

Environment, fertility, structural change, and growth

by

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CHAPTER 1. Introduction

Economic growth is a process of creating and utilizing physical, human, financial, and social assets to generate improved and broadly shared economic well-being and quality of life for a community or region. Traditionally, aggregate economic growth is measured in terms of gross national product (GNP) or gross domestic product (GDP), although alternative metrics are sometimes used.

Economic growth is concerned with the long-run trend in production due to structural causes such as technological growth and factor (capital and labor) accumulation. Also, there are other aspects that affecting economic growth such as human capital accumulation, political institutions, entrepreneurs and new products, structural change and environmental impact.

In this thesis, we analyze three interesting aspects that affecting economic growth, that is, the environment aspect, the fertility aspect, and the structural change aspect, and clarify their relationships with economic growth. The reasons for choosing these aspects and the abstracts of our work are as follows.

First, over the past 30 years, many developing countries have achieved rapid economic growth through the excessive use of resources, thus increasing the environmental burden and causing many environmental problems. In countries such as China, India, Mexico, and Brazil, air and water pollution has become a serious problem because of industrialization, economic growth, population growth, and urbanization. Furthermore, amid rapid economic and social transformations and the deepening economic

interdependence among countries, the environmental issues have gradually transcended national boundaries and become international issues. Therefore, it is important to examine the relationship between economic growth and environmental protection from an international perspective.

Accordingly, in order to analyze transboundary pollution issues in Chapter 2¹, we develop an OLG model, which considers the environmental interaction between two regions, representing developed and developing countries (or regions). We consider two models reflecting two different environmental relationships between the two regions. In one, the two regions interact and share an environment; this captures the situation where the two regions are contiguous. There are environmental externalities between the regions: the environmental maintenance of one will affect the shared environment. In the other model, the regions are isolated and do not share an environment.

By comparing the environmental quality of the steady states of the two models, we show that both the developing and developed regions have a lower environmental quality under a shared environment than they have under a non-shared environment. The model with the shared environment features a free rider problem whereby both regions reduce their environmental maintenance because they share environmental resources and thus benefit from the environmental maintenance of the other region without paying for it. In

¹ This chapter is published as “Human capital, life expectancy, and the environment”. (see Wu., C., 2017. Human capital, life expectancy, and the environment. *The Journal of International Trade and Economic Development* 26 (8), 885-906.)

contrast, unlike environmental quality, at least one of the regions has higher human capital (output) under the shared environment than under the non-shared environment.

When considering a two-way positive causation between environmental quality and life expectancy, our model shows the existence of an environmental poverty trap, in which the two regions sharing an environment may end up in a stable steady state with both lower environmental quality and shorter life expectancy than the independent regions, due to the free rider problem. Measures to improve environmental quality and life expectancy temporarily are not effective in the long-run.

Second, demographic transition refers to the historical shift from high fertility rates and high infant mortality rates in societies with minimal technology, education (especially of women) and economic development, to low fertility and mortality rates in societies with advanced technology, education and economic development. Actual changes of fertility are more complicated: fertility could change temporarily (for example, during the periods of baby boom and "baby bust") and show different movements depending on the age group (for example, increased fertility of older mothers and decreased fertility of younger mothers in recent years).

In Chapter 3, which is co-authored with Dr. Kazuhiro Yuki, in order to focus on two aspects of fertility: total fertility rate and childbearing age, we develop a three-period model with two periods of childbearing to consider the two aspects at the same time. Also, since theoretical and empirical researches show the correlations between educational (human capital) factors and fertility, we incorporate education into the model. We examine how

well the model can explain the long-run changes in fertility and education in countries such as the US and the UK qualitatively.

We focus on four periods of the long-run changes: the period of the rapid fertility decline roughly from the early 1920s to the early 1930s; the period of baby boom roughly from the early 1940s to the early 1960s; the period of “baby bust” roughly from the early or middle 1960s to the mid-1970s; and the period of rising fertility of elder mothers and falling fertility of younger mothers after the mid-1970s. Changes of fertility and education in the period roughly from the early 1920s to the early 1930s can be explained by the increased productivity of childhood education of the model; the changes in the period of baby boom can be explained by the decreased fixed time spent on household chores, the decreased relative cost of late childbearing and the decreased cost of child-rearing except the increased years of secondary education; the changes in the period of “baby bust” can be explained by the increased cost of child-rearing; the changes in the period after the mid-1970s can be explained by the increased productivity of adulthood education except the largely unchanged total fertility rate.

Finally, traditional structural change analysis typically uses the Cobb-Douglas production function in which the elasticity of substitution between capital and labor (ES) is 1. Although such setting makes analysis relatively simple, it is not able to explain a part of empirical facts, such as changes in factor income shares and sectoral capital–labor ratios. Also, evidence shows that the ESs vary between sectors and the ES of the macro economy changes over time. Therefore, we consider an additional source of technology-driven structural change, that is, the sectoral difference in the ES.

In Chapter 4, in order to consider the ES source of technology-driven structural change, we use a linear production function for the agricultural sector and a Cobb-Douglas production function for the non-agricultural sector, considering the fact that the ES of the former sector is much bigger than the latter sector and it allows theoretical analysis of the dynamics. Also, we consider sectoral differences in productivity growth rate and the capital elasticity of output. As for the utility function, we consider non-homothetic preferences in which the income elasticity of the non-agricultural good (the agricultural good) is greater (smaller) than 1.

Based on this model, we examine how the income ratio of capital to labor, the capital, labor share and value-added shares of the non-agricultural sector, and the ES of the economy change over time.

We show that the income ratio of capital to labor exhibits an inverted U-shaped trend, the capital share of the non-agricultural sector exhibits a U-shaped trend, and the macro-ES decreases over time in the early stage of development and increases over time in the late stage of development. The first result is consistent with empirical observations except recent periods in developed countries in which the income ratio is rising. By contrast, the remaining results agree with some studies but are only partially consistent with other studies. A likely reason for the discrepancy is that in our model, capital stands for equipment such as machines and power products, while in these works, capital includes structure as well as equipment.

CHAPTER 2. Human capital, life expectancy, and the environment

2.1 Introduction

The environmental issues have gradually transcended national boundaries and become international issues. For instance, on January 30, 2000, a dam containing toxic water from the Baia Mare Aurul gold mine in north western Romania burst and released 100,000 cubic meters of waste water heavily contaminated with cyanide into several tributaries of the river Danube. Hungary, Yugoslavia and other countries were affected, resulting in a large number of dead river fish, undrinkable water, and serious damage to the ecological environment basin (BBC news, January 31, 2001).

From January to February 2013, the density of PM 2.5 (atmospheric particulate matter with a diameter of 2.5 micrometers or less) in Beijing, China, was about 350 to 500 micrograms, about 14 to 20 times higher than the 25 micrograms considered safe by the World Health Organization. High PM 2.5 values were also reported in western Japan, due to the transboundary air pollution (The Guardian News, February 16, 2013).

Several prior studies, including Mariani, Pérez-Barahona, and Raffin (2010); Raffin (2009), and Bassetti, Benos, and Karagiannis (2010), have examined the relationship between economic development and environmental protection within single countries. As transboundary pollution has gradually become a matter of international importance, however, analyzing the relationship within single countries, without considering the interactions of environmental conditions between countries, is inadequate.

Accordingly, we analyze transboundary pollution issues by developing an overlapping generations (OLG) model that considers the environmental interactions between two regions, one representing developed countries (or regions) and the other representing developing countries (or regions).²

Each region contains overlapping generations of two-period lived agents who obtain utility from consumption, environmental quality, and the human capital of their children, which is a proxy for wealth.³ Agents live safely through the first period, while surviving the second is subject to uncertainty. During the young period, when all decisions are made, the agents work and allocate their income to consumption, environmental maintenance and human capital investment. Unlike agents in the developing region, those in the developed region value environmental protection more than wealth accumulation.

A single good is produced in the economy. Each agent has one firm producing the final good using human capital with a constant return to scale technology. The good's market is perfectly competitive.

² To emphasize the impact of the environmental pollution of developing countries on developed countries, we interpret the two regions as being developing and developed countries.

³ To consider a two-way causality between human capital accumulation and the environment, we use human capital investment to represent wealth accumulation. On one hand, human capital depends on the environment, since educational choices vary with life expectancy, which is assumed to be affected by environmental conditions; while on the other hand, environmental quality dynamics are affected by human capital through pollution flows (via the production of final goods) and environmental maintenance (through income).

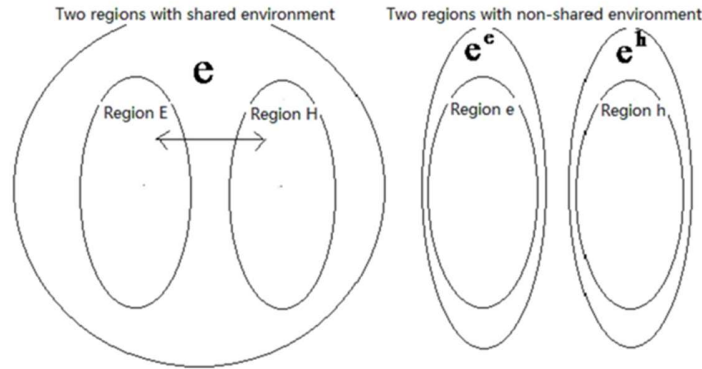


Figure 2.1 – Two regions with shared and non-shared environment

We consider two models with different environmental relationships between the two regions (see Figure 2.1). In one model, the two regions interact and share an environment, capturing the situation in which two regions are contiguous. There exist environmental externalities between the regions: the environmental maintenance of each region affects the shared environment. Agents in each region make decisions taking into account the strategies of the other region. The optimal choices constitute a Nash equilibrium. In the other model, the regions are isolated and do not share an environment.

The survival probability is exogenous until Section 2.5. By comparing the environmental quality of the steady states of the two models, we show that both the developing and developed regions have a lower environmental quality under a shared environment than they have under a non-shared environment. The model with the shared environment features a free rider problem whereby both regions reduce their environmental maintenance because they share environmental resources and thus benefit from the environmental maintenance of the other region without paying for it. In contrast, unlike

environmental quality, at least one of the regions has higher human capital (output) under the shared environment than under the non-shared environment.

In Section 2.5, we endogenize the survival probability. Survival until old age is probabilistic and depends on the environmental quality the agents enjoy when young. In turn, this survival probability influences the weights placed on future environmental quality and human capital in the utility function. A two-way positive causation between environmental quality and life expectancy arises. Our model shows the existence of an environmental poverty trap, in which the two regions sharing an environment may end up in a stable steady state with both lower environmental quality and shorter life expectancy than the independent regions, due to the free rider problem. Measures to improve environmental quality and life expectancy temporarily are not effective in the long-run.

