. To mitigate the concentrated impact of COVID-19 on GDP, a productivity growth rate of 0.70% is chosen for the model. The population growth rate ν is estimated at -0.11% , based on population data from 2003 to 2022. This negative value for ν indicates Japan's population decline over the past two decades.

The conditional survival probability ϕ_i , which is the probability that a household aged i will reach age $i + 1$, is calculated using data from the 23rd Life Tables (2020) published by the Ministry of Health, Labour and Welfare of Japan. The Ministry provides two types of life tables: Complete Life Tables based on the Annual Vital Statistics and the Population Census, and Abridged Life Tables based on the Provisional Annual Vital Statistics and the Population Estimates. Complete Life Tables are published every five years, while Abridged Life Tables are estimated annually. To ensure statistical accuracy, this study uses the 2020 Complete Life Tables. The gender ratio at age 20 is set at 105:100, and a weighted average is applied to calculate survival probabilities for each age, which are then used as ϕ_i .

1.3.2 Preferences

In this study, the coefficient of relative risk aversion is set to $\gamma = 3.0$. The Frisch elasticity of labor supply to wage changes for the average worker (i.e., a worker supplying average working hours), is given by the formula

$$\frac{1 - \tilde{h}}{\tilde{h}} \frac{1 - \theta(1 - \gamma)}{\gamma} = 0.5531. \quad (1.28)$$

Here, $\tilde{h} = 0.5809$ is the average working hours of the entire working-age population. By setting the consumption share parameter in the Cobb-Douglas utility function to $\theta = 0.6500$, the model yields a Frisch elasticity of 0.5531. However, introducing the extensive margin of labor supply makes the definition of average working hours less straightforward. The average working hours only over employed households is 0.6238, which results in a Frisch elasticity of 0.4624. Both values are close to 0.5.

1.3.3 Production Technology

This study calibrates the capital share α using statistical data from the Generation of Income Account in the National Accounts of Japan for 2022 (calendar year). In these accounts, operating surplus and mixed income are recorded as separate items, which makes them easier to use in analysis.

However, mixed income corresponds to proprietors' income, which combines the labor and capital contributions of sole proprietors, making it challenging to separate these components. To address this, compensation of employees is first added to the net operating surplus, and the ratio of compensation of employees to the total value is calculated. This ratio is then applied to net mixed income (excluding consumption of fixed capital) to estimate the labor contribution of sole proprietors to production. As a result, the labor income ratio to national income including consumption of fixed capital for 2022 is calculated to be 0.5876, which is used as the labor share, $1 - \alpha$. The capital share α is consequently set to 0.4124.

The average value of the capital coefficient (fixed capital stock/GDP) for Japan was calculated as 3.5194 for the period 2003 to 2022, and as 3.4355 for the period 2000 to 2019. Based on these calculations, the discount factor β is set to 0.9924 in the baseline model to ensure that the capital-to-output ratio (K/Y) equals 3.5. When $\theta = 0.6500$ and $\gamma = 3.0$, the growth-adjusted discount factor is

$$\tilde{\beta} = \beta (1 + \mu)^{\theta(1-\gamma)} = 0.9835. \quad (1.29)$$

The ratio of consumption of fixed capital to fixed capital stock for 2022 was calculated as 0.0629. To ensure that the interest rate r equals 0.0550 in the baseline, the depreciation rate δ is finely adjusted to 0.0628,

$$r = \alpha A \left(\frac{K_t}{L_t} \right)^{\alpha-1} - \delta = \alpha \frac{Y}{K} - \delta = 0.0549, \quad (1.30)$$

$$\delta = \alpha \frac{Y_t}{K_t} - 0.0550 = \frac{0.4124}{3.5} - 0.0550 = 0.0628. \quad (1.31)$$

To ensure that the wage rate w in the baseline economy equals 1.0, the total factor productivity (TFP)

is set to 0.8153,

$$w = (1 - \alpha) A \left(\frac{K_t}{L_t} \right)^\alpha = (1 - \alpha)^{1-\alpha} A \left(\frac{K_t}{Y_t} \right)^\alpha = 1, \quad (1.32)$$

$$A = \left[(1 - 0.4124)^{(1-0.4124)} (3.5)^{0.4124} \right]^{-1} = 0.8153. \quad (1.33)$$

1.3.4 Working Ability

Working ability calibration is based on data from Table 1 of the 2022 Basic Survey on Wage Structure, published by the Ministry of Health, Labour and Welfare of Japan. This dataset provides information on contractual cash earnings, actual number of scheduled hours worked, actual number of overtime worked, and annual special cash earnings by five-year age groups, covering public and private establishments across all industries. The data include all enterprise sizes with over 10 employees, total employees (male and female), and total educational backgrounds.

First, the average annual labor income by age group is calculated using contractual cash earnings and annual special cash earnings provided in the table. Employer-paid insurance contributions are considered part of employees' labor income in this study, and these contributions are added to the average amounts. Next, average annual labor hours are approximated using actual number of scheduled hours worked and overtime worked. Based on these calculations, the hourly labor income by age group is derived.

Using the hourly labor income data by age group, an OLS regression with a cubic function of age is performed to estimate the average hourly labor income by age. One important point to note is that the actual pension eligibility age in 2022 fell between 60 and 64 years old. As a result, the average hourly labor income in the data may underestimate the working ability of individuals aged 60 to 64, as well as those aged 65 to 69. This is because post-retirement re-employment often comes with lower wage levels than pre-retirement employment. Given that the policy experiment raises the pension eligibility age to 70, interpolation is used to estimate the average hourly labor income for individuals aged 60 to 69 to reflect the potential working ability, instead of relying on the actual data. Additionally, in the dataset, employees aged 70 and older are grouped together. Here, this study uses the actual data for this group as a proxy for individuals aged 70 to 74. Figure 1.1 shows the data and the estimation results. According to the estimation, the average hourly labor income for households reaches its peak at age 52 and declines to 0 by age 81.

Then, using the original data from the 2022 Basic Survey on Wage Structure, this study calculates the average hourly labor income for workers aged 20 to 64, and uses it to standardize the OLS estimates. The result is this model's average working ability by age, with an average value of 1.0 for ages 20 to 64.

Household labor productivity can fluctuate due to various events and external shocks. Therefore, the

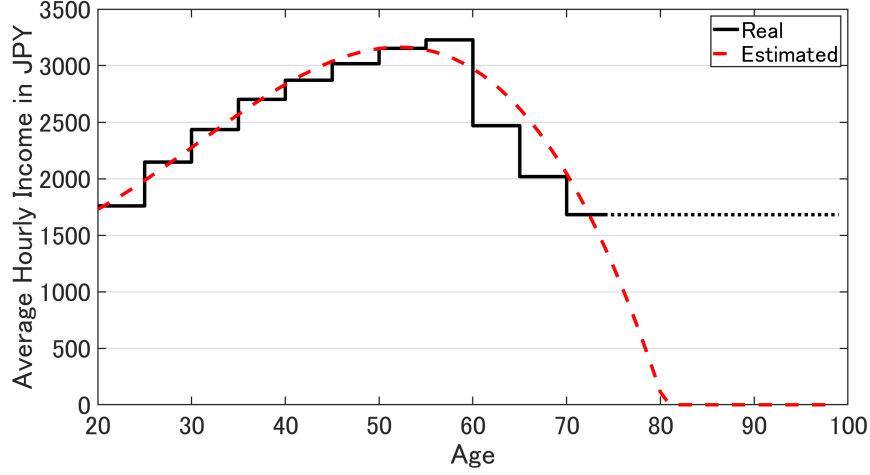


Figure 1.1: Average Hourly Income

working ability of an age i household in this model is assumed to satisfy:

$$\ln e_i = \ln \bar{e}_i + \ln z_i, \quad (1.34)$$

where i ranges from 1 to 61 as households' working ability becomes zero after $i = 61$ (age 80). Here, $\bar{e}_i$ is the previously estimated average working ability, and z_i is a persistent shock assumed to follow an AR(1) process. Specifically,

$$\ln z_i = \rho \ln z_{i-1} + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2), \quad (1.35)$$

where the autocorrelation parameter ρ is set to 0.95, and the standard deviation σ is 0.22. This study uses data from the 2022 Comprehensive Survey of Living Conditions, published by the Ministry of Health, Labour and Welfare of Japan, to calibrate the AR(1) process. The data includes non-executive employees aged 15 and above, categorized into 10-year age groups, with those under 29 grouped together and those aged 60 and above grouped together. The variance of logarithmic income is calculated for employees aged 20 to 59. These variances are then compared with the model's calculated variances of logarithmic labor income to adjust ρ and σ . Introducing the extensive margin reduces the variance of labor income among working households in the model because low-productivity households optimally choose not to work. The calibration, therefore, ensures that the model's variance of logarithmic labor income for working households falls within the empirical range, as shown in Figure 1.2.

The AR(1) process is approximated using a Markov chain. In this approach, $\ln z_i$ is discretized into five nodes, and the Markov transition matrix is constructed using Rouwenhorst's method. The unconditional probability distribution is given by $\pi_i = (0.0625, 0.2500, 0.3750, 0.2500, 0.0625)$ for $i =$

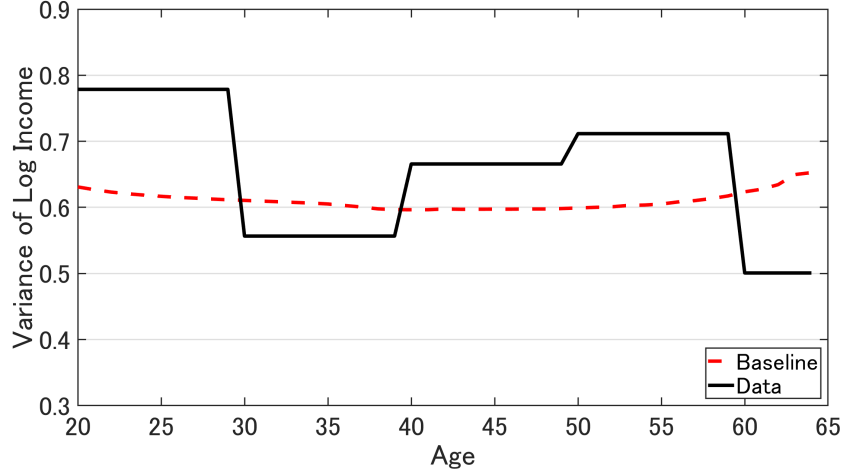


Figure 1.2: The Variance of Log Labor Income by Age

$1, \dots, 61$. The Markov transition matrix is

$$\Pi_i = \begin{pmatrix} 0.90369 & 0.02317 & 0.00059 & 0.00002 & 0.00000 \\ 0.09269 & 0.90547 & 0.04637 & 0.00178 & 0.00006 \\ 0.00357 & 0.06956 & 0.90607 & 0.06956 & 0.00357 \\ 0.00006 & 0.00178 & 0.04637 & 0.90547 & 0.09269 \\ 0.00000 & 0.00002 & 0.00059 & 0.02317 & 0.90369 \end{pmatrix}.$$

For households aged 65 and above, all elements above the diagonal of Π_i are set to zero. In other words, after reaching the pension eligibility age, households are assumed not to move to a higher working ability level. The remaining elements are rescaled to ensure that the total probability for each age is one. In addition, the lowest working ability level is set to zero for these households.

The fraction of households that choose to remain in the labor market is given by

$$\xi(\mathbf{s}, \mathbf{S}_t; \Psi_t) = \max \left[0.1, 1 - e^{-\lambda \max(0, v^h(\mathbf{s}, \mathbf{S}_t; \Psi_t) - v^0(\mathbf{s}, \mathbf{S}_t; \Psi_t))} \right], \quad (1.36)$$

where $v^h(\cdot)$ is the lifetime utility when the household remains in the labor market, and $v^0(\cdot)$ is the lifetime utility when the household exits employment for the current period. For numerical stability, the share of households with non-zero working ability that remain in the labor market $\xi(\cdot)$ is bounded below at 0.1. For households with working ability $e = 0$, $\xi(\cdot)$ is fixed at 0. λ controls the responsiveness of ξ and is set to $\lambda = 2.0$ so that the model matches the 2022 age-specific employment rates reported in the Labor Force Survey published by the Ministry of Health, Labour and Welfare of Japan. As shown in Figure 1.3, the calibrated profile closely matches the empirical rates for older households.

According to the 2021 Survey on the Redistribution of Income published by the Ministry of Health, Labour and Welfare of Japan, the Gini coefficient of pre-tax income among the surveyed households is

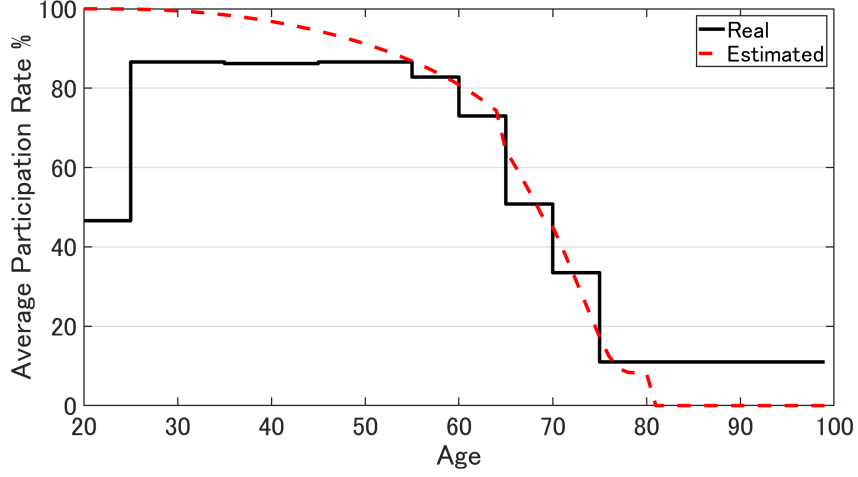


Figure 1.3: Average Participation Rate

0.5700. The baseline model generates a value of 0.6014 when calculated over all households, indicating that the model provides a reasonably close match to Japan's initial income inequality.

1.3.5 Taxation

In this model, income is divided into three categories: labor, interest, and pension income. A flat tax rate is applied to interest income, a combination of progressive deductions and a flat tax rate is used for pension income, and labor income tax follows the function proposed by Gouveia and Strauss (1994). In summary, the income tax function is

$$\begin{aligned}
 \tau_{I,t}(r_t a, w_t e h, tr_{SS,t}(\cdot)) &= \tilde{\tau}_{I,t}(r_t a, \tilde{y}, tr_{SS,t}(\cdot)) \\
 &= \tau_{K,t}(r_t a) + \varphi \left[\tilde{y} - (\tilde{y}^{-\varphi_1} + \varphi_2)^{-\frac{1}{\varphi_1}} \right] + \tau_{SS,t}(i, b) \\
 &\quad + \tau_{R,t}(r_t a, \tilde{y}, tr_{SS,t}(\cdot)),
 \end{aligned} \tag{1.37}$$

where

$$\tau_{K,t}(r_t a) = \bar{\tau}_{k,t} r_t a, \tag{1.38}$$

$$\tilde{y} = \max \{ w_t e h - d_1(w_t e h) - \tau_{H,t}(\cdot) - \tau_{P,t}(\cdot), 0 \}, \tag{1.39}$$

$$\tau_{SS,t}(i, b) = \tilde{\tau}_{SS,t}(tr_{SS,t}(\cdot)) = \bar{\tau}_{ss,t} [tr_{SS,t}(\cdot) - d_2(tr_{SS,t}(\cdot), \psi_t)], \tag{1.40}$$

$$\tau_{R,t}(r_t a, \tilde{y}, tr_{SS,t}(\cdot)) = \bar{\tau}_{r1,t} r_t a + \bar{\tau}_{r2,t} [tr_{SS,t}(\cdot) - d_2(\cdot) + \tilde{y}]. \tag{1.41}$$

The interest income tax rate is denoted by $\bar{\tau}_{k,t}$, $\tilde{y}$ is the taxable labor income, $d_1(\cdot)$ is the basic exemption and the employment income deduction, $\bar{\tau}_{ss,t}$ is the pension income tax rate, and $d_2(\cdot)$ is the progressive deduction used to calculate the pension income tax. In this model, the basic exemption is first applied to labor income, and if any amount remains, it is applied to pension income.

In addition to income tax, this model incorporates a simplified resident tax $\tau_{R,t}(\cdot)$, applying different resident tax rates based on income categories. In the real Japanese economy, resident tax consists of two components: a prefectural tax and a municipal tax. The tax payment also consists of two parts: a per capita portion and an income-proportional portion. However, since the model does not include local governments, the tax rates for prefectural and municipal taxes are unified. Additionally, for simplicity, the per capita portion is omitted in the model. It is also assumed that the same taxable income base is used as each income tax. The resident tax rates are set at $\bar{\tau}_{r1,t} = 5\%$ for interest income and $\bar{\tau}_{r2,t} = 10\%$ for labor income and pension income. The revenue from resident taxes is allocated to government expenditures as this model assumes the absence of local governments.

Given the complexity of Japan's intertwined income tax and social security systems, this study calibrates the labor income tax function using the following method. Salary nodes are set in 10,000 yen increments, ranging from a monthly employment salary of 50,000 yen to 3,880,000 yen, with additional nodes placed at statutory threshold points. Annual salary for each node is then calculated by aggregating monthly salary, and this value is treated as a household's gross employment salary. Based on these annual salary levels, basic exemption, employment income deduction, and full pension and health insurance contributions are determined. In reality, social insurance contributions are shared equally between employees and employers. This study considers employer-paid insurance contributions as part of employees' labor income. Accordingly, adjustments are made so that the modeled labor income, based on the model's wage rate, working ability, and working hours, corresponds to the total of actual salaries and employer-paid insurance contributions.

Taxable labor income nodes are also set in increments of 10,000 yen, with additional nodes placed at statutory threshold points. Using the income tax rate tables on the National Tax Agency Japan's website, the income tax amount is calculated for each taxable income level. The parameter $\varphi_t = 0.45$ is set to match the maximum marginal tax rate of 45% in Japan's current system.

Before estimating the labor income tax function used in the model, all monetary amounts are standardized. This study standardizes the model using the average annual labor income of employed households aged 20 to 64, which in the model is 0.63192. Using data from Table 1 of the 2022 Basic Survey on Wage Structure, including the social insurance contributions paid by employers, the average annual labor income of households aged 20 to 64 is calculated as 5,769,873 yen. Dividing this by the average labor income in the model, one unit of labor income in the model corresponds to 9,130,702 yen in reality, which is used for standardization. After standardizing the tax amounts and taxable income, the

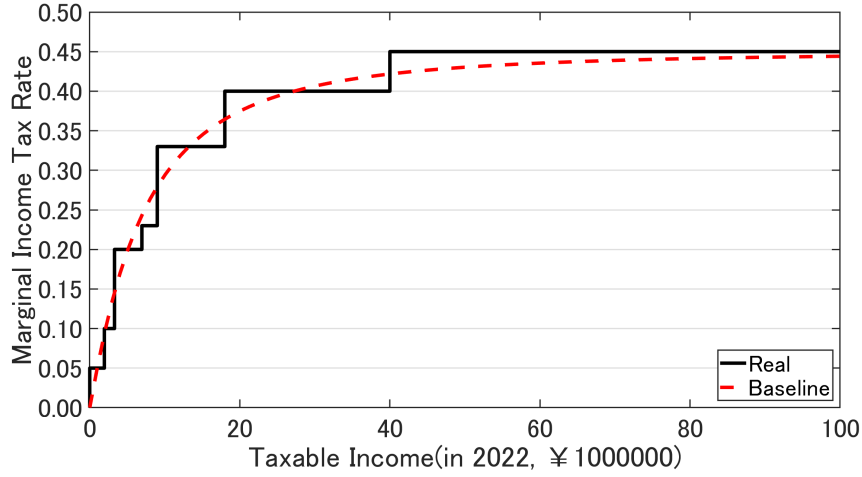


Figure 1.4: Marginal Income Tax Rates

OLS method is applied to estimate the remaining two parameters in the Gouveia and Strauss income tax formula, yielding $\varphi_1 = 1.0623$ and $\varphi_2 = 0.6579$. Here, φ_1 is the curvature of the income tax function, and φ_2 is the scale. Figure 1.4 compares the estimated marginal tax rates from the model with the actual marginal tax rates.

Calculating income tax, corporate tax, and inheritance tax revenues as a percentage of GDP in 2022 yields 7.2%. In this model, $\bar{\tau}_{k,t}$ is set to 0.202, which brings the ratio of income tax (excluding resident tax) to GDP to 7.2% in the baseline economy's steady-state equilibrium.

The basic exemption in the model is simplified to a uniform amount of 480,000 yen, while in the real economy, this exemption applies to individuals with a total income of 24 million yen or less. For higher income levels, the basic exemption is reduced in three steps and becomes zero for individuals with total income exceeding 25 million yen. The actual employment income deduction is a piecewise linear function of annual earnings, ranging from 550,000 yen to 1,950,000 yen. In this study, this deduction is approximated as a quadratic function of labor income. The lower limit of 550,000 yen is ignored for simplicity, which may result in an overestimation of taxes for low-income taxpayers. However, the effect is partially offset because the Gouveia and Strauss formula tends to underestimate taxes for low-income taxpayers.

For pension income, a deduction $d_2(\cdot)$ is applied based on pension income levels. In the actual system, for households aged 65 or above, this deduction is calculated using total income, with pension income excluded from the computation. Three different schemes are available. To simplify the analysis, this study assumes that all eligible households qualify for the scheme offering the highest deduction amount, which ranges from 1,100,000 yen to 1,950,000 yen. Since pension benefits are adjusted using the benefit adjustment factor ψ_t , the deductions are scaled accordingly to align with the adjusted benefits. A pension income tax rate of $\bar{\tau}_{ss,t} = 5\%$ is then applied to calculate the pension income tax.

1.3.6 Social Security System

This model includes a social insurance system with two components: a pay-as-you-go public pension system, consisting of the NP and EPI, and the simplified HI system. The public pension system employs a detailed structure with an actual contribution rate of 18.3%. Since the model does not analyze the impact of health insurance on household behavior, the healthcare insurance system is simplified, using a contribution rate of 10%, based on the average rate set by the Japan Health Insurance Association.

In the real economy, social insurance contributions are calculated using actual salaries, which is $w_t eh - \frac{1}{2} (\tau_{P,t}(\cdot) + \tau_{H,t}(\cdot))$ in this model. For computational feasibility, this study avoids the complexity of nested calculations by converting the contribution rates instead of labor income. The EPI contribution function is

$$\tau_{P,t}(i, w_t eh) = \mathbf{1}_{\{i < I_2\}} \bar{\tau}_{p,t} \min(w_t eh, \vartheta_{2max}), \quad (1.42)$$

where $\bar{\tau}_{p,t} = 0.1603$ is the converted EPI contribution rate. Working households must enroll in the EPI system until they reach $I_2 = 51$ (age 70), where ϑ_{2max} is the converted upper limit of the standard remuneration of the EPI system. In the actual system, the standard remuneration of the EPI system is predetermined based on the previous year's salaries and divided into 32 levels. In this model, however, the standard remuneration is simplified to depend on the current period's salaries, and only the upper limit is retained.

The converted upper limit of the standard remuneration used in calculating EPI contributions is set as $\vartheta_{2max} = 1.5380$. The upper limit $\vartheta_{1max} = 1.3471$ used in calculating average historical earnings b is the unconverted version of ϑ_{2max} , corresponding to the actual amount of 12,300,000 yen in 2022. In the actual system, the upper limit of the standard remuneration does not include bonuses. However, bonuses are subject to the same EPI contribution rate as standard remuneration, with a separate upper limit of 1,500,000 yen per occurrence, applicable up to three times per year. In this model, since bonuses are not explicitly considered, they are treated as part of labor income when setting the upper limit of standard remuneration.

The calculation of pension benefits differs between the NP and EPI systems. The NP provides a fixed pension benefit based on the contribution-paid periods, while EPI benefits depend on both the contribution-paid periods and average historical earnings. In this model, it is not feasible to precisely measure the number of years each household has paid pension contributions. For the EPI, this issue is addressed by updating the formula each period. For the NP, households with extremely low average historical earnings are treated as having made only partial National Pension contributions and receive 80% of the full NP benefits, while all other households receive the full amount (777,800 yen per year in 2022, the amount a household with 480 months of contribution-paid periods would receive). The

80-percent value is calibrated based on data reported in the Ministry of Health, Labour and Welfare's Overview of the Employees' Pension Insurance and National Pension Programs (FY 2022), which shows that NP benefits-only recipients receive approximately 80% of the full amount.

The EPI benefit function is

$$tr_{EP,t}(i, b) = \mathbf{1}_{\{i \geq I_1\}} \kappa b I_P (1 + \mu)^{I_1 - i}, \quad (1.43)$$

where $\kappa = 0.005481$, the benefit multiplier, reflects the actual multiplier in the Japanese EPI system and is used to calculate EPI benefits based on average historical earnings b and the contribution-paid period I_P .

The NP benefit function is

$$tr_{NP,t}(i, b) = \mathbf{1}_{\{i \geq I_1\}} np(b) (1 + \mu)^{I_1 - i}, \quad (1.44)$$

where $np(b)$ is the annual NP benefit received by households, taking either the full amount, 0.0852, or 80% of the full amount, depending on households' average historical earnings. In the current Japanese economy, this amount is determined annually by the pension sector.

This model incorporates the Old-Age Pension System for Working Retirees. Therefore, the total public pension benefits function is

$$tr_{SS,t}(i, b) = \mathbf{1}_{\{i \geq I_1\}} \psi_t \left[tr_{NP,t}(\cdot) + \max \left(0, tr_{EP,t}(\cdot) - \max \left\{ 0, \frac{1}{2} [tr_{EP,t}(\cdot) + y_P - \vartheta_{3max}] \right\} \right) \right], \quad (1.45)$$

which accounts for the adjustments to EPI benefits under the Old-Age Pension System for Working Retirees. Here, $y_P = \min \{ w_t eh - \frac{1}{2} [\tau_{H,t}(\cdot) + \tau_{P,t}(\cdot)], \vartheta_{1max} \}$ is a simplified function to calculate the household's standard remuneration used in the EPI. $\vartheta_{3max} = 0.6571$ is the annual threshold for the adjustment and suspension of EPI benefits under the Old-Age Pension System for Working Retirees. Due to a revision in the Old-Age Pension System for Working Retirees, the 2024 threshold of 500,000 yen per month is used instead of the 2022 threshold of 470,000 yen per month.

Under this system, if the sum of $tr_{EP,t}(\cdot)$ and y_P exceeds ϑ_{3max} , 50% of the excess amount is suspended from the EPI benefit. If the sum is sufficiently large, the EPI benefit would be fully withheld temporarily. For simplicity, this study assumes that the pension sector maintains a balanced budget in each period. Since the EPI contribution rate and the calculation method for pension benefits are fixed according to actual policies, a balance factor ψ_t is used to proportionately adjust the pension benefits to match the collected contributions, with $\psi_t = 0.9878$ in the baseline economy. ψ_t is fixed at the baseline

level in the policy experiments.

The ratio of total pension benefits to GDP is 8.7% in the baseline economy. According to the Ministry of Health, Labour, and Welfare's Overview of the Employees' Pension Insurance and National Pension Programs, total pension benefits paid to public pension recipients at the end of 2022 amounted to 55.7 trillion yen (including mutual aid pensions). Since survivor and disability pensions are not included in the model framework, this study excludes these components when calculating the actual pension benefits-to-GDP ratio. The adjusted amount is 47.9 trillion yen, which corresponds to a pension benefits-to-GDP ratio of 8.5%. Due to the lack of data on survivor and disability mutual aid pensions, their exact amounts could not be subtracted. Nevertheless, the model's pension benefits-to-GDP ratio remains quite close to the actual figure.

For health insurance, this study assumes that HI contributions are made from both labor income and pension income. The function of the healthcare insurance contribution is

$$\tau_{H,t}(w_t eh, tr_{SS,t}(\cdot)) = \bar{\tau}_{h,t} \min(w_t eh, \vartheta_{4max}) + 0.1 tr_{SS,t}(\cdot), \quad (1.46)$$

where $\bar{\tau}_{h,t} = 0.0876$ is the converted HI contribution rate. As in the pension system, $\vartheta_{4max} = 2.7003$ is the converted upper limit of the standard remuneration of the HI system, reflecting the actual upper limit of 22,410,000 yen in 2022. For simplicity, only the upper limit is retained. Similar to the EPI, under the actual system, the upper limit for standard remuneration for the HI in this model includes not only an individual's annual income but also an annual bonus of up to 5,730,000 yen.

For households whose labor income exceeds ϑ_{2max} in the EPI system, their marginal EPI contribution rate becomes 0; therefore, $\bar{\tau}_{h,t}$ is adjusted to 0.0952 accordingly. For working households aged 70 or older, since they are no longer required to pay EPI contributions, $\bar{\tau}_{h,t}$ is adjusted to 0.0952, and ϑ_{4max} is adjusted to 2.5771 accordingly. In reality, individuals aged 75 or older are covered under the Elderly Person's Medical Program, and health insurance contributions are paid on an individual basis, without employer co-contribution. However, for simplicity, the model assumes that even for households aged 75 or older, HI contributions arising from labor income continue to follow the employer co-contribution structure. Pension income is not subject to EPI contributions, nor does it involve employer co-contribution. Therefore, the HI contribution rate applied to pension income does not need to be converted in this model.

Since this study does not consider state variables representing the health status of households, it assumes that the government collects HI contributions to fund general government expenditures. In the baseline scenario, the government's net worth $W_{G,t}$ is set to zero, meaning that $C_{G,t} = T_{C,t} + T_{I,t} + T_{H,t} - TR_{LS,t}$. In the policy experiments, government consumption $C_{G,t}$ is fixed at the baseline level, and the government adjusts the consumption tax rate $\tau_{c,t}$ to achieve budget balance, where $W_{G,t} = W_{G,t+1}$.

1.4 Policy Experiments

This study analyzes the effects of three pension reform proposals, focusing on their impacts on the macroeconomy, long-run economic growth, and welfare. The first policy involves cutting public pension benefits proportionately by 15% while reducing pension contribution levels for working households to maintain the financial balance of the pension system. The second policy proposes gradually raising the pension eligibility age from 65 to 70, which reduces total pension payments for elderly households, and lowers pension contribution levels for working households to keep the system balanced. Considering that the Old-Age Pension System for Working Retirees likely reduce work incentives for elderly households, the third policy experiment examines the effects of removing pension payment adjustments and suspensions in this system.

The economy is at a baseline steady-state equilibrium in period $t = 0$. At the beginning of period $t = 1$, the government announces a permanent new policy, triggering adjustments in the economy that ultimately lead to a new steady-state equilibrium.

1.4.1 Cutting Public Pension Benefits Proportionately by 15%

Section 1.4.1 analyzes the effects of a policy that cuts public pension benefits proportionately by 15%. To implement the 15% reduction in the model, the NP benefit is multiplied by an adjustment factor of 85%. For the EPI, the policy is applied by adjusting the benefit multiplier κ in the pension calculation formula with the same factor of 85%. Since the pension system in this model operates with an independent budget, reduced expenditures allow the government to lower the public pension contribution rate. This, in turn, affects macroeconomic indicators, requiring the government to adjust tax rates to maintain a balanced general budget.

The policy is scheduled to be implemented in period $t = 1$. An immediate 15% reduction in public pension benefits would be too unfair to households that are already elderly in period $t = 1$. Therefore, households aged 60 and older ($i \geq 41$) in period $t = 1$ are not affected by the policy. Adjustment for future elderly households are applied gradually over 26 years, eventually reaches a 15% reduction in pension benefits.

To implement the policy in the model, two parameters, $\psi_{tr,t}$ and $\psi_{p,t}$, are introduced. $\psi_{tr,t}$ is the adjustment factor for reducing pension benefit amounts, while $\psi_{p,t}$ is the adjustment factor for reducing the pension contribution rate. Households are assumed to pay pension contributions equal to $\psi_{p,t}$ times the contributions calculated based on their labor income to ensure the pension sector's budget remains balanced.

Because the 15% reduction is phased in over 26 years, $\psi_{tr,t}$ is not a fixed value during this adjustment period. In the baseline economy, $\psi_{tr,0} = 1$, but from period $t = 1$ onward, it is adjusted based on the

entry time of households into the model. Similarly, $\psi_{p,t}$ is adjusted dynamically to reflect the pension sector's budgetary condition. To maintain a balanced general budget, the consumption tax rate $\tau_{c,t}$ is adjusted as necessary. The adjustment factor $\psi_{tr,t}$ during this process is determined by the following equation:

$$\psi_{tr,t}(i) = \mathbf{1}_{\{i \leq t+15\}} 0.85 + \mathbf{1}_{\{t+15 < i < t+40\}} \times \left\{ 0.85 + (i - t - 15) \frac{0.15}{25} \right\} + \mathbf{1}_{\{i \geq t+40\}} 1. \quad (1.47)$$

The adjustment of the pension contribution reduction parameter $\psi_{p,t}$ is determined by the following equation:

$$\psi_{p,t} = \frac{TR_{SS,t}(\psi_t, \psi_{tr,t})}{T_{P,t}}, \quad (1.48)$$

where $TR_{SS,t}(\cdot)$ is the total public pension benefits distributed to all elderly generations in each period, which has already been adjusted by ψ_t and $\psi_{tr,t}$. $T_{P,t}$ is the total pension contributions that can originally be collected from working households before any adjustments, serving as the financial resource for the pension system.

The adjustment of the consumption tax rate is determined by the following equation:

$$\tau_{c,t} = \frac{1}{C_{P,t}} (C_{G,t} + TR_{LS,t} - T_{I,t} - T_{H,t}), \quad (1.49)$$

where $C_{P,t}$ is the private consumption. In this model, the effects of health status and health insurance on household utility are disregarded. Therefore, it is assumed that the health insurance contributions are fully spent by the government within the same period they are collected.

1.4.1.1 Life-Cycle Effects in the Long Run

Figure 1.5 shows the long-run effects of cutting public pension benefits proportionately by 15% over the life cycle of households, displayed as age-specific averages. The solid black line represents the baseline equilibrium in period 0, while the dashed red line represents the new steady-state equilibrium after the policy change. The noticeable dent in the working hours, EPI contributions, and labor income tax curves between age 65 and 69 is attributed to the Old-Age Pension System for Working Retirees. Under this system, pension recipients adjust their working hours downward to reduce their labor income and thereby avoid reductions in their pension benefits.

In Japan's policy framework, social insurance contributions are deductible when calculating labor income taxes. After reaching age 70, households no longer need to pay EPI contributions, causing the EPI contributions curve to drop to 0. At the same time, the labor income tax curve rises because households no longer receive the deduction for pension contributions.

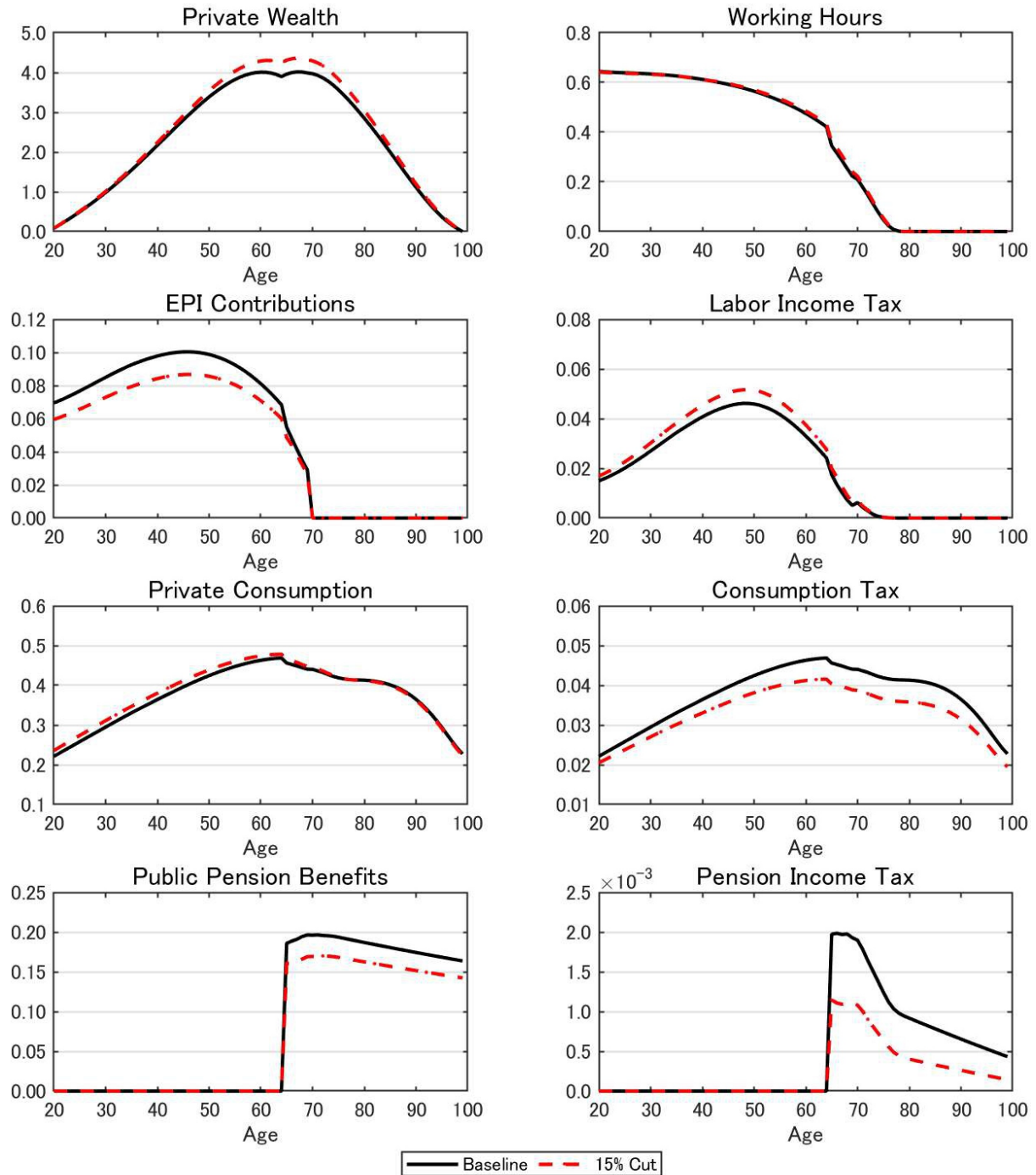


Figure 1.5: Life-Cycle Effects of Cutting Public Pension Benefits by 15%

Private wealth, particularly that of households aged 40 to 80s, would increase significantly after the policy change. This is because households anticipate reduced future pension benefits and adjust their behavior by increasing savings to compensate. To facilitate greater savings, working hours of households aged 40 and older would also increase slightly. The increase in private wealth is larger than the increase in labor supply, leading to a higher wage rate and a lower interest rate after the policy change.

As a result, labor income and labor income tax payments would increase across working households. The EPI contributions paid by households until they reach pension eligibility age ($i = 46$) would decrease

by approximately 15%. This would lead to higher labor income tax payments for households up to age 70, explaining the significant increase in their labor income tax payments relative to the change in working hours.

Higher income tax payments would boost government revenue. To maintain a balanced budget, the government would lower the consumption tax rate, leading to reduced consumption tax payments across all generations. Private consumption among working households would increase slightly, while consumption among households in their 90s would decrease slightly after the policy change. This difference arises from the reduction in EPI contributions, which decreases the disposable income of older households while boosting the disposable income of households up to age 70.

Pension income tax payments would decrease among eligible households due to the 15% reduction in pension benefits. Additionally, pension income tax payments would decrease with age, with the rate of decrease slowing around age 80. In this model, when households earn both pension and labor income, the basic exemption is applied to labor income first, with any remaining portion allocated to pension income. As households age and earn less labor income, a larger portion of the basic exemption would be allocated to pension income, resulting in a further reduction in pension income taxes.

1.4.1.2 Macroeconomic Effects in the Transition Path

In this study, a sufficiently long period of 200 years is used to capture the adjustment process as the economy transitions towards a new steady-state equilibrium following the policy change. The results of this transition process are summarized in Table 1.2 and Figure 1.6. Year $t = 0$ is the baseline economy before the policy change, while year $t = 201$ is the new steady-state equilibrium. As shown in Table 1.2 and Figure 1.6, by year $t = 151$, all variables have effectively converged to their new steady-state equilibrium values.

Cutting public pension benefits by 15% prompts households to increase their savings in preparation for retirement, resulting in a 0.2% increase in year $t = 6$ and a 6.1% increase in capital stock in the long run compared to the baseline level. Labor supply would increase by 0.1% in year $t = 6$ and by 0.7% in the long run. Since the model incorporates both intensive and extensive margins, the policy change raises labor supply not only because working households increase their working hours, but also because additional households enter the labor market.

Except for the early years of the transition, the growth in capital stock would consistently exceed the growth in labor supply. As a result, the interest rate would decrease slightly in year $t = 6$, and decrease by 0.4 percentage points in the long run. Wage rate would increase slightly in year $t = 6$, and increase by 2.1% in the long run.

The policy change would result in enhanced economic activity. GDP would increase by 0.2% in year

Table 1.2: Transition Effects of Cutting Pension Benefits by 15% (%ch. from the baseline economy)

Year	1	6	11	21	51	101	151	Long run
Capital Stock	0.0	0.2	0.5	1.3	4.7	6.1	6.0	6.1
Labor Supply	0.1	0.1	0.2	0.5	1.0	0.7	0.7	0.7
Gross Domestic Product	0.1	0.2	0.3	0.8	2.5	2.9	2.9	2.9
Private Consumption	-0.1	-0.1	-0.0	0.3	2.0	2.7	2.7	2.7
Interest Rate ^a	0.0	-0.0	-0.0	-0.1	-0.2	-0.4	-0.4	-0.4
Wage Rate	-0.0	0.0	0.1	0.3	1.5	2.2	2.1	2.1
Consumption Tax Rate ^a	-0.0	-0.1	-0.1	-0.3	-1.1	-1.3	-1.3	-1.3
EPI Contribution Adjustment Factor	-0.1	-0.2	-0.8	-3.9	-15.9	-15.7	-15.6	-15.6
Welfare of Age 20 Households	0.3	0.7	1.2	2.3	4.7	5.0	5.0	5.0

Note: ^a Changes in percentage points from the baseline economy.

$t = 6$ and by 2.9% in the long run. The consumption tax rate would decrease by 0.1 percentage points in year $t = 6$ and decrease by 1.3 percentage points in the long run. As a result, private consumption, which would decrease by 0.1% in year $t = 6$ because households anticipate lower pension benefits in the future and therefore reduce current spending, would increase by 2.7% in the long run.

The EPI contribution adjustment factor $\psi_{p,t}$, initially set to 1.0 in the baseline, would decrease gradually as the reform-affected cohorts begin receiving pension benefits, and would decrease by 15.6% in the long run. It should be noted that $\psi_{p,t}$ is a scaling factor applied to the converted contribution rate, reflecting the assumption that households bear the full burden of both employee and employer contributions. Under this assumption, any reduction in the contribution rate implies that the forgone employer contributions are fully returned to households through higher salaries. Consequently, a 15.6% reduction in the adjusted contribution rate corresponds to an effective 25.7% reduction in the actual contribution burden borne by employees in reality. This reduction indicates that, in the model economy, the policy would successfully alleviate the burden of public pension contributions on working households.

The results of this policy experiment, which take the opposite adjustment approach to the actual policy changes that increased pension contribution rates, suggest that lower contribution rates and lower benefit levels could provide a more positive stimulus to the economy. As a response to the fiscal imbalance in the pension system caused by population aging, maintaining contribution rates while reducing pension benefits might also be a desirable approach, in contrast to raising contribution rates to sustain benefit levels. Furthermore, this implies that if the pension reforms implemented between 2004 and 2017 had set the target contribution rate below 18.3% and adopted a lower replacement rate, the economy might have been more prosperous today.

1.4.1.3 The Impact on Welfare

In this study, the veil of ignorance measure is used to calculate consumption equivalence. As shown in Table 1.2, the welfare of newborn households would improve by the equivalent of a 0.7% increase in

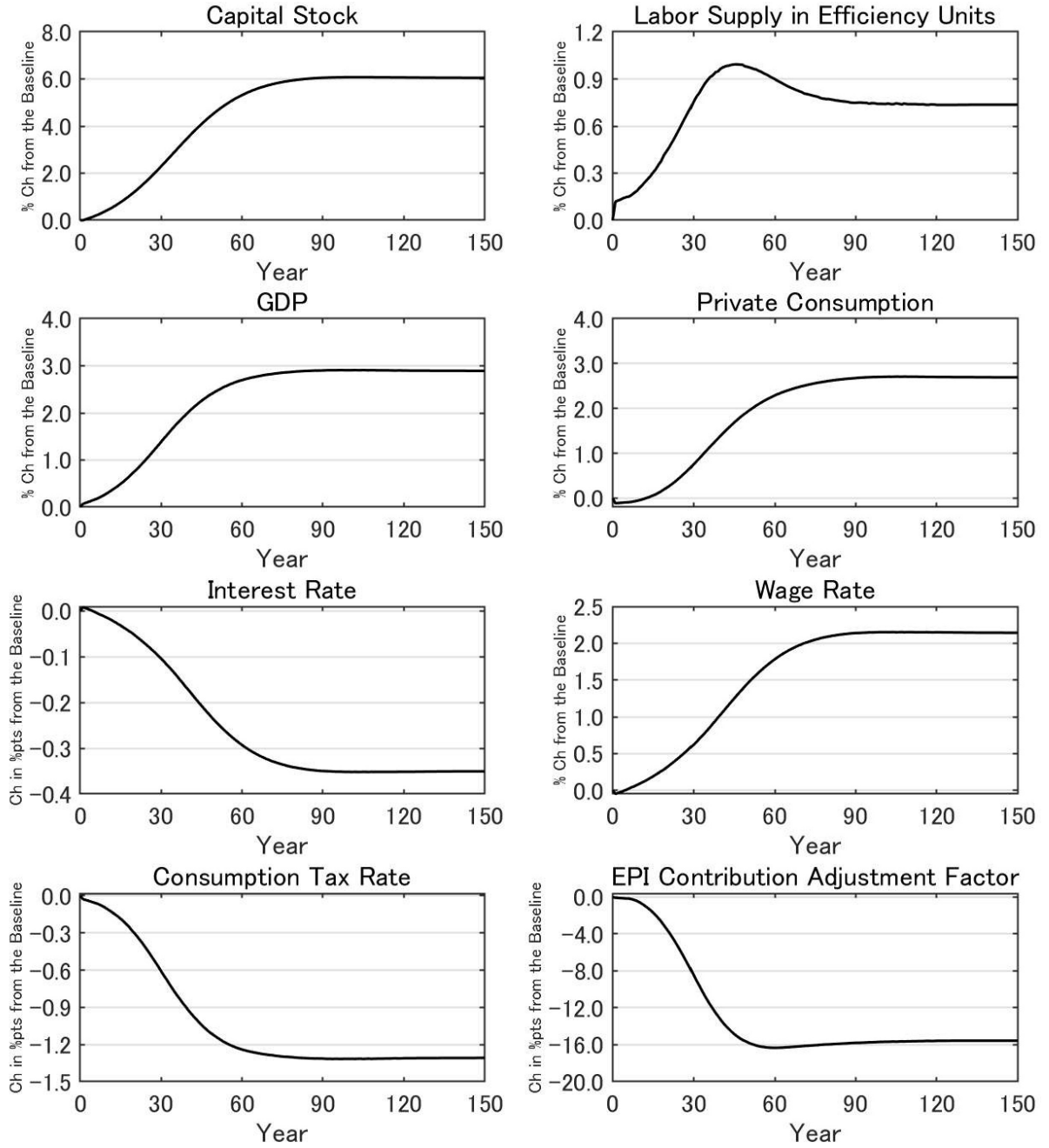


Figure 1.6: Transition Effects of Cutting Pension Benefits by 15% (%ch. from the baseline economy)

the baseline consumption path in year $t = 6$, and further improve by the equivalent of a 5.0% increase in the long run.

Figure 1.7 shows the consumption equivalence of welfare changes by age, capturing the changes in households' lifetime expected utility. The vertical axis is the consumption equivalence rate, while the horizontal axis is the age of households at the time of policy change ($t = 1$). The vertical line marks households aged 20 in year $t = 1$. The area to the left of the line corresponds to households existing in the economy at year $t = 1$, while the area to the right corresponds to newborn households entering the economy after the policy is implemented. Consequently, the right part of the horizontal axis contains

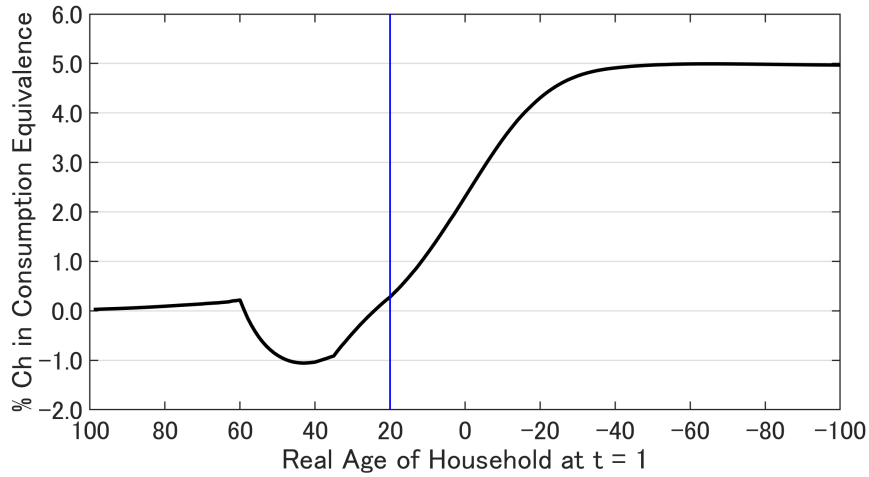


Figure 1.7: Welfare Effects of Cutting Pension Benefits by 15% (%ch. in consumption equivalence)

negative values, corresponding to the current age of these future newborn households in period $t = 1$.

The policy applies to households aged 59 and below in year $t = 1$. Although they are not directly affected by the reform, elderly households would benefit from the lower consumption tax rate. Moreover, because some households in their 60s and 70s still participate in the labor market, they would also benefit from higher wage rate. Although the interest rate would decrease after the policy change, its negative effect for older cohorts is dominated by the gains from lower consumption tax rate and higher wage rate. As a result, welfare changes for households aged 60 and above range from 0.02% to 0.22%, as shown in Figure 1.7.

Households aged 25 to 58 in year $t = 1$ would bear the negative impact of the policy change. These households face reduced future benefits despite having paid more into the system with a relatively high EPI contribution rate, resulting in a decline in their lifetime expected utility. Moreover, they have less time to prepare for the reduction in future benefits, which also negatively affects their well-being. Future households would experience welfare gains. In the long run, newborn households would be better off by 5.0%, as measured by consumption equivalence.

After the policy implementation, the Gini coefficient of pre-tax income would decrease from 0.6014 in the baseline to 0.5986. For cohorts aged 30 and above, the Gini coefficient would become slightly lower after the policy change, indicating a modest reduction in income inequality. In summary, while the policy negatively impacts the welfare of current households in the short run, it yields significant long-run benefits. From a welfare perspective, the results suggest that an economy with both lower contribution rates and lower pension benefits may achieve a higher level of social welfare than with higher contribution rates and higher benefits. This implies that maintaining contribution rates while reducing benefit levels could be a more favorable approach than sustaining benefit levels by increasing contribution rates.

1.4.2 Raising the Pension Eligibility Age by Five Years

As Japan's birthrate continues to decline, labor shortages and imbalances in the public pension system are expected to intensify, posing a threat to the sustainability of social security. Raising the pension eligibility age offers a direct policy solution to these challenges. In fact, the Japanese government has already begun taking steps in this direction, including initiatives to promote employment for individuals up to age 70.

Currently, Japan is transitioning the pension eligibility age from 60 to 65, with the age set to reach 65 for men by 2025 and for women by 2030. In the baseline economy, this study assumes age 65 as the pension eligibility age. This section examines the potential economic impacts of gradually raising the eligibility age to 70, focusing on labor supply, economic growth, and social welfare.

This model incorporates the Old-Age Pension System for Working Retirees, which introduces two key age thresholds. The first is the pension eligibility age, which is set at $I_1 = 46$ in the baseline, corresponding to age 65 in reality. The second is $I_2 = 51$, equivalent to age 70, when elderly households who continue working are no longer required to pay public pension contributions. Based on these assumptions, the policy experiment explores a gradual increase in the public pension eligibility age by age cohort. To maintain consistency with the first policy experiment, the eligibility age for households aged 60 and above in year $t = 1$ remains unchanged. For households aged 54–59, the eligibility age is set at $i = 47$ (corresponding to age 66 in reality). For households aged 48–53, the eligibility age is set at $i = 48$ (corresponding to age 67 in reality). For households aged 42–47, the eligibility age is set at $i = 49$ (corresponding to age 68 in reality). For households aged 36–41, the eligibility age is set at $i = 50$ (corresponding to age 69 in reality). For households aged 35 and younger, along with future generations, the eligibility age is set at $i = 51$ (corresponding to age 70 in reality). The second threshold $I_2 = 51$ remains fixed throughout the policy adjustment.

As the pension eligibility age increases, households aged 65 to 69 no longer receive pension benefits, and therefore, benefit adjustments and suspensions under the Old-Age Pension System for Working Retirees no longer apply to these households. However, for households aged 70 and above, adjustments will still apply if the sum of labor income and pension income exceeds ϑ_{3max} .

During the adjustment process, the pension eligibility age I_1 , which varies across different age cohorts, is defined as follows:

$$I_1 = \mathbf{1}_{\{i \leq t+15\}} 51 + \mathbf{1}_{\{t+16 \leq i \leq t+21\}} 50 + \mathbf{1}_{\{t+22 \leq i \leq t+27\}} 49 + \mathbf{1}_{\{t+28 \leq i \leq t+33\}} 48 \\ + \mathbf{1}_{\{t+34 \leq i \leq t+39\}} 47 + \mathbf{1}_{\{i \geq t+40\}} 46. \quad (1.50)$$

In the baseline economy, the NP benefit is modeled under the assumption that most households have contributed to the NP system for a 40-year contribution-paid period. However, as the pension eligibility

age is raised under this policy experiment, the contribution-paid period should also be extended. To simplify the analysis, this policy experiment assumes that the NP contribution-paid period is lengthened proportionally as the eligibility age increases, leading to a corresponding rise in the NP benefit. The updated formula for calculating the NP benefit is

$$tr_{NP,t}(i, b) = np(b) \frac{I_1 - 6}{40} (1 + \mu)^{I_1 - i}, \quad (1.51)$$

where np is the full amount of NP benefit (or 80% of the full amount for households with $b = 1$) in the baseline economy.

As in the previous policy experiment, the fiscal balance of the pension sector is maintained by adjusting the scaling factor $\psi_{p,t}$ applied to public pension contributions, ensuring the pension system's financial stability during the transition process. The general government budget is balanced by adjusting the consumption tax rate $\tau_{c,t}$.

1.4.2.1 Life-Cycle Effects in the Long Run

Figure 1.8 shows the long-run effects of raising the pension eligibility age from 65 to 70 over the life cycle of households, displayed as age-specific averages. The shift in pension eligibility age is evident from the public pension benefits curve. Furthermore, as the commencement of pension benefits is delayed, households stay enrolled in the public pension system for a longer period, and their extended contribution-paid periods lead to higher average pension benefit amounts.

The private wealth of households up to their late 60s would increase significantly after the policy change. In this model, households' working ability peaks at age 52 and gradually declines thereafter. Since they cannot receive pension benefits until age 70, households would save during the years when their working ability is relatively high and draw down their assets gradually during their 60s to finance consumption. Because households know that their future pension benefits would increase, they place greater weight on current consumption rather than saving, resulting in lower asset holdings after age 70.

From the curve of working hours, it can be observed that the distortions caused by the Old-Age Pension System for Working Retirees between ages 65 and 70, which were present in the baseline economy, are no longer observable in the new steady state. With the starting age for pension benefits raised to 70, households aged 65 to 69 would no longer be subject to pension benefit adjustments and suspensions, resulting in their labor supply being unaffected by the system.

The increase in private wealth is larger than the increase in labor supply, leading to a higher wage rate and lower interest rate after the policy change. As a result, labor income and labor income tax payments would increase across working households. Similar to the first policy experiment, raising the pension eligibility age by five years would reduce pension contributions, thereby weakening the deductible effect

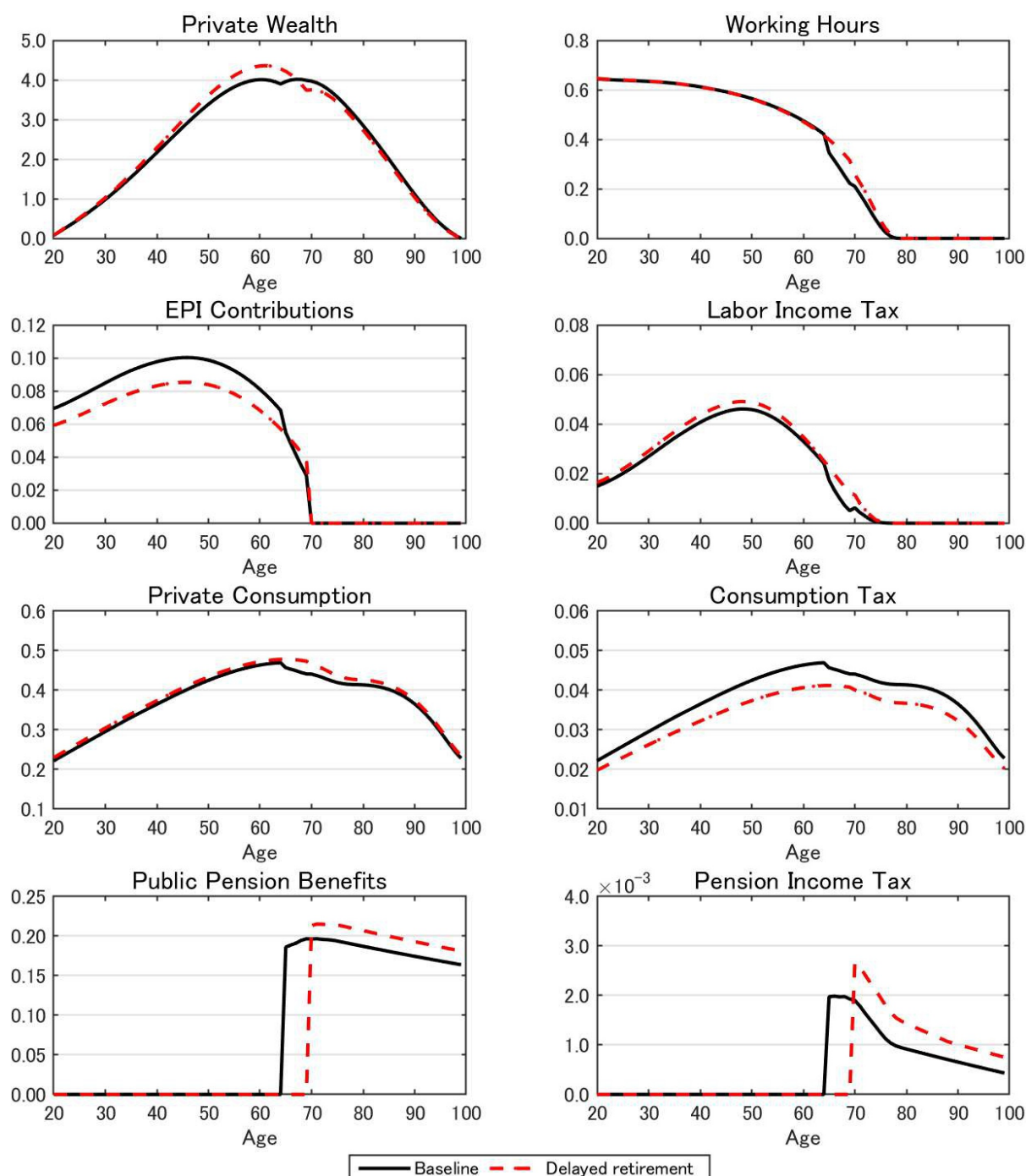


Figure 1.8: Life-Cycle Effects of Raising Public Pension Eligibility Age by Five Years

and leading to higher labor income taxes. Also, the increase in working hours among households in their 70s is driven by their lower asset holdings and the modest increase in wage rate.

Higher income tax payments would boost government revenue. To maintain a balanced budget, the government would lower the consumption tax rate, leading to reduced consumption tax payments across all generations. Unlike the first policy experiment, private consumption among elderly households would increase slightly. This is because eligible households receive more pension benefits after this policy change. After the policy change, pension income tax payments will be postponed and increase among

Table 1.3: Transition Effects of Raising Eligibility Age by Five Years (%ch. from the baseline economy)

Year	1	6	11	21	51	101	151	Long run
Capital Stock	0.0	0.0	0.2	0.8	2.9	3.1	3.1	3.1
Labor Supply	-0.1	-0.2	0.4	1.4	1.9	1.8	1.9	1.9
Gross Domestic Product	-0.1	-0.1	0.3	1.1	2.3	2.4	2.4	2.4
Private Consumption	-0.2	-0.2	0.2	1.1	2.8	3.0	3.0	3.0
Interest Rate ^a	-0.0	-0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1
Wage Rate	0.0	0.1	-0.1	-0.2	0.4	0.5	0.5	0.5
Consumption Tax Rate ^a	0.0	0.1	-0.2	-0.8	-1.4	-1.4	-1.4	-1.4
EPI Contribution Adjustment Factor	-0.0	0.1	-4.4	-12.5	-16.0	-15.2	-15.1	-15.2
Welfare of Age 20 Households	0.9	1.3	1.7	2.5	2.9	2.8	2.8	2.8

Note: ^a Changes in percentage points from the baseline economy.

eligible households, as expected under this policy experiment.

1.4.2.2 Macroeconomic Effects in the Transition Path

The results are summarized in Table 1.3 and Figure 1.9. Since the policy is applied by age cohort, some curves exhibit a step-shaped pattern. The policy of cutting pension benefits proportionately by 15% is implemented gradually over 26 years. To align with this approach, the policy experiment raising the eligibility age to 70 is also designed to be phased in over approximately 26 years, increasing the age by one year every six years.

As households anticipate working for a longer period in the future, the labor supply would decrease by 0.2% in year $t = 6$. Pension contributions from households become insufficient, and the EPI contribution rate increases by 0.1% in year $t = 6$. After year $t = 6$, households aged 65 to 70 gradually lose eligibility for pension benefits, and therefore participate more in the labor market. Labor supply would increase by 0.4% in year $t = 11$ and by 1.9% in the long run.

Capital stock would decrease slightly in year $t = 6$, and increase by 3.1% in the long run. The effect on capital accumulation is smaller than in the first policy experiment, because the expectation of higher pension benefits reduce the incentive to save, leading households to shift consume more during working ages.

Except for the first decades, the growth in capital stock would exceed the growth in labor supply. As a result, the interest rate, which would increase slightly in year $t = 11$, would decrease by 0.1 percentage points in the long run. The wage rate would decrease by 0.1% in year $t = 11$ and increase by 0.5% in the long run. Economic activity would slightly contract immediately after the policy change, and would be stimulated later, resulting in a GDP increase of 0.3% in year $t = 11$ and 2.4% in the long run. The consumption tax rate would decrease by 0.2 percentage points in year $t = 11$ and by 1.4 percentage points in the long run.

Private consumption, which would decrease by 0.2% in year $t = 6$, would increase by 0.2% in year

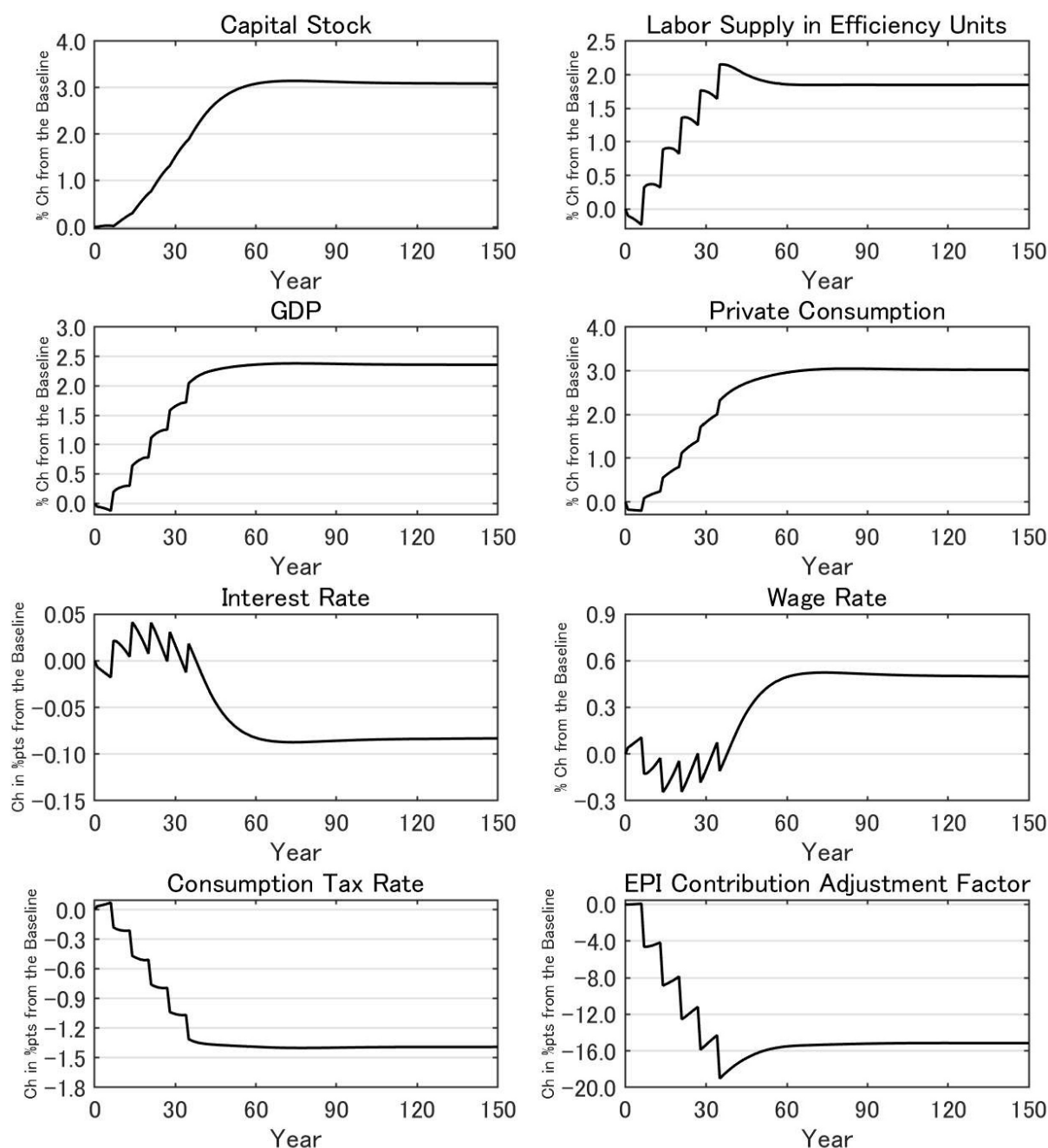


Figure 1.9: Transition Effects of Raising Eligibility Age by Five Years (%ch. from the baseline economy)

$t = 11$, and by 3.0% in the long run. The EPI contribution adjustment factor would decrease by 4.4% in year $t = 11$, and by 15.2% in the long run. This decrease occurs because the pension eligibility age is raised, resulting in fewer households receiving pensions during the same period, which leads to lower total pension expenditures. Moreover, working households, especially those aged between 65 and 70, pay more pension contributions than the baseline, which further reduces the financial burden on the pension system. If converted under the assumption of equal sharing between employers and employees, as in the first policy experiment, this corresponds to a maximum decrease of 25.1% in the pension contribution burden on employees.

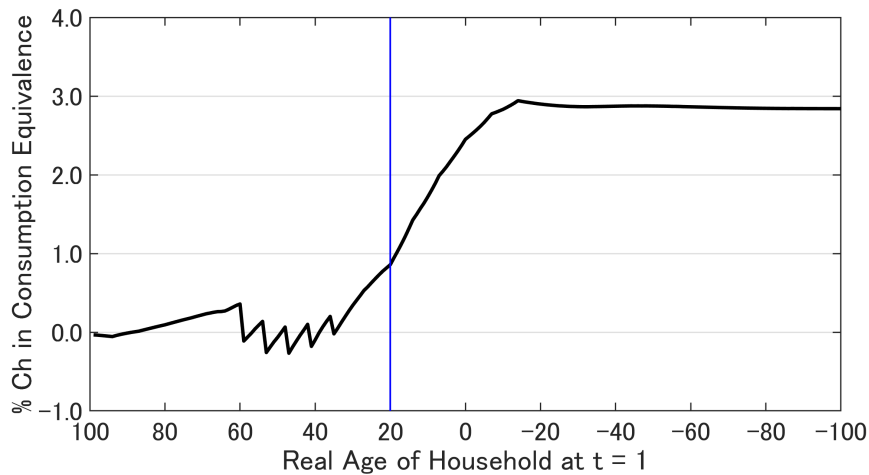


Figure 1.10: Welfare Effects of Raising Pension Eligibility Age (%ch. in consumption equivalence)

These results suggest that raising the pension eligibility age may impose short-term economic costs, but it is an effective policy in the long run, as it improves the financial sustainability of the pension system and stimulates the macroeconomy. However, it is important to note that health status is not considered in this model. As individuals age, some may face physical difficulty in remaining in the labor force, placing natural limits on the reform's applicability.

1.4.2.3 The Impact on Welfare

Next, the study examines how raising the pension eligibility age affects welfare. As in the first policy experiment, the veil of ignorance measure is used to calculate consumption equivalence for different age cohorts, and is summarized in Table 1.3 and Figure 1.10.

Since this policy experiment does not directly affect households aged 60 and older in year $t = 1$, Figure 1.10 clearly shows that the most significant effects are observed in households aged below 60. Households in their 90s would be slightly worse off, as they would be affected by the higher consumption tax rate immediately after the policy announcement and do not live long enough to benefit from the indirect effects of the policy change.

For other households beyond age 60, welfare would improve slightly, corresponding to an increase of less than 0.36% in the baseline consumption path. Although they would experience a temporarily decline in the wage rate and, for those in their 60s, a higher pension contribution burden, these households would still benefit overall because they benefit from lower consumption tax rate for a longer period.

Some households aged 35 to 59 in year $t = 1$ would experience the negative effects of the policy. Among them, those aged 59, 53, 47, 41, and 35 are the first cohorts subject to the new pension eligibility age, and would experience a noticeably larger decline in welfare. Households aged 54, 48, 42, 36, on the other hand, would be slightly better off as they are the last cohorts to face the new eligibility age. Future generations, particularly those born later, would show more significant improvements in welfare, with an

increase equivalent to a 2.8% rise in the baseline consumption path in the long run.

After the policy implementation, the Gini coefficient of pre-tax income would decrease from 0.6014 in the baseline to 0.5940. For households aged 40 to 64, the Gini coefficient would rise slightly. In contrast, a substantially larger decline in the Gini coefficient would be observed among senior households. Overall, although the impact on welfare is smaller than that of the first policy experiment, this policy can be considered beneficial for enhancing welfare particularly for future households.

1.4.3 Removing Pension Adjustments and Suspensions

The baseline economy in this study assumes the existence of the Old-Age Pension System for Working Retirees. This system allows elderly households to receive pension benefits while continuing to work and remain enrolled in the EPI system until age 70. However, if the combined total of labor income and pension benefits exceeds a certain threshold, monthly pension benefits may be reduced or even suspended entirely.

Before the introduction of this system, older individuals in Japan had to leave the labor market in order to receive pension benefits. However, older individuals who continued working often faced low wages. The system was therefore introduced to allow them to receive pension benefits while remaining in the labor market, and it incorporated benefit adjustments to maintain the financial sustainability of the public pension system. Today, however, as Japan experiences tightening labor market conditions, concerns have emerged that these adjustments and suspensions may discourage the labor supply of older households. This section focuses specifically on removing benefit adjustments and suspensions to examine the impact on labor supply, the economy and overall welfare.

To simulate the removal of benefit adjustments and suspensions under the Old-Age Pension System for Working Retirees, the formula for calculating employee pension benefits in the model is modified as follows:

$$tr_{SS,t}(i, b) = \mathbf{1}_{\{i \geq I_1\}} \psi_t \left[tr_{NP,t}(\cdot) + \max \left(0, tr_{EP,t}(\cdot) - \mathbf{1}_{\{i \geq t+40\}} \max \left\{ 0, \frac{1}{2} [tr_{EP,t}(\cdot) + y_P - \vartheta_{3max}] \right\} \right) \right], \quad (1.52)$$

the adjustments and suspensions of EPI benefits are assumed to be removed for households aged 59 and younger in year $t = 1$.