Our work is primarily related to Raffin (2009), Basseti, Benos, and Karagiannis (2010), and Mariani, Pérez-Barahona, and Raffin (2010), who show the possibility of multiple (i.e., good and bad) steady state equilibria through a two-way causality between one variable related to the environment and the other related to income. They also show, in a version of the model in which life expectancy is an endogenous variable, that an environmental poverty trap characterized by short life expectancy and low environmental quality exists.

These theoretical works consider only one region populated by representative agents. In practice, however, people in developed countries are more concerned about environmental conditions than are people in developing countries. Moreover, as already mentioned, environmental issues have gradually developed into international issues. To

consider the interactions of environmental conditions between countries, we examine a two-region economy populated by different types of agents in our model. One region represents a developing country which is populated by agents who value wealth (human capital) accumulation more, and the other one represents a developed country which is populated by agents who place greater value on environmental protection. We compare two cases when the two regions share and do not share the environment.

While the existing studies explain the possibility of an environmental poverty trap based on multiple steady-state equilibria, this work explains the trap based on a free rider problem. When the two regions share the environment, individuals spend less on environmental protection investment. Under certain conditions, the two regions fall into an environmental poverty trap under a shared environment, but they do not under a non-shared environment. The result highlights the possibly serious negative effect of transboundary pollution.

Our model is also related to papers that analyze environmental issues in an OLG framework. Among them, John and Pecchenino (1994) and Ikefuji and Horii (2007) show the existence of environmental poverty traps but without considering the role of life expectancy. Chakraborty (2004) mentions that life expectancy is a possible cause of underdevelopment traps but does not link it to the environment.

The rest of this chapter is organized as follows. Section 2.2 presents the model with a shared environment, and Section 2.3 presents the model with a non-shared environment. Section 2.4 examines the differences between the models when the survival probability is exogenously determined. Section 2.5 studies the difference between the models when the

survival probability is endogenously determined and shows the existence of an environmental poverty trap. Finally, Section 2.6 concludes this chapter. Appendix 2.1 includes calculations and proofs of propositions.

2.2 Model of two regions with a shared environment

There are two regions in the economy, each with one type of agent. We use i to express the type of region, and $i = E, H$ represents the region where, compared to the other region, agents value environmental quality more than human capital (or human capital more than environmental quality). In this section, the two regions share an environment.

2.2.1 Structure of the model

We start by setting up a model in which the economy has an infinite-horizon and is populated by overlapping generations of agents living during two periods: young and old. The agents allocate their income to consumption, environmental maintenance and human capital investment.

All decisions are made in the young period. Survival through the young period to the old is probabilistic, and there is no population growth.

Agents have the following utility function:

$$U_t^i(c_t^i, e_{t+1}, h_{t+1}^i) = \ln c_t^i + \pi_t(\alpha_i \ln e_{t+1} + \beta_i \ln h_{t+1}^i), \alpha_i > 0, \beta_i > 0, i = E, H. \quad (2.1)$$

Agents value their consumption (c_t^i), the environmental quality when they are old (e_{t+1}) and the human capital of their children (h_{t+1}^i). For simplicity, there is no time discounting. π_t denotes the probability of living to the old period, which is also the life

expectancy and is treated as an exogenous variable until Section 2.5. We assume that $\frac{\alpha_E}{\beta_E} > \frac{\alpha_H}{\beta_H}$ and $\alpha_E + \beta_E = \alpha_H + \beta_H$, which implies that $\alpha_E > \alpha_H$ and $\beta_E < \beta_H$. This means that, compared to the other region, the agents in region $E(H)$ value the environmental quality (human capital) more than human capital (environmental quality).

There is one good in our model. The production of this homogeneous good takes place according to the following function:

$$Y_t = h_t^E L_E + h_t^H L_H. \quad (2.2)$$

The total population of each young generation is L , and the young population of each region is L_E and L_H ; thus, $L = L_E + L_H$.

The budget constraint is written as

$$c_t^i + m_t^i + n_t^i = h_t^i, i = E, H. \quad (2.3)$$

Agents spend their income on consumption (c_t^i), environmental maintenance (m_t^i) and human capital investment (n_t^i). Environmental maintenance comprises all the actions the agents can undertake to preserve and boost the environmental conditions. Human capital investment denotes the education provided by parents to their children, implying that education is privately funded.

Parents invest in education and transform it into human capital for their children according to the following function:

$$h_{t+1}^i = \delta (h_t^i)^\theta (\mu + n_t^i)^{1-\theta}, \delta > 0, \mu > 0, i = E, H. \quad (2.4)$$

The accumulation of productive skills is accomplished by parents' human capital (h_t^i) and educational investment (n_t^i) depending on $\theta \in (0,1)$. The parameter μ prevents the human capital from being zero even when parents do not invest in education.

The following equation gives the law of motion of the environmental quality:

$$E_{t+1} = (1 - \eta)E_t + \gamma M_t - \sigma Y_t,$$

where E_t represents the gross environmental resources, a measure of environmental quality, and M_t is the gross environmental maintenance. The parameter $0 < \eta < 1$ denotes the natural rate of environmental deterioration, $\gamma > 0$ represents the effectiveness of environmental maintenance, and $\sigma > 0$ reflects the degree of pollution induced by production. We assume that $\gamma - \sigma > 0$, implying that the benefit produced by one unit of environmental maintenance is larger than the damage caused by one unit of production. Otherwise the environmental resources eventually converge to zero.

With $E_t = (1 + \pi_{t-1})Le_t$, $E_{t+1} = (1 + \pi_t)Le_{t+1}$ and $M_t = m_t^E L_E + m_t^H L_H$, we can transform the law of motion into the following expression:

$$(1 + \pi_t)Le_{t+1} = (1 - \eta)(1 + \pi_{t-1})Le_t + \gamma(m_t^E L_E + m_t^H L_H) - \sigma(h_t^E L_E + h_t^H L_H), \quad (2.5)$$

where e_t denotes the environmental resources per capita.

2.2.2 Optimal choices

Agent i ($i = E, H$) maximizes (2.1) subject to (2.2) to (2.5), knowing the strategy of agent j ($j \neq i$) in investing in environmental maintenance.

$$\max_{c_t^i, m_t^i, n_t^i} U_t^i = \ln c_t^i + \pi_t(\alpha_i \ln e_{t+1} + \beta_i \ln h_{t+1}^i)$$

$$\text{s.t.} \quad c_t^i + m_t^i + n_t^i = h_t^i$$

$$h_{t+1}^i = \delta (h_t^i)^\theta (\mu + n_t^i)^{1-\theta}$$

$$(1 + \pi_t)e_{t+1} = (1-\eta)(1 + \pi_{t-1})e_t + \gamma \left(m_t^i \frac{L_i}{L} + m_t^j \frac{L_j}{L} \right) - \sigma \left(\frac{L_i}{L} h_t^i + \frac{L_j}{L} h_t^j \right)$$

We focus on the inner solutions of the optimal choice problem. The analysis of corner solutions is presented in Appendix 2.1. The optimal choices are

$$m_t^i = \frac{1}{\gamma[1+\pi_t\alpha_i+(1-\theta)\pi_t\beta_i]L_i} \{ \gamma\pi_t\alpha_i L_i h_t^i + \sigma[1 + (1-\theta)\pi_t\beta_i](L_i h_t^i + L_j h_t^j) - \gamma[1 + (1-\theta)\pi_t\beta_i]L_j m_t^j - (1-\eta)(1 + \pi_{t-1})[1 + (1-\theta)\pi_t\beta_i]Le_t + \gamma\pi_t\alpha_i L_i \mu \}, \quad (2.6)$$

$$n_t^i = \frac{(1-\theta)\pi_t\beta_i}{\gamma[1+\pi_t\alpha_i+(1-\theta)\pi_t\beta_i]L_i} [(\gamma - \sigma)L_i h_t^i + L_j(\gamma m_t^j - \sigma h_t^j) + (1-\eta)(1 + \pi_{t-1})Le_t] - \frac{1 + \pi_t\alpha_i}{[1 + \pi_t\alpha_i + (1-\theta)\pi_t\beta_i]} \mu, \quad (2.7)$$

and

$$c_t^i = \frac{1}{\gamma[1+\pi_t\alpha_i+(1-\theta)\pi_t\beta_i]L_i} [(\gamma - \sigma)L_i h_t^i + L_j(\gamma m_t^j - \sigma h_t^j) + (1-\eta)(1 + \pi_{t-1})Le_t + \gamma L_i \mu], \quad (2.8)$$

with $i, j = E, H$ and $j \neq i$.

(2.6) gives the solutions of m_t^E and m_t^H (the full calculation is presented in Appendix 2.2):

$$m_t^i = \frac{1}{\gamma\{\pi_t\alpha_i\alpha_j + \alpha_i[1+(1-\theta)\pi_t\beta_j] + \alpha_j[1+(1-\theta)\pi_t\beta_i]\}L_i} \times (\{\gamma\pi_t\alpha_i\alpha_j + \gamma\alpha_i[1 + (1-\theta)\pi_t\beta_j] + \sigma\alpha_j[1 + (1-\theta)\pi_t\beta_i]\}L_i h_t^i$$

$$\begin{aligned}
& +\{\pi_t\alpha_i\alpha_jL_i + \alpha_i[1 + (1 - \theta)\pi_t\beta_j]L_i - \alpha_j[1 + (1 - \theta)\pi_t\beta_i]L_j\}\mu \\
& -(\gamma - \sigma)\alpha_j[1 + (1 - \theta)\pi_t\beta_i]L_jh_t^j - (1 - \eta)(1 + \pi_{t-1})\alpha_j[1 + (1 - \theta)\pi_t\beta_i]Le_t) \quad (2.9)
\end{aligned}$$

By substituting (2.9) into (2.7) and (2.8), we obtain

$$\begin{aligned}
n_t^i &= \frac{1}{\gamma\{\pi_t\alpha_i\alpha_j + \alpha_i[1 + (1 - \theta)\pi_t\beta_j] + \alpha_j[1 + (1 - \theta)\pi_t\beta_i]\}L_i} \\
&\times \left[(1 - \theta)\pi_t\alpha_j\beta_i[(\gamma - \sigma)(L_ih_t^i + L_jh_t^j) + (1 - \eta)(1 + \pi_{t+1})Le_t] \right. \\
&\quad \left. - \gamma\{\pi_t\alpha_i\alpha_jL_i + \alpha_i[1 + (1 - \theta)\pi_t\beta_j]L_i + \alpha_jL_i - (1 - \theta)\pi_t\alpha_j\beta_iL_j\}\mu \right] \quad (2.10)
\end{aligned}$$

and

$$\begin{aligned}
c_t^i &= \frac{\alpha_j}{\gamma\{\pi_t\alpha_i\alpha_j + \alpha_i[1 + (1 - \theta)\pi_t\beta_j] + \alpha_j[1 + (1 - \theta)\pi_t\beta_i]\}L_i} [(\gamma - \sigma)(L_ih_t^i + L_jh_t^j) \\
&\quad + (1 - \eta)(1 + \pi_{t-1})Le_t + \gamma L\mu] \text{ with } i, j = E, H, j \neq i. \quad (2.11)
\end{aligned}$$

Comparative statics

From (2.9), we can observe a negative association between environmental maintenance m_t^i and the current environmental quality e_t ; investment in maintenance is needed less if the inherited environment is less degraded. Further, the maintenance of type i is positively affected by their human capital, and negatively by the human capital of the other type. It depends on h_t^i positively because of the traditional income effect, and depends on h_t^j negatively because the human capital of type j raises their income and thus their environmental maintenance, which reduces the environmental maintenance of the free-riding type i .