After the policy change, households begin receiving public pensions at age 65 while being able to continue working without any reduction or suspension of their pension benefits. They could remain enrolled in the public pension system until age 70, during which they are required to pay pension contributions, and their pension benefits are periodically recalculated based on their contribution-paid periods and average historical earnings. Once households reach the age of 70, they are no longer covered by the

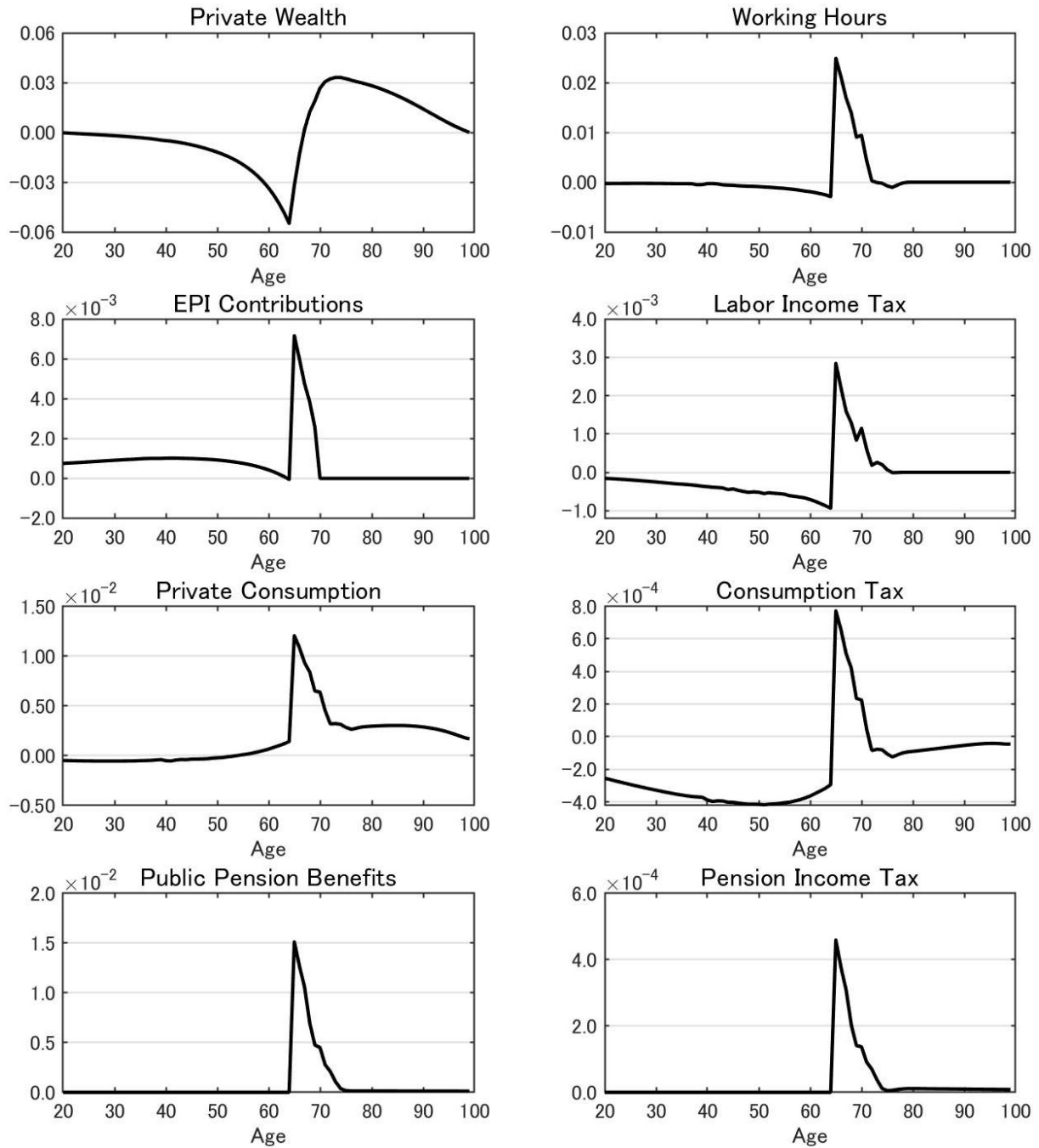


Figure 1.11: Life-Cycle Effects of Removing Pension Adjustments

public pension system, which means they no longer have to pay pension contributions, and their pension benefits are no longer subject to recalculation.

1.4.3.1 Life-Cycle Effects in the Long Run

Unlike the first two policy experiments, the impact of this policy on the macroeconomy is relatively small. To facilitate comparison, Figure 1.11 illustrates the the difference between the steady-state equilibrium after implementation and the baseline. It is evident that households aged 65 and older are more

responsive to this policy experiment, indicating that these households are the most affected by the pension adjustments and suspensions under the Old-Age Pension System for Working Retirees. This is particularly true for households aged 65 to 69, as after the policy implementation, working during this period not only increases their future pension benefits, but also does not lead to a reduction in their current pension income. Therefore, these elderly households are more willing to work and are consequently more significantly affected.

Working hours would increase for households between 65 and 72, while decreasing slightly for other working households. Private wealth, EPI contribution, labor income tax payment, and private consumption would all increase slightly for households aged 65 and older. Private wealth for households aged up to 64 would decrease, as they place less importance on current savings due to the expectation of higher future income. Economic activity would be stimulated. As a result, the government would lower the consumption tax rate to maintain the general government budget balance. This leads to a slight decrease in consumption tax payments across most households, with a increase observed for households aged 65 to 71 as they earn more and consume more. Because pension adjustments and suspensions would be removed after the reform, public pension benefits received at age 65 and older would increase. Pension income tax payments would also increase among eligible households.

In summary, removing benefit adjustments and suspensions under the Old-Age Pension System for Working Retirees improves macroeconomic outcomes for households aged 65 and older.

1.4.3.2 Macroeconomic Effects in the Transition Path

The results are summarized in Table 1.4 and Figure 1.12. Upon the policy announcement in year $t = 1$, households aged up to 59 anticipate that their pension benefits will no longer be reduced or suspended, even if they work more after turning 65. This expectation influences households' labor decisions and saving behavior. Working households would choose to enjoy more leisure time or temporarily leave the labor market, resulting in a 0.17% decrease in labor supply in year $t = 6$. In the long run, as households age 65 and beyond would increase their labor supply, resulting in a 0.35% increase compared to the baseline economy. This change in labor behavior would lower savings and thus the capital stock, which would decrease by 0.19% in year $t = 6$, and by 0.06% in the long run.

Except for the first years soon after the policy change, capital stock would shrink while labor supply would growth. As a result, the wage rate would increase by 0.04% in year $t = 1$, and decrease by 0.17% in the long run. The interest rate would decrease slightly in year $t = 1$, and increase by 0.03 percentage points in the long run. In the short run, the economy would contract slightly as labor supply decreases. In year $t = 6$, GDP would initially decrease by 0.18%. The consumption tax rate would increase by 0.10 percentage points, and private consumption would decrease by 0.11%. In the long run, GDP would

Table 1.4: Transition Effects of Removing Pension Adjustments (%ch. from the baseline economy)

Year	1	6	11	21	51	101	151	Long run
Capital Stock	0.00	-0.19	-0.29	-0.17	-0.09	-0.07	-0.07	-0.06
Labor Supply	-0.09	-0.17	0.24	0.33	0.35	0.35	0.35	0.35
Gross Domestic Product	-0.06	-0.18	0.02	0.12	0.17	0.18	0.18	0.18
Private Consumption	-0.08	-0.11	0.09	0.25	0.35	0.36	0.36	0.36
Interest Rate ^a	-0.01	0.00	0.04	0.04	0.03	0.03	0.03	0.03
Wage Rate	0.04	-0.01	-0.22	-0.21	-0.18	-0.17	-0.17	-0.17
Consumption Tax Rate ^a	0.05	0.10	-0.01	-0.07	-0.09	-0.10	-0.10	-0.10
EPI Contribution Adjustment Factor	1.45	1.55	1.28	1.32	1.26	1.23	1.23	1.23
Welfare of Age 20 Households	-0.20	-0.20	-0.18	-0.12	-0.07	-0.06	-0.06	-0.07

Note: ^a Changes in percentage points from the baseline economy.

increase by 0.18%. The consumption tax rate would decrease by 0.10 percentage points, and private consumption would increase by 0.36%.

The EPI contribution adjustment factor, $\psi_{p,t}$, would increase by 1.55% in year $t = 6$ and by 1.23% in the long run, which corresponds to a maximum increase of 2.33% in the pension contribution burden on employees. In the short run, households would reduce their current labor supply, leading to lower pension contributions, which contributes to a rise in $\psi_{p,t}$. In the long run, although households work more and contribute more to the pension system, the pension sector would still face a heavier financial burden compared to the baseline economy. This is because senior households would receive higher pension benefits, driven by higher labor supply and the removal of the pension adjustment. Overall, the results indicate that the Old-Age Pension System for Working Retirees has actually played a useful role in reducing the expenditure burden on the pension sector, although it discourages labor supply among elderly households.

1.4.3.3 The Impact on Welfare

The impact of removing pension adjustments and suspensions under the Old-Age Pension System for Working Retirees on welfare is limited compared to the previous two policies.

From Table 1.4, it can be seen that the welfare of future generations would be slightly worse off, with a decrease equivalent to 0.07% of the baseline consumption path. This decline is driven by the lower wage rate and the higher pension contribution burden. The policy change applies to households aged 59 and younger in year $t = 1$. As shown in Figure 1.13, welfare would fall for most households after the policy change. Households beyond age 77 would be worse off because they are exposed to the initial period of higher consumption tax rate and lower interest rate, but not live long enough to benefit from the later indirect effects. Households aged 61 to 76 experience a slight improvement in welfare, and the effect is strongest for those near age 70. This is partly because households in their 60s still pay pension contributions, and those closer to age 70 would face this higher contribution burden after the reform for

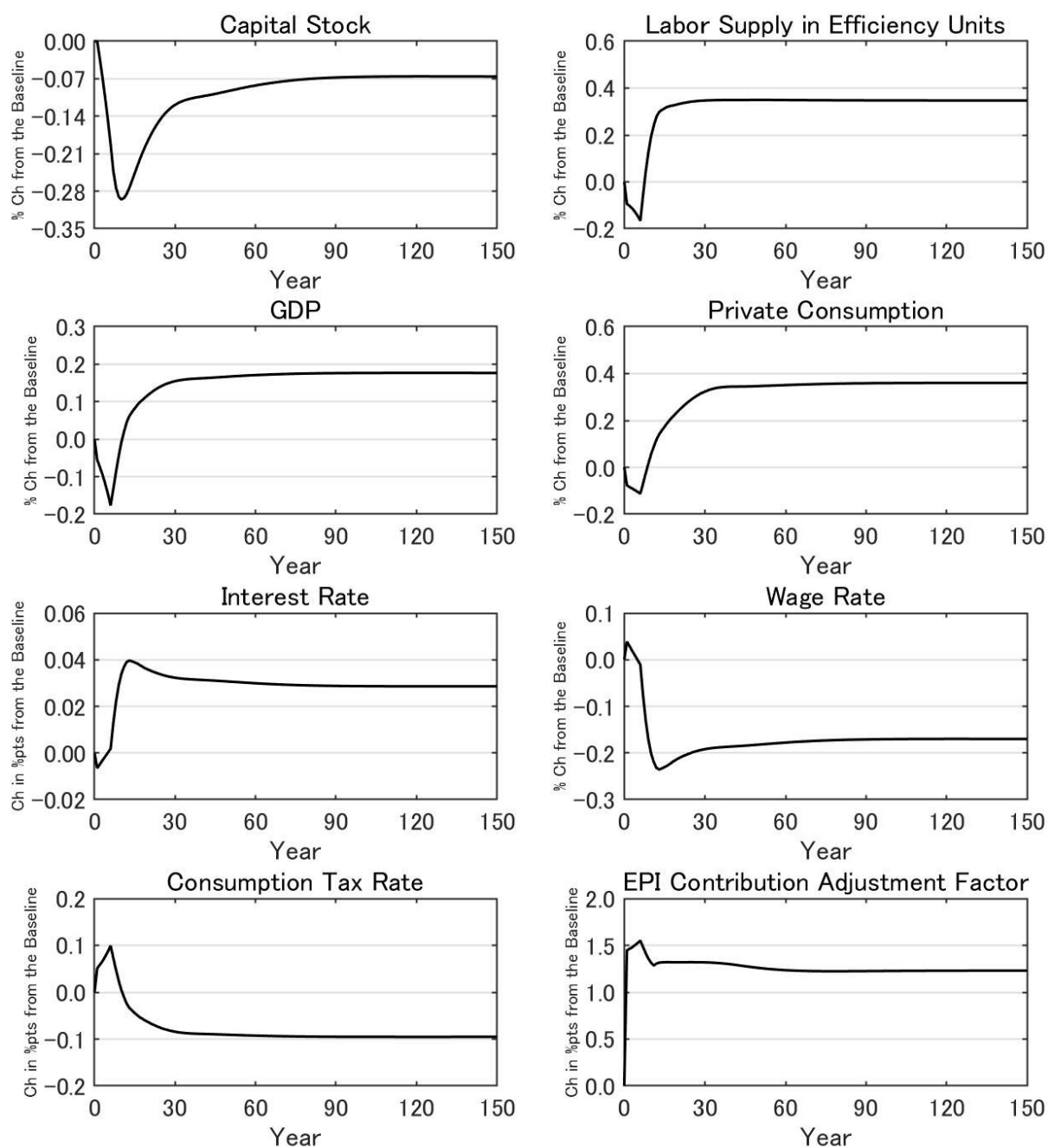


Figure 1.12: Transition Effects of Removing Pension Adjustments (%ch. from the baseline economy)

a shorter period.

In year $t = 1$, while the 59-year-old households experience the largest welfare gain of 0.18%, the magnitude of improvement declines with age and becomes negative for younger cohorts. The welfare of 20-year-old households born in year $t = 4$ would worsen the most under the policy reform, by 0.21% relative to the baseline consumption path. One possible explanation is that they would have missed the temporary increase in wage rate immediately after the policy announcement. Younger households would be less exposed to the initial period of higher consumption tax rate and lower interest rate gradually, resulting in smaller welfare loss.

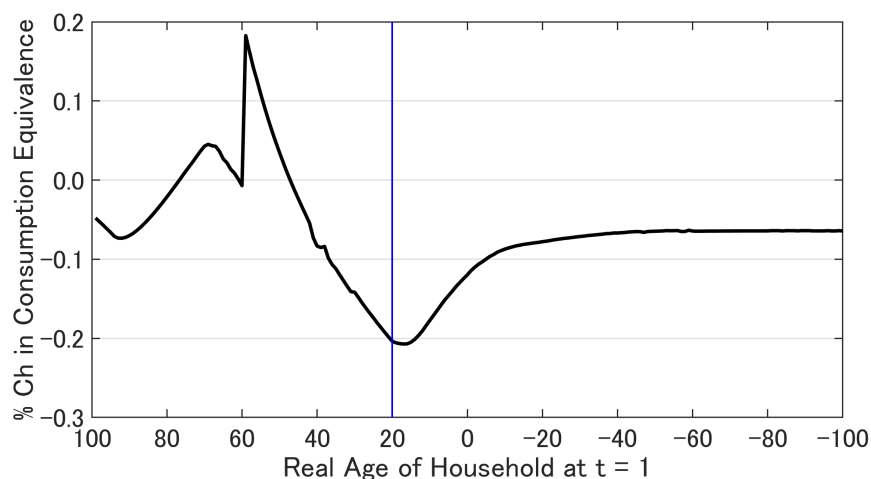


Figure 1.13: Welfare Effects of Removing Pension Adjustments (%ch. in consumption equivalence)

After the policy implementation, the Gini coefficient of pre-tax income decreases slightly from 0.6014 in the baseline to 0.6013. Most elderly households see a slight increase in inequality. Overall, it is somewhat surprising that removing pension adjustments and suspensions results in a slight reduction in welfare, rather than an improvement.

1.5 Concluding Remarks

This study extends a large-scale heterogeneous overlapping generations model to analyze the effects of three proposed reforms on labor supply, macroeconomy, and welfare in the Japanese pension system. The model developed in this study is calibrated based on the 2022 Japanese economy, capturing Japan's distinctive and complex income tax and social security systems. This approach not only provides a theoretically sound framework but also highlights the substantial quantitative impacts of the proposed reforms, ensuring more reliable simulation results.

The main findings of the present study are as follows. The first policy reform, a 15% reduction in public pension benefits, would increase capital stock by 6.1%, labor supply by 0.7%, and total output by 2.9% in the long run. As a result, employees' pension contribution burden would decline by up to 25.7% and the consumption tax rate would fall by 1.3 percentage points in the long run. Households aged between 25 and 58 at the time of policy change would be worse off on average, but future households would be better off on average by 5.0% in the consumption equivalence measure.

The second reform, a five-year increase in the pension eligibility age, would lead to a larger increase in labor supply, while its impacts on capital accumulation, output, consumption, and welfare would be smaller than those of the first reform. The reform would increase capital stock by 3.1%, labor supply by 1.9%, and total output by 2.4% in the long run, and it would reduce employees' pension contribution burden by 25.1% and the consumption tax rate by 1.4 percentage points. While labor supply increases

more than in the first reform, the impact on output and capital accumulation are smaller. One reason is that households tend to draw down their savings between age 65 and 70 to sustain consumption before becoming eligible for pensions, which dampens capital accumulation. Some households aged 35 to 59 at the time of policy change would be worse off on average, but future households would be better off on average by 2.8% in the consumption equivalence measure.

The third policy change, the removal of pension adjustments and suspensions in the Old-Age Pension System for Working Retirees, would decrease capital stock by 0.1%, increase labor supply by 0.4%, total output by 0.2% in the long run, and it would raise employees' pension contribution burden by 2.3% and reduce the consumption tax rate by 0.1 percentage points. Though the third reform generates a sizable impact to the labor supply of elderly households aged between 65 and 70, the overall effect on the economy is relatively small. Almost all households would be worse off in this pension reform, and future households would be slightly worse off on average by 0.1% in the consumption equivalence measure.

The results of the policy experiments show that cutting public pension benefits proportionately by 15% is desirable in terms of long-run economic growth and welfare. However, implementing such a policy in reality would be highly challenging due to political and social constraints, as it permanently reduces pension benefits for all households. In addition, this policy experiment suggests that if the Japanese government had set a lower target pension contribution rate during the 2004 pension reform, the economy might have been more prosperous today.

Raising the pension eligibility age by five years has a relatively smaller impact on economic growth and welfare, but leads to a larger increase in labor supply. It is also more politically acceptable, as the extension of the contribution-paid period results in higher pension benefits in the future. It is difficult to determine which of the two reforms is preferable. While the second reform may be desirable in helping to alleviate labor shortages, it is not a sustainable solution given health limitations among older households. As a more balanced approach, policies should aim to create an environment where households that are able and willing to work longer can continue working, instead of further raising the pension eligibility age. Moreover, when balancing pension finances, further increases in contribution rates to prevent benefit cuts may not always be required.

The third policy experiment, which removes pension adjustments and suspensions under the Old-Age Pension System for Working Retirees, has a relatively smaller impact compared with the other two reforms. Removing the adjustments and suspensions appears to increase labor supply among older households. However, the reform reduces households welfare, likely because it raises households' pension contribution burden.

To simplify the computation, this study adopts approximations for the basic exemption, employment income deduction, and pension income deduction. While this simplification introduces minor inaccuracy,

cies in labor income tax for low-income households and pension income tax for working elderly households, it ensures stable numerical convergence. Considering that social insurance contributions in Japan are deductible, this study incorporates a simplified version of health insurance in addition to public pension system. However, as the model does not account for the impact of health status on households' decision-making, health insurance primarily functions as a proportional tax in this framework.

This study does not incorporate gender heterogeneity. In Japan, wage disparities between men and women still exist, influencing pension income through differences in average historical earnings. As a result, households of different genders are likely to respond differently to public pension policy reforms. Incorporating marital status into the analysis would make it possible to capture Japan's complex spousal exemptions and the NP benefits for dependent spouses, which were not considered in this study but may significantly affect the results. Introducing gender and marital status, along with a more accurate reflection of deduction details, is left for future research.

While the model developed in this study focuses on the old-age pension system, it could also be applied to analyze the macroeconomic impacts of tax reforms, such as increases in the consumption tax rate. Additionally, the model could examine how the distribution of social insurance contribution rates between employers and employees, the effects of government cash transfers to households during the COVID-19 pandemic, and other policy changes influence the macroeconomy and welfare. If health status were incorporated, the model could be further extended to explore the impacts of health insurance reforms. In summary, expanding the model presented in this study would enable preliminary evaluations of various fiscal policy measures in Japan, providing valuable insights for policy recommendations.

1.A Small Open Economy

While the main text analyzes the model economy under a closed economy assumption, this appendix introduces a small open-economy environment in which international capital flows are allowed, and examines how the steady-state equilibrium changes after each policy reform. The Japanese economy lies between a closed and a small open economy, so both result serve as useful reference points for interpreting the policy effects.

In this small open economy, the domestic interest rate r is tied to the world interest rate r^* and therefore remains constant. This implies that the capital-labor ratio and the wage rate are also fixed, therefore the capital stock, which is the amount of capital required by domestic firms, labor supply and GDP change in the same rate. The policy outcomes are shown in Table A.

Here, CE and OE refer to the closed economy and open economy settings, respectively. Reforms 1-3 correspond to the the policy changes examined in the main text: a 15% reduction in pension benefits, a five-year increase in the pensionable age, and the removal of benefit adjustments under the Old-Age Pension System for Working Retirees.

In the open-economy setting, the wage rate changes observed under the closed economy disappear, leading to different labor supply responses in the long run. After the first policy experiment, the labor supply, which would originally increase by 0.7% under the closed economy compared to the baseline, would decrease by 0.3% instead. Private consumption and the welfare of future cohorts would increase, and the EPI contribution adjustment factor would decrease by 14.6%. Considering the overall macroeconomic effects, the reform is less desirable under the open economy assumption.

Table 1.A1: Long-run Effects Under Different Settings (%ch. from the baseline economy)

Variable	Reform 1		Reform 2		Reform 3	
	(CE)	(OE)	(CE)	(OE)	(CE)	(OE)
Capital Stock	6.1	-0.3	3.1	1.6	-0.06	0.43
Labor Supply	0.7	-0.3	1.9	1.6	0.35	0.43
Gross Domestic Product	2.9	-0.3	2.4	1.6	0.18	0.43
Private Consumption	2.7	2.5	3.0	3.0	0.36	0.39
Interest Rate ^a	-0.4	0.0	-0.1	0.0	0.03	0.00
Wage Rate	2.1	0.0	0.5	0.0	-0.17	0.00
Consumption Tax Rate ^a	-1.3	-1.1	-1.4	-1.3	-0.10	-0.11
EPI Contribution Adjustment Factor	-15.6	-14.6	-15.2	-14.9	1.23	1.15
Welfare of Age 20 Households	5.0	4.0	3.1	2.6	-0.07	0.01

Note: ^a Changes in percentage points from the baseline economy.

After the second policy experiment, although the magnitude would become smaller, its overall impact on the economy remained largely unchanged. Among the three policy experiments, only the third one performs better under the open economy assumption. One of the possible reasons is that, under the closed economy assumption, the wage rate would decrease by 0.17% in the long run, whereas this negative effect disappears under the open economy assumption. This leads to a 0.43% increase in labor supply in the long run. Households face a smaller increase in pension contribution burden, and the welfare would improve by the equivalent of a 0.01% increase in the baseline consumption path.

Chapter 2

Tax and Pension Reform, Female Labor Supply, and Economic Growth: A Dynamic General Equilibrium Approach

2.1 Introduction

In response to declining fertility rates and the associated labor shortages, as well as to advance gender equity, Japan has been pursuing policies to promote women's participation in the labor market. However, certain features of the current tax and social security systems may still discourage female labor supply. Examples include the Category 3 insured person system in the National Pension system and the (special) exemption for spouses under the income tax system. Under the Category 3 system, dependent spouses of employees are eligible to receive National Pension benefits without paying their own contributions. The (special) exemption for spouses provides a tax advantage to taxpayers with a dependent spouse whose income is below a certain threshold. Although these policies are gender-neutral in design, they tend to affect women more due to the wage gap between men and women. As a result, they may discourage women's labor force participation. The abolition of these two policies constitutes the first and second policy experiments in this study. A third policy experiment is the introduction of the family quotient system in income taxation. This system has recently attracted attention in Japan as a measure to address declining fertility. Under this system, household income is divided by an adjusted household size that includes children, with each child counted with weights lower than adults. While this policy implies an effective tax reduction for households with children, it may affect women's labor supply through changes in marginal tax rates within the household. This study quantitatively examines the macroeconomic effects of these policy reforms, with a particular focus on female labor supply.

Many studies since Auerbach and Kotlikoff (1987) have used large-scale OLG models to evaluate policy effects, however few studies have incorporated gender and marital status as dimensions of household heterogeneity. Guner et al. (2012) use a deterministic large-scale general equilibrium OLG model with gender and marital status to analyze the effects of gender-based tax reforms on female labor supply

and social welfare in the United States. Nishiyama (2019) uses a Bewley-type model to examine how the survivor pension system in the United States affects the labor supply of married households and the macroeconomy.

Regarding research on Japan, Kitao and Mikoshiba (2020) incorporate gender differences in labor force participation into a large-scale deterministic general equilibrium OLG model. Their study shows that higher labor productivity and greater participation among women and older individuals play a key role in mitigating the adverse macroeconomic effects of labor shortages. However, their study does not account for marital status. Kitao et al. (2023) further incorporate marital status and its transitions into the model and evaluate the macroeconomic effects of population aging in Japan, as well as several counterfactual scenarios. Labor supply from households are exogenous in this model. Kitao and Mikoshiba (2024) examine the effects of tax and social security policies—including the (special) exemption for spouses, the Category 3 insured status, and survivor pensions—on female labor force participation, employment type, and human capital accumulation. However, their study is conducted in a partial equilibrium framework.

In contrast, the present study employs a large-scale Bewley-type OLG model with general equilibrium, in which households are heterogeneous with respect to assets, age, working ability, average historical earnings, and marital status. It extends the model developed in Chapter 1 by incorporating gender, marital status, the survivors' pension system, and children. To account for features of Japanese labor market, the model also incorporates the extensive margin of labor supply for married women. By embedding Japanese tax and social security institutions in detail, the model can capture the effects of policy reforms on women's decisions and the macroeconomy more accurately.

Three pension reform policies are analyzed in this study: the abolition of the Category 3 insured person system under the National Pension scheme, the abolition of the (special) exemption for spouses in the income tax system, and introducing the family quotient system in income taxation. In the long run, abolishing the Category 3 insured person system would increase female labor supply by 1.33%. It would also increase capital stock by 0.60%, and GDP by 0.58%. Removing the (special) exemption for spouses would increase female labor supply by 0.81% but would decrease capital stock and GDP by 3.00% and 1.16%, respectively. Introducing the family quotient system would decrease female labor supply by 0.80%. It would also decrease capital stock by 1.16%, and GDP by 0.45%.

The rest of this study is structured as follows. Section 2.2 introduces the large-scale heterogeneous-agent OLG model employed in this study and defines the recursive competitive equilibrium of the model economy. Section 2.3 describes the model calibration and outlines the baseline economy. Section 2.4 presents the results of three policy experiments and analyzes their effects on the model economy. Finally, Section 2.5 concludes the study.

2.2 The Model

The model economy consists of a large number of heterogeneous and OLG households, a representative firm operating in a perfectly competitive market, and a government. The economy is on a balanced-growth path with a productivity growth rate μ and a population growth rate ν .

2.2.1 The Households

Households are heterogeneous with respect to their marital status, $m = \{0, 1, 2\}$; wealth, $a \in A = [0, a_{max}]$; the average historical earnings of wives (or single women), $b_1 \in B = [0, b_{max}]$; the average historical earnings of husbands (or single men), $b_2 \in B$; the working ability of wives (or single women), $e_1 \in E = [0, e_{max}]$; the working ability of husbands (or single men), $e_2 \in E$; and age, $i = 1, \dots, I$. Households enter the economy and begin working at $i = 1$, which corresponds to age 20 in the real world. Households retire from work and begin receiving old-age pension benefits at age $I_R = 46$ (correspond to a real age 65), and may live up to age $I = 80$ (correspond to a real age 99).

When entering the model, a fraction of households are assumed to marry and form married households. The others are assumed to remain single throughout their lifetime. The model incorporates age-specific mortality rates separately for men and women, which generate the possibility of widowhood in married households. If a spouse dies, the household becomes single and does not enter a new marriage. Divorce is not considered. For simplicity, individuals are assumed to survive with certainty until age 65. Given the low mortality rates for young households, the impact of this assumption is negligible. In this model, $m = 0$ is married households, $m = 1$ is single or widowed women, $m = 2$ is single or widowed men. Consumption c and wealth a are assumed to be shared within household members.

The individual state of a household is represented by $\mathbf{s} = (i, e_1, e_2, a, b_1, b_2, m)$, while the aggregate state of the economy in period t is represented by $\mathbf{S}_t = (x(\mathbf{s}), W_{G,t})$. Here, $x(\mathbf{s})$ is the population density function of households, and $W_{G,t}$ is the government's net worth at the beginning of period t .

The government's policy schedule at the beginning of period t is

$$\Psi_t = \{C_{G,s}, tr_{LS,s}, \tau_{c,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), \tau_{H,s}(\cdot), tr_{SS,s}(\cdot), q_s, W_{G,s+1}\}_{s=t}^{\infty},$$

where $C_{G,t}$ is the government's general expenditure, $tr_{LS,t}$ is lump-sum transfers to each household. $\tau_{c,t}$ is a flat tax rate on consumption, $\tau_{P,t}(\cdot)$ is the function that calculate household's contributions to Employees' Pension Insurance (EPI), $\tau_{H,t}$ is the function that calculate household's contributions to Health Insurance (HI). $tr_{SS,t}(\cdot)$ is the function that calculates the social security benefits households receive after reaching retirement age. $\tau_{I,t}(\cdot)$ is the function that calculate income tax collected from households. q_t is the uniform transfer received by each household, which in this model is accidental

bequests collected and distributed by the government. $W_{G,t+1}$ is the government's net worth at the beginning of period $t + 1$.

2.2.1.1 The Optimization Problem

Let $v(\mathbf{s}, \mathbf{S}_t; \Psi_t)$ be the household's value function at the beginning of period t . The optimization problem then is

$$v(\mathbf{s}, \mathbf{S}_t; \Psi_t) = \max_{c, h_1, h_2} \left\{ u(c, h_1, h_2; m, n_{m,i}) + \tilde{\beta} \sum_{m'=0}^2 \int_{E^2} v(\mathbf{s}', \mathbf{S}_{t+1}; \Psi_{t+1}) d\Pi_i(e'_1, e'_2, m' | e_1, e_2, m) \right\} \quad (2.1)$$

subject to the constraints for the decision variables,

$$\begin{aligned} c &> 0, \\ 0 &\leq h_1 \leq 1 - \eta_m \zeta n_{m,i}, \\ 0 &\leq h_2 \leq 1 - (1 - \eta_m) \zeta n_{m,i}. \end{aligned}$$

The laws of motion of the individual state variables are

$$\mathbf{s}' = (i + 1, e'_1, e'_2, a', b'_1, b'_2, m'), \quad (2.2)$$

$$\begin{aligned} a' &= \frac{1}{1 + \mu} \{ (1 + r_t) a + w_t e_1 h_1 + w_t e_2 h_2 + tr_{LS,t} + tr_{SS,t}(i, b_1, b_2; m) \\ &\quad - \tau_{P,t}(w_t e_1 h_1, w_t e_2 h_2; m, i) - \tau_{H,t}(w_t e_1 h_1, w_t e_2 h_2, tr_{SS,t}(\cdot); m) \\ &\quad - \tau_{I,t}(w_t e_1 h_1, w_t e_2 h_2, r_t a, tr_{SS,t}(\cdot); m) + (1 + \mathbf{1}_{\{m=0\}}) q_t - (1 + \tau_{c,t}) c \}, \end{aligned} \quad (2.3)$$

$$\begin{aligned} b'_j &= \mathbf{1}_{\{i \geq I_R\}} b_j + \mathbf{1}_{\{i < I_R, w_t e_j h_j > \vartheta_{min}\}} \frac{1}{i} \left[(i - 1) b_j \right. \\ &\quad \left. + \min \left\{ w_t e_j h_j - \frac{1}{2} [\tau_{P,t}(\cdot) + \tau_{H,t}(\cdot)], \vartheta_{1max} \right\} \right] \quad \text{if } j = 1 \text{ or } j = 2. \end{aligned} \quad (2.4)$$

Here, $u(\cdot)$ is the household's period utility function, h_1 is the working hour of female, h_2 is the working hour of male, $n_{m,i}$ is the number of children under age 6, varies depending on the household's marital status and age, $\tilde{\beta}$ is a growth-adjusted discount factor defined as $\tilde{\beta} = \beta (1 - \mu)^{\theta(1-\gamma)}$, where μ is the productivity growth rate. $\Pi_i(\cdot | \cdot)$ is the state transition function of households, which depends on the survival probability of household members at the end of age i given that they are alive at the beginning of age i . ζ is the average time a household spends in each period on raising one preschool-aged child,

For married household, child-care time is shared between spouses, η_m is the share of childcare time undertaken by females. For single households, this study set $\eta_{m \neq 0} = 1$, which implies that for single women all child-care time is allocated to herself, whereas for single men childcare time is not considered, since they are assumed not to have children, i.e., $n_{2,i} = 0$.

In this model, average historical earnings are used in calculating the benefits for employees' pensions. Household members earning less than ϑ_{min} in a given period are considered not enrolled in the employees' pension system, and their average historical earnings are not updated for that period. ϑ_{1max} is the upper limit of the standard remuneration used to calculate EPI benefits under the pension system.

The utility function of a single (unmarried) household is as follows:

$$\begin{aligned} u(c, h_1, h_2; m = j, n_{m,i}) &= \tilde{u}(c, h_j; n_{j,i}) \\ &= \begin{cases} \frac{\{c^\theta [1 - h_1 - \eta_j \zeta n_{j,i}]^{1-\theta}\}^{1-\gamma}}{1-\gamma} & \text{if } j = 1, \\ \frac{\{c^\theta [1 - h_2 - (1 - \eta_j) \zeta n_{j,i}]^{1-\theta}\}^{1-\gamma}}{1-\gamma} & \text{if } j = 2. \end{cases} \end{aligned} \quad (2.5)$$

The utility function of a married household is given by an additively separable form,

$$u(c, h_1, h_2; m = 0, n_{m,i}) = \tilde{u}\left(\frac{c}{1 + \lambda}, h_1; n_{0,i}\right) + \tilde{u}\left(\frac{c}{1 + \lambda}, h_2; n_{0,i}\right), \quad (2.6)$$

where married households operate under the assumption of joint consumption, the degree of joint consumption is $1 - \lambda \in [0, 1]$.

The optimization problem described above applies to single households and dual-earner households. To reflect features of the Japanese labor market, the model also allows married women to choose to leave the labor market and become full-time homemakers. In this case, married women's working hours are fixed at $h_1 = 0$, all child-care time is assigned to married women, with $\eta_0 = 1$. Households evaluate the lifetime utility they would obtain as dual-earner or full-time homemaker households. By comparing these two lifetime utilities, the share of married households in which the wife stay in the labor market is determined endogenously for each relevant state. The details of this mechanism are explained in Section 2.3.4. The state-specific outcome for married households are defined as the weighted average of the two types.

2.2.1.2 The State Transition Function

The state transition function considers changes in the household's status, including changes in working ability, e_j , and marital status, m . In this model, changes in marital status depend only on the survival probabilities of individuals within married households. Additionally, it is assumed that the survival probabilities of the husband and wife are independent of each other, and the same assumption applies to their

working abilities. The state transition function is

$$\Pi_i (e'_1, e'_2, m' | e_1, e_2, m) = \begin{cases} \pi_{e_1, i} (e'_1 | e_1) \pi_{e_2, i} (e'_2 | e_2) p_{m, i} (m' | m) & \text{if } m = 0, \\ \pi_{e_1, i} (e'_1 | e_1) p_{m, i} (m' | m) & \text{if } m = 1, \\ \pi_{e_2, i} (e'_2 | e_2) p_{m, i} (m' | m) & \text{if } m = 2, \end{cases} \quad (2.7)$$

where $\pi_{e_1, i} (\cdot)$ and $\pi_{e_2, i} (\cdot)$ are transition probability functions of working ability. $p_{m, j} (\cdot)$ are the transition probabilities of marital status,

$$\begin{aligned} p_{m, i} (0|0) &= \phi_{1, i} \phi_{2, i}, & p_{m, i} (1|0) &= \phi_{1, i} (1 - \phi_{2, i}), & p_{m, i} (2|0) &= (1 - \phi_{1, i}) \phi_{2, i}, \\ p_{m, i} (0|1) &= 0, & p_{m, i} (1|1) &= \phi_{1, i}, & p_{m, i} (2|1) &= 0, \\ p_{m, i} (0|2) &= 0, & p_{m, i} (1|2) &= 0, & p_{m, i} (2|2) &= \phi_{2, i}. \end{aligned}$$

Here, $\phi_{1, i}$ and $\phi_{2, i}$ are the conditional survival probabilities of female and male at the end of age i , respectively, given that the household member is alive at the beginning of age i .

2.2.1.3 The Household's Decision Rules

To solve the household's optimization problem for all possible states $\mathbf{s}$, this study first computes the household's decision rules $c(\mathbf{s}, \mathbf{S}_t; \Psi_t)$, $h_1(\mathbf{s}, \mathbf{S}_t; \Psi_t)$ and $h_2(\mathbf{s}, \mathbf{S}_t; \Psi_t)$ numerically. For married households, the decision rules are computed for both type of households described above. The remaining decision rules a' , b'_j are then calculated using the obtained values of c , h_1 and h_2 ,

$$\begin{aligned} a'(\mathbf{s}, \mathbf{S}_t; \Psi_t) &= \frac{1}{1 + \mu} \{ (1 + r_t) a + tr_{LS, t} + tr_{SS, t}(i, b_1, b_2; m) \\ &\quad + w_t e_1 h_1(\mathbf{s}, \mathbf{S}_t; \Psi_t) + w_t e_2 h_2(\mathbf{s}, \mathbf{S}_t; \Psi_t) \\ &\quad - \tau_{P, t}(w_t e_1 h_1(\mathbf{s}, \mathbf{S}_t; \Psi_t), w_t e_2 h_2(\mathbf{s}, \mathbf{S}_t; \Psi_t); m, i) \\ &\quad - \tau_{H, t}(w_t e_1 h_1(\mathbf{s}, \mathbf{S}_t; \Psi_t), w_t e_2 h_2(\mathbf{s}, \mathbf{S}_t; \Psi_t), tr_{SS, t}(\cdot); m) \\ &\quad - \tau_{I, t}(w_t e_1 h_1(\mathbf{s}, \mathbf{S}_t; \Psi_t), w_t e_2 h_2(\mathbf{s}, \mathbf{S}_t; \Psi_t), r_t a, tr_{SS, t}(\cdot); m) \\ &\quad + (1 + \mathbf{1}_{\{m=0\}}) q_t - (1 + \tau_{c, t}) c(\mathbf{s}, \mathbf{S}_t; \Psi_t) \}, \end{aligned} \quad (2.8)$$

$$\begin{aligned} b'_j(\mathbf{s}, \mathbf{S}_t; \Psi_t) &= \mathbf{1}_{\{i \geq I_R\}} b_j + \mathbf{1}_{\{i < I_R, w_t e_j h_j(\mathbf{s}, \mathbf{S}_t; \Psi_t) > \vartheta_{min}\}} \frac{1}{i} \left[(i - 1) b_j \right. \\ &\quad \left. + \min \left\{ w_t e_j h_j(\mathbf{s}, \mathbf{S}_t; \Psi_t) - \frac{1}{2} [\tau_{P, t}(\cdot) + \tau_{H, t}(\cdot)], \vartheta_{1max} \right\} \right] \quad \text{if } j = 1 \text{ or } j = 2. \end{aligned} \quad (2.9)$$

2.2.1.4 The Distribution of Households

Let $x_t(\mathbf{s})$ be the population density function of households at period t , and $X_t(\mathbf{s})$ be the cumulative distribution function. Households entering the economy at age 20 have no assets or earning history, thus $a = b_1 = b_2 = 0$. However, each household has different work abilities e_j . A proportion ω of households enter the economy in a married state, while the remaining $1 - \omega$ stay single throughout their lives. Furthermore, the total population of 20-year-old females and males are each normalized to unity. The number of 20-year-old households with marital status $m = 0$, $m = 1$ and $m = 2$ are

$$\int_{E^2 \times A \times B^2} dX_t(1, e_1, e_2, a, b_1, b_2, m = 0) = \int_{E^2} dX_t(1, e_1, e_2, 0, 0, 0, 0) = \omega, \quad (2.10)$$

$$\int_{E^2 \times A \times B^2} dX_t(1, e_1, e_2, a, b_1, b_2, m = 1) = \int_E dX_t(1, e_1, 0, 0, 0, 0, 1) = 1 - \omega, \quad (2.11)$$

$$\int_{E^2 \times A \times B^2} dX_t(1, e_1, e_2, a, b_1, b_2, m = 2) = \int_E dX_t(1, 0, e_2, 0, 0, 0, 2) = 1 - \omega. \quad (2.12)$$

Considering the population growth rate ν and the state transition function $\Pi_i(e'_1, e'_2, m' | e_1, e_2, m)$, the law of motion of the growth-adjusted population distribution is

$$x_{t+1}(\mathbf{s}') = \frac{1}{1 + \nu} \sum_{m=0}^2 \int_{E^2 \times A \times B^2} \mathbf{1}_{\{a'=a'(\mathbf{s}, \mathbf{S}_t; \Psi_t), b'_1=b'_1(\mathbf{s}, \mathbf{S}_t; \Psi_t), b'_2=b'_2(\mathbf{s}, \mathbf{S}_t; \Psi_t)\}} \pi_i(e'_1, e'_2, m' | e_1, e_2, m) dX_t(\mathbf{s}), \quad (2.13)$$

where $\pi_i(e'_1, e'_2, m' | e_1, e_2, m)$ is the transition probability density function of working abilities e_1, e_2 and marital state m from age i to age $i + 1$.

2.2.1.5 The Supply of Capital and Labor

This study assumes a closed economy, where the capital stock K_t is the sum of total private wealth $W_{P,t}$ and the government's net worth $W_{G,t}$. The total labor supply is the aggregate efficient labor provided by all households. Their functional forms are

$$W_{P,t} = \sum_{m=0}^2 \sum_{i=1}^I \int_{E^2 \times A \times B^2} a dX_t(\mathbf{s}), \quad K_t = W_{P,t} + W_{G,t}, \quad (2.14)$$

$$L_t = \sum_{m=0}^2 \sum_{i=1}^{I_R-1} \int_{E^2 \times A \times B^2} (e_1 \bar{h}_1(\mathbf{s}, \mathbf{S}_t; \Psi_t) + e_2 \bar{h}_2(\mathbf{s}, \mathbf{S}_t; \Psi_t)) dX_t(\mathbf{s}), \quad (2.15)$$

where $\bar{h}_j$ are the weighted average working hours over the two employment-status types of wives for married households.

2.2.2 The Representative Firm

The representative firm employs capital and labor as production factors and operates according to a Cobb-Douglas production function

$$Y_t = A \tilde{K}_t^\alpha \tilde{L}_t^{1-\alpha}, \quad (2.16)$$

where A is the total factor productivity, $\tilde{K}$ and $\tilde{L}$ are the capital input and labor input in efficiency units chosen by the firm. Letting δ be the depreciation rate, the firm's profit maximizing conditions are

$$\alpha A \left(\frac{\tilde{K}_t}{\tilde{L}_t} \right)^{\alpha-1} - (r_t + \delta) = 0, \quad (2.17)$$

$$(1 - \alpha) A \left(\frac{\tilde{K}_t}{\tilde{L}_t} \right)^\alpha - w_t = 0. \quad (2.18)$$

Given the assumption of a closed economy, the factor markets clear such that $\tilde{K}_t = K_t$ and $\tilde{L}_t = L_t$. Gross domestic product, which is here identical to gross national product Y_t is

$$Y_t = F(K_t, L_t) = (r_t + \delta)(W_{P,t} + W_{G,t}) + w_t L_t. \quad (2.19)$$

2.2.3 The Government

2.2.3.1 Government General Budget

This section defines the government revenue and expenditure in this model. For married households, weighted averages are first taken over the two employment-status types of wives, and then aggregated over all states. The public pension system contribution revenue $T_{P,t}$ is

$$T_{P,t}(\bar{\tau}_{p,t}) = \sum_{m=0}^2 \sum_{i=1}^{I_R-1} \int_{E^2 \times A \times B^2} \tau_{P,t}(w_t e_1 h_1(\mathbf{s}, \mathbf{S}_t; \Psi_t), w_t e_2 h_2(\mathbf{s}, \mathbf{S}_t; \Psi_t); m, i, \bar{\tau}_{p,t}) dX_t(\mathbf{s}), \quad (2.20)$$

where $\bar{\tau}_{p,t}$ is the contribution rate of Employees' Pension Insurance. The public pension system benefit expenditure $TR_{SS,t}$ is

$$TR_{SS,t}(\psi_t) = \sum_{m=0}^2 \sum_{i=I_R}^I \int_{E^2 \times A \times B^2} tr_{SS,t}(i, b_1, b_2; m, \psi_t) dX_t(\mathbf{s}), \quad (2.21)$$

where ψ_t is the pension sector balance factor, adjusted to ensure that the pension sector budget is always balanced.

The Healthcare Insurance contribution revenue $T_{H,t}$ is

$$T_{H,t}(\bar{\tau}_{h,t}) = \sum_{m=0}^2 \sum_{i=1}^I \int_{E^2 \times A \times B^2} \tau_{H,t}(w_t e_1 h_1(\mathbf{s}, \mathbf{S}_t; \Psi_t), w_t e_2 h_2(\mathbf{s}, \mathbf{S}_t; \Psi_t), tr_{SS,t}(\cdot); m, \bar{\tau}_{h,t}) dX_t(\mathbf{s}), \quad (2.22)$$

where $\bar{\tau}_{h,t}$ is the contribution rate of Health Insurance. The government's income tax revenue $T_{I,t}$ is

$$T_{I,t}(\varphi_t, \bar{\tau}_{k,t}, \bar{\tau}_{ss,t}) = \sum_{m=0}^2 \sum_{i=1}^I \int_{E^2 \times A \times B^2} \tau_{I,t}(w_t e_1 h_1(\mathbf{s}, \mathbf{S}_t; \Psi_t), w_t e_2 h_2(\mathbf{s}, \mathbf{S}_t; \Psi_t), r_t a, tr_{SS,t}(\cdot); m, \varphi_t, \bar{\tau}_{k,t}, \bar{\tau}_{ss,t}) dX_t(\mathbf{s}), \quad (2.23)$$

where φ_t is a parameter of labor income tax function, representing the highest progressive tax rate faced by households. $\bar{\tau}_{k,t}$ is the interest income tax rate, $\bar{\tau}_{ss,t}$ is the pension income tax rate. The consumption tax revenue $T_{C,t}$ is

$$T_{C,t}(\tau_{c,t}) = \tau_{c,t} \sum_{m=0}^2 \sum_{i=1}^I \int_{E^2 \times A \times B^2} c(\mathbf{s}, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}), \quad (2.24)$$

and the lump-sum transfer spending $TR_{LS,t}$ is

$$TR_{LS,t}(tr_{LS,t}) = \sum_{m=0}^2 \sum_{i=1}^I \int_{E^2 \times A \times B^2} tr_{LS,t} dX_t(\mathbf{s}). \quad (2.25)$$

The law of motion of the government's net worth $W_{G,t+1}$ is

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} \left[(1+r_t) W_{G,t} + T_{I,t} + T_{C,t}(\tau_{c,t}) - TR_{LS,t}(tr_{LS,t}) + T_{P,t} - TR_{SS,t} + T_{H,t} - C_{G,t} \right]. \quad (2.26)$$

2.2.3.2 Accidental Bequests

To simplify the analysis, it is assumed that when a household with assets decreases, those assets are collected by the government and distributed as a lump sum transfer to all households within the same period. Let Q_t be the total accidental bequests collected by the government in period t , then

$$Q_t = \sum_{m=0}^2 \sum_{i=1}^I \int_{E^2 \times A \times B^2} (1 - \phi_{1,i}) (1 - \phi_{2,i}) (1 + \mu) \bar{a}'(\mathbf{s}, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}). \quad (2.27)$$

For married households, $\bar{a}'(\mathbf{s}, \mathbf{S}_t; \Psi_t)$ is the weighted average of end-of-period wealth over households in state $\mathbf{s}$. The accidental bequest q_t each household receives is

$$q_t = \left(\sum_{m=0}^2 \sum_{i=1}^I \int_{E^2 \times A \times B^2} (1 + \mathbf{1}_{\{m=0\}}) dX_t(\mathbf{s}) \right)^{-1} Q_t. \quad (2.28)$$

2.2.4 Recursive Competitive Equilibrium

The recursive competitive equilibrium in this economy is defined as follows.

Definition: Recursive Competitive Equilibrium. Let $\mathbf{s} = (i, e_1, e_2, a, b_1, b_2, m)$ be the individual state of households, $\mathbf{S}_t = (x(\mathbf{s}), W_{G,t})$ be the aggregate state of the economy, Ψ_t be the government policy schedule at the beginning of period t ,

$$\Psi_t = \{C_{G,s}, tr_{LS,s}, \tau_{c,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), \tau_{H,s}(\cdot), tr_{SS,s}(\cdot), q_s, W_{G,s+1}\}_{s=t}^{\infty},$$

A time series of the factor prices and government policy variables,

$$\Omega_t = \{r_s, w_s, \varphi_s, \tau_{c,s}, \bar{\tau}_{k,s}, \bar{\tau}_{ss,s}, \bar{\tau}_{p,s}, \bar{\tau}_{h,s}, tr_{LS,s}, \psi_s, q_s, C_{G,s}, W_{G,s+1}\}_{s=t}^{\infty},$$

the value functions of households, $\{v(\mathbf{s}, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty}$, the distribution of households, $\{x_s(\mathbf{s})\}_{s=t}^{\infty}$, and the decision rules of households

$$d(\mathbf{s}, \mathbf{S}_s; \Psi_s) = \{c(\mathbf{s}, \mathbf{S}_s; \Psi_s), h_1(\mathbf{s}, \mathbf{S}_s; \Psi_s), h_2(\mathbf{s}, \mathbf{S}_s; \Psi_s), \\ a'(\mathbf{s}, \mathbf{S}_s; \Psi_s), b'_1(\mathbf{s}, \mathbf{S}_s; \Psi_s), b'_2(\mathbf{s}, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty},$$

are in a recursive competitive equilibrium if, for all $s = t, \dots, \infty$,

1. each household chooses $d(\mathbf{t}, \mathbf{S}_t; \Psi_t)$ to maximize her lifetime utility, taking factor prices and the government policy as given;
2. firm chooses its factor inputs $(\tilde{K}_t, \tilde{L}_t)$ and maximizes its own profit;

3. factor markets clear under the factor prices (r_t, w_t) ;
4. the government budget constraint is satisfied;
5. by Walras's Law, the aggregate resource constraint is satisfied.

The economy is in a steady-state equilibrium, thus on the balanced-growth path, if $\mathbf{S}_s = \mathbf{S}_{s+1}$ and $\Psi_s = \Psi_{s+1}$ for all $s = t, \dots, \infty$.

2.3 Calibration

The baseline economy is calibrated using statistics from Japan in 2022. The retirement age is set at 65, other institutional features follow the current Japanese system. Key parameter values are summarized in Table 2.1.

2.3.1 Demographics

In Japan, individuals join the public pension system when they turn 20. Reflecting this, households in this model enter the economy at age 20 ($i = 1$). They are assumed to live for a maximum of 80 periods ($I_{max} = 80$), which means the model includes individuals up to age 99. Households retire and start to receive pension benefits at $I_1 = 46$ (age 65).

For the productivity growth rate μ and the population growth rate ν , this study follows Chapter 1 and sets $\mu = 0.70\%$ and $\nu = -0.11\%$. According to the Population Statistics (2023 Edition) published by the National Institute of Population and Social Security Research, the lifetime unmarried rate in 2020 was approximately 28% for men and 18% for women. Based on this information, the share of individuals who are married at age 20 in the model is set to $\omega = 0.8$. The conditional survival probability $\phi_{j,i}$ for $j = 1$ and $j = 2$, which is the probability that an individual of gender j at age i survives to age $i + 1$, is derived from the 23rd Life Tables (2020) published by the Ministry of Health, Labour and Welfare of Japan.

2.3.2 Preference

In this study, the coefficient of relative risk aversion is set to $\gamma = 3.0$. The Frisch elasticity of labor supply with respect to wage changes is calibrated on the working hours of the average worker. An average worker is defined as an individual aged 20 to 64 in this model who supplies the average labor hours. Let $\bar{h}$ be average working hours. The Frisch elasticity is determined using the following condition

$$\frac{1 - \bar{h}}{\bar{h}} \frac{1 - \theta(1 - \gamma)}{\gamma} = 0.4955, \quad (2.29)$$

Table 2.1: Parameters and Values

Parameter		Value	Comment
Maximum Possible Age	I	80	Corresponds to a real-life age of 99
Retirement Age	I_1	46	
Productivity Growth Rate	μ	0.0070	See Chapter 1
Population Growth Rate	ν	-0.0011	See Chapter 1
Individual Survival Rates	$\phi_{j,i}$		Based on the 23rd Life Tables 2020
Labor Participation Responsiveness	χ	2.0000	Match the share of full-time housewife
Marriage Rate of Newborn households	ω	0.8000	Based on the 2020 lifetime unmarried rate
Number of Children	$n_{m,i}$		Based on the 2022 age-specific fertility rates
Share Parameter of Capital Stock	α	0.4124	See Chapter 1
Discount Factor	β	0.9966	$K/Y = 3.5$ in the baseline
Coefficient of Relative Risk Aversion	γ	3.0000	
Depreciation Rate of Capital Stock	δ	0.0628	SNA data $r = 0.0550$ in the baseline
Share Parameter of Consumption	θ	0.6500	Frisch elasticity 0.4955 for the average worker
Autocorrelation Parameter of Log Income	ρ	0.9500	See Chapter 1
Standard Deviation of Log Income Shocks	σ	0.2200	See Chapter 1
Childcare Time per Preschool-Aged Child	κ	0.3125	Based on the 2022 time use data
Childcare Time Share by Women	η_m		Based on the 2022 time use data ^c
Average Earning Ability	$\bar{e}_{j,i}$		Estimated by OLS with 2022 wage data
Full Amount of NP Benefit	np	0.0867	¥777,800 in 2022 ^a
Lower Limit of the Standard	ϑ_{min}	0.1177	$12 \times ¥88,000 = ¥1,056,000$ in 2022 ^a
Remuneration y_P			
Upper Limit of the Standard	ϑ_{1max}	1.3708	$12 \times ¥650,000 + 3 \times ¥1,500,000$ $= ¥12,300,000$ in 2022 ^a
Remuneration y_P			
Converted ϑ_{1max}	ϑ_{2max}	1.5651	
Converted Upper Limit of the Standard	ϑ_{3max}	2.7479	$12 \times ¥1,390,000 + ¥5,730,000$ $= ¥22,410,000$ in 2022 ^a
Remuneration of the HI ^b			
Benefit Multiplier of the EPI	ξ	0.0055	
Income Tax Parameters: Tax Rate Limit	φ_t	0.4500	Maximum individual income tax rate
: Curvature	φ_1	1.0621	} Estimated by OLS
: Scale	φ_2	0.6457	
Consumption Tax Rate	$\tau_{c,t}$	0.1000	
Interest Income Tax Rate	$\bar{\tau}_{k,t}$	0.1920	$T_I/Y = 7.2\%$ in the Baseline
Pension Income Tax Rate	$\bar{\tau}_{ss,t}$	0.0500	
Converted EPI Contribution Rate	$\bar{\tau}_{p,t}$	0.1603	18.3% in reality
National Pension Contribution	$\bar{\tau}_{np,t}$	0.0215	
Converted HI Contribution Rate ^b	$\bar{\tau}_{h,t}$	0.0876	10% average contribution rate
Government Consumption	$C_{G,t}$	17.4276	$C_G = T_C + T_I + T_H - TR_{LS}$ in the baseline
Lump-sum Transfers	$tr_{LS,t}$	0.0000	
Pension Sector Balance Factor	ψ_t	1.5541	
Total Factor Productivity	A	0.8153	$w = 1.0$ in the baseline
Accidental Bequests per Households	q_t	0.0102	

Note: ^a One unit of income in the model economy equals to 8,972,695 Japanese Yen.

^b Varies depending on the household's age and labor income.

^c Varies depending on marital status.

Here, $\bar{h} = 0.6074$ is the model-implied average working hours, computed as the average working hours of men and working women. The consumption share parameter in the Cobb-Douglas utility function is set to $\theta = 0.6500$ to make the Frisch elasticity close to 0.5.

The number of children $n_{m,i}$ in each household refers to the number of preschool-aged children. This

is calculated using the age-specific fertility rates for women in 2022, defined as the number of births per female population, published by the National Institute of Population and Social Security Research. Based on Table 7-1 of the 2021 Survey on Time Use and Leisure Activities published by the Statistics Bureau of Japan, this study obtains the average childcare time by gender and five-year age group, where childcare refers specifically to the time spent caring for preschool-aged children. As a result, the time required per preschool-aged child is set to $\xi = 0.3125$, corresponding to 5 hours per day. Using the same data, this study computes the ratio of average childcare time between women and men among married individuals. The resulting ratios indicate that childcare time among women in their 20s is more than five times that of men, while among those in their 30s it is approximately 3.5 times that of men. Based on these figures, the female share of childcare time in married households is set to $\eta_{m=0} = 0.8$.

Assuming that the number of children differs between married women (wives) and single women (including never-married, divorced, and widowed), this study calculates the average childcare time separately for all women of child-rearing age and for single women only, still using the same data source. The results are 41 minutes and 5 minutes per day, respectively. Taking the marriage rate $\omega = 0.8$ into account, this study assigns households the following number of children: married households are assumed to have 1.225 times the number of children derived from the age-specific fertility rate, while single women are assumed to have 0.1 times the number. This adjustment preserves the age-specific fertility rate across female population at the data level.

2.3.3 Production Technology

For the capital share parameter α , this study sets $\alpha = 0.4124$ follows Chapter 1. The capital coefficient, defined as the ratio of fixed capital stock to GDP, averaged 3.5194 between 2003 and 2022, and 3.4355 between 2000 and 2019. Reflecting these values, the baseline model sets the discount factor β at 0.9966, which yields a steady-state capital-output ratio of 3.5. With $\theta = 0.6500$ and $\gamma = 3.0$, the growth-adjusted discount factor is

$$\tilde{\beta} = \beta (1 + \mu)^{\theta(1-\gamma)} = 0.9876. \quad (2.30)$$

The consumption of fixed capital as a proportion of the fixed capital stock was 0.0629 in 2022. To ensure that the interest rate r equals 0.0550 in the baseline, the depreciation rate δ is finely adjusted to 0.0628,

$$r = \alpha A \left(\frac{K_t}{L_t} \right)^{\alpha-1} - \delta = \alpha \frac{Y}{K} - \delta = 0.0549, \quad (2.31)$$

$$\delta = \alpha \frac{Y_t}{K_t} - 0.0550 = \frac{0.4124}{3.5} - 0.0550 = 0.0628. \quad (2.32)$$

To normalize the baseline wage rate w to 1.0, the total factor productivity (TFP) parameter A is set to 0.8153, as it satisfies the following condition

$$w = (1 - \alpha) A \left(\frac{K_t}{L_t} \right)^\alpha = (1 - \alpha)^{1-\alpha} A \left(\frac{K_t}{Y_t} \right)^\alpha = 1, \quad (2.33)$$

$$A = \left[(1 - 0.4124)^{(1-0.4124)} (3.5)^{0.4124} \right]^{-1} = 0.8153. \quad (2.34)$$

2.3.4 Working Ability

The calibration of age-dependent working ability is based on data from Table 1 of the 2022 Basic Survey on Wage Structure, conducted by the Ministry of Health, Labour and Welfare of Japan. The calibration method follows exactly the same as in Chapter 1. The only difference is that this study computes hourly labor income separately for men and for women. Using the hourly labor income data by age group, separate OLS regressions by gender are conducted, where hourly labor income is regressed on a cubic polynomial in age. Since households in this study are assumed to retire at age 65, the working ability profile is adjusted so that it reaches zero at the retirement age for both males and females. Figure 2.1 shows the raw data and the estimated age-efficiency profiles.

Next, this study computes the average hourly labor income for individuals aged 20 to 64 by pooling male and female data from the original 2022 Basic Survey on Wage Structure. This pooled average is used to normalize the age-specific OLS estimates, which produces the working ability profiles in this model. The average working ability over ages 20 to 64 is 0.8 for females and 1.1 for males.

Household labor productivity is subject to fluctuations arising from individual circumstances and external shocks. In this model, the working ability of a household member at age i is modeled as:

$$\ln e_{j,i} = \ln \bar{e}_{j,i} + \ln z_i, \quad (2.35)$$

where i ranges from 1 to 45. $\bar{e}_{j,i}$ is the previously estimated average working ability where $j = 1$ for female and $j = 2$ for male. z_i is a persistent shock assumed to follow an AR(1) process. Specifically,

$$\ln z_i = \rho \ln z_{i-1} + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2). \quad (2.36)$$

Follows Chapter 1, the autocorrelation parameter ρ is set to 0.95, and the standard deviation σ is 0.22. Within married households, working abilities evolve independently. However, when newborn cohort at age 20 forms married households, 80% of couples are formed by spouses with the same working ability

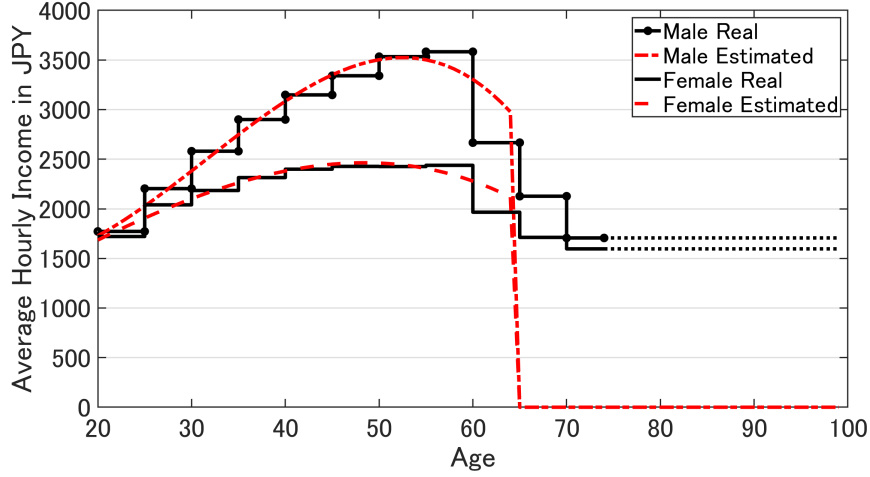


Figure 2.1: Average Hourly Income

level, while the remaining couples are matched randomly.

The AR(1) process is approximated using a Markov chain. $\ln z_i$ is discretized into five nodes, and the Markov transition matrix is constructed using Rouwenhorst's method. The unconditional probability distribution is given by $\pi_i = (0.0625, 0.2500, 0.3750, 0.2500, 0.0625)$ for $i = 1, \dots, 45$. The Markov transition matrix is

$$\Pi_i = \begin{pmatrix} 0.90369 & 0.02317 & 0.00059 & 0.00002 & 0.00000 \\ 0.09269 & 0.90547 & 0.04637 & 0.00178 & 0.00006 \\ 0.00357 & 0.06956 & 0.90607 & 0.06956 & 0.00357 \\ 0.00006 & 0.00178 & 0.04637 & 0.90547 & 0.09269 \\ 0.00000 & 0.00002 & 0.00059 & 0.02317 & 0.90369 \end{pmatrix}.$$

The fraction of households in which wives choose to remain in the labor market is given by

$$p_r(\mathbf{s}, \mathbf{S}_t; \Psi_t) = \max \left[0.1, 1 - e^{-\chi \max(0, v^h(\mathbf{s}, \mathbf{S}_t; \Psi_t) - v^0(\mathbf{s}, \mathbf{S}_t; \Psi_t))} \right], \quad (2.37)$$

where $v^h(\cdot)$ is the lifetime utility of a household in which the wife remains in the labor market, and $v^0(\cdot)$ is the lifetime utility of a household in which the wife chooses to be a housewife in the current period. For numerical stability, $p_r(\cdot)$ is bounded below by 0.1. For households that reach retirement age, $p_r(\cdot)$ is fixed at 1. Using the data from the 2022 Labour Force Survey published by the Statistics Bureau of Japan, the housewife rate, defined as the share of married households with an working husband and a non-employed wife aged 15-64, is calculated as 25.3%. In the model, the parameter χ , which controls the responsiveness of p_r , is set to $\chi = 2.0$, yielding a housewife rate of 24.7% in the model, which is close to the value observed in the data.

2.3.5 Taxation

In this model, household income is categorized into three sources: labor income, capital income, and pension benefits. Capital income is taxed at a uniform rate, while pension benefits are subject to a uniform tax rate combined with progressive deductions. Labor income taxation follows the function introduced by Gouveia and Strauss (1994). The income tax function is

$$\begin{aligned}
 \tau_{I,t}(w_t e_1 h_1, w_t e_2 h_2, r_t a, tr_{SS,t}(\cdot); m) &= \tilde{\tau}_{I,t}(\tilde{y}_1, \tilde{y}_2, r_t a, tr_{SS,t}(\cdot); m) \\
 &= \tau_{K,t}(r_t a) + \tau_{SS,t}(i, b_1, b_2; m) \\
 &\quad + \varphi_t \left[\tilde{y}_1 - \left(\tilde{y}_1^{-\varphi_1} + \varphi_2 \right)^{-\frac{1}{\varphi_1}} \right] \\
 &\quad + \varphi_t \left[\tilde{y}_2 - \left(\tilde{y}_2^{-\varphi_1} + \varphi_2 \right)^{-\frac{1}{\varphi_1}} \right],
 \end{aligned} \tag{2.38}$$

where

$$\tau_{K,t}(r_t a) = \bar{\tau}_{k,t} r_t a, \tag{2.39}$$

$$\begin{aligned}
 \tilde{y}_1 &= \max \{ w_t e_1 h_1 - d_1(w_t e_1 h_1) - d_2(w_t e_1 h_1, w_t e_2 h_2; m) \\
 &\quad - \tilde{\tau}_{P,t}(w_t e_1 h_1) - \tilde{\tau}_{H,t}(w_t e_1 h_1), 0 \},
 \end{aligned} \tag{2.40}$$

$$\begin{aligned}
 \tilde{y}_2 &= \max \{ w_t e_2 h_2 - d_1(w_t e_2 h_2) - d_2(w_t e_1 h_1, w_t e_2 h_2; m) \\
 &\quad - \tilde{\tau}_{P,t}(w_t e_2 h_2) - \tilde{\tau}_{H,t}(w_t e_2 h_2), 0 \}.
 \end{aligned} \tag{2.41}$$

Here, $\bar{\tau}_{k,t}$ is the capital income tax rate, $\tilde{y}_j$ is the taxable labor income of household member j . For a single household, since there is only one member, only that individual's labor income is counted for income tax calculation. $d_1(\cdot)$ is the combination of the basic exemption and the employment income deduction, $d_2(\cdot)$ is the (special) exemption for spouses, determined by both the household member j 's own income and the spouse's income within this framework.

Since survivors' pension is tax-exempt, it is excluded from the calculation of taxable pension income. Hence, the pension income tax for a one-adult household is

$$\begin{aligned}
 \tau_{SS,t}(i, b_1, b_2; m = j) &= \tilde{\tau}_{SS,t}(i, b_j) \\
 &= \bar{\tau}_{ss,t} \left[\mathbf{1}_{\{i \geq I_R\}} \psi_t (np + \xi b_j I_{Pj}) (1 + \mu)^{I_R - i} \right. \\
 &\quad \left. - d_3 \left(\mathbf{1}_{\{i \geq I_R\}} (np + \xi b_j I_{Pj}) (1 + \mu)^{I_R - i}, \psi_t \right) \right],
 \end{aligned} \tag{2.42}$$

where $\bar{\tau}_{ss,t}$ is the pension income tax rate, $d_3(\cdot)$ is the progressive deduction applied to pension benefits.

Define $\tilde{tr}_{SS,t}(i, b_j; m)$ as the pension benefits received by an individual, the pension income tax for a married household is

$$\begin{aligned} \tau_{SS,t}(i, b_1, b_2; m = 0) = & \sum_{j=1}^2 \bar{\tau}_{ss,t} [\tilde{tr}_{SS,t}(i, b_j; m = 0) \\ & - d_2(\tilde{tr}_{SS,t}(i, b_j; m = 0), \tilde{tr}_{SS,t}(i, b_{3-j}; m = 0)) \\ & - d_3(\tilde{tr}_{SS,t}(i, b_j; m = 0))] \end{aligned} \quad (2.43)$$

Here, the (special) exemption for spouses $d_2(\cdot)$ also applies to pension income.

To capture the deductibility of social insurance contributions, this study adopts the same procedure as Chapter 1, which discretizes earnings into a series of income nodes and calculates the corresponding contribution amounts at each level. Labor income in this model is defined as the sum of salaries and both the employee and employer shares of social insurance contributions. A series of taxable labor income nodes are set following the same procedure as Chapter 1. Using the official income tax rate table provided by Japan's National Tax Agency, the corresponding income tax amount is determined for each nodes. The parameter $\varphi_t = 0.45$ is chosen to replicate the top marginal tax rate of 45% under the current Japanese income tax system.

All monetary amounts are standardized before estimating the labor income tax function. This study uses the average annual labor income of individuals combining both female and male, aged 20 to 64, which in the model is 0.6435. Based on Table 1 of the 2022 Basic Survey on Wage Structure, and counting employer-paid social insurance contributions as part of labor income, the average annual earnings for households combining both female and male aged 20 to 64 is estimated to be 5,774,288 yen. Dividing this by the average annual labor income in the model, it is determined that one model unit corresponds to 8,972,695 yen in reality. This factor is then used for standardization.

Using the standardized values, the two remaining parameters in the Gouveia and Strauss (1994) income tax function are estimated via the OLS method, resulting in $\varphi_1 = 1.0621$ for the curvature and $\varphi_2 = 0.6457$ for the scale. Figure 2.2 shows the estimated marginal tax rates alongside the actual marginal tax rates.

Calculating income tax, corporate tax, and inheritance tax revenues as a percentage of GDP in 2022 yields 7.2%. In this model, $\bar{\tau}_{k,t}$ is set to 0.1920, which brings the ratio of income tax (excluding resident tax) to GDP to 7.2% in the baseline economy's steady-state equilibrium.

This model incorporates four types of income deductions: the basic exemption, the employment income deduction, the (special) exemption for spouses, and the pension income deduction. The basic exemption of 480,000 yen and the employment income deduction, which is approximated as a function of labor income with an upper limit of 1,950,000 yen, follow the same method as in Chapter 1. To reflect

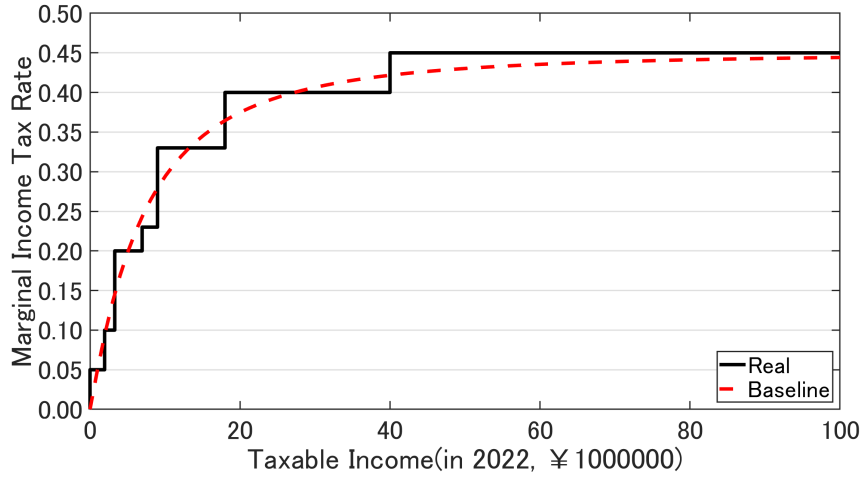


Figure 2.2: Marginal Income Tax Rates

what is known as the income wall in Japan's tax system, this model keeps the lower bound of 550,000 yen on the employment income deduction.

The (special) exemption for spouses in this model reflects the actual system, under which a married household can claim a deduction when one spouse financially supports the other. The deduction amount depends on both the taxpayer's and the spouse's total income. If the supported spouse's total income exceeds 480,000 yen but does not exceed 1,330,000 yen, and the taxpayer's total income is 9,000,000 yen or less, the deduction ranges from 380,000 yen to 30,000 yen. If the taxpayer's income is between 9,000,000 and 9,500,000 yen, the deduction ranges from 260,000 yen to 20,000 yen. If the taxpayer's income is between 9,500,000 and 10,000,000 yen, the deduction ranges from 130,000 yen to 10,000 yen. Each of these routes is approximated as a quadratic function of the supported spouse's labor income. The (special) exemption for spouses is applied to the spouse with the higher total amount of income. The pension income deduction $d_3(\cdot)$ is set follow the same method as in Chapter 1. A flat pension income tax rate of $\bar{\tau}_{ss,t} = 5\%$ is then applied to compute the pension income tax.

2.3.6 Social Security System

The social insurance system in this model consists of two components: a public pension system and a healthcare insurance system (HI). The public pension system adopts a detailed schedule. The HI system is simplified by applying a flat contribution rate of 10% as the model does not consider the behavioral effects of health insurance. The contribution rate is set based on the average rate provided by the Japan Health Insurance Association.

In the real economy, social insurance contributions are calculated using actual salaries, which is $w_{teh} - \frac{1}{2}(\tau_{P,t}(\cdot) + \tau_{H,t}(\cdot))$ in this model. For the simplicity of calculation, this model converts the contribution rate to a simplified form, avoiding the complexity of nested calculations. Japan's public pension system consists of two tiers: the National Pension (basic pension) and the Employees' Pension

Insurance. Households enrolled in the EPI are also covered by the NP, but they are not required to pay the NP contribution. To capture the structure of the Category 3 insured person system, this model assumes that individuals with monthly earnings of 88,000 yen or less cannot enroll in the EPI and instead pay only the fixed contribution for the NP. Individuals that have only enrolled in the NP only receive NP benefits after retirement, while individuals enrolled in the EPI can receive benefits from both the NP and the EPI.

The public pension contribution of a single household is

$$\begin{aligned}\tau_{P,t}(w_t e_1 h_1, w_t e_2 h_2; m = j, i) &= \tilde{\tau}_{P,t}(w_t e_j h_j; i) \\ &= \mathbf{1}_{\{i < I_R, m=j\}} \left[\mathbf{1}_{\{w_t e_j h_j \geq \vartheta_{min}\}} \bar{\tau}_{p,t} \min(w_t e_j h_j, \vartheta_{2max}) \right. \\ &\quad \left. + \mathbf{1}_{\{w_t e_j h_j < \vartheta_{min}\}} \tau_{np,t} \right] \quad \text{if } j = 1 \text{ or } j = 2,\end{aligned}\quad (2.44)$$

where $\vartheta_{min} = 0.1177$ is the threshold corresponding to the monthly earnings requirement of 88,000 yen for enrollment in the EPI. Following Chapter 1, $\vartheta_{2max} = 1.5651$ is the converted upper limit of the standard remuneration of the EPI system. The upper limit $\vartheta_{1max} = 1.3708$ used in calculating average historical earnings b is the unconverted version of ϑ_{2max} , corresponding to the actual amount of 12,300,000 yen in 2022. $\bar{\tau}_{p,t} = 0.1603$ is the converted EPI contribution rate, which reflects the 18.3% contribution rate in reality. $\tau_{np,t}$ is the fixed contribution for the NP. To eliminate regressivity, this study sets $\tau_{np,t}$ equal to the lowest EPI contribution amount, which is 193,248 yen in 2022 and converted to 0.0215 in model units. The function for calculating the public pension contribution of a married household is

$$\begin{aligned}\tau_{P,t}(w_t e_1 h_1, w_t e_2 h_2; m = 0, i) &= \sum_{j=1}^2 \left(\tilde{\tau}_{P,t}(w_t e_j h_j; i) \right. \\ &\quad \left. - \mathbf{1}_{\{w_t e_j h_j < \vartheta_{min}, w_t e_{3-j} h_{3-j} \geq \vartheta_{min}\}} \tau_{np,t} \right),\end{aligned}\quad (2.45)$$

which reflects the Category 3 insured persons system. Under this system, when one member of a married household is enrolled in the EPI while the other is only enrolled in the NP, the spouse covered only by the NP is treated as a dependent and is not required to pay the NP contribution.