From (2.10), parental investment in education depends positively on the current environmental quality because, if the inherited environment is less degraded, investment in maintenance is needed less, and thus investment in education can be increased. Educational investment also depends positively on the human capital of the two types. It depends on h_t^i because of the traditional income effect, and on h_t^j because the human capital of type j raises their income and thus their environmental maintenance, which in turn lowers the maintenance and raises the educational investment of type i . These comparative static results will be used later.

Differentiating m_t^i with respect to π_t produces

$$\frac{\partial m_t^i}{\partial \pi_t} = \frac{\alpha_i \alpha_j [(\gamma - \sigma)(L_i h_t^i + L_j h_t^j) + \gamma L \mu + (1 - \eta)(1 + \pi_{t-1}) L e_t] [\alpha_j + (\beta_j - \beta_i)(1 - \theta)]}{\gamma L_i \{\pi_t \alpha_i \alpha_j + \alpha_i [1 + (1 - \theta) \pi_t \beta_j] + \alpha_j [1 + (1 - \theta) \pi_t \beta_i]\}^2} \quad (2.12)$$

It is clear that $\frac{\partial m_t^E}{\partial \pi_t} > 0$. For $\frac{\partial m_t^H}{\partial \pi_t} > 0$ to hold, however, $\alpha_E > (\beta_H - \beta_E)(1 - \theta)$

must hold. Without this condition, we always have

$$\frac{\partial M_t}{\partial \pi_t} = \frac{\partial (L_E m_t^E + L_H m_t^H)}{\partial \pi_t} > 0. \quad (2.13)$$

Differentiating n_t^i with respect to π_t produces

$$\frac{\partial n_t^i}{\partial \pi_t} = \frac{(\alpha_i + \alpha_j) \alpha_j \beta_i (1 - \theta) [(\gamma - \sigma)(L_i h_t^i + L_j h_t^j) + \gamma L \mu + (1 - \eta)(1 + \pi_{t-1}) L e_t]}{\gamma L_i \{\pi_t \alpha_i \alpha_j + \alpha_i [1 + (1 - \theta) \pi_t \beta_j] + \alpha_j [1 + (1 - \theta) \pi_t \beta_i]\}^2} > 0 \quad (2.14)$$

From (2.13) and (2.14), if the life expectancy becomes longer, the gross environmental maintenance and individual and gross human capital investment increase because the increase in π_t increases the weights on the future environmental quality and

human capital in the utility function. Thus, environmental quality and human capital are positively affected by life expectancy.

2.2.3 Steady state

We have (2.4) and (2.5) to describe the dynamics of our model. By substituting (2.9) to (2.11) into (2.4) and (2.5), we obtain the following system of three difference equations.

$$h_{t+1}^i = \delta(h_t^i)^\theta \left(\frac{(1-\theta)\pi_t\alpha_j\beta_i[(\gamma-\sigma)(L_i h_t^i + L_j h_t^j) + (1-\eta)(1+\pi_{t-1})Le_t + \gamma L\mu]}{\gamma L_i \{\pi_t\alpha_i\alpha_j + \alpha_i[1+(1-\theta)\pi_t\beta_j] + \alpha_j[1+(1-\theta)\pi_t\beta_i]\}} \right)^{1-\theta}, \quad (2.15)$$

and

$$e_{t+1} = \frac{\pi_t\alpha_i\alpha_j[(\gamma-\sigma)(L_i h_t^i + L_j h_t^j) + (1-\eta)(1+\pi_{t-1})Le_t + \gamma L\mu]}{(1+\pi_t)\{\pi_t\alpha_i\alpha_j + \alpha_i[1+(1-\theta)\pi_t\beta_j] + \alpha_j[1+(1-\theta)\pi_t\beta_i]\}L}, \quad (2.16)$$

with $i, j = E, H, j \neq i$.

A steady-state equilibrium is defined as a fixed point such that $h_{t+1}^i = h_t^i = h^i$, $e_{t+1} = e_t = e$ and $\pi_{t+1} = \pi_t = \pi$. By substituting these conditions into (2.15) and (2.16), we can solve for the steady-state values of h^E , h^H , and e :

$$h^i = \frac{\gamma(1-\theta)\pi\alpha_j\beta_i L\mu}{\left(\gamma\{\eta\pi\alpha_i\alpha_j + \alpha_i[1+(1-\theta)\pi\beta_j] + \alpha_j[1+(1-\theta)\pi\beta_i]\}\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)(\alpha_i\beta_j + \alpha_j\beta_i) \right) L_i}, \quad (2.17)$$

and

$$e = \frac{\gamma^2\pi\alpha_i\alpha_j\delta^{\frac{1}{\theta-1}}\mu}{(1+\pi)\left(\gamma\{\eta\pi\alpha_i\alpha_j + \alpha_i[1+(1-\theta)\pi\beta_j] + \alpha_j[1+(1-\theta)\pi\beta_i]\}\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)(\alpha_i\beta_j + \alpha_j\beta_i) \right)}, \quad (2.18)$$

with $i, j = E, H, j \neq i$ (the full calculation is presented in Appendix 2.2).

The necessary and sufficient condition for the steady state values to be strictly positive is

$$\begin{aligned}
& \gamma\{\eta\pi\alpha_E\alpha_H + \alpha_E[1 + \beta_H\pi(1 - \theta)] + \alpha_H[1 + \beta_E\pi(1 - \theta)]\}\delta^{\frac{1}{\theta-1}} \\
& -\pi(1 - \theta)(\gamma - \sigma)(\alpha_E\beta_H + \alpha_H\beta_E) > 0 \\
& \Leftrightarrow \gamma(\eta\pi\alpha_E\alpha_H + \alpha_E + \alpha_H)\delta^{\frac{1}{\theta-1}} \\
& +\pi(1 - \theta)(\alpha_E\beta_H + \alpha_H\beta_E)\left[\gamma\delta^{\frac{1}{\theta-1}} - (\gamma - \sigma)\right] > 0. \tag{2.19}
\end{aligned}$$

Notice that

$$\gamma > (\gamma - \sigma)\delta^{\frac{1}{1-\theta}} \tag{2.20}$$

is a sufficient condition for the steady-state values to be strictly positive. When $\delta \leq 1$, it always holds; and when $\delta > 1$, it holds only if $\frac{\gamma}{\sigma} < \frac{\delta^{\frac{1}{1-\theta}}}{\delta^{\frac{1}{1-\theta}} - 1}$. We assume it to hold henceforth.

The condition requires that the weight attached to human capital (δ and θ in the human capital production function) must be small enough so that $\delta^{\frac{1}{1-\theta}} < \frac{1}{\frac{\gamma}{\sigma} - 1}$ holds. The sufficient condition becomes more feasible, if the difference between the maintenance effectiveness (γ) and the degree of pollution induced by production (σ) is smaller.

Assumption (2.20) does not appear very restrictive because, when $\delta > 1$ and $\frac{\gamma}{\sigma} \geq \frac{\delta^{\frac{1}{1-\theta}}}{\delta^{\frac{1}{1-\theta}} - 1}$, the steady state with positive values of endogenous variables exist only when

$\gamma(\eta\pi\alpha_E\alpha_H + \alpha_E + \alpha_H) > \left[(\gamma - \sigma)\delta^{\frac{1}{1-\theta}} - \gamma \right] \pi(1 - \theta)(\alpha_E\beta_H + \alpha_H\beta_E)$ (see Appendix 2.3 for details).

Steady-state comparative statics

Differentiating e with respect to π produces

$$\frac{\partial e}{\partial \pi} = \frac{\gamma^2 \pi \alpha_i \alpha_j \delta^{\frac{1}{\theta-1}} \mu \left\{ \gamma [\alpha_i + \alpha_j - \eta \pi^2 \alpha_i \alpha_j - (1-\theta) \pi^2 (\alpha_i \beta_j + \alpha_j \beta_i)] \delta^{\frac{1}{\theta-1}} + \pi^2 (1-\theta) (\gamma - \sigma) (\alpha_i \beta_j + \alpha_j \beta_i) \right\}}{(1+\pi)^2 \left(\gamma \{ \eta \pi \alpha_i \alpha_j + \alpha_i [1 + (1-\theta) \pi \beta_j] + \alpha_j [1 + (1-\theta) \pi \beta_i] \} \delta^{\frac{1}{\theta-1}} - \pi (1-\theta) (\gamma - \sigma) (\alpha_i \beta_j + \alpha_j \beta_i) \right)^2} \quad (2.21)$$

We can show that $\frac{\partial e}{\partial \pi} > 0$ when π is sufficiently small,⁴ and the value of $\frac{\partial e}{\partial \pi}$ falls with π .⁵ This can be explained as follows. When π increases, according to the utility function, agents pay more attention to environmental maintenance, which has a positive effect on the environmental resources per capita. At the same time, however, the total population increases, which has a negative effect on per capita resources. When π is sufficiently small, the positive effect dominates and $\frac{\partial e}{\partial \pi} > 0$; when π increases, the negative (positive) effect becomes greater (smaller) and the total effect is ambiguous.

However, we always have

⁴ From (2.21), $\frac{\partial e}{\partial \pi} > 0$ when $\pi < \sqrt{\frac{(\alpha_E + \alpha_H) \gamma \delta^{\frac{1}{\theta-1}}}{\eta \gamma \alpha_E \alpha_H \delta^{\frac{1}{\theta-1}} + (1-\theta) (\alpha_E \beta_H + \alpha_H \beta_E) \left[\gamma \delta^{\frac{1}{\theta-1}} - (\gamma - \sigma) \right]}} \equiv \widetilde{\pi^e}$.

⁵ The sign of $\lim_{\pi \rightarrow 1} \frac{\partial e}{\partial \pi}$ is ambiguous.

$$\frac{\partial E}{\partial \pi} = \frac{\partial [(1+\pi)Le]}{\partial \pi} > 0. \quad (2.22)$$

From (2.17), it is obvious that h^i depends on π positively.

Hence, gross environmental resources and individual and gross human capital are positively affected by life expectancy in the steady-state.