This study incorporates Japan's survivors' pension system into the model. The benefits of a one-adult household received from the public pension system is

$$\begin{aligned}tr_{SS,t}(i, b_1, b_2; m = j) &= \tilde{tr}_{SS,t}(i, b_j; m = j) \\ &= \mathbf{1}_{\{i \geq I_R\}} \psi_t \left[\max \left(\xi b_j I_{P,j}, \mathbf{1}_{\{m \neq 0\}} \max \left(\frac{3}{4} \xi b_{3-j} I_{P,3-j}, \right. \right. \right. \\ &\quad \left. \left. \left. \frac{1}{2} (\xi b_j I_{P,j} + \xi b_{3-j} I_{P,3-j}) \right) \right) + np \right] (1 + \mu)^{I_R - i} \quad \text{if } j = 1 \text{ or } j = 2.\end{aligned}\quad (2.46)$$

Because mortality occurs only from age 65, the survivors' pension is simplified while reflecting the

actual system. First, the following two amounts are calculated: one-half of the sum of both spouses' EPI benefits, or three-fourths of the deceased spouse's EPI benefit. Then, the larger one is taken. If this amount is higher than the survivor's own EPI benefit, the difference is paid as survivor's pension. Otherwise, no survivors' pension is paid. For computational tractability, equation (2.46) provides a simplified representation of this calculation. $np = 0.0867$ represents the full amount of National Pension benefits received by an individual, corresponding to 777,800 yen per year in 2022. Since all households in the model are assumed to contribute to the NP throughout their entire working lives, they all receive the full benefit amount. b_j is the average historical earning, $I_{P,j}$ is the contribution-paid period, $\xi = 0.005481$ is the benefit multiplier, reflects the actual multiplier in the Japanese EPI system. If the household is single or if the deceased spouse had no EPI benefit, the survivors' pension component automatically becomes zero. The function for calculating the benefits of married household receive from the system is

$$tr_{SS,t}(i, b_1, b_2; m = 0) = \sum_{j=1}^2 \tilde{tr}_{SS,t}(i, b_j; m = 0). \quad (2.47)$$

This study assumes that HI contributions are made from both labor income and pension income. The function of the healthcare insurance (HI) contribution of a single household is

$$\begin{aligned} \tau_{H,t}(w_t e_1 h_1, w_t e_2 h_2, tr_{SS,t}(\cdot); m = j) &= \tilde{\tau}_{H,t}(w_t e_j h_j, \tilde{tr}_{SS,t}(\cdot)) \\ &= \mathbf{1}_{\{w_t e_j h_j \geq \vartheta_{min}\}} \bar{\tau}_{h,t} \min(w_t e_j h_j, \vartheta_{3max}) \\ &\quad + 0.1 \tilde{tr}_{SS,t}(\cdot) \quad \text{if } j = 1 \text{ or } j = 2, \end{aligned} \quad (2.48)$$

where $\bar{\tau}_{h,t} = 0.0876$ is the converted HI contribution rate. As in the pension system, $\vartheta_{3max} = 2.7479$ is the converted upper limit of the standard remuneration for the HI system, corresponding to the actual cap of 22,410,000 yen in 2022. The function of the healthcare insurance contribution of a married household is

$$\begin{aligned} \tau_{H,t}(w_t e_1 h_1, w_t e_2 h_2, tr_{SS,t}(\cdot); m = 0) &= \sum_{j=1}^2 (\tilde{\tau}_{H,t}(w_t e_j h_j, \tilde{tr}_{SS,t}(\cdot)) \\ &\quad - \mathbf{1}_{\{w_t e_j h_j < \vartheta_{min}, w_t e_{3-j} h_{3-j} \geq \vartheta_{min}\}} \\ &\quad \bar{\tau}_{h,t} \min(w_t e_j h_j, \vartheta_{2max})), \end{aligned} \quad (2.49)$$

where the Category 3 insured person system is considered, under which HI contributions for dependent spouses are also exempted. For simplicity, it is assumed that there are no dependent spouses in the health insurance system for the elderly.

Since this study does not incorporate household health status, it is assumed that health insurance (HI) contributions are collected by the government to finance general public expenditures. In the baseline

Table 2.2: Long Run Effects of the Three Pension Reform Policies (% ch. from the baseline economy)

Variable		Scenario 1	Scenario 2	Scenario 3
Capital Stock	K	0.60	-3.00	-1.16
Labor Supply	L	0.57	0.15	0.05
(female only)		(1.33)	(0.81)	(-0.80)
Gross Domestic Product	Y	0.58	-1.16	-0.45
Private Consumption	C	0.72	-0.51	-0.09
Interest Rate ^a	r	-0.00	0.22	0.08
Wage Rate	w	0.01	-1.31	-0.50
Consumption Tax Rate ^a	τ_c	-0.33	0.18	1.54
Pension Contribution Adjustment Factor	ψ_p	-3.76	0.52	0.48

Note: ^a Changes in percentage points from the baseline economy.

scenario, the government's net worth $W_{G,t}$ is set to zero, implying that government consumption is determined as $C_{G,t} = T_{C,t} + T_{I,t} + T_{H,t} - TR_{LS,t}$. In the policy experiments, government consumption $C_{G,t}$ is held constant at its baseline level, and the government adjusts the consumption tax rate $\tau_{c,t}$ to satisfy the intertemporal budget constraint, such that $W_{G,t} = W_{G,t+1}$.

2.4 Policy Experiments

This section examines the long-run effects of three policy experiments: the abolition of the Category 3 insured person system, the elimination of the (special) exemption for spouses, and the introduction of the family quotient system in income taxation. The study quantitatively evaluates how these reforms affect macroeconomic variables, particularly female labor supply. The long-run effects of each policy are summarized in Table 2.2. Scenario 1 is the abolition of the Category 3 insured person system, Scenario 2 is the abolition of the (special) exemption for spouses, and Scenario 3 is the introduction of the family quotient system.

2.4.1 Abolition of the Category 3 Insured Person System

Although Category 3 insured persons are defined under the public system, they are also treated as dependents under the health insurance system. In this study, the abolition of the Category 3 insured person system is defined as its elimination from both the pension system and the health insurance system.

When the Category 3 insured person system is abolished, the public pension contribution of a married household is given by

$$\tau_{P,t}(w_t e_1 h_1, w_t e_2 h_2; m = 0, i) = \sum_{j=1}^2 \tilde{\tau}_{P,t}(w_t e_j h_j; i), \quad (2.50)$$

where $\tilde{\tau}_{P,t}(\cdot)$ is the individual public pension contribution. The function of the healthcare insurance

contribution of a married household becomes

$$\tau_{H,t}(w_te_1h_1, w_te_2h_2, tr_{SS,t}(\cdot); m=0) = \sum_{j=1}^2 \tilde{\tau}_{H,t}(w_te_jh_j, \tilde{tr}_{SS,t}(\cdot)), \quad (2.51)$$

where $\tilde{\tau}_{H,t}(\cdot)$ is the individual HI contribution.

The model considers an independent pension budget. To allow for a comparison between the baseline and the long run, the pension sector balance factor ψ_t is fixed. The abolition of the Category 3 insured person system is expected to increase public pension contributions by broadening the base of contributors, and the pension contribution rate is adjusted through a new factor $\psi_{p,t}$ to maintain budget balance. The pension contribution adjustment parameter $\psi_{p,t}$ is determined by

$$\psi_{p,t} = \frac{TR_{SS,t}(\psi_t)}{T_{P,t}}, \quad (2.52)$$

where $TR_{SS,t}(\cdot)$ is the total public pension benefits distributed to all elderly generations, which has already been adjusted by ψ_t . $T_{P,t}$ is the total pension contributions that can originally be collected from working households before any adjustments.

General government expenditures are balanced through adjustments in the consumption tax rate $\tau_{c,t}$. The adjustment is determined by

$$\tau_{c,t} = \frac{1}{C_{P,t}} (C_{G,t} + TR_{LS,t} - T_{I,t} - T_{H,t}), \quad (2.53)$$

where $C_{P,t}$ is the private consumption.

2.4.1.1 Life-Cycle Effects in the Long Run

Figure 2.3 shows the long-run effects of the abolition of the Category 3 insured person system over the life cycle of married households, displayed as age-specific averages. Some variables become zero after retirement, therefore they are omitted here for clarity. Only married households are shown in this figure, since the policy does not directly affect single households. Households between age 20 and 54 are childcare cohorts in this model. In particular, households aged 28 to 40 have more preschool-aged children, resulting in lower average working hours for wives and a higher share of Category 3 insured persons. The solid black line represents the baseline equilibrium, while the dashed red line represents the new steady-state equilibrium after the policy change.

The overall effects of the policy are modest. As shown in Figure 2.3, changes in married households' private wealth over the life cycle are barely noticeable. Private consumption of married households would increase slightly in the long run. The average working hours of wives would increase among

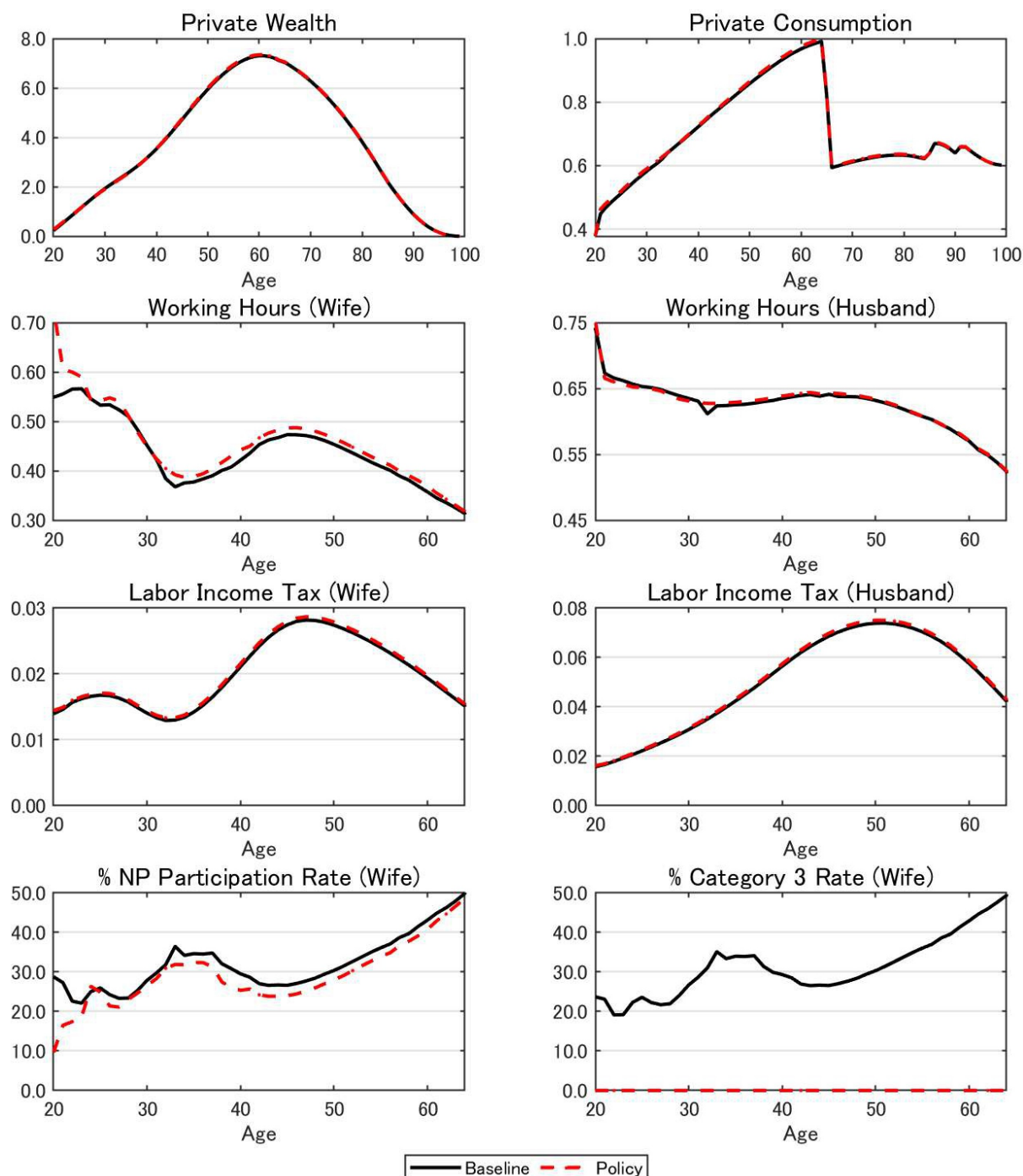


Figure 2.3: Life-Cycle Effects of Eliminating the Category 3 System (married households)

almost all cohorts in the long run, with a particularly pronounced increase among households in their 20s. The Category 3 insured person system provides an incentive for spouses with lower labor income to reduce working hours and remain dependents. In this model, women have lower average working ability than men, making them more likely to become dependent spouses. Abolishing the system would impose additional pension contribution burdens on households with dependent spouses, and would alter households' labor supply decisions. Also, as the expected lifetime utility of remaining as a housewife declines, some housewives would optimally enter the labor market. Together, these effects would lead to

a larger increase in wives' average working hours.

For households in their 20s, a relatively small gender gap in average working ability combined with limited asset holdings leads to a substantial increase in wives' average working hours after the policy change, bringing them close to those of husbands. As shown by the Category 3 rate of wives curve, the Category 3 insured person system would be completely eliminated after the policy change. As wives work more, the share of wives enrolled only in the NP would decrease noticeably across almost all working age groups.

Husbands' working hours would decline slightly in their 20s as wives' working hours increase, while increase slightly in their 30s and 40s. As households in their 30s and 40s face higher childcare burdens, a relatively large share of wives remain in low-income status and pay the NP contribution after the policy reform. As a result, the NP contribution would become an expenditure burden for a larger share of these households, which would lead to a slightly increase in husbands' average working hours. For households in their 50s and beyond, the gender gap in working ability is most pronounced in this model. With the increase in wage rate following the policy reform (as explained below), households can cover the additional NP contribution burden without a noticeable increase in husbands' working hours. Labor income tax payment made by both spouses would increase slightly, however for wives it would rise less than working hours, particularly among those in their 20s. This is because pension contributions are tax deductible. With the abolition of the Category 3 insured person system, previously dependent wives would work more and start paying their own pension contributions, which partially offset the increase in tax liabilities.

Table 2.2 indicates that aggregate capital stock would increase by approximately 0.60% in the new steady state. Labor supply would increase by 0.57% across all households, while women's labor supply alone would increase by 1.33%. This shows that the Category 3 insured person system discourages women from working. The increase in private wealth is slightly larger than the increase in labor supply, leading to a 0.01% higher wage rate and slightly lower interest rate after the policy change. The policy change results in enhanced economic activity, leading to a 0.58% increase in GDP and a 0.33 percentage points decrease in the consumption tax rate in the long run. Private consumption would increase by 0.72%. The pension contribution adjustment factor ψ_p , initially set to 1.0 in the baseline, would decrease by 3.76% after the policy change, as a result of the improvement in pension financing brought about by the abolition of the Category 3 insured person system.

In summary, the abolition of the Category 3 insured person system would increase women's labor supply, and slightly stimulates the economy.

2.4.2 Abolition of the (Special) Exemption for Spouses

In the absence of the (special) exemption for spouses, the taxable labor income is calculated as follows:

$$\tilde{y}_1 = \max \{w_t e_1 h_1 - d_1(w_t e_1 h_1) - \tilde{\tau}_{P,t}(w_t e_1 h_1) - \tilde{\tau}_{H,t}(w_t e_1 h_1), 0\}, \quad (2.54)$$

$$\tilde{y}_2 = \max \{w_t e_2 h_2 - d_1(w_t e_2 h_2) - \tilde{\tau}_{P,t}(w_t e_2 h_2) - \tilde{\tau}_{H,t}(w_t e_2 h_2), 0\}, \quad (2.55)$$

where $d_1(\cdot)$ is the combination of the basic exemption and the employment income deduction. The pension income tax for married households is calculated as

$$\tau_{SS,t}(i, b_1, b_2; m = 0) = \sum_{j=1}^2 \bar{\tau}_{ss,t} [\tilde{tr}_{SS,t}(i, b_j; m = 0) - d_3(\tilde{tr}_{SS,t}(i, b_j; m = 0))], \quad (2.56)$$

where $d_3(\cdot)$ is the pension income deduction.

As in the previous policy experiment, the fiscal balance of the pension sector is maintained by adjusting the scaling factor $\psi_{p,t}$ applied to public pension contributions. The general government budget is balanced by adjusting the consumption tax rate $\tau_{c,t}$.

2.4.2.1 Life-Cycle Effects in the Long Run

Figure 2.4 shows the long-run effects of the abolition of the (special) exemption for spouses over the life cycle of married households. Same as the first policy experiment, the overall effects of the policy are limited; however, the impacts move in a different direction compared to the first case.

As shown in Figure 2.4, private wealth would decrease for married households up to their 70s, as the abolition of the (special) exemption for spouses would raise the effective income tax burden and reduce disposable income, thereby weakening incentives for saving. Private consumption would decrease for married households aged 30 to 80. The consumption of retired households would decrease because without the (special) exemption for spouses, some households' pension income tax payment would increase, which would reduce their disposable income.

The working hours curves show that wives' working hours clearly increase for almost all married households. In the baseline economy, the presence of the (special) exemption for spouses induces some individuals with relatively low working ability to optimally limit their labor supply in order to generate a tax benefit for their spouses. Because female's average working ability is assumed to be lower than that of males in this model, this distortion primarily affects wives. Once the exemption is abolished, the distortion is removed, leading wives to supply more labor. For husbands, because they are typically the main recipients of the (special) exemption for spouses, its abolition would increase their effective

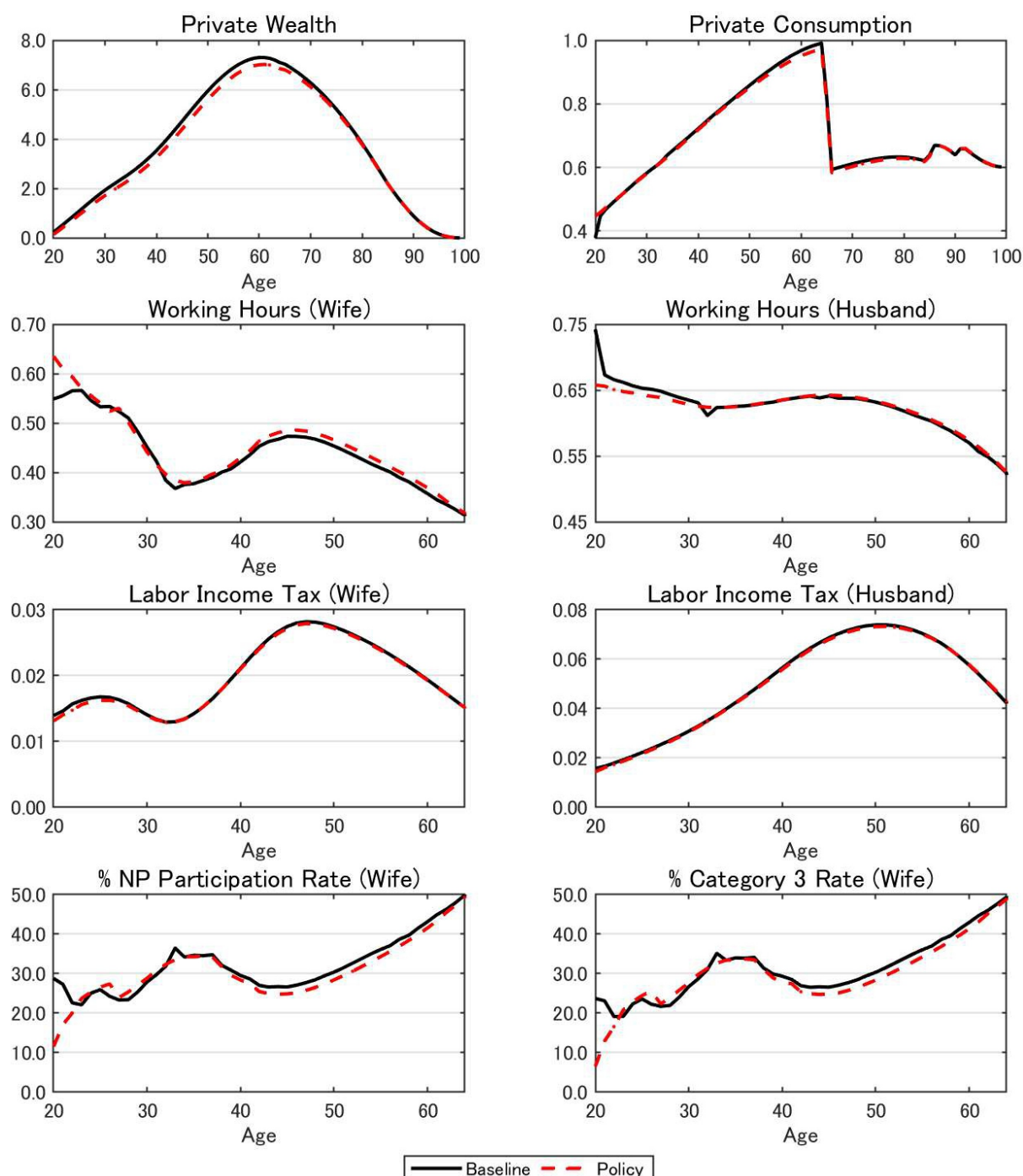


Figure 2.4: Life-Cycle Effects of Eliminating the Exemption for Spouses (married households)

income tax rate, leading to a reduction in working hours after the policy change. As in the abolition of the Category 3 insured person system, labor supply adjustments are largest among couples in their 20s, since the gender gap in average working ability is relatively small for these cohorts. For households in their 30s, the increase in wives' working hours is limited, reflecting heavier childcare responsibilities.

The curves of the NP participation rate and the Category 3 rate show that, as wives would work more, the share of wives enrolled only in the NP would become smaller, and the share of Category 3 wives would also decline. Table 2.2 shows that women's labor supply alone would increase by 0.81% after the

policy change, which shows that the (special) exemption for spouses discourages women from working. Capital stock would decrease significantly by 3.00%, while labor supply of the whole economy would increase by 0.15%. The decrease in capital stock and increase in labor supply would lead to a 1.31% lower wage rate and 0.22 percentage points higher interest rate.

Driven by the decline in the wage rate, the labor income tax payment made by both spouses would fall slightly after the policy change, which moves in the opposite direction of what would be expected from eliminating the (special) exemption for spouses. For husbands in their 40s and older, as childcare-related time constraints ease, after the policy change their working hours would increase slightly in response to the tighter household budget constraint. The overall economy would contract, leading to a 1.16% decrease in GDP and a 0.18 percentage points increase in the consumption tax rate in the long run. Private consumption would decrease by 0.51% after the policy change. As households earn less, the financial condition of the pension system would worsen, the pension contribution adjustment factor $\psi_{p,t}$ would increase by 0.52%.

In summary, the abolition of the (special) exemption for spouses would increase women's labor supply, but may worsen overall economic outcomes.

2.4.3 Family Quotient System in Income Taxation

Although there are rules related to household members, such as the (special) exemption for spouses, Japan's income tax system is basically applied on an individual basis. In recent years, there have been discussions about adopting a household-based taxation system as a measure to address the declining birth rate. This approach is known as the family quotient system. This section examines how the system would affect macroeconomic variables in the long run.

After the policy change, the income tax function would be

$$\begin{aligned}\tau_{I,t}(w_t e_1 h_1, w_t e_2 h_2, r_t a, tr_{SS,t}(\cdot); m, i) &= \tilde{\tau}_{I,t}(\tilde{y}, r_t a, tr_{SS,t}(\cdot); m) \\ &= \tau_{K,t}(r_t a) + \tau_{SS,t}(i, b_1, b_2; m) \\ &\quad + \varphi_t \left[\tilde{y} - (\tilde{y}^{-\varphi_1} + \varphi_2)^{-\frac{1}{\varphi_1}} \right] \left(1 + \mathbf{1}_{\{m=0\}} + \frac{2}{3} n_{m,i} \right),\end{aligned}\tag{2.57}$$

where $\tilde{y}$ is taxable income per tax unit,

$$\tilde{y} = (\tilde{y}_1 + \tilde{y}_2) / \left(1 + \mathbf{1}_{\{m=0\}} + \frac{2}{3} n_{m,i} \right).\tag{2.58}$$

This study refers to the family quotient system implemented in France. Taxable income is adjusted for the number of children, so households with more children face a lower income tax burden. Under

the original system in France, the first and second children count as 0.5 units each, while the third child and beyond count as one unit each. The adjustment factor is set to $2/3$ for simplicity, as the number of children in this model is continuous. Because the care of preschool children affects parents' working hours as well as labor income more significantly, this model uses the number of preschool children $n_{m,i}$ to compute taxable income $\tilde{y}$. Based on $\tilde{y}$, labor income tax per tax unit is computed, and multiplying it by the number of tax units yields the final income tax payment. Since this policy change applies joint taxation, the (special) exemption for spouses applied to labor income is also abolished, while the exemption applied to pension income remains.

The fiscal balance of the pension sector is maintained by adjusting the scaling factor $\psi_{p,t}$ applied to public pension contributions. The general government budget is balanced by adjusting the consumption tax rate $\tau_{c,t}$.

2.4.3.1 Life-Cycle Effects in the Long Run

Figure 2.5 shows the long-run effects of introducing the family quotient system in income taxation of married households. In this policy experiment, the income tax shifts from an individual-based system to a household-based system. Therefore, the labor income tax payment is the average per-person payment across the couple.

In this model, because women's average working ability is lower than men's, they are more likely to face a lower marginal tax rate under individual taxation. When the system switches to joint taxation, household labor incomes are pooled, which would raise the marginal tax rate applied to women while lowering the marginal tax rate faced by men, thereby generating an overall tax reduction at the household level. Here, taxable income is adjusted considering the number of preschool children. Even so, the tax advantage arising from the gender gap in working ability would become more pronounced with age. Households expect to benefit more from joint taxation later in the life cycle. This would shift the intertemporal allocation toward higher current consumption and postpones savings. As shown in Figure 2.5, married households would save less and consume slightly more in their younger age after switch to the family quotient system.

Higher marginal labor income tax rate weaken wives' incentive to work and thereby reduce their working hours, except for wives in their 20s. Because the policy change also removes the (special) exemption for spouses, the labor supply distortion faced by households in their 20s would disappear. As a result, wives' working hours would increase, while husbands' working hours would decline accordingly. For other age groups, husbands' working hours would increase slightly.

The shares of wives enrolled only in the NP and the shares of Category 3 wives would both increase as they work less after the policy change. Table 2.2 shows that labor supply would increase by 0.05%

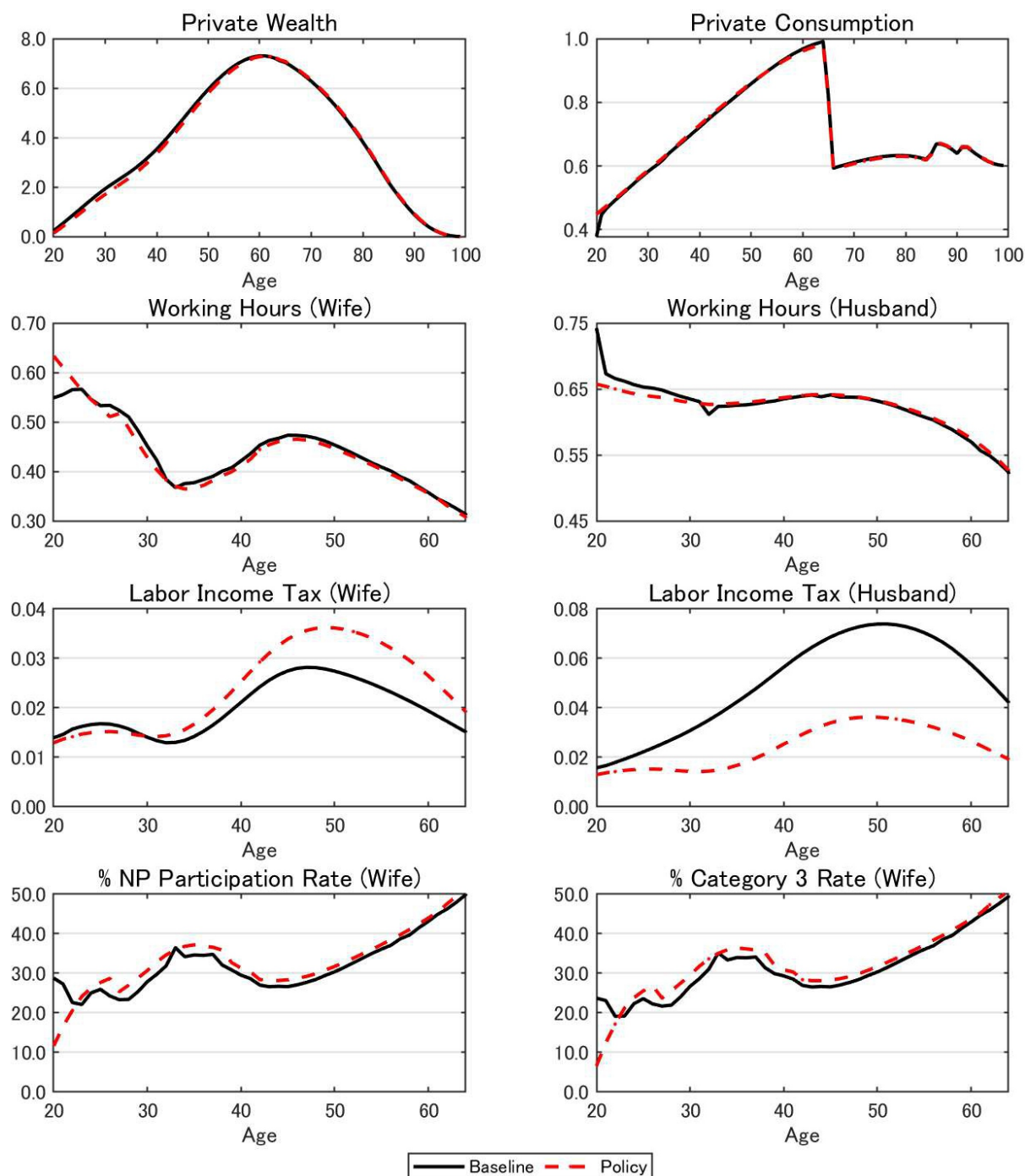


Figure 2.5: Life-Cycle Effects of Family Quotient System (married households)

in the long run across all households, while women's labor supply would decrease by 0.80%. Capital stock would decrease by 1.16%. The decrease in capital stock and increase in labor supply would lead to a 0.50% lower wage rate and 0.08 percentage points higher interest rate. The overall economy would contract, leading to a 0.45% decrease in GDP. This policy would effectively reduce income tax payments. To maintain budget balance, the government would raise the consumption tax rate. In the long run, consumption tax rate would increase by 1.54 percentage points, while private consumption would decrease by 0.09%. The pension budget deteriorates, households would face a 0.48% higher pension contribution

burden. The results suggest that the introduction of the family quotient system would possibly worsen economic outcomes and has a negative effect on women's labor supply.

2.5 Concluding Remarks

This study develops a large-scale heterogeneous overlapping generations model that incorporates gender, marital status, and institutional details of Japan's tax and pension systems to evaluate the long-run effects of three reform scenarios: the abolition of the Category 3 insured person system under the National Pension, the (special) exemption for spouses under the income tax system, and the introduction of the family quotient system in income taxation.

Abolishing the Category 3 insured person system would increase labor supply by 0.57%, with a particularly large increase of 1.33% for women. Capital stock would increase by 0.60%, and GDP would increase by 0.58% in the long run. Private consumption would increase by 0.72%, and the pension contribution adjustment factor would decrease by 3.76%, indicating improved pension financing. These results indicate that the Category 3 insured person system discourages women's labor supply, and removing it may lead to improvements in the overall economy.

Removing the (special) exemption for spouses would generate a smaller impact on labor supply. Labor supply would increase by 0.15%, with a 0.81% increase for women. However, this policy would decrease capital stock by 3.00%, and decreases GDP by 1.16% in the long run. Private consumption would decrease by 0.51%, and the pension contribution adjustment factor would increase by 0.52%. Although the (special) exemption for spouses is often argued to distort labor supply and should therefore be abolished, the quantitative results indicate that its removal does increase women's labor supply, but may not lead to better overall economic performance.

The introduction of the family quotient system would increase labor supply slightly by 0.05%, but with an 0.80% decrease for women. Capital stock would decrease by 1.16%, and GDP would decrease by 0.45% in the long run. Private consumption would decrease by 0.09%, and the pension contribution adjustment factor would increase by 0.48%. This policy experiment shows a particularly negative effect on women's labor supply and worsens overall economic outcomes. From the perspective of promoting women's labor market participation, joint taxation may warrant careful consideration.

One limitation of this study lies in the computational complexity of the model. Due to the large number of state variables, the model is computationally intensive, and the analysis is limited to steady-state comparisons. As a result, the transitional dynamics following policy implementation are not captured. Incorporating the transition path enables tracking short- and medium-term effects of reforms and allows for evaluating welfare changes across cohorts. A second limitation concerns the treatment of childcare time. In the current model, the allocation of childcare time is fixed between husbands and wives. Allow-

ing married households to choose this allocation endogenously could capture their labor supply decisions more accurately. These limitations are left for future research.

This modeling framework offers several possibilities for future applications. While the present study focuses on tax and pension reforms, this modeling framework can be extended to examine childcare support and education policies on household decisions and the macroeconomy. Such extensions could broaden the scope of policy analysis in Japan, offering valuable insights for evidence-based policymaking.

Chapter 3

Fertility, Education, Inequality, and Economic Growth

with Shinichi Nishiyama*

3.1 Introduction

Over the past several decades, numerous child-related policies have been introduced in many countries. These policies aim to improve the welfare of children and young households, reduce poverty rates and inequality, and promote economic growth. In some countries, they also aim to prevent further declines in birth rates. However, the outcomes of these diverse child-related policies are difficult to predict empirically, except for short-term direct effects. Therefore, the purpose of this study is to provide a comprehensive theoretical and quantitative analysis of child-related policies to complement existing empirical research.

The present study extends the Barro–Becker model (Becker and Barro, 1988; Barro and Becker, 1989) on endogenous fertility and intergenerational transfers by incorporating education expenditure or human capital investment in children and life-cycle saving decisions (De la Croix and Doepke, 2003). Furthermore, this study transforms this extended model into a Bewley-type incomplete market model by incorporating idiosyncratic labor income shocks, human capital shocks, and wealth shocks.¹ Calibrating the model to the U.S. economy, this study quantitatively analyzes the potential effects of child-related policies—such as child allowances, education subsidies, and estate (inter vivos transfer) taxes—on heterogeneous household decision-making, economic growth, and the distribution of income and wealth.

The model economy in this study consists of heterogeneous households living through two adult periods, a representative firm, and the government. Households are heterogeneous regarding initial assets, human capital (labor productivity), and idiosyncratic labor income shocks. In each period, young households choose their consumption, savings, number of children (fertility), inter vivos transfers to children, and education expenditures (human capital investment) for children to maximize their lifetime utility

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¹Children do not inherit labor income shocks but partially inherit human capital shocks of their parents.

and dynastic value. Child-related policies first affect each household's number of children, inter vivos transfers, and education expenditure. Second, the decisions of young (parent) households influence the initial assets and human capital of the next generation (children), amplifying or dampening the policy effects in the second period and beyond.

The contributions of this study are as follows. First, it constructs a Barrow–Becker–Bewley model incorporating human capital investment to serve as a foundation for comprehensively analyzing various child-related policies concerning fertility rates, intergenerational transfers, and education expenditures. Second, it empirically verifies that both the income elasticity and working ability elasticity of the number of children are positive, though not particularly high. Third, by combining this model with empirical findings, it quantitatively demonstrates the potential short-run and long-run effects of child-related policies on household behavior and the aggregate economy.

The policy experiments in the present study suggest that behavioral responses to child-related policies in the model economy vary across households depending on their human capital and wealth. However, the average impact of these policies on income and wealth inequality is limited. This is due to the low income (working ability) elasticity of fertility and the fact that altruistic households' optimal decisions involve intergenerational risk sharing, which is likely to mitigate income and wealth inequality. Policy experiments also suggest that any child-related policy aimed at promoting economic growth and improving the welfare of future generations will ultimately raise fertility rates, not only in the long run but even in the short run. Furthermore, because the calibrated economy is dynamically efficient, child-related policies that promote capital accumulation, physical capital, human capital, or both are likely to raise fertility rates in the second period and beyond.

The previous literature that the present study is based on are as follows. Loury (1981) and Laitner (1988, 1991) construct Bewley-type OLG-dynasty models with intergenerational transfers. Becker and Barro (1988) and Barro and Becker (1989) construct deterministic dynasty models of optimal fertility, consumption, and intergenerational transfers in a small-open economy and a closed economy, respectively. Alvarez (1999) combines the above features and constructs a Bewley-type dynasty model, in which households choose their optimal fertility, consumption, and intergenerational transfers.

Nishiyama (2002) constructs a altruistically linked four-period OLG-dynasty model, in which the parents and their (adult) children are altruistically linked, and these households strategically choose the optimal sizes of inter vivos transfers (in both directions) and bequests. De la Croix and Doepke (2003) construct a two-period deterministic OLG model, in which the young households (parents) receive a warm-glow utility from the children and choose the optimal number of children and education spending for children.