2.3 Model of two regions with non-shared environment

In Section 2.2, we analyzed the model with a shared environment. In this section, we present the model with a non-shared environment to compare the environmental qualities of the steady states of the two models in Section 2.4.

The two regions are the same as in the model with a shared environment, but now we use $i = e, h$ to represent the region where agents care more about human capital than environmental quality (or more about environmental quality than human capital).

Agent $i = e, h$ maximizes (2.23) subject to (2.24) to (2.26):

$$\text{Max}_{c_t^i, m_t^i, n_t^i} U_t^i = \ln c_t^i + \pi_t (\alpha_i \ln e_{t+1} + \beta_i \ln h_{t+1}^i) \quad (2.23)$$

s.t.

$$c_t^i + m_t^i + n_t^i = h_t^i \quad (2.24)$$

$$h_{t+1}^i = \delta (h_t^i)^\theta (\mu + n_t^i)^{1-\theta} \quad (2.25)$$

$$(1 + \pi_t) e_{t+1}^i = (1 - \eta)(1 + \pi_{t-1}) e_t^i + \gamma m_t^i - \sigma h_t^i \quad (2.26)$$

The optimal choices are:

$$m_t^i = \frac{\{\gamma\pi_t\alpha_i + \sigma[1+(1-\theta)\pi_t\beta_i]\} - (1-\eta)(1+\pi_{t-1})[1+(1-\theta)\pi_t\beta_i]e_t^i + \gamma\pi_t\alpha_i\mu}{\gamma[\pi_t\alpha_i + 1 + (1-\theta)\pi_t\beta_i]},$$

$$n_t^i = \frac{(1-\theta)\pi_t\beta_i}{\gamma[1+\pi_t\alpha_i+(1-\theta)\pi_t\beta_i]}[(\gamma-\sigma)h_t^i + (1-\eta)(1+\pi_{t-1})e_t^i] - \frac{1+\pi_t\alpha_i}{[1+\pi_t\alpha_i+(1-\theta)\pi_t\beta_i]}\mu$$

and

$$c_t^i = \frac{1}{\gamma[\pi_t\alpha_i + 1 + (1-\theta)\pi_t\beta_i]}[(\gamma-\sigma)h_t^i + (1-\eta)(1+\pi_{t-1})Le_t^i + \gamma\mu],$$

with $i = e, h$.

By substituting the solutions of optimal choices, $h_{t+1}^i = h_t^i = h^i$, $e_{t+1}^i = e_t^i = e^i$ and $\pi_{t+1} = \pi_t = \pi$ into (2.25) and (2.26), we have:

$$h^i = \frac{\gamma(1-\theta)\pi\beta_i\mu}{\gamma[\eta\pi\alpha_i + 1 + (1-\theta)\pi\beta_i]\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)\beta_i} \quad (2.27)$$

and

$$e^i = \frac{\gamma^2\pi\alpha_i\delta^{\frac{1}{\theta-1}}\mu}{(1+\pi)\left\{\gamma[\eta\pi\alpha_i + 1 + (1-\theta)\pi\beta_i]\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)\beta_i\right\}}, \quad (2.28)$$

with $i = e, h$.

From assumption (2.20), h^i and e^i are strictly positive.⁶ We can show that $\frac{\partial e^i}{\partial \pi} >$

0, $i = e, h$, when π is sufficiently small, and the value of $\frac{\partial e^i}{\partial \pi}$ falls with π .

2.4 Difference between shared and non-shared environments

We now compare the environmental qualities when the two regions are interconnected (e) and when they are independent (e^e and e^h).

By transforming (2.18) and (2.28) so that their numerators are equal, we can compare them with their denominators.

The denominator of e^h minus the denominator of e^e is

$$\begin{aligned} & \gamma[\alpha_E - \alpha_H + \alpha_E\beta_H\pi(1 - \theta) - \alpha_H\beta_E\pi(1 - \theta)]\delta^{\frac{1}{\theta-1}} \\ & - \pi(1 - \theta)(\gamma - \sigma)(\alpha_E\beta_H - \alpha_H\beta_E), \end{aligned} \quad (2.29)$$

and the denominator of e minus that of e^i is

$$\gamma\delta^{\frac{1}{\theta-1}}\alpha_i(1 + (1 - \theta)\pi\beta_j) - \pi(1 - \theta)(\gamma - \sigma)\alpha_i\beta_j, i = e, h. \quad (2.30)$$

From assumption (2.20), (2.29) and (2.30) are positive, which means that $e^e > e^h > e$.

⁶ From (2.28), $\frac{\partial e^i}{\partial \pi} = \frac{\gamma^2\alpha_i\delta^{\frac{1}{\theta-1}}\mu\left(\gamma\delta^{\frac{1}{\theta-1}} - \pi^2\left\{\gamma\delta^{\frac{1}{\theta-1}}[\eta\alpha_i + (1-\theta)\beta_i] - (1-\theta)(\gamma-\sigma)\beta_i\right\}\right)}{(1+\pi)^2\left\{\gamma[\eta\pi\alpha_i + 1 + (1-\theta)\pi\beta_i]\delta^{\frac{1}{\theta-1}} - (1-\theta)(\gamma-\sigma)\pi\beta_i\right\}^2}$, $i = e, h$. $\frac{\partial e^i}{\partial \pi} > 0$ only if $\pi < \sqrt{\frac{\gamma\delta^{\frac{1}{\theta-1}}}{\gamma\delta^{\frac{1}{\theta-1}}[\eta\alpha_i + (1-\theta)\beta_i] - (1-\theta)(\gamma-\sigma)\beta_i}} \equiv \widehat{\pi}_i^e$.

We can then propose the following:⁷

Proposition 2.1 *In the steady state, under certain conditions,⁸ the regions with a shared environment have lower environmental quality than the regions with a non-shared environment. Among the latter regions, the region populated by agents concerned about human capital (wealth) has lower environmental quality than the other region. Thus, $e^e > e^h > e$.*

The two connected regions have lower environmental quality than the two independent regions have, and independent region h has lower environmental quality than the independent region e ($e^e > e^h > e$) has, for several reasons. First, when the regions do not share an environment, the environment of region e is better than that of region h ($e^e > e^h$) because the agents in the former region value the environment more than do those in the latter region. Second, when the regions share an environment, a free rider problem arises, broadly speaking, due to environmental externalities between the regions. According to (2.6), (2.7), and the comparative statics analysis in Section 2.2, the environmental maintenance of one region has a direct negative effect on the maintenance of the other and a positive indirect effect on maintenance through positive dynamic effects

⁷ Appendix 2.1 shows that even when corner solutions constitute the steady-state equilibrium, Proposition 2.1 still holds.

⁸ These conditions constitute assumption (2.20), $\frac{\alpha_E}{\beta_E} > \frac{\alpha_H}{\beta_H}$ and $\alpha_E + \beta_E = \alpha_H + \beta_H$.

on educational investment and thus income.⁹ In the steady state with a shared environment, the negative effect dominates; hence, the environmental quality under the shared environment is lower than that under the non-shared environment.

For simplicity, we did not consider the free rider problem between agents within the same region. Proposition 2.1 still holds when we consider this case because, unlike the free rider problem between agents from different regions, that between agents within the same region occurs under both shared and non-shared environment conditions. It also holds even in some extreme cases (for example, when α_E and β_H are much greater than β_E and α_H) under the assumption of (2.20), because (2.29) and (2.30) are always positive.

In practice, the shared environment case corresponds to the situation in which a developed country and a developing country are contiguous. The developing country will pay more attention to economic development than environmental protection, and the developed country will focus more on environmental protection. The two countries share some environmental resource. Due to the free rider problem in environmental maintenance, the environmental conditions of the two countries will be worse than if they did not share the environment, and the negative impact will be greater for the developed country than that for the developing country.

⁹ The higher environmental maintenance of one region lowers the maintenance of the other, which increases the disposable income in the other region and thus the educational investment. As a result, the income of future generations in the other region increases, and thus their environmental maintenance also increases because of the income effect.

We can also compare human capital accumulation and thus the output when the two regions are interconnected (h^E and h^H) and independent (h^e and h^h).

By transforming (2.17) and (2.27) so that their numerators are equal, we can compare them with their denominators.

The denominator of h^E minus that of h^e is

$$\begin{aligned} & -\gamma\{\eta\pi\alpha_E\alpha_H L_H - \alpha_E[1 + (1 - \theta)\pi\beta_H]L_E + \alpha_H[1 + (1 - \theta)\pi\beta_E]L_H\}\delta^{\frac{1}{\theta-1}} \\ & -\pi(1 - \theta)(\gamma - \sigma)(\alpha_E\beta_H L_E - \alpha_H\beta_E L_H), \end{aligned} \quad (2.31)$$

and the denominator of h^H minus that of h^h is

$$\begin{aligned} & -\gamma\{\eta\pi\alpha_E\alpha_H L_E + \alpha_E[1 + (1 - \theta)\pi\beta_H]L_E - \alpha_H[1 + (1 - \theta)\pi\beta_E]L_H\}\delta^{\frac{1}{\theta-1}} \\ & +\pi(1 - \theta)(\gamma - \sigma)(\alpha_E\beta_H L_E - \alpha_H\beta_E L_H). \end{aligned} \quad (2.32)$$

Except for the terms $\gamma\eta\pi\alpha_E\alpha_H L_H$ and $\gamma\eta\pi\alpha_E\alpha_H L_E$, the signs of the remaining terms in (2.31) and (2.32) are reversed. Depending on the size of $\frac{L_E}{L_H}$, the following three cases occur.

$$\text{From assumption (2.20), when } \frac{L_E}{L_H} > \frac{\gamma\eta\pi\alpha_E\alpha_H + \gamma\alpha_H[1 + (1 - \theta)\pi\beta_E]\delta^{\frac{1}{\theta-1}} - \pi(1 - \theta)(\gamma - \sigma)\alpha_H\beta_E}{\gamma\alpha_E[1 + (1 - \theta)\pi\beta_H]\delta^{\frac{1}{\theta-1}} - \pi(1 - \theta)(\gamma - \sigma)\alpha_E\beta_H},$$

(2.31) is positive and (2.32) is negative, which means that $h^E < h^e$ and $h^H > h^h$;

$$\text{when } \frac{L_E}{L_H} < \frac{\gamma\alpha_H[1 + (1 - \theta)\pi\beta_E]\delta^{\frac{1}{\theta-1}} - \pi(1 - \theta)(\gamma - \sigma)\alpha_H\beta_E}{\gamma\eta\pi\alpha_E\alpha_H + \gamma\alpha_E[1 + (1 - \theta)\pi\beta_H]\delta^{\frac{1}{\theta-1}} - \pi(1 - \theta)(\gamma - \sigma)\alpha_E\beta_H},$$

(2.31) is negative and (2.32) is positive, which means that $h^E > h^e$ and $h^H < h^h$;

and when

$$\frac{L_E}{L_H} \in \left[\frac{\gamma\alpha_H[1+(1-\theta)\pi\beta_E]\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)\alpha_H\beta_E}{\gamma\eta\pi\alpha_H\alpha_E + \gamma\alpha_E[1+(1-\theta)\pi\beta_H]\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)\alpha_E\beta_H}, \right. \\ \left. \frac{\gamma\eta\pi\alpha_E\alpha_H + \gamma\alpha_H[1+(1-\theta)\pi\beta_E]\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)\alpha_H\beta_E}{\gamma\alpha_E[1+(1-\theta)\pi\beta_H]\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)\alpha_E\beta_H} \right],$$

both (2.31) and (2.32) are negative, which means that $h^E > h^e$ and $h^H > h^h$.

Proposition 2.2 *In the steady-state, under certain conditions,¹⁰ depending on the ratio of the population sizes of the two regions ($\frac{L_E}{L_H}$), the magnitude relations of human capital accumulation under the shared environment (h^E and h^H) and the non-shared environment (h^e and h^h) differ: when $\frac{L_E}{L_H}$ is sufficiently small, $h^E > h^e$ and $h^H < h^h$; when $\frac{L_E}{L_H}$ is sufficiently large, $h^E < h^e$ and $h^H > h^h$; and when $\frac{L_E}{L_H}$ is in the middle range, $h^E > h^e$ and $h^H > h^h$.*

Unlike for environmental quality, both regions cannot have lower human capital (output) under the shared environment than under the non-shared environment; that is, $h^E < h^e$ and $h^H < h^h$ do not hold simultaneously. The relationship between human capital accumulation under the shared and non-shared environments varies with the ratio of the population sizes of the two regions for the following reason.

Since the regions do not share human capital, there is no free rider problem for human capital investment under the shared environment, which means that there are no direct effects between the human capital investments of the two regions. The human capital

¹⁰ These conditions are the assumption of (2.20), $\frac{\alpha_E}{\beta_E} > \frac{\alpha_H}{\beta_H}$ and $\alpha_E + \beta_E = \alpha_H + \beta_H$.

investment of the two regions can indirectly affect each other only through environmental maintenance. According to (2.6) and (2.7), the human capital investment of one region has a positive dynamic effect and a negative static effect on its own environmental maintenance and thus a positive static effect and a negative dynamic effect on the other region's investment.^{11, 12}

Which effect dominates, depends on the ratio of the population sizes because the positive effect on the investment of the other region increases with the ratio of the population size of the region to that of the other region. In the steady state with a shared environment, when $\frac{L_E}{L_H}$ is in the middle range, the positive effects dominate in both regions, which means that the human capital accumulations of the two regions under a shared environment are greater than that under a non-shared environment. When $\frac{L_E}{L_H}$ is sufficiently large, the positive (negative) effect dominates in region E (H), where agents care more about the environmental quality (human capital) than about human capital (environmental quality), which means that the human capital investment of region E (H) has a positive (negative) effect on that of region H (E). Thus, the human capital of region H (E) under a shared environment is greater (smaller) than under a non-shared environment. Similarly,

¹¹ The negative effect arises because higher human capital investment lowers the remaining disposable income and thus the environmental maintenance. The positive dynamic effect arises because higher human capital investment increases the income of future generations and consequently their environmental maintenance.

¹² As mentioned in the analysis of Proposition 2.1, the higher (lower) environmental maintenance of one region lowers (raises) the environmental maintenance of the other region, which raises (lowers) the remaining disposable income of the other region. Consequently, the human capital investment of the other region also increases (decreases).

when $\frac{L_E}{L_H}$ is sufficiently small, the negative (positive) effect dominates in region E (H), which means that the investment of region E (H) has a negative (positive) effect on that of region H (E). Thus, the human capital of region H (E) under a shared environment is smaller (greater) than that under a non-shared environment. Therefore, the magnitude relations of human capital accumulation under shared and non-shared environments differ depending on the ratio of the population sizes of the two regions.

We can examine specific examples to understand the implication of the result. For example, when $\frac{L_E}{L_H}$ is in the middle range, regions E and H could represent Greece and Bulgaria respectively. According to Syrakov et al. (2002), Bulgaria has the worst air pollution in the EU and about 4% of the sulfur oxides emitted by Bulgaria were deposited over the Greek territory in 1995. When $\frac{L_E}{L_H}$ is sufficiently large, the regions E and H could represent the neighboring regions in South Korea and North Korea, such as Seoul and Pyongyang. According to Tenenbaum (2005), air pollution in Pyongyang (North Korea), largely due to combustion of coal, and the pollution of rivers and streams were reported as being severe and adversely affecting the health of the residents of Seoul (South Korea). When $\frac{L_E}{L_H}$ is sufficiently small, the regions E and H could represent Korea and China, respectively. The most harmful source of air pollution in China is particulates known as PM2.5, and its effect on Korea is serious. An OECD report warned in June, 2016 that air pollution will cause millions of deaths in Korea by 2060, and deal a heavy blow to the country's economy unless it makes more efforts to improve the air quality (Chosun Media, June 13, 2016). According to Proposition 2.1, in all the cases, the environments of both regions are worse under a shared environment. However, according to Proposition 2.2, the

human capital (output) of both regions in the first case and that of the smaller region in the second and third cases are higher under the shared environment.

2.5 Life expectancy and the environment

In Section 2.4, we showed the adverse effects of sharing an environment on the environmental quality, when the survival probability π_t is exogenous, but it would not be realistic to assume that π_t is exogenous. In this section, we endogenize π_t and show the adverse effects of sharing an environment on both environmental quality and survival probability and the possibility of an environmental poverty trap.

We introduce the following specific functional form relating the survival probability to environmental conditions.

$$\pi_t(e_t) = \begin{cases} \underline{\pi} & \text{for } e_t < \hat{e} \\ \overline{\pi} & \text{for } e_t \geq \hat{e}, \end{cases} \quad (2.33)$$

where $\overline{\pi} > \underline{\pi}$, $\hat{e}$ is an exogenous threshold value of environmental quality, and the following condition is assumed to hold:

$$e(\overline{\pi}) < \hat{e} < e^h(\underline{\pi}).^{13} \quad (2.34)$$

As shown in Appendix 2.4, for (2.34) to hold, the difference between α_E and α_H needs not be very large (note $\alpha_E > \alpha_H$).

From the comparative statics analysis in Section 2.2 and (2.33), a two-way positive causation between life expectancy and environmental quality arises, along the transition

¹³ As is clarified later, this is a necessary condition for the existence of the low environmental poverty trap.

path, and in the steady state. From (2.33), life expectancy depends on the environmental quality positively; from (2.1), environmental quality depends on life expectancy positively because the preference for higher environmental quality increases with life expectancy.

From (2.21) and (2.28), when π is sufficiently small,¹⁴ $\frac{\partial e}{\partial \pi} > 0$, $\frac{\partial e^e}{\partial \pi} > 0$ and $\frac{\partial e^h}{\partial \pi} > 0$ hold. Then, from (2.33), (2.34) and $e^e > e^h$ (Proposition 2.1), the steady-state values of environmental quality under shared and non-shared environments become $e = e(\underline{\pi})$, $e^e = e^e(\bar{\pi})$ and $e^h = e^h(\bar{\pi})$, and $e(\underline{\pi}) < e^h(\underline{\pi}) < e^h(\bar{\pi}) < e^e(\bar{\pi})$ holds. Thus, environmental quality and life expectancy are lower under a shared environment than they are under a non-shared environment.

We can also show the existence of an environmental poverty trap. Under the shared environment, if an exogenous shock that improves health and medical conditions occurs, and thus $\underline{\pi}$ increases to $\bar{\pi}$ temporarily, the environmental quality also increases temporarily. In the long run, however, because of condition (2.34), π and e decrease back to $\underline{\pi}$ and $e(\underline{\pi})$. Although the environmental quality and life expectancy improve temporarily through the exogenous shock, the improved environmental quality does not exceed the exogenous threshold value $\hat{e}$, and the environmental maintenance of the agents is lower than the depletion of the environment. Eventually, the environmental quality and life expectancy decrease back to the original state. In contrast, under the non-shared environment, even if

¹⁴ π must be sufficiently small so that $\pi < \tilde{\pi} \equiv \min(\tilde{\pi}^e, \tilde{\pi}^h)$, $i = e, h$, is satisfied, where $\tilde{\pi}^e$ is π such that $\frac{\partial e}{\partial \pi} = 0$ (see footnote 4) and

$\tilde{\pi}^h$, $i = e, h$, is π such that $\frac{\partial e^i}{\partial \pi} = 0$ (see footnote 6).

an exogenous shock that worsens the health and medical conditions occurs, the environmental quality and life expectancy will eventually increase back to the original good state in the long-run (according to (2.34)). Thus, we propose the following:

Proposition 2.3 *Under certain conditions,¹⁵ the economy of two regions with a shared environment enters an environmental poverty trap,¹⁶ characterized by both lower environmental quality and shorter life expectancy than in two regions with a non-shared environment, due to the free rider problem.*

The proposition has the following policy implications. If neighboring developed and developing countries (or regions) are in an environmental poverty trap, measures to improve life expectancy and environmental quality temporarily, such as health and/or medical assistances from international organizations or developed countries, are not effective in the long-run. As long as individuals maintain their preferences, the free rider problem exists, and the two regions cannot escape from the trap. Rather, defraying the environmental maintenance expenditures of developing countries or helping agents in developing countries enhance their environmental consciousness would be more effective.

¹⁵ These conditions are the assumption of a sufficiently small π , (2.20), (2.34), $\frac{\alpha_E}{\beta_E} > \frac{\alpha_H}{\beta_H}$ and $\alpha_E + \beta_E = \alpha_H + \beta_H$.

¹⁶ Commonly, a poverty trap implies that an economy is trapped in a bad steady state characterized by relatively low per capita income, environmental degradation and low life expectancy. However, according to Propositions 2.1 and 2.2, under the shared environment, two regions enter a state with lower environmental quality and shorter life expectancy, but at least one of them has higher human capital (wealth). Therefore, in our model, the environmental poverty trap does not necessarily imply material poverty trap.

Unlike our model, Raffin (2009), Basseti, Benos, and Karagiannis (2010), and Mariani, Pérez-Barahona, and Raffin (2010) use a homogeneous agent model to demonstrate a two-way positive causation between life expectancy and environmental quality, and an environmental poverty trap. Only one region populated by one type of agents exists; thus, the free rider problem does not exist in their model. Because of the two-way positive causation, multiple steady-state equilibria exist under certain conditions. An economy could be trapped in an equilibrium that has poorer environmental quality and lower life expectancy than the other equilibria. This could be characterized as an environmental poverty trap.