More recent studies that construct an OLG model or a dynasty model with fertility decision, intergenerational transfers, and/or education spending include the following. Stantcheva (2015) constructs a

dynasty model, in which the households are heterogeneous with respect to their initial wealth and human capital, and they choose the education spending and bequests; and this study analyzes the effects of income, education, and bequest taxes. Córdoba et al. (2016) constructs a dynasty model, in which the households are heterogeneous with respect to their initial wealth and labor productivity, and they choose the optimal number of children and bequests per child; and this study evaluates the effect of endogenous fertility on social mobility and long-run inequality.

Daruich and Kozłowski (2020) construct a altruistically linked OLG model, in which the adult households in each period are heterogeneous with respect to their wealth, human capital, education attainment, human capital shocks, and they choose the number of children, children's consumption, and inter vivos transfers that affect the children's education choices; and this study evaluates the effect of endogenous fertility and transfers on intergenerational mobility. Zhou (2022) constructs a warm-glow OLG model, in which the adult households in each period are heterogeneous with respect to their wealth and human capital, and they choose the number of children, at-home childcare, market childcare, and education spending for their children to maximize the warm-glow utility from children; and this study evaluates the effect of endogenous fertility, childcare, and education spending on macroeconomic variables.

The rest of this paper is presented as follows: Section 3.2 provides the empirical background of the quantitative model, Section 3.3 describes the Barro–Becker–Bewley model with human capital investment, Section 3.4 describes the calibration strategy to the U.S. economy and shows the baseline economy, Section 3.5 explains the possible effects of six child-related policies in equilibrium transition paths, and Section 3.6 concludes the paper. Appendix describes the computational algorithms used to find the optimal decisions of heterogeneous households and to solve the model for equilibrium transition paths.

3.2 Empirical Background

This section first discusses the empirical relationship between the number of children in each married household and the household's total income, labor income, or hourly wages. Next, this section discusses the effects of hourly wages and the number of children on the working hours of husbands and wives.

3.2.1 Income and Hourly Wages on the Number of Children

We first estimate the effects of total income, labor income, and hourly wages on the number of children in each married household, using the combined cross-section data from the Survey of Consumer Finances (SCF) of years 2016 and 2019. We only use the data of married households with wives' ages

Table 3.1: Summary Statistics of the Subsets of SCF 2016 and 2019

	Mean	SD	Min	Max	N
<i>Married Households with Wives' Ages between 35 and 54</i>					
<i>kids</i>	1.5541	1.2449	0.	7.	2606
$\ln income$	11.7285	0.8479	7.7666	18.8776	2606
$\ln wageinc$	11.5352	0.9126	2.4683	16.1175	2606
$\ln hwage$	3.2668	0.6848	2.3026	8.1069	2606
$\ln hwage_m$	3.2376	0.7630	2.3026	8.4780	2606
$\ln hwage_f$	2.9109	0.6540	2.3026	7.5104	2606
age_f	44.1600	5.7351	35.	54.	2606
$educ.hs_m$	0.8834	0.3210	0.	1.	2606
$educ.hs_f$	0.9144	0.2798	0.	1.	2606
$race.wh$	0.6440	0.4789	0.	1.	2606
<i>Married Households with Wives' Ages between 25 and 54</i>					
$hours_m$	2151.95	882.62	0.	6760.	3563
$hours_f$	1473.30	994.98	0.	5200.	3563
<i>kids</i>	1.5010	1.2598	0.	7.	3563
$\ln hwage_m$	3.1769	0.7125	2.3026	9.0628	3563
$\ln hwage_f$	2.8697	0.6178	2.3026	7.5104	3563
$health_m$	1.9564	0.7190	1.	4.	3563
$health_f$	1.8770	0.7230	1.	4.	3563
age_m	41.9362	9.8801	21.	86.	3563
age_f	39.3668	8.3633	25.	54.	3563
$educ.hs_m$	0.8910	0.3116	0.	1.	3563
$educ.hs_f$	0.9236	0.2658	0.	1.	3563
$race.wh$	0.6391	0.4803	0.	1.	3563

Note: Numbers of mean and standard deviation are calculated with the SCF weights. Numbers of income and wage income are bottom coded at \$1, and numbers of hourly wages are bottom coded at \$10. Hourly wages of those with zero working hours are set to \$10. The health states are set to 2 (Good) if they are 0 (inappropriate).

between 35 and 54 and with strictly positive income and labor income. The models we estimate are

$$kids = \beta_0 + \beta_1 \ln income + \gamma \mathbf{X} + u, \quad (3.1)$$

$$kids = \beta_0 + \beta_1 \ln wageinc + \gamma \mathbf{X} + u, \quad (3.2)$$

$$kids = \beta_0 + \beta_1 \ln hwage + \gamma \mathbf{X} + u, \quad (3.3)$$

$$kids = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \gamma \mathbf{X} + u, \quad (3.4)$$

where *kids* is the number of children, *income* is the household's total income, *wageinc* is the household's total labor income, *hwage* is the household's hourly wage, and *hwage_m* and *hwage_f* are the hourly wages of husband and wife, respectively; and **X** are the controls,

$$\gamma \mathbf{X} = \gamma_1 age_f + \gamma_2 age_f^2 + \gamma_3 educ.hs_m + \gamma_4 educ.hs_f + \gamma_5 race.wh + \gamma_6 year_{2016}, \quad (3.5)$$

Table 3.2: The Effects of Income, Labor Income, and Hourly Wages on the Number of Children and Working Hours

	<i>kids</i>				<i>hours_m</i>	<i>hours_f</i>
	(1)	(2)	(3)	(4)	(6)	(7)
$\ln income$	0.1613 (0.0238)					
$\ln wage_{inc}$		0.1279 (0.0220)				
$\ln hwage$			0.1947 (0.0287)			
$\ln hwage_m$				0.1628 (0.0260)	356.94 (19.48)	-275.17 (19.73)
$\ln hwage_f$				-0.0145 (0.0335)	-125.87 (19.86)	758.63 (26.95)
<i>kids</i>					34.27 (8.85)	-113.77 (10.91)
<i>health_m</i>					-155.86 (18.05)	35.68 (19.81)
<i>health_f</i>					56.36 (18.94)	-57.40 (19.21)
<i>age_m</i>					56.41 (8.24)	
<i>age_m²</i>					-0.75 (0.10)	
<i>age_f</i>	0.2344 (0.0762)	0.2355 (0.0759)	0.2472 (0.0763)	0.2426 (0.0767)		62.99 (14.77)
<i>age_f²</i>	-0.0034 (0.0009)	-0.0034 (0.0009)	-0.0035 (0.0009)	-0.0035 (0.0009)		-0.81 (0.18)
<i>educ_hs_m</i>	-0.2941 (0.0682)	-0.2758 (0.0669)	-0.2782 (0.0643)	-0.2834 (0.0648)	172.14 (42.59)	92.06 (44.61)
<i>educ_hs_f</i>	-0.0910 (0.0769)	-0.0703 (0.0766)	-0.0553 (0.0756)	-0.0157 (0.0757)	-78.51 (54.12)	409.50 (50.76)
<i>race_wh</i>	-0.1518 (0.0462)	-0.1361 (0.0457)	-0.1383 (0.0453)	-0.1372 (0.0454)	109.70 (26.11)	-26.03 (24.29)
<i>year₂₀₁₆</i>	0.0135 (0.0426)	0.0116 (0.0425)	0.0300 (0.0430)	0.0233 (0.0432)	19.47 (20.89)	15.99 (22.26)
<i>intercept</i>	-3.5009 (1.6765)	-3.1816 (1.6558)	-2.6097 (1.6696)	-2.4116 (1.6754)	385.62 (187.71)	-1244.96 (282.36)
<i>N</i>	2606	2606	2606	2606	3559	3559
<i>R²</i>	0.1145	0.1122	0.1149	0.1136	0.1518	0.2953
<i>SD</i>	0.0004	0.0003	0.0003	0.0002	0.0011	0.0015

Note: All regression results are estimated by weighted least squares (WLS) using the merged data set of SCF 2016 and 2019.

where age_f is the wife's age, $educ_hs_m$ and $educ_hs_f$ are dummy variables for the husband and wife having a high school diploma (or higher), respectively, $race_wh$ takes the value 1 if the husband is white and non-Hispanic, and $year_{2016}$ takes the value 1 if the household is from the 2016 SCF.

The top panel of Table 3.1 shows the summary statistics of the SCF subset of married households with wives' ages between 35 and 54, and the first four columns of Table 3.2 show the results of the weighted least squares (WLS) regressions on the number of children. The income semi-elasticity of (the effect of the rate of change in income on) the number of children is 0.1613, the wage income semi-

elasticity is 0.1279, the hourly wage semi-elasticity is 0.1947, and the semi-elasticity with respect to the husband's hourly wage is 0.1628. These estimates imply that children are normal goods, and the household's income effects are in general positive. The coefficient of the wife's hourly wage is negative but insignificant. This is partly because a larger share of wives, relative to husbands, are not working in the market and their hourly wages are imputed to be \$10. Yet, this is also possibly because wives tend to spend more time for childcare, and the wives' working ability is positively related to the cost of raising children.

When we divide the above semi-elasticities by the average number of children, 1.5541, in Table 3.1, the income elasticity of the number of children is 0.1038, the wage income elasticity is 0.0823, the hourly wage elasticity is 0.1253, and the husband's wage elasticity is 0.1048. This study uses one of these elasticities of the average number of children to construct the initial steady-state (baseline) economy in Section 3.4.

3.2.2 Hourly Wages and the Number of Children on Working Hours

Next, we estimate the effects of the couple's hourly wages and the number of children on each spouse's working hours, using the cross-section data from the Survey of Consumer Finances of years 2016 and 2019. We only use the data of married households with wives' ages between 25 and 54 and with strictly positive income and labor income. The models we estimate are

$$hours_m = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \beta_3 kids + \delta \mathbf{X}_m + u, \quad (3.6)$$

$$hours_f = \beta_0 + \beta_1 \ln hwage_m + \beta_2 \ln hwage_f + \beta_3 kids + \delta \mathbf{X}_f + u, \quad (3.7)$$

where $hours_m$ and $hours_f$ are annual working hours of husbands and wives, respectively, and $\mathbf{X}_m$ and $\mathbf{X}_f$ are the controls,

$$\begin{aligned} \delta \mathbf{X}_m = & \delta_1 health_m + \delta_2 health_f + \delta_3 age_m + \delta_4 age_m^2 + \delta_5 educ_{hs_m} + \delta_6 educ_{hs_f} \\ & + \delta_7 race_{wh} + \delta_8 year_{2016}, \end{aligned} \quad (3.8)$$

$$\begin{aligned} \delta \mathbf{X}_f = & \delta_1 health_m + \delta_2 health_f + \delta_3 age_f + \delta_4 age_f^2 + \delta_5 educ_{hs_m} + \delta_6 educ_{hs_f} \\ & + \delta_7 race_{wh} + \delta_8 year_{2016}, \end{aligned} \quad (3.9)$$

where age_m is the husband's age, and $health_m$ and $health_f$ are the health conditions of husband and wife, respectively, each of which takes 1 when the condition is excellent, 2 when it is good, 3 when it is fair, and 4 when it is poor.

The bottom panel of Table 3.1 shows the summary statistics of the SCF subset of married households with wives' ages between 25 and 54, and last two columns of Table 3.2 show the results of the WLS

regressions on the working hours of husband and wife. The hourly wage semi-elasticity of husband's working hours 356.94 hours, and the hourly wage semi-elasticity of wife's working hours 758.63 hours. The cross elasticity of the working hours with respect to their spouses' hourly wages are both negative, which supports the assumption that the married households determine their labor supply jointly and altruistically. The effect of an additional child on the husband's working hours is +34.27 hours and that on the wife's working hours is -133.77.

If there were no time costs of raising children at all but only financial costs of that, this study presumes, both the husband and wife would increase their labor supply equally by 34.27 hours. However, the wife would reduce her working hours by 113.77, according to the estimate, and the wife would spend the difference, $34.27 + 113.77 = 148.07$ hours, for the at-home childcare of the additional child. It might be the case that the husband would also spend some additional hours for the childcare in addition to working longer hours. It is also likely that they would purchase some additional childcare services from the market. Therefore, this study assumes that the lower bound of the time cost of childcare per child to be 148 hours or 4.1% of the average working hours of married households, $2152 + 1473 = 3625$ hours, in Table 3.1.

3.3 Model Economy

The economy consists of a large number of heterogeneous and overlapping households that are altruistically linked to their descendants, a large number of representative firms with constant-returns-to-scale production technology, and a government that can commit to its fiscal policy schedule. Time is discrete, and all variables are growth adjusted with the long-run growth rate ρ in this model economy. Households live through two periods, and there are two types of households, young (working-age) and old (partially-retired), in each period in the economy. Households are heterogeneous with respect to their initial wealth, a_t , human capital, h_t , and a working (earning) ability shock, ε_t . These households can work for $1 + \zeta$ periods, where $\zeta \in [0, 1]$. Representative firms, in each period, choose the capital input and the labor input to maximize their profits. The government follows the fiscal policy rule from period t and onward.

Let Ω_t be a time series of the government policy variables and the factor prices,

$$\Omega_t = \{\tau_{k,s}, \tau_{h,s}, \tau_{c,s}, \tau_{n,s}, \tau_{b,s}, \tau_{e,s}, \tau_{p,s}, tr_{p,s}, \vartheta_s, \psi_s, g_s, r_s, w_s\}_{s=t}^{\infty}, \quad (3.10)$$

where $\tau_{k,t}$ is a capital income tax rate, $\tau_{h,t}$ is a labor income tax rate, $\tau_{c,t}$ is a consumption tax rate, $\tau_{n,t}$ is a child tax (allowance if negative) rate, $\tau_{b,t}$ is an estate (inter vivos transfer) tax rate, $\tau_{e,t}$ is an education tax (subsidy if negative) rate, $\tau_{p,t}$ is a payroll tax rate for public pension, $tr_{p,s}$ is a public pension benefit,

ϑ_t is public education spending per child, ψ_t is childcare and paid leave subsidies (in hours) per child, g_t is the other government spending per household, r_t is the real interest rate, and w_t is the real wage rate per efficiency unit of labor.

3.3.1 Heterogeneous Households' Problem

Let's consider a young household born in period t . Let $a_t \in A = [0, \infty)$ be the initial wealth of this household, let $h_t \in H = (0, \infty)$ be the human capital, let $\varepsilon_t > 0$ be the i.i.d. working ability shock, where the expected value of $\exp(\varepsilon_t)$ is normalized to unity. Let c_t be the consumption per unit of family members in the first period, let s_t be the wealth at the end of the first period, let d_{t+1} be growth-adjusted consumption in the second period, let n_t be the number of children of this household, let b_t be the inter vivos transfer per unit of children, and let e_t be the education spending per unit of children.² Then, the household's optimization problem is

$$v(a_t, h_t, \varepsilon_t; \Omega_t) = \max_{c_t, s_t, n_t, b_t, e_t} \{u(c_t) + \tilde{\beta}u(d_{t+1}) + \tilde{\gamma}\Phi(n_t)\mathbb{E}_t[v(a_{t+1}, h_{t+1}, \varepsilon_{t+1}; \Omega_{t+1})]\} \quad (3.11)$$

subject to the budget constraint and the laws of motion of the state variables,

$$(1 + \tau_{c,t})(1 + n_t)^\xi c_t + s_t + \tau_{n,t}n_t + (1 + \tau_{b,t})b_t n_t + (1 + \tau_{e,t})e_t n_t \quad (3.12)$$

$$= (1 + (1 - \tau_{k,t})r_t)a_t + (1 - \tau_{h,t} - \tau_{p,t})w_t h_t \exp(\varepsilon_t)(1 - (\phi - \psi_t)n_t),$$

$$(1 + \tau_{c,t+1})d_{t+1} \quad (3.13)$$

$$= (1 + (1 - \tau_{k,t+1})r_{t+1})\frac{s_t}{1 + \rho} + (1 - \tau_{h,t+1} - \tau_{p,t+1})w_{t+1}\frac{h_t \exp(\varepsilon_t)\zeta}{1 + \rho} + tr_{p,t+1},$$

$$a_{t+1} = \frac{1}{1 + \rho} b_t \exp(\epsilon_{a,t+1}) \geq 0, \quad \epsilon_{a,t+1} \sim N(-\sigma_{\epsilon_a}^2/2, \sigma_{\epsilon_a}^2), \quad (3.14)$$

$$h_{t+1} = \frac{1}{1 + \rho} \Lambda(e_t, h_t, \vartheta_t; \bar{h}_t) \exp(\epsilon_{h,t+1}) > 0, \quad \epsilon_{h,t+1} \sim N(-\sigma_{\epsilon_h}^2/2, \sigma_{\epsilon_h}^2), \quad (3.15)$$

$$\varepsilon_{t+1} \sim N(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2), \quad (3.16)$$

$$c_t > 0, \quad s_t \geq 0, \quad n_t \geq 0, \quad b_t \geq 0, \quad e_t \geq 0, \quad (3.17)$$

where $v(a_t, h_t, \varepsilon_t)$ is a value function, $u(c_t)$ is a period utility function, $\tilde{\beta} \in (0, 1)$ is a growth-adjusted discount factor, $\tilde{\gamma} \in (0, \tilde{\beta}]$ is the growth-adjusted and discounted degree of altruism, $\Phi(n_t)$ is a scale function of n_t children, $(1 + n_t)^\xi$ is the adult-equivalent number of family members, ϕ is the time cost of childcare per dependent child, $\Lambda(e_t, h_t, \vartheta_t; \bar{h}_t)$ is a human capital accumulation function; and $\epsilon_{a,t}$ and $\epsilon_{h,t}$ are the i.i.d. shocks to inter vivos transfers to their children and the human capital of their children, respectively.

²The unit of children, n_t , in the model economy is 2.0 in the real economy.

The period utility function is one of constant relative risk aversion,

$$u(c_t) = \frac{c_t^{1-\sigma} - c_{\min}^{1-\sigma}}{1-\sigma}, \quad u'(c_t) > 0, \quad u''(c_t) < 0, \quad (3.18)$$

where σ is the coefficient of relative risk aversion, and $c_{\min}$ is a minimum (existing) level of consumption.³ When the discount factors per period are β and γ , the growth-adjusted discount factors are

$$\tilde{\beta} = (1 + \rho)^{1-\sigma} \beta, \quad \tilde{\gamma} = (1 + \rho)^{1-\sigma} \gamma. \quad (3.19)$$

The scaling function of n_t children is a standardized version of exponential child discounting as in Córdoba et al. (2016),

$$\Phi(n_t) = \frac{1 - \exp(-\mu n_t)}{1 - \exp(-\mu)}, \quad \Phi(0) = 0, \quad \Phi(1) = 1, \quad \lim_{n_t \rightarrow \infty} \Phi(n_t) = \frac{1}{1 - \exp(-\mu)}, \quad (3.20)$$

where μ is inversely related to the elasticity of $\Phi(n_t)$ with respect to n_t , and for all $n_t > 0$,

$$\lim_{\mu \rightarrow 0} \Phi(n_t) = n_t, \quad \lim_{\mu \rightarrow \infty} \Phi(n_t) = 1.$$

The inter-generational law of motion of human capital is one in De la Croix and Doepke (2003),

$$\Lambda(e_t, h_t, \vartheta_t; \bar{h}_t) = B(\vartheta_t + e_t)^\eta h_t^\tau \bar{h}_t^\kappa, \quad (3.21)$$

where B is the scale parameter of human capital and $\bar{h}_t$ is the average level of human capital in period t .

Appendix to this paper describes the Kuhn–Tucker conditions and the complementarity problem to solve the household's optimization problem, (3.11)–(3.17). Solving this problem for the optimal decisions yields the decision rules, $c_t(a_t, h_t, \varepsilon_t; \Omega_t)$, $s_t(a_t, h_t, \varepsilon_t; \Omega_t)$, $n_t(a_t, h_t, \varepsilon_t; \Omega_t)$, $b_t(a_t, h_t, \varepsilon_t; \Omega_t)$, $e_t(a_t, h_t, \varepsilon_t; \Omega_t)$, and

$$d_{t+1}(a_t, h_t, \varepsilon_t; \Omega_t) = \frac{1}{1 + \tau_{c,t+1}} \times \left[(1 + (1 - \tau_{k,t+1})r_{t+1}) \frac{s_t(a_t, h_t, \varepsilon_t; \Omega_t)}{1 + \rho} + (1 - \tau_{h,t+1} - \tau_{p,t+1})w_{t+1} \frac{h_t \exp(\varepsilon_t)\zeta}{1 + \rho} + tr_{p,t+1} \right]. \quad (3.22)$$

The value function of the household born in period t is also obtained as

$$v(a_t, h_t, \varepsilon_t; \Omega_t) = u(c_t(a_t, h_t, \varepsilon_t; \Omega_t)) + \tilde{\beta} u(d_{t+1}(a_t, h_t, \varepsilon_t; \Omega_t)) \quad (3.23)$$

³The elasticity of inter-temporal substitution, $1/\sigma$, must be less than one; otherwise, the household's total consumption would be negatively correlated to the number of family members. Yet, the utility value, $u(c_t)$, must be mostly positive; otherwise the optimal number of children would be zero.

$$+ \tilde{\gamma} \Phi(n_t(a_t, h_t, \varepsilon_t; \Omega_t)) \mathbb{E}_t[v(a_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{a,t+1}; \Omega_t), h_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{h,t+1}; \Omega_t), \varepsilon_{t+1}; \Omega_{t+1})],$$

where

$$a_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{a,t+1}; \Omega_t) = \frac{1}{1+\rho} b_t(a_t, h_t, \varepsilon_t; \Omega_t) \exp(\epsilon_{a,t+1}), \quad (3.24)$$

$$h_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{h,t+1}; \Omega_t) = \frac{1}{1+\rho} \Lambda(e_t(a_t, h_t, \varepsilon_t; \Omega_t), h_t, \vartheta_t; \bar{h}_t) \exp(\epsilon_{h,t+1}). \quad (3.25)$$

3.3.2 The Joint Distribution of Households

The population of young households is growth adjusted and normalized to unity in each period. Let $X_t(a_t, h_t, \varepsilon_t)$ be a cumulative distribution function of the heterogeneous households born in period t , and let $x_t(a_t, h_t, \varepsilon_t)$ be the corresponding density function, where

$$\int_{A \times H \times \mathbf{R}} dX_t(a_t, h_t, \varepsilon_t) = 1. \quad (3.26)$$

Let $\bar{n}_t$ be the total (and average) number of children of young households in period t ,

$$\bar{n}_t = \int_{A \times H \times \mathbf{R}} n_t(a_t, h_t, \varepsilon_t; \Omega_t) dX_t(a_t, h_t, \varepsilon_t), \quad (3.27)$$

which is equal to the gross population growth rate from period t to period $t+1$. Then, the law of motion of $x_t(a_t, h_t, \varepsilon_t)$ is

$$\begin{aligned} x_{t+1}(a_{t+1}, h_{t+1}, \varepsilon_{t+1}) &= \frac{1}{\bar{n}_t} \int_{A \times H \times \mathbf{R}^3} 1_{\{a_{t+1}=a_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{a,t+1}; \Omega_t)\}} 1_{\{h_{t+1}=h_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{h,t+1}; \Omega_t)\}} \\ &\quad \times n_t(a_t, h_t, \varepsilon_t; \Omega_t) dF_a(\epsilon_{a,t+1}) dF_h(\epsilon_{h,t+1}) dX_t(a_t, h_t, \varepsilon_t), \end{aligned} \quad (3.28)$$

where $F(\epsilon_{a,t})$ and $F(\epsilon_{h,t})$ are the cumulative distribution functions of inter vivos transfer shock, $\epsilon_{a,t}$, and human capital shock, $\epsilon_{h,t}$, respectively.

Then, total private consumption, other than education spending, in period t , C_t , is

$$\begin{aligned} C_t &= \int_{A \times H \times \mathbf{R}} (1 + n_t(a_t, h_t, \varepsilon_t; \Omega_t))^\xi c_t(a_t, h_t, \varepsilon_t; \Omega_t) dX_t(a_t, h_t, \varepsilon_t) \\ &\quad + \frac{1}{\bar{n}_{t-1}} \int_{A \times H \times \mathbf{R}} d_t(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1}) dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}), \end{aligned} \quad (3.29)$$

where $1/\bar{n}_{t-1}$ is the population of old households in this period. Total private wealth at the beginning of

period t , W_t , is

$$W_t = \int_{A \times H \times \mathbf{R}} a_t dX_t(a_t, h_t, \varepsilon_t) + \frac{1}{\bar{n}_{t-1}} \int_{A \times H \times \mathbf{R}} \frac{s_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}; \Omega_{t-1})}{1 + \rho} dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}), \quad (3.30)$$

and total labor supply in efficiency units in period t , L_t^s , is

$$L_t^s = \int_{A \times H \times \mathbf{R}} h_t \exp(\varepsilon_t) (1 - (\phi - \psi_t) n_t(a_t, h_t, \varepsilon_t; \Omega_t)) dX_t(a_t, h_t, \varepsilon_t) + \frac{1}{\bar{n}_{t-1}} \int_{A \times H \times \mathbf{R}} \frac{h_{t-1} \exp(\varepsilon_{t-1}) \zeta}{1 + \rho} dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}). \quad (3.31)$$

Total inter vivos transfers at the end of period t , B_t , is

$$B_t = \int_{A \times H \times \mathbf{R}} b_t(a_t, h_t, \varepsilon_t; \Omega_t) n_t(a_t, h_t, \varepsilon_t; \Omega_t) dX_t(a_t, h_t, \varepsilon_t), \quad (3.32)$$

and total private education spending in period t , E_t , is

$$E_t = \int_{A \times H \times \mathbf{R}} e_t(a_t, h_t, \varepsilon_t; \Omega_t) n_t(a_t, h_t, \varepsilon_t; \Omega_t) dX_t(a_t, h_t, \varepsilon_t). \quad (3.33)$$

3.3.3 The Representative Firm's Problem

The production function is assumed to be one of Cobb–Douglas,

$$Y_t = F(K_t, L_t) = A K_t^\alpha L_t^{1-\alpha}, \quad (3.34)$$

where Y_t is total output, K_t is the capital input, and L_t is the labor input, all of which are population and productivity adjusted, and A is total factor productivity.

The representative firm's profit maximization problem is

$$\max_{K_t, L_t} A K_t^\alpha L_t^{1-\alpha} - (r_t + \delta) K_t - w_t L_t, \quad (3.35)$$

where δ is the depreciation rate of the capital stock, and the first order conditions are

$$r_t = \alpha A \left(\frac{K_t}{L_t} \right)^{\alpha-1} - \delta, \quad w_t = (1 - \alpha) A \left(\frac{K_t}{L_t} \right)^\alpha. \quad (3.36)$$

The market clearing conditions of the capital and labor markets are

$$K_t = W_t, \quad L_t = L_t^s. \quad (3.37)$$

3.3.4 The Government's Budget

Total government consumption, other than public education spending in period t , G_t , is

$$G_t = \left(1 + \frac{1}{\bar{n}_{t-1}}\right) g_t, \quad (3.38)$$

total public education spending in period t , Θ_t , is

$$\Theta_t = \int_{A \times H \times \mathbf{R}} \vartheta_t n_t(a_t, h_t, \varepsilon_t; \Omega_t) dX_t(a_t, h_t, \varepsilon_t) = \vartheta_t \bar{n}_t, \quad (3.39)$$

total cost of childcare and paid leave subsidies in period t , Ψ_t , is

$$\Psi_t = \int_{A \times H \times \mathbf{R}} w_t \bar{h}_t \psi_t n_t(a_t, h_t, \varepsilon_t; \Omega_t) dX_t(a_t, h_t, \varepsilon_t) = w_t \bar{h}_t \psi_t \bar{n}_t, \quad (3.40)$$

where $\bar{h}_t$ is the average human capital of the young households,

$$\bar{h}_t = \int_{A \times H \times \mathbf{R}} h_t dX_t(a_t, h_t, \varepsilon_t), \quad (3.41)$$

and total public pension benefits to old households in period t , P_t , is

$$P_t = \frac{1}{\bar{n}_{t-1}} \int_{A \times H \times \mathbf{R}} tr_{p,t} dX_{t-1}(a_{t-1}, h_{t-1}, \varepsilon_{t-1}) = \frac{tr_{p,t}}{\bar{n}_{t-1}}. \quad (3.42)$$

Let's assume, for simplicity, that the government's net wealth or debt is zero. Then, the government's budget constraint of the general account is

$$\tau_{c,t} C_t + \tau_{k,t} r_t K_t + \tau_{h,t} w_t L_t + \tau_{b,t} B_t + \tau_{e,t} E_t + \tau_{n,t} \bar{n}_t = G_t + \Theta_t + \Psi_t, \quad (3.43)$$

and the budget constraint of the public pension account is

$$\tau_{p,t} w_t L_t = P_t. \quad (3.44)$$

When the government budget constraints hold, gross investment in period t , I_t , is

$$I_t = Y_t - C_t - E_t - G_t - \Theta_t - \Psi_t, \quad (3.45)$$

and the law of motion of growth-adjusted capital stock is

$$K_{t+1} = \frac{1}{1 + \rho} \frac{1}{\bar{n}_t} [I_t + (1 - \delta) K_t]. \quad (3.46)$$

3.3.5 Recursive Competitive Equilibrium

The recursive competitive equilibrium of this model economy is defined as follows.

DEFINITION Recursive Competitive Equilibrium: Let $(a_t, h_t, \varepsilon_t)$ be the individual state of households, and let Ω_t be a time series of the government policy variables and the factor prices,

$$\Omega_t = \{\tau_{k,s}, \tau_{h,s}, \tau_{c,s}, \tau_{n,s}, \tau_{b,s}, \tau_{e,s}, \tau_{p,s}, tr_{p,s}, \vartheta_s, \psi_s, g_s, r_s, w_s\}_{s=t}^{\infty}.$$

The value functions of young households, $\{v(a_s, h_s, \varepsilon_s; \Omega_s)\}_{s=t}^{\infty}$, the decision rules of households,

$$\begin{aligned} \{d(a_s, h_s, \varepsilon_s; \Omega_s)\}_{s=t}^{\infty} = \{ & c(a_s, h_s, \varepsilon_s; \Omega_s), s(a_s, h_s, \varepsilon_s; \Omega_s), \\ & n(a_s, h_s, \varepsilon_s; \Omega_s), b(a_s, h_s, \varepsilon_s; \Omega_s), e(a_s, h_s, \varepsilon_s; \Omega_s)\}_{s=t}^{\infty}, \end{aligned}$$

and the distribution of households, $\{x_s(a_s, h_s, \varepsilon_s)\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, for all $s = t, \dots, \infty$, each household solves the optimization problem, (3.11)–(3.17), taking Ω_s as given; the firm solves its profit maximization problem, (3.35)–(3.36); the government policy schedule satisfies conditions (3.38)–(3.44); and the factor markets are cleared as shown in equation (3.37). The economy is in a steady-state equilibrium, and thus on the balanced growth path, if, in addition, $\Omega_{s+1} = \Omega_s$ and $x_{s+1}(a_{s+1}, h_{s+1}, \varepsilon_{s+1}) = x_s(a_s, h_s, \varepsilon_s)$ for all $s = t, \dots, \infty$.

3.4 Calibration and the Baseline Economy

We assume that the baseline economy is in a steady-state equilibrium, so that the economy is on a balanced-growth path with the current-law tax and pension systems.

3.4.1 Main Parameter Values and Government Policy Assumptions

Table 3.3 shows the main parameter values and baseline government policy values of the model economy. The capital income share, α , of the production function is set to 0.36. The annual capital depreciation rate is assumed to be 5%, and the capital depreciation rate per period, δ , is set to $1 - 0.95^{30} = 0.7854$. The annual productivity growth rate is assumed to be 1%, and the productivity growth rate per period, ρ , is set to $1.01^{30} - 1 = 0.3478$. The households in this economy are assumed to work for 40 years, and the old households work for 10 years out of 30 years. Thus, the share of working years in the old period, ζ , is set to $10/30 = 0.3333$.

The annual capital–output ratio is assumed to be 3.0, and the capital–output ratio, K_t/Y_t , is targeted to be $3.0/30 = 0.10$ in the baseline (initial steady-state) economy. The subjective discount factor of time,

Table 3.3: Main Parameter Values and Baseline Government Policy Values

<i>Main Parameters</i>			
Share parameter of capital income	α	0.3600	
Discount factor	β	0.5277	$K/Y = 3.0/30$ (baseline)
Growth-adjusted discount factor	$\tilde{\beta}$	0.4545	$(1 + \rho)^{1-\sigma} \beta$
Growth-adjusted discount and altruism factor	$\tilde{\gamma}$	0.3636	$0.8 \tilde{\beta}$
Capital depreciation rate	δ	0.7854	$1 - 0.95^{30}$ (5% annual rate)
Share of working years in the old period	ζ	0.3333	10/30 (retire at age 65)
Education elasticity of human capital	η	0.5000	
Parents' human capital elasticity of human capital	τ	0.2000	De la Croix and Doepke (2003)
Average human capital elasticity of human capital	κ	0.0000	
Hazard rate of exponential child discount factor	μ	1.7397	Wage elasticity of $n_t = 0.125$
Adult equivalent scale for consumption	ξ	0.5000	
Long run productivity growth rate	ρ	0.3478	$1.01^{30} - 1$ (1% annual rate)
Coefficient of relative risk aversion	σ	1.5000	
Time cost of childcare per unit of children	ϕ	0.1000	
<i>Scaling and Other Parameters</i>			
Scaling parameter (TFP) of production function	A	3.0482	$w_t = 1.0$ (baseline)
Scaling parameter of human capital function Λ	B	3.2810	$\bar{h}_t = 1.0$ (baseline)
Shifting parameter of utility function u	$c_{\min}$	0.1014	$\bar{n}_t = 1.0$ (baseline)
SD of log labor income shocks ε_t	σ_ε	0.2000	
SD of log wealth shocks $\varepsilon_{a,t}$	σ_a	0.1000	
SD of log human capital shocks $\varepsilon_{h,t}$	σ_h	0.1000	
<i>Baseline Government Policies</i>			
Capital income tax rate	$\tau_{k,t}$	0.1000	
Labor income tax rate	$\tau_{h,t}$	0.1000	
Consumption tax rate	$\tau_{c,t}$	0.1000	
Child tax rate	$\tau_{n,t}$	0.0000	
Estate (gift inter vivos) tax rate	$\tau_{b,t}$	0.0000	
Education tax rate	$\tau_{e,t}$	0.0000	
PAYG public pension (payroll) tax rate	$\tau_{p,t}$	0.1000	
PAYG public pension benefit	$tr_{p,t}$	0.1146	
Public education spending per child	ϑ_t	0.0716	$\vartheta_t \bar{n}_t = 0.04 Y_t$
Childcare and paid leave subsidies per child	ψ_t	0.0000	
Other government consumption	g_t	0.1069	

Note: A unit of the number of children, n_t , in this study is 2 children in the real economy. Thus, $\bar{n}_t = 1$ in the baseline economy is corresponding to the total fertility rate of 2.0. The child tax and the education tax are considered as allowance and subsidy, respectively, when the tax rates are negative.

β , is set to 0.5277 so that $K_t/Y_t = 0.10$. The coefficient of relative risk aversion, σ , of the utility function is assumed to be 1.5. Then, the growth-adjusted discount factor, $\tilde{\beta}$, is set to $(1 + \rho)^{1-\sigma} \beta = 0.4545$. The annual discount factor is $0.4545^{1/30} = 0.9741$. The degree of parental altruism toward their children is assumed to be 0.8, that is, the parents care about their children 20% less than they care about themselves. Then, the growth-adjusted discount and altruism factor, $\tilde{\gamma}$, is set to $0.8 \tilde{\beta} = 0.3636$.

The education elasticity of child's human capital, η , is set to 0.5. The parents' human capital elasticity of children's human capital, τ , is set to 0.2, following the estimates in De la Croix and Doepke (2003). The average human capital elasticity of child's human capital, κ , is set to 0, that is, the model economy abstracts from the externality of human capital investment. The hazard rate, μ , of the exponential dis-

count factor, $\Phi(n_t)$, of children's value is set to 1.7397 so that the hourly wage (working ability) elasticity of the number of children is 0.125, which is equal to the estimated income semi-elasticity, 0.1613, divided by the average number of children, 1.5541, in Section 3.2.⁴ When μ is 1.7397, $\lim_{n \rightarrow \infty} \Phi(n_t) = 1.2130$. The parameter, ξ , of the adult-equivalent number of family members for consumption is set to 0.5. The time cost of childcare per unit of children, ϕ , is set to 0.10.⁵

The wage rate per effective labor, w_t , the average level of human capital, $\bar{h}_t$, and the average number of children, $\bar{n}_t$, are all normalized to unity in the baseline economy. The scaling parameter, A , of the production function is set so that $w_t = 1.0$ when $K_t/Y_t = 0.10$ in the baseline economy. The scaling parameter, B , of the human capital function is set so that $\bar{h}_t = 1.0$ in the baseline economy, and the shifting parameter, $c_{\min}$, of the utility function is set to 0.1014 so that $\bar{n}_t = 1.0$ in the baseline economy. The average consumption of young households per unit of adult family members, $\bar{c}_t$, is 0.4244 as in Table 4. So, in this economy, parents do not want to raise children if they expect their children's future consumption to be less than 24% of the average consumption.