Short life spans and poor environment conditions arise for a different reason in our model: they arise because a free rider problem appears under a shared environment. Both regions reduce their environmental maintenance to benefit from the environmental maintenance of the other region without paying for it. The possibility of the two regions to end up in a stable steady state with both lower environmental quality and shorter life expectancy is greater than that of the independent region. When life expectancy is sufficiently low and the difference in the preferences for environmental quality of agents in the two regions is not very large, they fall into an environmental poverty trap under a shared environment, but not under a non-shared one.

2.6 Conclusions

This work develops an OLG model, which considers the environmental interaction between two regions, representing developed and developing countries (or regions). We consider two models reflecting two different environmental relationships between the two

regions. In one, the two regions interact and share an environment; this captures the situation where the two regions are contiguous. There are environmental externalities between the regions: the environmental maintenance of one will affect the shared environment. In the other model, the regions are isolated and do not share an environment.

By comparing the environmental qualities of the steady states of the two models with exogenous survival probability, we show that both the developing and developed regions have lower environmental quality under the shared environment than they have under the non-shared environment. In the model with the shared environment, a free rider problem arises: both regions reduce their environmental maintenance because they can benefit from the environmental maintenance of the other region without paying for it.

Finally, we endogenize the survival probability. Survival until old age is probabilistic and depends on the environmental quality level when the agents were young. In turn, this survival probability influences the weights placed on the future environmental quality and future human capital in the utility function. A two-way positive causation between life expectancy and environmental quality arises. Our model can explain the existence of an environmental poverty trap based on the shape of the survival probability function: two regions sharing an environment may end up in the trap, characterized by both lower environmental quality and shorter life expectancy, due to the free rider problem.

The finding that two regions with a shared environment have lower environmental quality and shorter life expectancy than regions with a non-shared environment have, and that they could fall into an environmental poverty trap has several real-world implications. According to the model, under a shared environment, the transboundary pollution caused

by the pursuit of economic growth in developing countries could lead to a bad state with lower environmental quality and shorter life expectancy than under the non-shared environment, not only in these countries but also in the bordering developed countries. Measures to improve life expectancy and environmental quality temporarily are not effective in the long-run. Rather, defraying the environmental maintenance expenditures of developing countries would be more effective. Developed countries can also help agents in developing countries enhance their environmental consciousness.

The transborder air pollution on the Mexico-U.S. border is a good example (Geo-Mexico, May 26, 2011). The worst air pollution along the Mexico-U.S. border occurs in the border cities of Ambos Nogales (Both Nogales) regions, which share many air quality issues. Most pollution, including dust and vehicle emissions, comes from the south of the border. Air quality is also adversely impacted by natural hazards such as wildfires in the region. The air in the US side of the region regularly fails to meet the U.S. Environmental Protection Agency's limits for coarse particulate (PM10) pollution. Air pollution adversely affects health, resulting in a higher incidence of respiratory diseases and premature mortality.

However, steps are being taken to ameliorate the air pollution in Ambos Nogales by authorities on both sides of the border. On the Mexican side, along with the strengthening of the awareness for environmental protection, reliable air quality monitoring systems are being installed, with the financial help of international agencies. On the US side, the Arizona Department of Environmental Quality is trying an innovative approach, using federal funds to help Mexican owners replace their old and polluting

equipment. For example, they pay Mexican truck owners to replace their old, leaky mufflers with state-of-the-art catalytic converters.

Appendix Calculations and Proofs of Propositions in chapter 2

A2.1 The corner solutions

In Section 2.2.2, we focused on the inner solutions of the optimal choice problem. Let us now consider the corner solutions of the problem:

$$\text{Max}_{c_t^i, m_t^i, n_t^i} U_t^i = \ln c_t^i + \pi_t(\alpha_i \ln e_{t+1} + \beta_i \ln h_{t+1}^i)$$

$$\text{s.t.} \quad c_t^i + m_t^i + n_t^i = h_t^i$$

$$h_{t+1}^i = \delta (h_t^i)^\theta (\mu + n_t^i)^{1-\theta}$$

$$(1 + \pi_t)e_{t+1} = (1-\eta)(1 + \pi_{t-1})e_t + \gamma \left(m_t^i \frac{L_i}{L} + m_t^j \frac{L_j}{L} \right) - \sigma \left(\frac{L_i}{L} h_t^i + \frac{L_j}{L} h_t^j \right)$$

It is obvious that in the steady state, $c_t^i > 0$, $c_t^j > 0$, $m_t^i + m_t^j > 0$, so we assume that $m_t^i > 0$ and $m_t^j = 0$, with $i, j = E, H$, $j \neq i$.

The optimal choice problems become

$$\text{Max}_{c_t^i, m_t^i, n_t^i} U_t^i = \ln c_t^i + \pi_t(\alpha_i \ln e_{t+1} + \beta_i \ln h_{t+1}^i) \quad (\text{A2.1})$$

s.t.

$$c_t^i + m_t^i + n_t^i = h_t^i \quad (\text{A2.2})$$

$$h_{t+1}^i = \delta (h_t^i)^\theta (\mu + n_t^i)^{1-\theta} \quad (\text{A2.3})$$

$$(1 + \pi_t)e_{t+1} = (1-\eta)(1 + \pi_{t-1})e_t + \gamma m_t^i \frac{L_i}{L} - \sigma \left(\frac{L_i}{L} h_t^i + \frac{L_j}{L} h_t^j \right) \quad (\text{A2.4})$$

$$\text{Max}_{c_t^j, n_t^j} U_t^j = \ln c_t^j + \pi_t(\alpha_j \ln e_{t+1} + \beta_j \ln h_{t+1}^j) \quad (\text{A2.5})$$

s.t.

$$c_t^j + n_t^j = h_t^j \quad (A2.6)$$

$$h_{t+1}^j = \delta(h_t^j)^\theta (\mu + n_t^j)^{1-\theta} \quad (A2.7)$$

with $i, j = E, H, j \neq i$.

We have

$$m_t^i = \frac{\gamma \pi_t \alpha_i L_i h_t^i + \sigma [1 + \pi_t \beta_i (1 - \theta)] (L_i h_t^i + L_j h_t^j) - (1 - \eta) (1 + \pi_{t-1}) [1 + \pi_t \beta_i (1 - \theta)] e_t + \gamma \pi_t \alpha_i L_i \mu}{\gamma [\pi_t \alpha_i + 1 + \pi_t \beta_i (1 - \theta)] L_i} \text{ and } m_t^j = 0.$$

By substituting the solutions of the problem into (A2.3), (A2.4) and (A2.7), and then substituting $h_{t+1}^i = h_t^i = h^i$, $h_{t+1}^j = h_t^j = h^j$, $e_{t+1} = e_t = e'$ and $\pi_{t+1} = \pi_t = \pi$ into them, we have

$$e' = \frac{\gamma^2 \pi \alpha_i \delta^{\frac{1}{\theta-1}} L_i \mu}{(1 + \pi) \left[\gamma (\eta \pi \alpha_i + 1 + (1 - \theta) \pi \beta_i) \delta^{\frac{1}{\theta-1}} - \pi (1 - \theta) (\gamma - \sigma) \beta_i \right] L} - \frac{\gamma \sigma^2 \alpha_i \beta_j \delta^{\frac{1}{\theta-1}} L_j \mu}{(1 + \pi) \left\{ \gamma [\eta \pi \alpha_i + 1 + (1 - \theta) \pi \beta_i] \delta^{\frac{1}{\theta-1}} - \pi (1 - \theta) (\gamma - \sigma) \beta_i \right\} \left\{ [1 + (1 - \theta) \pi \beta_j] \delta^{\frac{1}{\theta-1}} - \pi (1 - \theta) \beta_j \right\} L} \quad (A2.8)$$

with $i, j = E, H, j \neq i$.

By comparing (2.28) and (A2.8), we have

$$e' < e^i, i = E, H, j \neq i. \quad (A2.9)$$

From (A2.9), even when corner solutions constitute the steady-state equilibrium, Proposition 2.1 still holds. ■

A2.2 Derivation of important solutions

The solutions of m_t^E and m_t^H , (2.9)

From (2.6), we have

$$m_t^E = \frac{\gamma\pi_t\alpha_E + \sigma[1 + \beta_E\pi_t(1 - \theta)]}{\gamma[\pi_t\alpha_E + 1 + \beta_E\pi_t(1 - \theta)]} h_t^E + \frac{[1 + \beta_E\pi_t(1 - \theta)][(\sigma h_t^H - \gamma m_t^H)L_H - (1 - \eta)(1 + \pi_{t-1})Le_t]}{\gamma[\pi_t\alpha_E + 1 + \beta_E\pi_t(1 - \theta)]L_E} \\ + \frac{\pi_t\alpha_E\mu}{\pi_t\alpha_E + 1 + \beta_E\pi_t(1 - \theta)}$$

and

$$m_t^H = \frac{\gamma\pi_t\alpha_H + \sigma[1 + \beta_H\pi_t(1 - \theta)]}{\gamma[\pi_t\alpha_H + 1 + \beta_H\pi_t(1 - \theta)]} h_t^H + \frac{[1 + \beta_H\pi_t(1 - \theta)][(\sigma h_t^E - \gamma m_t^E)L_E - (1 - \eta)(1 + \pi_{t-1})Le_t]}{\gamma[\pi_t\alpha_H + 1 + \beta_H\pi_t(1 - \theta)]L_H} \\ + \frac{\pi_t\alpha_H\mu}{\pi_t\alpha_H + 1 + \beta_H\pi_t(1 - \theta)}.$$

They are rewritten as

$$m_t^E = -\frac{[1 + \beta_E\pi_t(1 - \theta)]L_H}{[\pi_t\alpha_E + 1 + \beta_E\pi_t(1 - \theta)]L_E} m_t^H + \frac{\gamma\pi_t\alpha_E + \sigma[1 + \beta_E\pi_t(1 - \theta)]}{\gamma[\pi_t\alpha_E + 1 + \beta_E\pi_t(1 - \theta)]} h_t^E + \frac{\pi_t\alpha_E\mu}{\pi_t\alpha_E + 1 + \beta_E\pi_t(1 - \theta)} \\ + \frac{[1 + \beta_E\pi_t(1 - \theta)][\sigma h_t^H L_H - (1 - \eta)(1 + \pi_{t-1})Le_t]}{\gamma[\pi_t\alpha_E + 1 + \beta_E\pi_t(1 - \theta)]L_E}, \quad (A2.10)$$

which we label ME;

and

$$m_t^E = -\frac{[\pi_t\alpha_E + 1 + \beta_E\pi_t(1 - \theta)]L_H}{[1 + \beta_E\pi_t(1 - \theta)]L_E} m_t^H + \frac{\gamma\pi_t\alpha_H + \sigma[1 + \beta_H\pi_t(1 - \theta)]}{\gamma[1 + \beta_H\pi_t(1 - \theta)]L_E} L_H h_t^H + \frac{\pi_t\alpha_H L_H \mu}{[1 + \beta_H\pi_t(1 - \theta)]L_E} \\ + \frac{\sigma w_E h_t^E L_E - (1 - \eta)(1 + \pi_{t-1})Le_t}{\gamma L_E}. \quad (A2.11)$$

which we label MH.

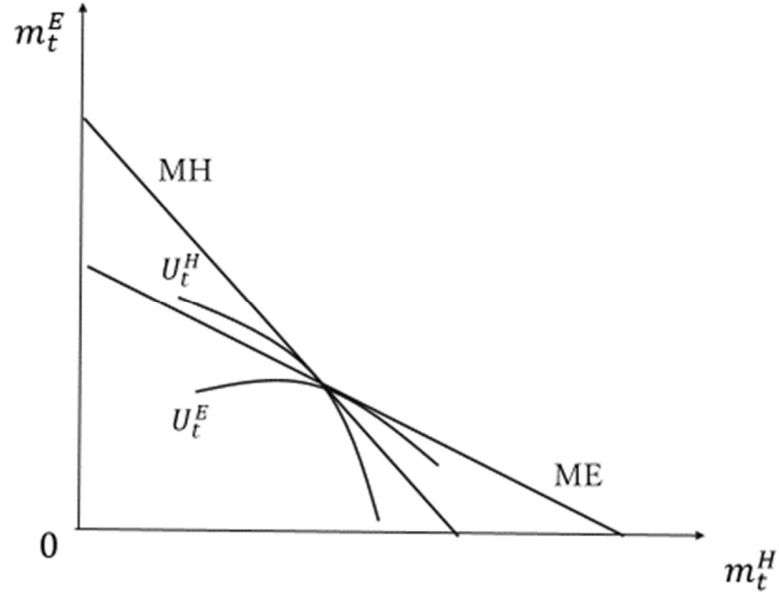


Figure A2.1 – Nash equilibrium

This allows us to depict the utility function U_t^E and U_t^H , ME and MH in Figure A2.1.

The point of intersection gives us the solutions of m_t^E and m_t^H .

$$m_t^i = \frac{1}{\gamma\{\pi_t\alpha_i\alpha_j + \alpha_i[1 + (1-\theta)\pi_t\beta_j] + \alpha_j[1 + (1-\theta)\pi_t\beta_i]\}L_i}$$

$$(\{\gamma\pi_t\alpha_i\alpha_j + \gamma\alpha_i[1 + (1-\theta)\pi_t\beta_j] + \sigma\alpha_j[1 + (1-\theta)\pi_t\beta_i]\}L_i h_t^i$$

$$-(\gamma - \sigma)\alpha_j[1 + (1-\theta)\pi_t\beta_i]L_j h_t^j - (1-\eta)(1 + \pi_{t-1})\alpha_j[1 + (1-\theta)\pi_t\beta_i]Le_t$$

$$+ \{\pi_t\alpha_i\alpha_j L_i + \alpha_i[1 + (1-\theta)\pi_t\beta_j]L_i - \alpha_j[1 + (1-\theta)\pi_t\beta_i]L_j\}\mu).$$

with $i, j = E, H, j \neq i$.

■

Steady-state values of e , h^E , h^H , (2.17), and (2.18)

From (2.15), we have the relationship between h^E and h^H .

$$\frac{h^E}{h^H} = \frac{\alpha_H \beta_E L_H}{\alpha_E \beta_H L_E} \quad (A2.12)$$

We substitute this relation (A2.12) into (2.15) and (2.16).

From (2.15), we get

$$e = \frac{\gamma\{\pi\alpha_E\alpha_H + \alpha_E[1 + \beta_H\pi(1 - \theta)] + \alpha_H[1 + \beta_E\pi(1 - \theta)]\}\delta^{\frac{1}{\theta-1}} - \pi(1 - \theta)(\alpha_E\beta_H + \alpha_H\beta_E)}{(1 - \theta)(1 - \eta)(1 + \pi)\pi\alpha_H\beta_E L} L_E h^E - \frac{\gamma\mu}{(1 - \eta)(1 + \pi)}, \quad (A2.13)$$

which we label HE.

From (2.16), we have

$$e = \frac{\pi \frac{\alpha_E}{\beta_E} (\gamma - \sigma)(\alpha_E\beta_H + \alpha_H\beta_E)L_E h^E + \gamma\pi\alpha_E\alpha_H L\mu}{(1 + \pi)\{\eta\pi\alpha_E\alpha_H + \alpha_E[1 + \beta_H\pi(1 - \theta)] + \alpha_H[1 + \beta_E\pi(1 - \theta)]\}L}, \quad (A2.14)$$

which we label EE.

■

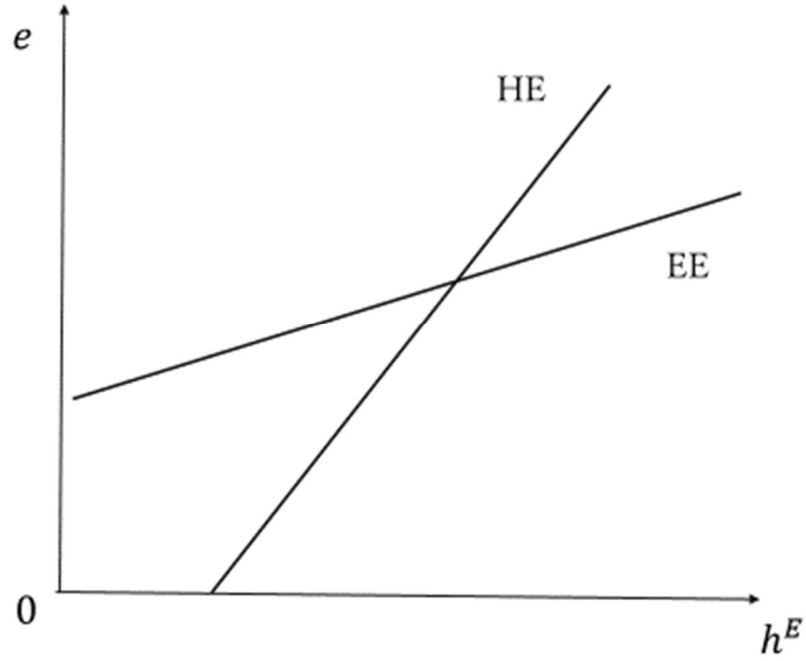


Figure A2.2 – The steady state

We depict HE and EE in Figure A2.2.

The point of intersection gives us the solution of h^i and e :

$$h^i = \frac{\gamma(1-\theta)\pi\alpha_j\beta_i L\mu}{\left(\gamma\{\eta\pi\alpha_i\alpha_j + \alpha_i[1+(1-\theta)\pi\beta_j] + \alpha_j[1+(1-\theta)\pi\beta_i]\}\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)(\alpha_i\beta_j + \alpha_j\beta_i)\right)L_i}$$

$$e = \frac{\gamma^2\pi\alpha_i\alpha_j\delta^{\frac{1}{\theta-1}}\mu}{(1+\pi)\left(\gamma\{\eta\pi\alpha_i\alpha_j + \alpha_i[1+(1-\theta)\pi\beta_j] + \alpha_j[1+(1-\theta)\pi\beta_i]\}\delta^{\frac{1}{\theta-1}} - \pi(1-\theta)(\gamma-\sigma)(\alpha_i\beta_j + \alpha_j\beta_i)\right)}$$

with $i, j = E, H, j \neq i$.

■

A2.3 Existence of steady-state equilibrium when (2.20) does not hold

According to (2.15) and (2.16), $\frac{h_{t+1}^E}{h_{t+1}^H} = \left(\frac{h_t^E}{h_t^H}\right)^\theta \left(\frac{\alpha_H \beta_E L_H}{\alpha_E \beta_H L_E}\right)^{1-\theta}$. In the long-run, we have

$$\frac{h_{t+1}^E}{h_{t+1}^H} = \frac{h_t^E}{h_t^H} = \frac{\alpha_H \beta_E L_H}{\alpha_E \beta_H L_E}. \quad (\text{A2.15})$$

From (2.15), (2.16) and (A2.15), we can derive dynamic equations of h_t^E and e_t .

$$h_{t+1}^E = \delta(h_t^E)^\theta \left(\frac{(1-\theta)\pi_t \alpha_H \beta_E \left[(\gamma-\sigma) \left(L_E h_t^E + \frac{\alpha_E \beta_H h_t^E}{\alpha_H \beta_E} \right) + (1-\eta)(1+\pi_{t-1})L e_t + \gamma L \mu \right]}{\gamma L_E \{ \pi_t \alpha_E \alpha_H + \alpha_E [1+(1-\theta)\pi_t \beta_H] + \alpha_H [1+(1-\theta)\pi_t \beta_E] \}} \right)^{1-\theta} \quad (\text{A2.16})$$

$$e_{t+1} = \frac{\pi_t \alpha_E \alpha_H \left[(\gamma-\sigma) \left(L_E h_t^E + \frac{\alpha_E \beta_H h_t^E}{\alpha_H \beta_E} \right) + (1-\eta)(1+\pi_{t-1})L e_t + \gamma L \mu \right]}{(1+\pi_t) \{ \pi_t \alpha_E \alpha_H + \alpha_E [1+(1-\theta)\pi_t \beta_H] + \alpha_H [1+(1-\theta)\pi_t \beta_E] \}}. \quad (\text{A2.17})$$

From (A2.16) and (A2.17), we have

$$h_{t+1}^E = \delta(h_t^E)^\theta \left[\frac{(1+\pi_t)(1-\theta)\beta_E e_{t+1}}{\gamma \alpha_E L_E} \right]^{1-\theta},$$

which means that

$$h_t^E = \delta(h_{t-1}^E)^\theta \left[\frac{(1+\pi_{t-1})(1-\theta)\beta_E e_t}{\gamma \alpha_E L_E} \right]^{1-\theta}. \quad (\text{A2.18})$$

From (A2.16) and (A2.18), we have

$$\begin{aligned} & \gamma \delta^{\frac{1}{\theta-1}} \left\{ \pi_t \alpha_E \alpha_H \left[\left(\frac{h_{t+1}^E}{h_t^E} \right)^{\frac{1}{1-\theta}} h_t^E - (1-\eta) \left(\frac{h_t^E}{h_{t-1}^E} \right)^{\frac{1}{1-\theta}} h_{t-1}^E \right] + (\alpha_E + \alpha_H) \left(\frac{h_{t+1}^E}{h_t^E} \right)^{\frac{1}{1-\theta}} h_t^E \right\} L_E \\ & + \pi_t (1-\theta) (\alpha_E \beta_H + \alpha_H \beta_E) h_t^E \left[\left(\frac{h_{t+1}^E}{h_t^E} \right)^{\frac{1}{1-\theta}} \gamma \delta^{\frac{1}{\theta-1}} - (\gamma - \sigma) \right] L_E \\ & = (1-\theta) \pi_t \alpha_H \beta_E \gamma L \mu. \end{aligned} \quad (\text{A2.19})$$