The standard deviation of log labor income shocks, ε_t , is assumed to be 0.20. The standard deviations of log wealth (inter vivos transfer) shocks, $\epsilon_{a,t}$, and log human capital shocks, $\epsilon_{h,t}$, are both assumed to be 0.10. Without the latter two shocks, the household's wealth, a_t , and human capital, h_t , are perfect substitutes, that is, either inter vivos transfers, b_t , or education spending, e_t , is binding at the lower bound 0. The sizes of the standard deviations of these shocks determines the elasticity of substitution between b_t and e_t .

The capital income tax rate, $\tau_{k,t}$, the labor income tax rate, $\tau_{h,t}$, and the consumption tax rate, $\tau_{c,t}$, are all set to 10% in the baseline economy. The child tax rate, $\tau_{n,t}$, the estate (inter vivos transfer) tax rate, $\tau_{b,t}$, and the education tax rate, $\tau_{e,t}$, are all normalized to 0% in the baseline economy. The payroll tax rate, $\tau_{p,t}$, for the pay-as-you-go public pension is assumed to be 10%, and the public pension benefit per old household, $tr_{p,t}$, is set to 0.1146 so that the government pension budget is balanced in the baseline economy. Public education spending per child, ϑ_t , is set to 0.0716 so that total public education spending, $\vartheta_t \bar{n}_t$, is 4.0% of GDP.⁶ Childcare and paid leave subsidies (in hours) per child, ψ_t , are normalized to 0. The other government consumption per household is set to 0.1069 so that the rest of the government budget is also balanced in the baseline economy.

⁴If we calibrate the model so that the income elasticity of the number of children is 0.104 instead, the hazard rate, μ , is increased to 1.9560. Yet, the effects of this parameter change on policy responses are modest. For example, introducing the child allowance of 0.01 would increase the number of children by 2.4% and 2.1% in the first two periods rather than 2.6% and 2.2% as in Table 3.5 in Section 3.5.

⁵The lower bound of the time cost of childcare per child is estimated to be about 140 hours per child or 4.1% of the average working hours of married households, as discussed in Section 3.2, and it is 8.2% per unit of children in the model economy. Considering a possible depreciation of human capital during the childcare, ϕ is set to 10.0% or 0.10 in this study.

⁶Total expenditures for public elementary and secondary schools in the United States were \$870 billion in 2019-20 and \$927 billion in 2020-21, according to Institute of Education Sciences (2023, 2024), which are about 4.0% of GDP in these years.

3.4.2 Baseline Economy

Table 3.4 shows the averages of individual variables, aggregate variables, and inequality statistics in the baseline economy. The average initial (beginning-of-period) wealth of young households, $\bar{a}_t$, is 0.0801, which is 8.0% of the average human capital, $w_t \bar{h}_t$, in the baseline economy. The average consumption of young households per unit of adult family members, $\bar{c}_t$, is 0.4244, and the average saving is 0.1335. The average number of children, $\bar{n}_t$, is normalized to unity, and one unit of children in the model economy is corresponding to two children in the real economy. The average inter vivos transfer to children, $\bar{b}_t$, is 0.1040 per unit of children, and the average private education spending, $\bar{e}_t$, is 0.0964 per unit of children.

The average consumption of old households, $\bar{d}_t$, is 0.6022, and it is larger than $\bar{c}_t$, because the equilibrium interest rate is higher than the subjective discount rate of time. The average initial wealth of children, $\bar{a}_{t+1}$, is 0.0771 per unit of children, and the average human capital of children, $\bar{h}_{t+1}$, is 0.9921. Both $\bar{a}_{t+1}$ and $\bar{h}_{t+1}$ are smaller than $\bar{a}_t$ and $\bar{h}_t$ because the former two averages are calculated without considering the number of children of each household. The value or the lifetime dynastic utility of young households, $\bar{v}_t$, is 7.4946. Total consumption (excluding education spending), C_t , is 1.2043, total capital stock, K_t , is 0.1791, total labor supply in efficiency units, L_t , is 1.1463, and total output, Y_t , is 1.7912. The interest rate, r_t , is 2.8146 per period or 4.56% annual rate, and the wage rate, w_t , is normalized to unity in the baseline economy.

Regarding inequalities, the Gini coefficient of consumption (including private education spending) is 0.1628, the Gini coefficient of labor income (both young and old households) is 0.3053, and the Gini coefficient of beginning-of-period wealth of young and old households is 0.3318. The wealth Gini coefficient is much smaller than that observed in the data because the present study assumes that all intergenerational transfers are inter vivos transfers rather than bequests. If we calculate the wealth Gini coefficient just before inter vivos transfers are made, the Gini coefficient is calculated as 0.6698 in the baseline economy. Regarding intergenerational mobility, the intergenerational correlation of human capital is 0.7707, and the correlation of labor income of young households is calculated as 0.4324. The income elasticity of the number of children is 0.1489, the wage income elasticity is 0.1235, and the working ability (hourly wage) elasticity is targeted to 0.1250 in the baseline economy.

Figure 3.1 shows the averages of individual variables of the young households conditional on the human capital, h_t , as well as the marginal distribution of human capital in the baseline economy. The average consumptions of young and old households are both increasing and slightly concave in h_t . The average number of children is also increasing in h_t , but it is more strongly concave by the assumption of $\Phi(n_t)$ and μ . The upper bound of the average number of children by h_t is around 1.25 in the baseline

Table 3.4: Main Variables and Inequality Statistics in the Baseline Economy

<i>Individual Variables</i> (population weighted average)			
Initial wealth (young)	$\bar{a}_t$	0.0801	
Human capital	$\bar{h}_t$	1.0000	normalized to unity
Consumption (young)	$\bar{c}_t$	0.4244	
Saving	$\bar{s}_t$	0.1335	
Number of children	$\bar{n}_t$	1.0000	normalized to unity
Inter vivos transfer per unit of children	$\bar{b}_t$	0.1040	
Education expenditure per unit of children	$\bar{e}_t$	0.0964	
Consumption (old)	$\bar{d}_{t+1}$	0.6022	
Initial wealth of children	$\bar{a}_{t+1}$	0.0771	
Human capital of children	$\bar{h}_{t+1}$	0.9921	
Value (lifetime utility)	$\bar{v}_t$	7.4946	
<i>Aggregate Variables</i> (per young household)			
Consumption	C_t	1.2043	$\bar{c}_t + \bar{d}_t/\bar{n}_t$
Capital stock	K_t	0.1791	$\bar{a}_t + \bar{s}_t/(\bar{n}_t(1 + \rho))$
Labor supply	L_t	1.1463	
Output	Y_t	1.7912	
Gross interest rate	$1 + r_t$	3.8146	4.56% annual rate
Wage rate	w_t	1.0000	normalized to unity
<i>Gini Coefficients</i>			
Consumption	(c_t, d_t)	0.1763	
Consumption and education	$(c_t + e_t n_t, d_t)$	0.1628	
Labor income (young)	$(\tilde{h}_t l_t)$	0.1511	
Labor income (young and old)	$(\tilde{h}_t l_t, \tilde{h}_{t-1} \zeta)$	0.3053	
Wealth (young and old)	(a_t, s_{t-1})	0.3318	
Wealth before inter vivos transfers	$(0, b_{t-1} + s_{t-1})$	0.6698	
<i>Intergenerational Correlations</i>			
Human capital	(h_t, h_{t+1})	0.7707	
Working ability	$(\tilde{h}_t, \tilde{h}_{t+1})$	0.4573	
Labor income	$(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.4324	
<i>OLS Coefficients (Semi-elasticities)</i>			
Income elasticity of n_t	$(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.1489	
Labor income elasticity of n_t	$(n_t, \ln \tilde{h}_t l_t)$	0.1235	
Working ability elasticity of n_t	$(n_t, \ln \tilde{h}_t)$	0.1250	

Note: The aggregate consumption, C_t , includes the consumption of dependent children. The average total consumption of young households, $\tilde{c}_t$, is the population weighted average of $(1 + n_t)^\varepsilon c_t$. The working ability is defined as $\tilde{h}_t = h_t e^{\varepsilon_t}$, and the labor income of a young household is $\tilde{h}_t l_t$, where $l_t = 1 - (\phi - \psi_t)n_t$. The variables in $t + 1$ and $t - 1$ are productivity growth adjusted.

economy or 2.5 children in the real economy.⁷ As Table 3.2 in Section 3.2 shows, the number of children is increasing in the husband's working ability but decreasing in the wife's working ability. Yet, it is increasing in combined human capital, as the permanent income measure of a household. Thus, children are normal goods in this model economy.

The inter vivos transfer, b_t , and education spending, e_t , are highly substitutable. The cost of the inter vivos transfer is independent of the parent's h_t , but the return to education spending is increasing in h_t

⁷In the model economy, the household's wealth, a_t , and working ability shocks, ε_t , affects the number of children. In the real economy, in addition, the preference heterogeneity would affect the number of children as well.

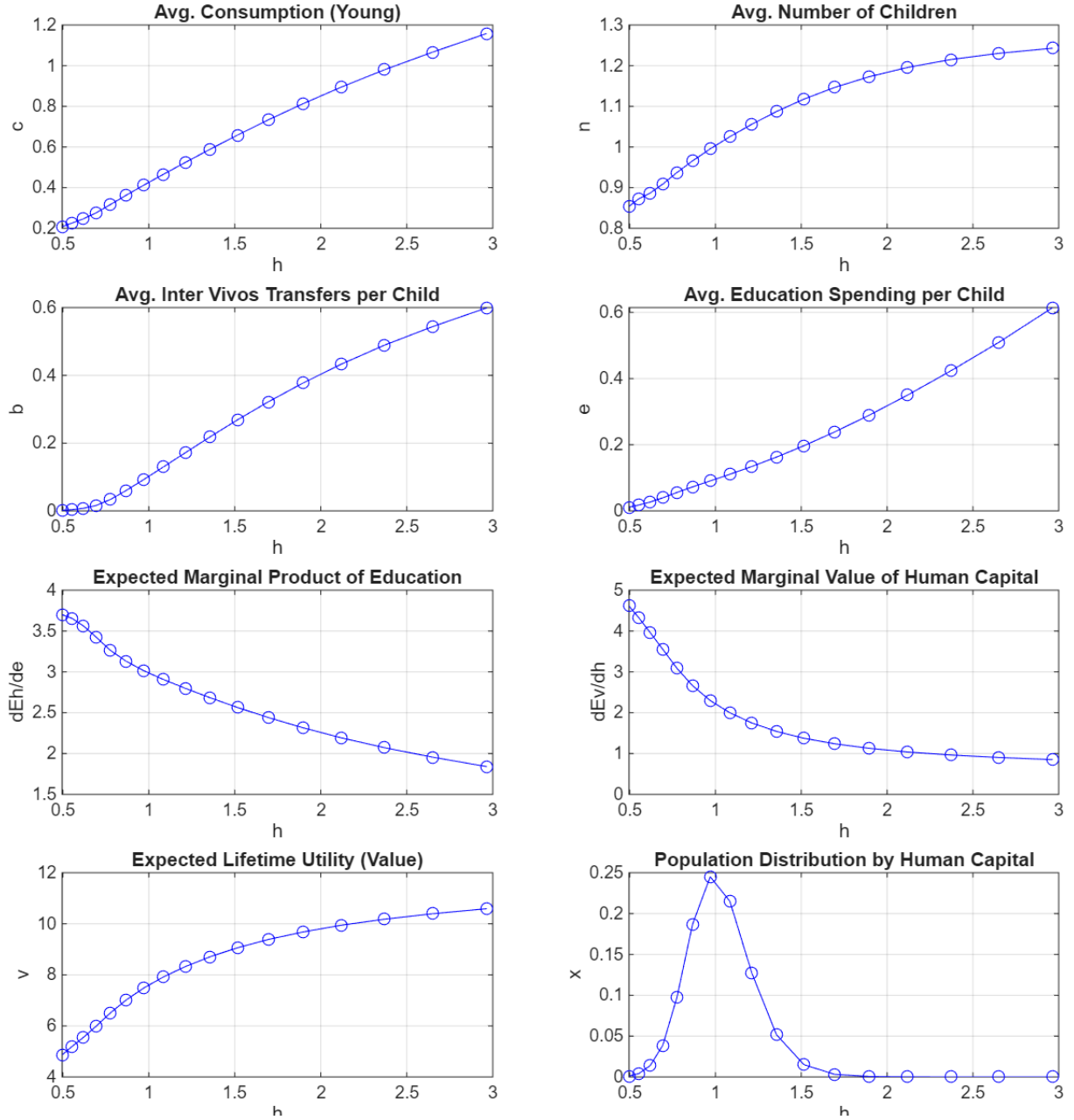


Figure 3.1: The Household Behaviors by Human Capital in the Baseline Economy

(or the cost of children's human capital is decreasing in h_t). Therefore, inter vivos transfers per unit of children, b_t , are on average increasing and slightly concave in h_t . The average education spending per unit of children, e_t , is increasing and slightly convex in h_t . The parents' education spending generates a skewed distribution of human capital in the long run.

3.5 Policy Experiments

The altruistic (child-related) part of the household's optimization problem is, abstracting from all shocks, productivity growth, and human capital externality,

$$\max_{n_t, b_t, e_t} \tilde{\gamma} \Phi(n_t) v(a_{t+1}, h_{t+1}) \quad (3.47)$$

subject to the budget constraint,

$$\begin{aligned} (1 + \tau_{c,t}) [(1 + n_t)^\xi - 1] c_t \\ + [\tau_{n,t} + (1 + \tau_{b,t}) b_t + (1 + \tau_{e,t}) e_t + (1 - \tau_{h,t} - \tau_{p,t}) w_t h_t (\phi - \psi_t)] n_t \\ = (1 - \varsigma) [(1 + (1 - \tau_{k,t}) r_t) a_t + (1 - \tau_{h,t} - \tau_{p,t}) w_t h_t] \equiv \text{inc}_t, \end{aligned} \quad (3.48)$$

and the laws of motion of the state variables,

$$a_{t+1} = b_t \geq 0, \quad h_{t+1} = B(\vartheta_t + e_t)^\eta h_t^\tau > 0, \quad (3.49)$$

where ς is the share of disposable income and wealth used for the parents' own consumption and saving.

The first order conditions for the interior solution are

$$\begin{aligned} n_t : \quad \tilde{\gamma} \Phi'(n_t) v(a_{t+1}, h_{t+1}) &= \lambda_t [(1 + \tau_{c,t}) \xi (1 + n_t)^{\xi-1} c_t \\ &+ \tau_{n,t} + (1 + \tau_{b,t}) b_t + (1 + \tau_{e,t}) e_t + (1 - \tau_{h,t} - \tau_{p,t}) w_t h_t (\phi - \psi_t)] \equiv \lambda_t \text{mc}_{n,t}, \end{aligned} \quad (3.50)$$

$$b_t : \quad \tilde{\gamma} \Phi(n_t) v_a(a_{t+1}, h_{t+1}) = \lambda_t (1 + \tau_{b,t}) n_t \equiv \lambda_t \text{mc}_{b,t}, \quad (3.51)$$

$$e_t : \quad \tilde{\gamma} \Phi(n_t) v_h(a_{t+1}, h_{t+1}) B \eta (\vartheta_t + e_t)^{\eta-1} h_t^\tau = \lambda_t (1 + \tau_{e,t}) n_t \equiv \lambda_t \text{mc}_{e,t}, \quad (3.52)$$

where $\lambda_t > 0$ is the Lagrange multiplier and

$$\lambda_t = \frac{u'(c_t)}{(1 + \tau_{c,t})(1 + n_t)^\xi}$$

in the full model. Let's call the left-hand sides the marginal values of n_t , b_t , and e_t , and call the right-hand sides the marginal costs of n_t , b_t , and e_t . Note that the marginal cost of n_t is increasing in b_t , e_t and h_t , and that the marginal costs of b_t and e_t are both increasing in n_t . This property generates the trade off between the quantity and quality of children (Becker and Lewis, 1973).

This section analyzes various policy reform plans aimed to reduce the marginal costs (or increase the marginal values) of raising children, making inter vivos transfers, and providing education spending on the individual decisions and the aggregate economy. More specifically, this section conducts the

following three sets of policy experiments:

1. increasing child allowances (reducing $\tau_{n,t}$ by 0.01 or 1% of baseline $w_t \bar{h}_t$) versus increasing education subsidies (reducing $\tau_{e,t}$ by 10.10 percentage points);
2. increasing inter vivos transfer subsidies (reducing $\tau_{b,t}$ by 9.10 percentage points) versus decreasing labor income taxes (reducing $\tau_{h,t}$ by 1.10 percentage points);
3. increasing public education expenditure (increasing ϑ_t by 0.01 from 0.0716) versus increasing childcare and paid leave subsidies (increasing ψ_t by 0.0127 or 1.27% of time endowment).

The costs of each policy reform are assumed to be financed by raising the consumption tax rate, $\tau_{c,t}$, so that the government's combined budget, the sum of equations (3.43) and (3.44), is balanced each period, that is,

$$\tau_{c,t} = [(G_t + \Theta_t(\vartheta_t) + \Psi_t(\psi_t) + P_t(tr_{p,t})) - (\tau_{k,t}r_tK_t + \tau_{h,t}w_tL_t + \tau_{b,t}B_t + \tau_{e,t}E_t + \tau_{n,t}\bar{n}_t)] C_t^{-1}. \quad (3.53)$$

The sizes of the policy changes are set so that the policy costs during the first period are the same, a 1.09 percentage point increase in $\tau_{c,t}$.

3.5.1 Increasing Child Allowances vs. Education Subsidies

This subsection compares the possible effects of increasing child allowances (reducing $\tau_{n,t}$ by 0.01, or 1.0% of the average human capital of young households in the baseline) and increasing education subsidies (reducing $\tau_{e,t}$ by 10.10 percentage points), both financed by increasing consumption taxes (adjusting $\tau_{c,t}$ to balance the government's budget in each period). Figure 3.2 shows the behavioral effects of these two policy changes in the first period by human capital, and Table 3.5 shows the individual, aggregate, and distributional effects of these policy changes over the transition paths.

When the child tax rate, $\tau_{n,t}$, was reduced permanently by 0.01, the marginal cost of children, $mc_{n,t}$, fell by 0.01 (on average by 1.9%), and the average number of children, $\bar{n}_t$, would increase by 2.6% in the first period. The increase in the number of children, n_t , however, is heterogeneous and decreasing in the household's human capital, h_t . The increase in n_t is on average around 5.9% when $h_t = 0.6$, and it is 0.8% when $h_t = 2.0$. The increase in n_t would increase the marginal costs of inter vivos transfers per child, $mc_{b,t}$, and education spending per child, $mc_{e,t}$, proportionately, and it would decrease both $\bar{b}_t$ and $\bar{e}_t$ by on average 1.7% and 2.5%, respectively, in the first period. The decreases in b_t and e_t are also heterogeneous and positively correlated to the increases in n_t for each h_t .

These changes in b_t and e_t in the first period would decrease the initial wealth of children, a_{t+1} , and the human capital of children, h_{t+1} . The averages of these, $\bar{a}_t$, and $\bar{h}_t$, at the beginning of the second

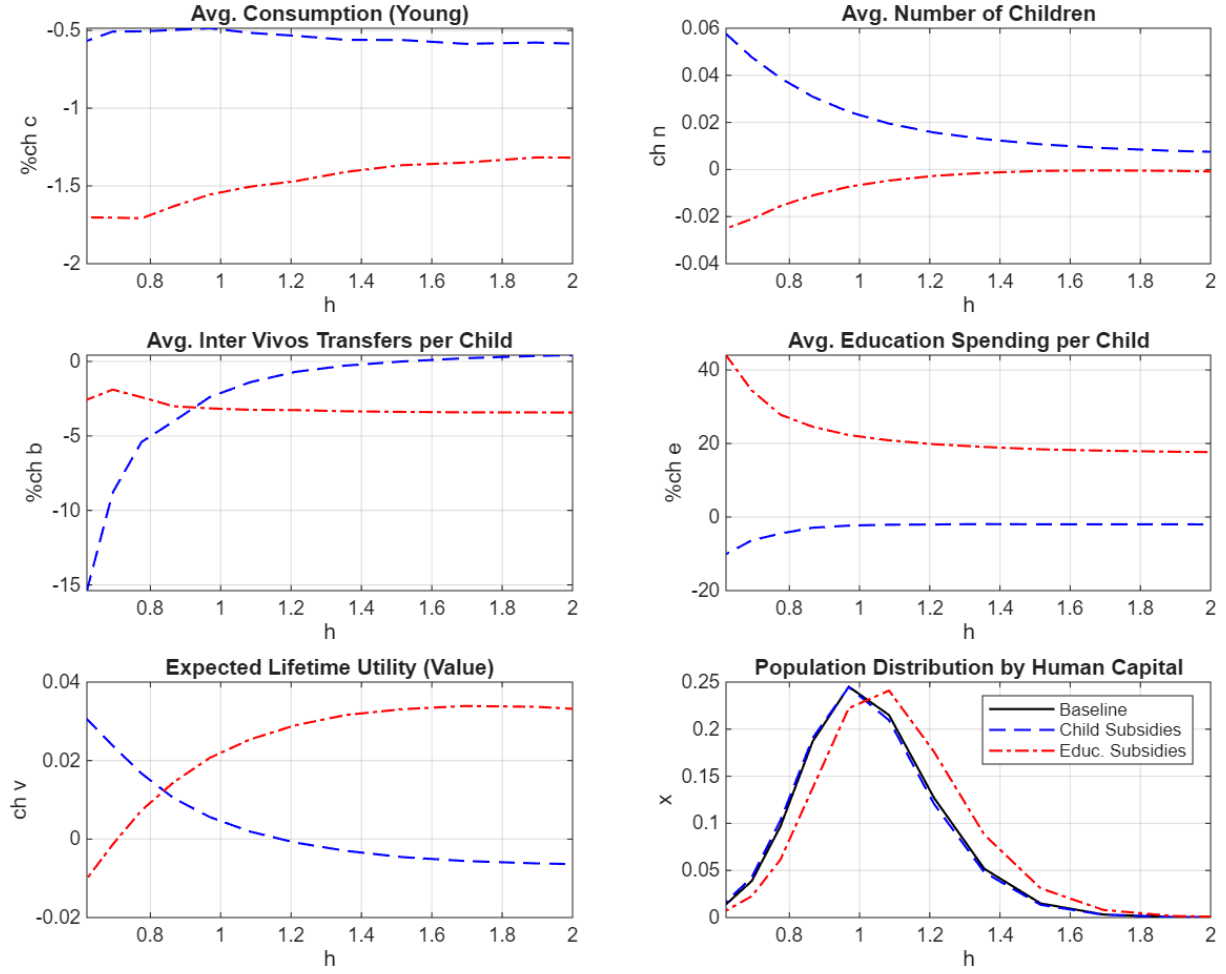


Figure 3.2: Policy Experiments: Child Allowances vs. Education Subsidies (policy effects by human capital in the first period)

period would decrease by 2.5% and 0.9%, respectively. Because of the negative income effect, inc_t , of decreasing $\bar{a}_t$ and $\bar{h}_t$ in the second period, the population growth rate or the increase in the average number of children, $\bar{n}_t$, would fall to 2.2% (from the baseline economy) in the second period. Total capital stock (per young household) at the beginning of the second and third periods would decrease by 1.6% and 2.6%, respectively. Total effective labor (per young household) in the second and third periods would decrease by 1.4% and 2.2%, respectively, and total output (per young household) would decrease by 1.5% and 2.3%, respectively, in these periods. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, and it would increase by 1.06 and 1.41 percentage points (from the baseline economy) in the second and third periods.

The policy reform would improve the welfare of young households with $h_t < 1.2$ and decline the welfare of those with $h_t > 1.2$ slightly in the first period. The welfare of young households would improve by 0.2% in period 1 and decline by 0.1% as a percentage of total wealth (capital stock) per young household under the compensating variation measure. The welfare of initial old households would decline on average by 0.9% in the same welfare measure because of the increase in the consumption tax

Table 3.5: Policy Experiments: Child Allowances vs. Education Subsidies

	Baseline	Child Allowances				Education Subsidies			
Period t	0	1	2	3	∞	1	2	3	∞
<i>Individual Variables</i> (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	-2.5	-3.6	-6.9	-0.0	-2.7	1.5	15.6
$\bar{h}_t$	1.000	0.0	-0.9	-1.8	-4.3	0.0	6.2	9.0	19.9
$\bar{c}_t$	0.424	-0.5	-1.5	-2.5	-5.3	-1.5	1.4	4.4	15.7
$\bar{s}_t$	0.133	1.7	0.3	-0.5	-3.1	0.2	1.6	4.4	14.5
$\bar{n}_t$	1.000	2.6	2.2	1.8	0.9	-0.8	0.2	1.3	4.7
$\bar{b}_t$	0.104	-1.7	-2.9	-3.8	-6.4	-3.2	1.2	4.2	15.7
$\bar{e}_t$	0.096	-2.5	-4.8	-6.4	-11.2	22.0	27.6	33.8	58.9
$\bar{d}_{t+1}$	0.602	-0.0	-1.3	-2.2	-4.9	1.0	4.7	7.7	19.1
$\bar{a}_{t+1}$	0.077	-1.7	-2.9	-3.8	-6.4	-3.2	1.2	4.2	15.7
$\bar{h}_{t+1}$	0.992	-0.7	-1.6	-2.2	-4.1	6.1	8.9	11.1	19.7
$\bar{v}_t$	7.495	0.1	-0.5	-1.0	-2.4	0.3	1.8	3.2	7.7
$\bar{c}v_t$	0.000	0.2	-0.1	-0.8	-2.6	1.6	2.4	4.0	11.0
<i>Initial Old Households</i> (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-1.0			
$\bar{v}_{t-1}$	7.495	-0.0				0.0			
$\bar{c}v_{t-1}$	0.000	-0.9				0.5			
<i>Aggregate Variables</i> (per young household, % ch. from the baseline)									
C_t	1.204	-0.5	-1.8	-2.7	-5.5	-1.3	1.7	4.6	15.4
K_t	0.179	0.0	-1.6	-2.6	-5.3	0.0	-0.6	1.5	12.2
L_t	1.146	-0.2	-1.4	-2.2	-4.5	0.1	5.0	8.2	18.2
Y_t	1.791	-0.1	-1.5	-2.3	-4.8	0.0	2.9	5.7	16.0
$1+r_t$	3.815	-0.1	0.1	0.3	0.5	0.0	3.4	4.0	3.2
w_t	1.000	0.1	-0.1	-0.2	-0.3	-0.0	-2.0	-2.3	-1.9
<i>Government Policy Variables</i> (% ch. in p.p. from the baseline)									
$\tau_{n,t}$	0.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00
$\tau_{e,t}$	0.00	0.00	0.00	0.00	0.00	-10.10	-10.10	-10.10	-10.10
$\tau_{c,t}$	0.00	1.09	1.06	1.41	2.52	1.09	0.32	-0.66	-3.88
<i>Gini Coefficients</i>									
$gini(c_t, d_t)$	0.176	0.177	0.179	0.180	0.181	0.179	0.176	0.177	0.183
$gini(\tilde{h}_t l_t)$	0.151	0.152	0.152	0.153	0.153	0.151	0.151	0.152	0.157
$gini(a_t, s_{t-1})$	0.332	0.332	0.338	0.342	0.353	0.332	0.330	0.324	0.315
$gini(0, b_{t-1} + s_{t-1})$	0.670	0.670	0.674	0.676	0.678	0.670	0.668	0.667	0.672
<i>Intergenerational Correlations</i>									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.466	0.468	0.469	0.469	0.461	0.467	0.475	0.531
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.443	0.445	0.446	0.445	0.435	0.442	0.449	0.508
<i>OLS Coefficients</i>									
$\hat{\beta}(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.149	0.111	0.112	0.112	0.113	0.170	0.168	0.167	0.163
$\hat{\beta}(n_t, \ln \tilde{h}_t l_t)$	0.124	0.083	0.084	0.085	0.087	0.146	0.145	0.143	0.145
$\hat{\beta}(n_t, \ln \tilde{h}_t)$	0.125	0.085	0.086	0.087	0.089	0.147	0.146	0.145	0.146

Note: The compensating variation in wealth, $\bar{c}v_t$, in each period is the negative of population-weighted average as a percentage of baseline wealth per young household, K_0 . The compensating variations of initial old households are measured at the beginning of period 1.

rate. The effect of this policy reform on labor income inequality is small, and the Gini coefficient of labor income of young households would increase only from 0.151 to 0.152 in the first and second periods. The intergenerational correlation of human capital would increase from 0.457 to 0.466 in the first period and to 0.468 in the second period. The Gini coefficient of household wealth would increase from 0.670 to 0.674 in the second period and 0.676 in the third period. This is because the inter vivos transfers of households with $h_t < 1.5$ would decrease but those with $h_t > 1.5$ would slightly increase.

When the education tax rate, $\tau_{e,t}$, was reduced permanently by 10.1 percentage points, the marginal cost of education, $mc_{e,t}$, fell by 10.1%, and the marginal cost of children, $mc_{n,t}$, would fall on average by 1.8%. The average education spending per child, $\bar{e}_t$, would increase by 22.0%, the average inter vivos transfers per child, $\bar{b}_t$, would decrease by 3.2%, and the average number of children, $\bar{n}_t$, would decrease by 0.8% in the first period. The percent increase in education spending, e_t , is heterogeneous and decreasing in the household's human capital, h_t . The increase in e_t is on average 44% when $h_t = 0.6$, and it is 17% when $h_t = 2.0$. The number of children, n_t , would on average decrease slightly from the baseline economy.

These changes in b_t and e_t in the first period would decrease the initial wealth of children, a_{t+1} , and increase the human capital of children, h_{t+1} . The average wealth of children, $\bar{a}_t$, would decrease by 2.7%, and the average human capital of children, $\bar{h}_t$, would increase by 6.2%, at the beginning of the second period. Because of the positive income effect, inc_t , of increasing $\bar{a}_t$ and $\bar{h}_t$ combined in the second period, the average number of children, $\bar{n}_t$, would increase by 0.2% (from the baseline economy) in the second period. Total capital stock at the beginning of the second period would decrease by 0.6% and that at the beginning of third period would increase by 1.5%. Total effective labor in the second and third periods would increase by 5.0% and 8.2%, respectively, and total output would increase by 2.9% and 5.7%, respectively, in these periods. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, but it would increase only by 0.32 percentage points (from the baseline economy) in the second period and decrease by 0.66 percentage points in the third period.

The policy reform would decline the welfare of young households with $h_t < 0.7$ and improve the welfare of those with $h_t > 0.7$ in the first period. The welfare of young households in period 1 would improve on average by 1.6% as a percentage of total wealth per young household under the compensating variation measure. The welfare of initial old households would also improve on average by 0.5% in the same welfare measure because of the welfare gains of the children. The effect of this policy reform on labor income inequality is small, and the Gini coefficient of labor income of young households would increase from 0.151 to 0.152 in the third period. The intergenerational correlation of human capital would increase more significantly from 0.457 to 0.461 in the first period and to 0.467 in the second period. The Gini coefficient of household wealth would decrease only slightly from 0.670 to 0.668 in the second period and to 0.667 in the third period.

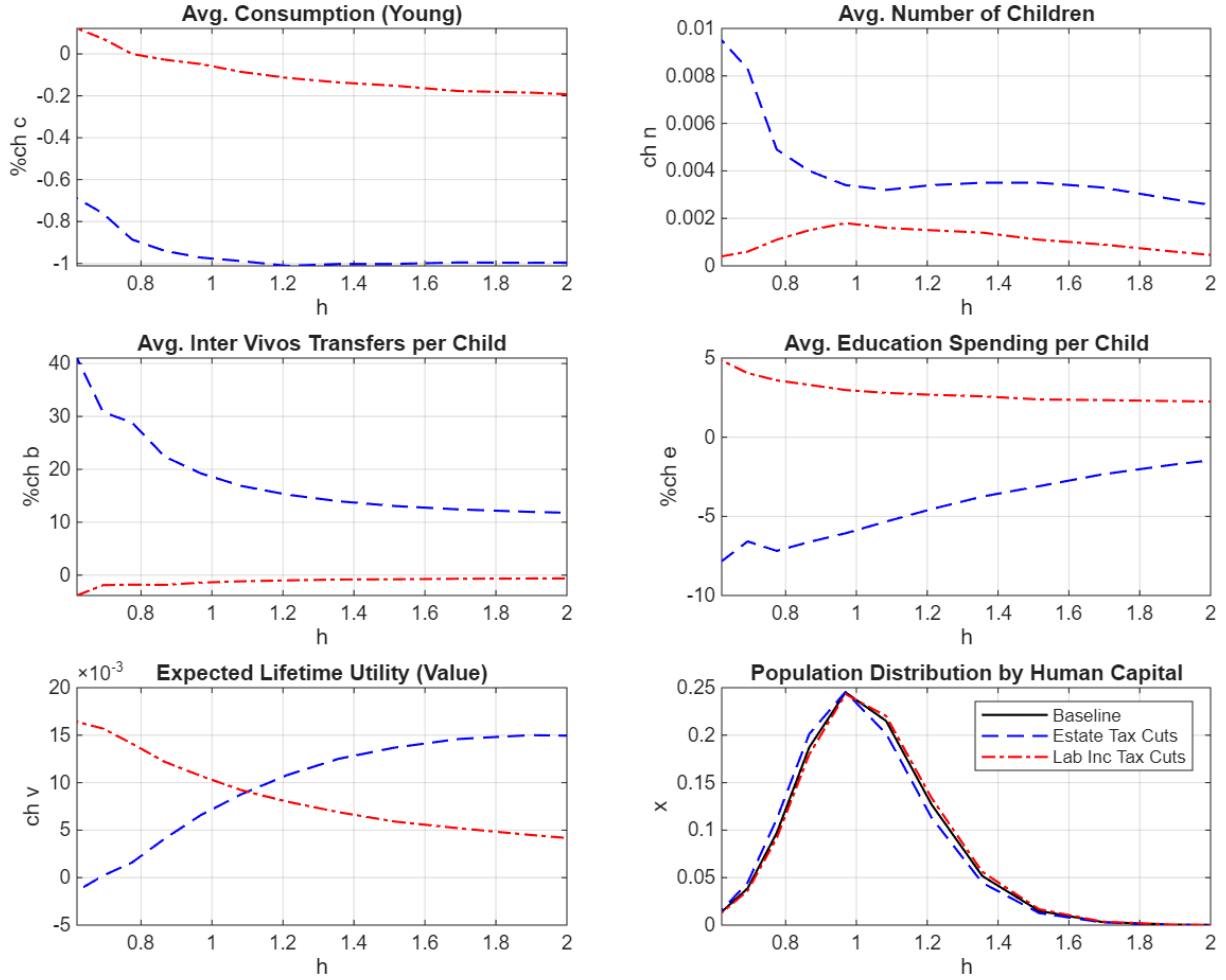


Figure 3.3: Policy Experiments: Estate Tax Cuts vs. Labor Income Tax Cuts (policy effects by human capital in the first period)

The Winner: Education Subsidies

3.5.2 Cutting Estate Taxes vs. Labor Income Taxes

This subsection compares the possible effects of cutting estate (inter vivos transfer) taxes (reducing $\tau_{b,t}$ by 9.10 percentage points) and cutting labor income taxes (reducing $\tau_{h,t}$ by 1.10 percentage points), both financed by increasing consumption taxes. Figure 3.3 shows the behavioral effects of these two policy changes in the first period by human capital, and Table 3.6 shows the individual, aggregate, and distributional effects of these policy changes over the transition paths.

When the estate tax rate, $\tau_{b,t}$, was reduced permanently by 9.10 percentage points, the marginal cost of inter vivos transfers, $mc_{b,t}$, fell by 9.10%, and the marginal cost of children, $mc_{n,t}$, would fall on average by 1.8%. The average inter vivos transfers per child, $\bar{b}_t$, would increase by 17.6%, the average education spending per child, $\bar{e}_t$, would decrease by 5.5%, and the average number of children, $\bar{n}_t$, would increase by 0.4% in the first period. The percent increase in inter vivos transfers, b_t , is heterogeneous and decreasing in the household's human capital, h_t . The increase in b_t is on average 41% when $h_t = 0.6$,

and it is 12% when $h_t = 2.0$.

Then, these changes in b_t and e_t in the first period would increase the initial wealth of children, a_{t+1} , and decrease the human capital of children, h_{t+1} . The average wealth of children, $\bar{a}_t$, would increase by 17.3%, and the average human capital of children, $\bar{h}_t$, would decrease by 1.6%, at the beginning of the second period. Because of the positive income effect, inc_t , of increasing $\bar{a}_t$ and $\bar{h}_t$ combined in the second period, the average number of children, $\bar{n}_t$, would increase by 1.8% (from the baseline economy) in the second period. Total capital stock at the beginning of the second and third periods would increase by 6.0% and 9.3%, respectively. Total effective labor in the second and third periods would decrease by 1.5% and 0.7%, respectively, and total output would increase by 1.2% and 2.8%, respectively, in the second and third periods. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, and it would increase by 0.89 and 0.16 percentage points (from the baseline economy) in the second and third periods.

The policy reform would decline on average the welfare of young households with $h_t < 0.7$ and improve the welfare of those with $h_t > 0.7$ in the first period. The welfare of young households would improve on average by 0.6% and 2.2%, respectively, in the first and second periods, as a percentage of total wealth per young household under the compensating variation measure. The welfare of initial old households would decline on average by 0.6% in the same welfare measure because of the increase in the consumption tax rate. The effect of this policy reform on labor income inequality is very small, and the Gini coefficient of labor income of young households would increase slightly from 0.151 in the baseline economy to 0.152 in the third period. The intergenerational correlation of human capital would increase steadily from 0.457 to 0.460 and 0.465 in the first and second periods, respectively. The Gini coefficient of household wealth before making inter vivos transfers would increase from 0.670 to 0.673 and 0.674 in the second and third period, and the Gini coefficient of household wealth after making transfers would decrease temporarily from 0.332 to 0.328 in the second period.

When the labor income tax rate, $\tau_{h,t}$, was reduced permanently by 1.10 percentage points, the disposable labor income for child-related expense, $(1 - \tau_{h,t} - \tau_{p,t})w_t h_t(1 - \varsigma)$, would increase by 1.4%, and the marginal cost of children, $\text{mc}_{n,t}$, would *increase* on average by 0.6%. In addition, the marginal values of education and children would both rise. The average education spending per child, $\bar{e}_t$, and the average number of children, $\bar{n}_t$, would increase by 2.9% and 0.1%, respectively, and the inter vivos transfers per child, $\bar{b}_t$, would decrease by 1.2% in the first period. The percent increase in education spending, e_t , is decreasing modestly in the household's human capital, h_t . The change in the number of children, n_t , and the percent change in inter vivos transfers, b_t , do not depend on the household's human capital level very much.

These changes in b_t and e_t in the first period would decrease the initial wealth of children, a_{t+1} , and increase the human capital of children, h_{t+1} . The average wealth of children, $\bar{a}_t$, would decrease

Table 3.6: Policy Experiments: Estate Tax Cuts vs. Labor Income Tax Cuts

Period t	Baseline	Estate Tax Cuts				Labor Income Tax Cuts			
	0	1	2	3	∞	1	2	3	∞
<i>Individual Variables</i> (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	17.3	19.3	30.7	-0.0	-1.2	-0.7	0.6
$\bar{h}_t$	1.000	0.0	-1.6	0.2	7.3	0.0	0.8	1.1	2.0
$\bar{c}_t$	0.424	-1.0	2.1	4.1	12.1	-0.1	0.2	0.5	1.6
$\bar{s}_t$	0.133	-2.7	3.0	4.8	12.0	1.1	1.1	1.4	2.3
$\bar{n}_t$	1.000	0.4	1.8	2.4	4.8	0.1	0.2	0.3	0.7
$\bar{b}_t$	0.104	17.6	19.6	22.0	31.2	-1.2	-0.7	-0.4	0.5
$\bar{e}_t$	0.096	-5.5	2.1	5.9	21.0	2.9	3.2	3.8	5.7
$\bar{d}_{t+1}$	0.602	-3.6	-1.0	0.9	8.5	0.3	0.6	0.9	1.9
$\bar{a}_{t+1}$	0.077	17.6	19.6	22.0	31.2	-1.2	-0.7	-0.4	0.5
$\bar{h}_{t+1}$	0.992	-1.6	0.3	1.7	7.2	0.8	1.1	1.3	2.0
$\bar{v}_t$	7.495	0.1	1.4	2.3	5.6	0.1	0.3	0.4	0.9
$\bar{c}v_t$	0.000	0.6	2.2	4.0	9.4	0.7	0.9	1.0	1.6
<i>Initial Old Households</i> (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-0.6			
$\bar{v}_{t-1}$	7.495	-0.0				0.0			
$\bar{c}v_{t-1}$	0.000	-0.6				0.0			
<i>Aggregate Variables</i> (per young household, % ch. from the baseline)									
C_t	1.204	-0.9	-0.7	1.0	8.5	-0.3	0.2	0.5	1.5
K_t	0.179	0.0	6.0	9.3	17.5	0.0	0.0	0.2	1.1
L_t	1.146	-0.0	-1.5	-0.7	5.8	-0.0	0.6	1.0	1.8
Y_t	1.791	-0.0	1.2	2.8	9.8	-0.0	0.4	0.7	1.6
$1+r_t$	3.815	-0.0	-4.3	-5.6	-6.1	-0.0	0.4	0.5	0.4
w_t	1.000	0.0	2.7	3.5	3.9	0.0	-0.2	-0.3	-0.2
<i>Government Policy Variables</i> (% ch. in p.p. from the baseline)									
$\tau_{b,t}$	0.00	-9.10	-9.10	-9.10	-9.10	0.00	0.00	0.00	0.00
$\tau_{h,t}$	10.00	0.00	0.00	0.00	0.00	-1.10	-1.10	-1.10	-1.10
$\tau_{c,t}$	0.00	1.09	0.89	0.16	-2.29	1.09	0.92	0.82	0.46
<i>Gini Coefficients</i>									
$gini(c_t, d_t)$	0.176	0.178	0.168	0.169	0.174	0.176	0.176	0.176	0.177
$gini(\tilde{h}_t l_t)$	0.151	0.151	0.151	0.152	0.156	0.151	0.151	0.151	0.152
$gini(a_t, s_{t-1})$	0.332	0.332	0.328	0.327	0.322	0.332	0.333	0.332	0.333
$gini(0, b_{t-1} + s_{t-1})$	0.670	0.670	0.673	0.674	0.678	0.670	0.670	0.670	0.671
<i>Intergenerational Correlations</i>									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.460	0.465	0.474	0.519	0.459	0.460	0.462	0.467
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.435	0.441	0.449	0.496	0.434	0.436	0.437	0.442
<i>OLS Coefficients</i>									
$\hat{\beta}(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.149	0.146	0.147	0.147	0.146	0.150	0.150	0.150	0.150
$\hat{\beta}(n_t, \ln \tilde{h}_t l_t)$	0.124	0.121	0.119	0.121	0.125	0.125	0.125	0.125	0.126
$\hat{\beta}(n_t, \ln \tilde{h}_t)$	0.125	0.122	0.121	0.122	0.126	0.126	0.126	0.127	0.128

Note: The compensating variation in wealth, $\bar{c}v_t$, in each period is the negative of population-weighted average as a percentage of baseline wealth per young household, K_0 . The compensating variations of initial old households are measured at the beginning of period 1.

by 1.2%, and the average human capital of children, $\bar{h}_t$, would increase by 0.8%, at the beginning of the second period. The average number of children, $\bar{n}_t$, would increase by 0.2% (from the baseline economy) in the second period. Total capital stock would not change in the second period and would increase by 0.2% in the third period. Total effective labor would increase by 0.6% and 1.0%, respectively, in the second and third periods, and total output would increase by 0.4% and 0.7%, respectively, in these periods. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, and it would increase by 0.92 and 0.82 percentage points in the second and third periods.

The welfare of young households in the first period would improve on average by 0.7% as a percentage of total wealth per young household under the compensating variation measure. The welfare of initial old households would not change. The effect of this policy reform on labor income inequality is very small, and the Gini coefficient of labor income of young households would stay at 0.151 through the third period. The intergenerational correlation of human capital would increase a little from 0.457 to 0.459 and 0.460 in the first and second periods. The Gini coefficient of household wealth would stay at 0.670 through the third period.

The Winner: Cutting Estate Taxes

3.5.3 Increasing Public Education vs. Childcare Paid Leave Subsidies

This subsection compares the possible effects of increasing public education (increasing ϑ_t by 0.01, or 1.0% of the average human capital of young households in the baseline) and increasing childcare and paid leave subsidies (increasing ψ_t by 0.0127, or 1.27% of the time endowment of young households), both financed by increasing consumption taxes. Figure 3.4 shows the behavioral effects of these two policy changes in the first period by human capital, and Table 3.7 shows the individual, aggregate, and distributional effects of these policy changes over the transition paths.

When the public education per child, ϑ_t , was increased permanently by 0.01, it would generate almost the same effect of decreasing the child tax rate per child, $\tau_{n,t}$, by the same amount. This is because the public education, ϑ_t , and private education, e_t , are perfect substitutes in the model economy, and the private education spending is larger than 0.01 for all households except for those with very small wealth, a_t , and very low human capital, h_t , in the baseline economy. So, the increase in public education would mostly be canceled out by the decrease in private education, and the reduction in the cost of private education would have the same effect of child subsidies.⁸

When childcare and paid leave subsidies per child, ψ_t , were increased permanently by 0.0127, the marginal cost of children, $mc_{n,t}$, would fall by on average 1.9%. So, the average number of children, $\bar{n}_t$, would increase by 1.8% in the first period. The increase in $\bar{n}_t$ would increase the marginal costs of inter

⁸Public education as well as many other paternalistic policies would be more important if the degree of parental altruism is heterogeneous and positively correlated to the parents' income or human capital.

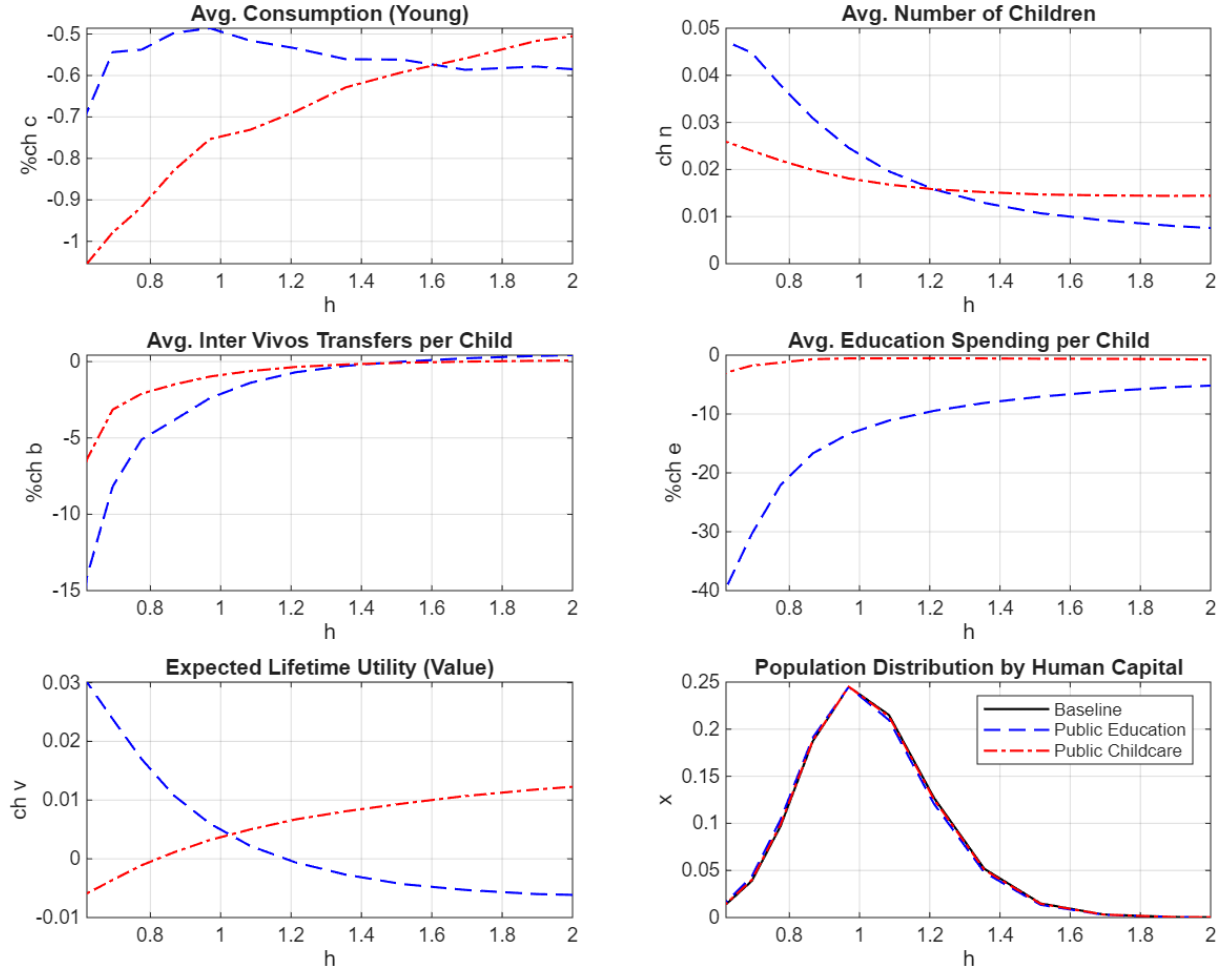


Figure 3.4: Policy Experiments: Public Education vs. Childcare Paid Leave Subsidies (policy effects by human capital in the first period)

vivos transfers, $mc_{b,t}$, and the education spending, $mc_{e,t}$, and $\bar{b}_t$ and $\bar{e}_t$ would both decrease by 0.7% in the first period. Then, these decreases would lower the average wealth of children, $\bar{a}_t$, by 1.0% and the average human capital of children, $\bar{h}_t$, by 0.3% in the second period. Because the negative income effect, inc_t , of decreasing $\bar{a}_t$ and $\bar{h}_t$ in the second period, the population growth rate, $\bar{n}_t$, would fall slightly to 1.7% in the second period.

Total capital stock at the beginning of the second and third periods would decrease by 0.7% and 1.0%, respectively. Total effective labor would increase by 1.0% and 0.4% (from the baseline economy) in the first and second periods, respectively. Total output would increase by 0.6% in the first period and decrease very slightly in the second period. The consumption tax rate, $\tau_{c,t}$, would increase by 1.09 percentage points in the first period, and it would increase by 0.96 and 1.06 percentage points in the second and third periods.

The policy reform would on average decline the welfare of young households with $h_t < 0.8$ and improve the welfare of those with $h_t > 0.8$ in the first period. The welfare of young households would improve on average by 0.3% in the first period as a percentage of total wealth per young household

Table 3.7: Policy Experiments: Public Education vs. Childcare Paid Leave Subsidies

	Baseline	Public Education				Childcare Paid Leave Subsidies			
Period t	0	1	2	3	∞	1	2	3	∞
<i>Individual Variables</i> (population-weighted average, % ch. from the baseline)									
$\bar{a}_t$	0.080	-0.0	-2.5	-3.5	-6.6	-0.0	-1.0	-1.3	-2.3
$\bar{h}_t$	1.000	0.0	-0.9	-1.7	-4.0	0.0	-0.3	-0.5	-1.3
$\bar{c}_t$	0.424	-0.5	-1.5	-2.4	-5.1	-0.8	-1.0	-1.3	-2.2
$\bar{s}_t$	0.133	1.6	0.4	-0.5	-2.9	1.3	0.9	0.6	-0.2
$\bar{n}_t$	1.000	2.5	2.1	1.8	0.9	1.8	1.7	1.6	1.3
$\bar{b}_t$	0.104	-1.7	-2.8	-3.7	-6.2	-0.7	-1.1	-1.4	-2.2
$\bar{e}_t$	0.096	-12.8	-15.0	-16.6	-21.1	-0.7	-1.4	-1.9	-3.4
$\bar{d}_{t+1}$	0.602	-0.0	-1.2	-2.1	-4.7	0.1	-0.3	-0.6	-1.5
$\bar{a}_{t+1}$	0.077	-1.7	-2.8	-3.7	-6.2	-0.7	-1.1	-1.4	-2.2
$\bar{h}_{t+1}$	0.992	-0.7	-1.5	-2.1	-3.9	-0.2	-0.5	-0.7	-1.2
$\bar{v}_t$	7.495	0.1	-0.5	-0.9	-2.3	0.0	-0.1	-0.3	-0.7
$\bar{c}v_t$	0.000	0.2	-0.1	-0.7	-2.4	0.3	0.3	0.1	-0.4
<i>Initial Old Households</i> (population-weighted average, % ch. from the baseline)									
$\bar{d}_t$	0.602	-1.0				-0.8			
$\bar{v}_{t-1}$	7.495	-0.0				-0.0			
$\bar{c}v_{t-1}$	0.000	-0.8				-0.6			
<i>Aggregate Variables</i> (per young household, % ch. from the baseline)									
C_t	1.204	-0.5	-1.8	-2.7	-5.2	-0.6	-1.2	-1.5	-2.3
K_t	0.179	0.0	-1.6	-2.5	-5.1	0.0	-0.7	-1.0	-1.9
L_t	1.146	-0.2	-1.4	-2.1	-4.3	1.0	0.4	0.2	-0.5
Y_t	1.791	-0.1	-1.4	-2.2	-4.6	0.6	-0.0	-0.3	-1.0
$1+r_t$	3.815	-0.1	0.1	0.3	0.5	0.6	0.7	0.7	0.8
w_t	1.000	0.1	-0.1	-0.2	-0.3	-0.4	-0.4	-0.4	-0.5
<i>Government Policy Variables</i> (% ch. in p.p. from the baseline)									
ϑ_t	7.16	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
ψ_t	0.00	0.00	0.00	0.00	0.00	1.27	1.27	1.27	1.27
$\tau_{c,t}$	0.00	1.09	1.05	1.39	2.44	1.09	0.96	1.06	1.38
<i>Gini Coefficients</i>									
$gini(c_t, d_t)$	0.176	0.177	0.179	0.180	0.180	0.178	0.179	0.179	0.179
$gini(\tilde{h}_t l_t)$	0.151	0.152	0.152	0.152	0.152	0.151	0.152	0.152	0.152
$gini(a_t, s_{t-1})$	0.332	0.332	0.337	0.342	0.351	0.332	0.334	0.336	0.338
$gini(0, b_{t-1} + s_{t-1})$	0.670	0.670	0.674	0.675	0.678	0.670	0.673	0.673	0.674
<i>Intergenerational Correlations</i>									
$corr(\tilde{h}_t, \tilde{h}_{t+1})$	0.457	0.464	0.464	0.464	0.461	0.460	0.461	0.461	0.460
$corr(\tilde{h}_t l_t, \tilde{h}_{t+1} l_{t+1})$	0.432	0.441	0.441	0.441	0.437	0.439	0.439	0.439	0.439
<i>OLS Coefficients</i>									
$\hat{\beta}(n_t, \ln(\tilde{h}_t l_t + r_t a_t))$	0.149	0.115	0.116	0.116	0.117	0.140	0.140	0.140	0.140
$\hat{\beta}(n_t, \ln \tilde{h}_t l_t)$	0.124	0.087	0.088	0.090	0.091	0.117	0.118	0.118	0.119
$\hat{\beta}(n_t, \ln \tilde{h}_t)$	0.125	0.089	0.090	0.091	0.093	0.118	0.119	0.119	0.120

Note: The compensating variation in wealth, $\bar{c}v_t$, in each period is the negative of population-weighted average as a percentage of baseline wealth per young household, K_0 . The compensating variations of initial old households are measured at the beginning of period 1.

under the compensating variation measure. The welfare would improve by 0.3% and 0.1% in the second and third periods. The welfare of initial old households would decline on average by 0.6% in the same welfare measure because of the increase in the consumption tax rate. The effect of this policy reform on labor income inequality is very small, and the Gini coefficient of labor income would increase from 0.151 to 0.152 in the second and third periods. The intergenerational correlation of human capital would increase from 0.457 to 0.460 in the first period and to 0.461 in the second period. The Gini coefficient of household wealth would increase a little from 0.670 to 0.673 in the second and third periods. This is because childcare and paid leave subsidies would benefit high income households more than low income households.

The Winner: Childcare and Paid Leave Subsidies (with a small margin)

3.6 Concluding Remarks

The present study constructs a Barro–Becker–Bewley model with parental human capital investment of De la Croix and Doepke (2003). In the model economy, heterogeneous households choose the number of children, education spending for children, and inter vivos transfers to children, in addition to they choose their own consumption and saving. Calibrating the model to the U.S. economy, this study then analyzes the possible effects of family-related policies.

- Increasing child allowances reduces the marginal cost of raising children. This policy increases the number of children but decreases per-child education expenditures and inter vivos transfers. Therefore, this policy declines the average welfare of both existing initial old households and future young households.
- Increasing education subsidies reduces both the marginal cost of child-rearing and the marginal cost of education expenditure. While this policy increases education spending in the short run, it reduces the number of children and inter vivos transfers. However, this policy improves the average welfare of both current households (including early elderly households) and future households.
- Increasing inter vivos transfer subsidies reduces both the marginal cost of child rearing and the marginal cost of inter vivos transfers. This policy increases inter vivos transfers and the number of children while reducing education expenditures in the short run. This policy improves the average welfare of current and future young households but declines the average welfare of initial old households.

Because the number of children, education spending for children, and inter vivos transfer to children are highly substitutable to each other, known as the quantity–quality tradeoff, the optimal policy implied by the quantitative analysis would be a combination of the above three policy changes.

For example, if child, education, and inter vivos transfer subsidy rates are set to $(\tau_{n,t}, \tau_{e,t}, \tau_{b,t}) = (-0.003, -0.047, -0.030)$, the marginal cost of children, education, and inter vivos transfers would decrease, more proportionately, by 2.0%, 4.7%, and 3.0%, respectively. Then, this policy would increase the number of children, education spending, and inter vivos transfers both in the short run and in the long run, and it would also improve the welfare, on average, of all current and future households.

The model developed in the present study is highly stylized and, to keep its transparency, it intentionally abstracts from several aspects that are possibly important to analyze the behaviors of married households.

- The model does not consider the timings of childbirths, education spending, and intergenerational transfers. One period in the model economy is one generation, and a married couple determine the number of children, education spending, and inter vivos transfers.
- The model does not incorporate the human capital heterogeneity (earning ability disparity) between a husband and wife. Accordingly, the model does not consider the allocation of working and childcare hours between them either.
- The model does not incorporate single households and decisions on marriages and divorces. Most single households without children would be worse off from any child-related policies unless they were appropriately compensated.

These aspects of the household's decisions will be addressed in the future. Regarding the lack of single households in the model economy, the average number of children is normalized to unity in the baseline economy, that is, the average total fertility rate is normalized to 2.0. Suppose, more realistically, that the 50% of all households are married and the other 50% are single (male or female) households. Then, the two thirds of women in the economy are married, and the total fertility rate will be 1.33. Even though this study abstracts from all the above aspects, the policy implications provided in the present study would not change the policy implications significantly.

3.A Solving the Bewley-type Dynastic OLG Model

This study solves the Bewley-type dynastic OLG model for a steady state equilibrium and an equilibrium transition paths with Gauss-Jacobi iterations to obtain the fixed point of Ω_t . The study solves the altruistically dynastic households' optimization problem by using the marginal value function iteration. The individual household's optimal decisions at each state in each period are obtained by solving the Kuhn-Tucker conditions (or the corresponding complementarity problem) with a nonlinear equation solver, `hybrd1`, in `MINPACK` of `Fortran90`. For more details of the computational algorithm and methods, see Nishiyama and Smetters (2014).

3.A.1 The Complementarity Problem of Each Household

Let $\mathbf{s}_t = (a_t, h_t, \varepsilon_t)$ be the household's state vector, and let $\mathbf{d}_t = (c_t, s_t, n_t, b_t, e_t)^\top$ be its decision vector. Let $f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)$ be the objective function on the right-hand-side of equation (3.11), that is,

$$f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) = u(c_t) + \tilde{\beta}u(d_{t+1}) + \tilde{\gamma} \Phi(n_t) \mathbb{E}_t[v(a_{t+1}, h_{t+1}, \varepsilon_{t+1}; \Omega_{t+1})], \quad (3.54)$$

and let $g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) = 0$ be the budget constraint in equation (3.12), that is,

$$g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) = (1 + \tau_{c,t})(1 + n_t)^\xi c_t + s_t + \tau_{n,t}n_t + (1 + \tau_{b,t})b_t n_t + (1 + \tau_{e,t})e_t n_t \\ - (1 + (1 - \tau_{k,t})r_t)a_t - (1 - \tau_{h,t} - \tau_{p,t})w_t h_t (1 - (\phi - \psi_t)n_t). \quad (3.55)$$

Let λ_t be the Lagrange multiplier for the budget constraint, let $\mu_t = (\mu_{c,t}, \mu_{s,t}, \mu_{n,t}, \mu_{b,t}, \mu_{e,t})^\top$ be the Lagrange multipliers for the inequality constraints in equation (3.17), and let $\mathbf{d}_{\min} = (\epsilon, \epsilon, 0, 0, 0)^\top$ be the lower bound of $\mathbf{d}_t$, where ϵ is a small number, such as 10^{-3} . Then, the Lagrangian of this one period problem is

$$\mathcal{L}(\mathbf{d}_t, \lambda_t, \mu_t; \mathbf{s}_t, \Omega_t) = f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) - \lambda_t g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) - \mu_t^\top (\mathbf{d}_{\min} - \mathbf{d}_t) \quad (3.56)$$

subject to equations (3.13)–(3.15), and the Kuhn-Tucker conditions are the first order (stationary) condition,

$$\nabla f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)^\top - \nabla g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)^\top \lambda_t + \mu_t = 0, \quad (3.57)$$

and the complementarity slackness conditions,

$$g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) \leq 0, \quad \lambda_t \geq 0, \quad \lambda_t g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) = 0, \quad (3.58)$$

$$\mathbf{d}_t \geq \mathbf{d}_{\min}, \quad \mu_t \geq 0, \quad \mu_{i,t}(\mathbf{d}_{i,\min} - \mathbf{d}_{i,t}) = 0 \quad \forall i. \quad (3.59)$$

The above complementarity problem can be expressed compactly as the nonlinear system of equations,

$$\min \left\{ \max \left[\begin{pmatrix} \nabla f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)^\top - \nabla g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)^\top \lambda_t \\ g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t) \end{pmatrix}, \begin{pmatrix} \mathbf{d}_{\min} - \mathbf{d}_t \\ 0 - \lambda_t \end{pmatrix} \right], \begin{pmatrix} \mathbf{d}_{\max} - \mathbf{d}_t \\ \lambda_{\max} - \lambda_t \end{pmatrix} \right\} \\ = 0, \quad (3.60)$$

where $\mathbf{d}_{\max}$ and $\lambda_{\max}$ are the upper bounds of $\mathbf{d}_t$ and λ_t . Adding non-binding constraints, $\mathbf{d}_t \leq \mathbf{d}_{\max}$ and $\lambda_t \leq \lambda_{\max}$, usually improves the computational stability when we solve the problem of wide-range of heterogeneous households with a Newton-type nonlinear equation solver. We also replace the $\min(a, b)$ and $\max(a, b)$ operators with the Fischer-Burmeister functions,

$$\phi^-(a, b) \equiv a + b - \sqrt{a^2 + b^2}, \quad \phi^+(a, b) \equiv a + b + \sqrt{a^2 + b^2}, \quad (3.61)$$

respectively, to make the above system of equations differentiable without affecting the solutions.

3.A.2 The first Order Conditions

The first order conditions for an interior solution, $\nabla f(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)^\top - \nabla g(\mathbf{d}_t; \mathbf{s}_t, \Omega_t)^\top \lambda_t = 0$, of the household's problem in each period are

$$c_t : \quad u'(c_t) - \lambda_t(1 + \tau_{c,t})(1 + n_t)^\xi = 0, \quad (3.62)$$

$$s_t : \quad \tilde{\beta}u'(d_{t+1}) \frac{\partial d_{t+1}}{\partial s_t} - \lambda_t = 0, \quad (3.63)$$

$$n_t : \quad \tilde{\gamma}\Phi'(n_t)\mathbb{E}_t[v(a_{t+1}, h_{t+1}, \varepsilon_{t+1}; \Omega_{t+1})] - \lambda_t[(1 + \tau_{c,t})\xi(1 + n_t)^{\xi-1}c_t + \tau_{n,t} + (1 + \tau_{b,t})b_t + (1 + \tau_{e,t})e_t + (1 - \tau_{h,t} - \tau_{p,t})w_t h_t \exp(\varepsilon_t)(\phi - \psi_t)] = 0, \quad (3.64)$$

$$b_t : \quad \tilde{\gamma}\Phi(n_t)\mathbb{E}_t\left[v_a(a_{t+1}, h_{t+1}, \varepsilon_{t+1}; \Omega_{t+1})\frac{\partial a_{t+1}}{\partial b_t}\right] - \lambda_t(1 + \tau_{b,t})n_t = 0, \quad (3.65)$$

$$e_t : \quad \tilde{\gamma}\Phi(n_t)\mathbb{E}_t\left[v_h(a_{t+1}, h_{t+1}, \varepsilon_{t+1}; \Omega_{t+1})\frac{\partial h_{t+1}}{\partial e_t}\right] - \lambda_t(1 + \tau_{e,t})n_t = 0, \quad (3.66)$$

where the value function is (3.23), the marginal value functions are (3.70) and (3.71), and

$$\frac{\partial d_{t+1}}{\partial s_t} = \frac{1}{1 + \rho} \frac{1 + (1 - \tau_{k,t+1})r_{t+1}}{1 + \tau_{c,t+1}}, \quad (3.67)$$

$$\frac{\partial a_{t+1}}{\partial b_t} = \frac{1}{1 + \rho} \exp(\epsilon_{a,t+1}), \quad (3.68)$$

$$\frac{\partial h_{t+1}}{\partial e_t} = \frac{1}{1 + \rho} B\eta(\vartheta_t + e_t)^{\eta-1} h_t^\tau \bar{h}_t^\kappa \exp(\epsilon_{h,t+1}). \quad (3.69)$$

Note that the marginal cost of n_t includes the terms with b_t and e_t , and the marginal costs of b_t and e_t are both proportional to n_t . These create the trade-off between the quantity and quality of children.

The marginal value functions of the household born in period t are

$$v_a(a_t, h_t, \varepsilon_t; \Omega_t) = (1 + (1 - \tau_{k,t})r_t)\lambda_t(a_t, h_t, \varepsilon_t; \Omega_t), \quad (3.70)$$

$$v_h(a_t, h_t, \varepsilon_t; \Omega_t) = (1 - \tau_{h,t} - \tau_{p,t})w_t \exp(\varepsilon_t)(1 - (\phi - \psi_t)n_t(a_t, h_t, \varepsilon_t; \Omega_t))\lambda_t(a_t, h_t, \varepsilon_t; \Omega_t) + \tilde{\beta}(1 - \tau_{h,t+1} - \tau_{p,t+1})w_{t+1} \frac{\exp(\varepsilon_t)\zeta}{1 + \rho} \frac{u'(d_{t+1}(a_t, h_t, \varepsilon_t; \Omega_t))}{1 + \tau_{c,t+1}} + \tilde{\gamma}\Phi(n_t(a_t, h_t, \varepsilon_t; \Omega_t)) \frac{1}{1 + \rho} \Lambda_h(e_t(a_t, h_t, \varepsilon_t; \Omega_t), h_t, \vartheta_t; \bar{h}_t) \times \mathbb{E}_t[\exp(\epsilon_{h,t})v_h(a_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{a,t+1}; \Omega_t), h_{t+1}(a_t, h_t, \varepsilon_t, \epsilon_{h,t+1}; \Omega_t), \varepsilon_{t+1}; \Omega_{t+1})]. \quad (3.71)$$

where

$$\lambda_t(a_t, h_t, \varepsilon_t; \Omega_t) = \frac{u'(c_t(a_t, h_t, \varepsilon_t; \Omega_t))}{(1 + \tau_{c,t})(1 + n_t(a_t, h_t, \varepsilon_t; \Omega_t))^\xi}. \quad (3.72)$$

3.A.3 Solving the Bewley Model for an Equilibrium

Steady-State Equilibrium. The computational algorithm for solving the Bewley dynastic model for a steady-state equilibrium with a time-invariant government policy schedule, Ω , is as follows.

1. Set the initial guesses of government's policy variables and factor prices,

$$\Omega^0 = (\tau_k^0, \tau_h^0, \tau_c^0, \tau_n^0, \tau_b^0, \tau_e^0, \tau_p^0, tr_p^0, \vartheta^0, \psi^0, g^0, r^0, w^0).$$

2. Given Ω^0 , find the decision rules and value function of households and the distribution of households.

- (a) Set the initial guesses of the value and marginal value functions,

$$\mathbf{v}^0(a, h, \varepsilon; \Omega^0) = (v^0(a, h, \varepsilon; \Omega^0), v_a^0(a, h, \varepsilon; \Omega^0), v_h^0(a, h, \varepsilon; \Omega^0)).$$

(b) Given $\mathbf{v}^0(a, h, \varepsilon; \Omega^0)$, find the decision rules of households and the state transition functions,

$$\mathbf{d}(a, h, \varepsilon; \Omega^0) = (c(a, h, \varepsilon; \Omega^0), s(a, h, \varepsilon; \Omega^0), n(a, h, \varepsilon; \Omega^0), b(a, h, \varepsilon; \Omega^0), \\ e(a, h, \varepsilon; \Omega^0), d'(a, h, \varepsilon; \Omega^0), a'(a, h, \varepsilon; \Omega^0), h'(a, h, \varepsilon; \Omega^0)).$$

- (c) Find the value and marginal value functions, $\mathbf{v}^1(a, h, \varepsilon; \Omega^0)$, that are corresponding to the decision rules, $\mathbf{d}(a, h, \varepsilon; \Omega^0)$, as fixed points by policy function iteration.⁹
- (d) If the relative difference, $\|\mathbf{v}^1(a, h, \varepsilon; \Omega^0) - \mathbf{v}^0(a, h, \varepsilon; \Omega^0)\|_\infty / \|1 + \mathbf{v}^0(a, h, \varepsilon; \Omega^0)\|_\infty$, is small enough, then move to Step 3. Otherwise, update $\mathbf{v}^0(a, h, \varepsilon; \Omega^0)$ by using $\mathbf{v}^1(a, h, \varepsilon; \Omega^0)$ and return to 2 (b).

3. Find the steady-state distribution of households $x(a, h, \varepsilon)$ by fixed-point iteration.
4. Compute aggregate variables, $(C, W, L^s, B, E, K, L, Y, G, \Theta, \Psi, P)$, and the government's policy variables and factor prices,

$$\Omega^1 = (\tau_k^1, \tau_h^1, \tau_c^1, \tau_n^1, \tau_b^1, \tau_e^1, \tau_p^1, tr_p^1, \vartheta^1, \psi^1, g^1, r^1, w^1).$$

5. If the relative difference, $\|\Omega^1 - \Omega^0\|_\infty / (1 + \|\Omega^0\|_\infty)$, is small enough, then stop. Otherwise, update Ω^0 by using Ω^1 and return to Step 2.

It will suffice to iterate the economy with $(K/L)^0$ instead of (r^0, w^0) in Step 5.

Equilibrium Transition Path. Assume that the economy is in the initial steady-state equilibrium in period 0 and that a new policy schedule, Ω_1 , is introduced at the beginning of period 1. Choose a sufficiently large number T such that the economy is said to reach the new steady-state equilibrium within T period. Let Ω_t be the time series of government's policy variables and factor prices as of period t ,

$$\Omega_t = \{\tau_{k,s}, \tau_{h,s}, \tau_{c,s}, \tau_{n,s}, \tau_{b,s}, \tau_{e,s}, \tau_{p,s}, tr_{p,s}, \vartheta_s, \psi_s, g_s, r_s, w_s\}_{s=t}^T.$$

Then, the computational algorithm for solving the model for an equilibrium transition path, from the initial steady state to the final steady state, is as follows.

1. Set the initial guesses of government's policy variables and factor prices,

$$\Omega_1^0 = \{\tau_{k,t}^0, \tau_{h,t}^0, \tau_{c,t}^0, \tau_{n,t}^0, \tau_{b,t}^0, \tau_{e,t}^0, \tau_{p,t}^0, tr_{p,t}^0, \vartheta_t^0, \psi_t^0, g_t^0, r_t^0, w_t^0\}_{t=1}^T.$$

2. Given Ω_T^0 , compute the final steady-state equilibrium in period T .
3. For period $t = T - 1, T - 2, \dots, 1$, compute *backward* the decision rules of households and the state transition functions,

$$\mathbf{d}_1(a, h, \varepsilon; \Omega_t^0) = \{c_t(a, h, \varepsilon; \Omega_t^0), s_t(a, h, \varepsilon; \Omega_t^0), n_t(a, h, \varepsilon; \Omega_t^0), b_t(a, h, \varepsilon; \Omega_t^0), \\ e_t(a, h, \varepsilon; \Omega_t^0), d_{t+1}(a, h, \varepsilon; \Omega_t^0), a_{t+1}(a, h, \varepsilon; \Omega_t^0), h_{t+1}(a, h, \varepsilon; \Omega_t^0)\}_{t=1}^{T-1},$$

and update the value and marginal value functions,

$$\mathbf{v}_1(a, h, \varepsilon; \Omega_t^0) = \{v_t(a, h, \varepsilon; \Omega_t^0), v_{a,t}(a, h, \varepsilon; \Omega_t^0), v_{h,t}(a, h, \varepsilon; \Omega_t^0)\}_{t=1}^{T-1}.$$

⁹We need to update the value function with policy function iteration, in each iteration step, because the Kuhn–Tucker conditions include not only the marginal value functions but also the value function itself.

4. Set the initial distribution of households $x_1(a, h, \varepsilon) = x_0(a, h, \varepsilon)$. For period $t = 1, 2, \dots, T - 1$, compute *forward* aggregate variables, $\{C_t, W_t, L_t^s, B_t, E_t, K_t, L_t, Y_t, G_t, \Theta_t, \Psi_t, P_t\}_{t=1}^{T-1}$, and the government's policy variables and factor prices,

$$\Omega_1^1 = \{\tau_{k,t}^1, \tau_{h,t}^1, \tau_{c,t}^1, \tau_{n,t}^1, \tau_{b,t}^1, \tau_{e,t}^1, \tau_{p,t}^1, tr_{p,t}^1, \vartheta_t^1, \psi_t^1, g_t^1, r_t^1, w_t^1\}_{t=1}^T,$$

by computing the distribution function, $\{x_{t+1}(a, h, \varepsilon)\}_{t=1}^{T-2}$, recursively.

5. If the relative difference, $\|\Omega_1^1 - \Omega_1^0\|_\infty / (1 + \|\Omega_1^0\|_\infty)$, is small enough, then stop. Otherwise, update Ω_1^0 by using Ω_1^1 and return to Step 2.

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