If h_t^E converges to a steady state value under time invariant π , that is, $h_{t+1}^E = h_t^E = h_{t-1}^E$, we have

$$\begin{aligned} & \gamma \delta^{\frac{1}{\theta-1}} (\eta \pi \alpha_E \alpha_H + \alpha_E + \alpha_H) h^E L_E + \pi (1 - \theta) (\alpha_E \beta_H + \alpha_H \beta_E) \left[\gamma \delta^{\frac{1}{\theta-1}} - (\gamma - \sigma) \right] h^E L_E \\ & = (1 - \theta) \pi \alpha_H \beta_E \gamma L \mu. \end{aligned}$$

For the steady state value to be positive,

$$\gamma (\eta \pi \alpha_E \alpha_H + \alpha_E + \alpha_H) \delta^{\frac{1}{\theta-1}} + \pi (1 - \theta) (\alpha_E \beta_H + \alpha_H \beta_E) \left[\gamma \delta^{\frac{1}{\theta-1}} - (\gamma - \sigma) \right] > 0 \quad \text{must hold.}$$

If

$$\begin{aligned} & \gamma (\eta \pi \alpha_E \alpha_H + \alpha_E + \alpha_H) \delta^{\frac{1}{\theta-1}} + \pi (1 - \theta) (\alpha_E \beta_H + \alpha_H \beta_E) \left[\gamma \delta^{\frac{1}{\theta-1}} - (\gamma - \sigma) \right] \leq 0 \\ & \Leftrightarrow \gamma (\eta \pi \alpha_E \alpha_H + \alpha_E + \alpha_H) \leq \left[(\gamma - \sigma) \delta^{\frac{1}{1-\theta}} - \gamma \right] \pi (1 - \theta) (\alpha_E \beta_H + \alpha_H \beta_E), \end{aligned}$$

from the equation above, h_t^E converges to a negative value or does not converge to any value, which means that the steady state equilibrium with positive values does not exist. ■

A2.4 The condition for (2.34) to hold

We substitute $\underline{\pi}$ and $\bar{\pi}$ into (2.34), and we have

$$e(\bar{\pi}) = \frac{\gamma^2 \bar{\pi} \alpha_E \alpha_H \delta^{\frac{1}{\theta-1}} \mu}{(1 + \bar{\pi}) \left(\gamma \{ \eta \bar{\pi} \alpha_E \alpha_H + \alpha_E [1 + (1 - \theta) \bar{\pi} \beta_H] + \alpha_H [1 + (1 - \theta) \bar{\pi} \beta_E] \} \delta^{\frac{1}{\theta-1}} - \bar{\pi} (1 - \theta) (\gamma - \sigma) (\alpha_E \beta_H + \alpha_H \beta_E) \right)}$$

and

$$e^h(\underline{\pi}) = \frac{\gamma^2 \underline{\pi} \alpha_H \delta^{\frac{1}{\theta-1}} \mu}{(1 + \underline{\pi}) \left(\gamma \{ \eta \underline{\pi} \alpha_H + [1 + (1 - \theta) \underline{\pi} \beta_H] \} \delta^{\frac{1}{\theta-1}} - \underline{\pi} (1 - \theta) (\gamma - \sigma) \beta_H \right)}.$$

In order to compare their values by their denominators, we multiply the numerator and denominator of $e^h(\underline{\pi})$ with $\frac{\bar{\pi}}{\underline{\pi}}\alpha_E$ so that their numerators become the same. The denominator of $e(\bar{\pi})$ minus the denominator of $e^h(\underline{\pi})$ is

$$\begin{aligned} & (\bar{\pi} - \underline{\pi}) \left\{ \gamma \delta^{\frac{1}{\theta-1}} \eta \bar{\pi} \alpha_E \alpha_H + \bar{\pi} (1 - \theta) \alpha_E \beta_H \left[\gamma \delta^{\frac{1}{\theta-1}} - (\gamma - \sigma) \right] \right\} \\ & + (1 + \bar{\pi}) \bar{\pi} (1 - \theta) \alpha_H \beta_E \left[\gamma \delta^{\frac{1}{\theta-1}} - (\gamma - \sigma) \right] + (1 + \bar{\pi}) \gamma \delta^{\frac{1}{\theta-1}} \left(\alpha_H - \frac{\bar{\pi} - \underline{\pi}}{\underline{\pi}} \alpha_H \right), \end{aligned}$$

where the first two terms are positive from (2.20).

For (2.34) to hold, the last term $(1 + \bar{\pi}) \gamma \delta^{\frac{1}{\theta-1}} \left(\alpha_H - \frac{\bar{\pi} - \underline{\pi}}{\underline{\pi}} \alpha_E \right)$, which could be negative, needs to be large enough, which means that the difference between α_E and α_H need not be very large (note $\alpha_E > \alpha_H$).

CHAPTER 3. Fertility, childbearing age, and education

3.1 Introduction

Theoretical analyses on the interactions between economic development and fertility behavior mainly focus on two aspects of fertility: total fertility rate and childbearing age. Preceding works argue that the change of total fertility rate results from factors such as the decrease of child mortality (Doepke, 2005), the effect of technological change on the return to education (Galor and Weil, 2000) and innovations in contraceptive technology (Strulik, 2017). The change of childbearing age is explained through various channels such as the opportunity cost of starting a family (Happel, Hill and Low, 1984), child benefits (Cigno and Ermisch, 1989) and career planning of women (Gustafsson, 2001).

Several existing studies, including Pestieau and Ponthiere (2014, 2015), consider the two aspects of fertility at the same time. Pestieau and Ponthiere (2014) show that, depending on the childbearing age, the economy with a given fertility rate can exhibit quite different (stationary or cyclical) dynamics. Pestieau and Ponthiere (2015) show the existence of a stationary equilibrium with a stationary total fertility rate and use a rise in the cost of childbearing and a decline in the degree of family altruism to explain the observed delay of childbearing numerically.

In this work, besides the two aspects of fertility, we also consider education and the interactions between education and the fertility variables. The main objective of this work is to develop a simple analytically tractable model that can explain the observed long-run changes in total fertility rate, age-specific fertility rate and education in developed countries. There are two reasons for considering the interactions between education and fertility.

First, empirical researches find the correlations between educational (human capital) factors and fertility. Heckman and Walker (1990) show that higher female wages delay

childbearing and reduce total fertility rate using Swedish data.¹⁷ Meisenberg (2008) shows, based on cross-sectional data of 78 countries, a negative correlation between education and fertility, which is stronger for females than males.

Second, theoretical papers, such as Happel, Hill and Low (1984), Cigno and Ermisch (1989) and Gustafsson (2001), show the mechanism of how fertility decisions are related to education (human capital). A low level of education leads to a low relative cost of early childbearing, which hastens childbearing. An increase in educational investment raises the relative cost of early childbearing, which delays childbearing. When the level of education becomes sufficiently high, the decrease of early childbearing exceeds the increase of late childbearing and thus total fertility rate decreases.

Iyigun (2000) and d'Albisa, Greulich and Ponthiere (2018) introduce education in the analysis of the relationship between total fertility rate and childbearing age. Iyigun (2000) focuses on the postponement of childbearing, and d'Albisa, Greulich and Ponthiere (2018) focus on the long-run decrease in total fertility rate and the U-shaped trend of childbearing age (increases first and then decreases). However, the observed fertility trends in developed countries are more complicated. In countries such as the United States and the United Kingdom, there exists four periods of distinctive fertility trends: the period of the rapid fertility decline from no later than the early 1920s until the early 1930s; the period of baby boom from the early 1940s until the early 1960s; the period of “baby bust” until the mid-1970s; and the period of rising fertility of elder mothers and falling fertility of younger mothers afterwards. Although these works replicate some parts of the fertility trend, they do not consider the remaining parts.

In this work, in order to explain the observed long-run fertility trends and changes in educational levels, we develop a three-period model with two periods of childbearing. In the first period (childhood), individuals do not make any decisions and are taken care of by parents. In each of the remaining two periods (adulthood), they decide how much time to

¹⁷ Higher male income hastens childbearing and raise total fertility rate. However, models that include marital and cohabitational status variables show much weaker effects of male income.

work and how many children they have. In order to study the interactions between fertility and education, we also assume that individuals choose how much time to spend on their own education in early adulthood, which corresponds to tertiary education in real life, and how much time to spend on their children' education in both periods, which corresponds to primary and secondary education in real life. We also consider the influences of various factors that affect total fertility rate, age-specific fertility rate and education, that is, the cost of household chores, the productivity and cost of adulthood (late adolescence) education, those of education of children, and the cost of childbearing in each period. This simple framework allows us to explain the observed long-run changes in total fertility rate, age-specific fertility rate and education in developed countries.

We examine how well the model can explain the long-run changes in fertility and education in countries such as the US and the UK qualitatively. First, as for the period roughly from the early 1920s to the early 1930s, data show that age-specific and total fertility rates fell sharply, average years of education of young generations rose rapidly, while their average years of tertiary education (especially of women) changed little. Evidence suggests that the demand for skilled workers, who corresponded to high school graduates at the time, increased greatly (Goldin and Katz, 1998). When the model captures the increased demand of skilled workers by the increased productivity of (or increased return to) childhood education, it can explain these facts.

Second, as for the period of baby boom roughly from the early 1940s to the early 1960s, observed facts are that age-specific and total fertility rates rose rapidly, and average years of secondary education and of tertiary education of young generations rose moderately. Researchers argue that the baby boom was caused by the increased productivity of the household sector from the diffusion of home appliances (Greenwood, Seshadri, and Vandenbroucke, 2005), a large fall in maternal mortality, which is a proxy for the overall maternal cost of having children (Albanesi and Olivetti, 2014), and a decrease in per space housing costs due to suburbanization (Tamura and Simon, 2017). When the model captures the increased productivity of the household sector by the decreased fixed time spent on household chores, it can explain these facts except the increased years of secondary education; when it captures the decreased maternal mortality

mainly by the decreased relative cost of late childbearing, it can explain these facts except the increased fertility rate of elder mothers and years of secondary education; when it captures the decreased housing costs mainly by the decreased cost of child-rearing, it can explain these facts except the increased years of secondary and tertiary education. Hence, the model can explain the facts except the increased years of secondary education, as long as the effects of the changes of the relative cost of late childbearing and the cost of child-rearing are not very large.

Third, as for the period of "baby bust" roughly from the early or middle 1960s to the mid-1970s, age-specific and total fertility rates fell rapidly, while average years of education, particularly of tertiary education, of young generations rose moderately. Researchers argue that the labor demand for young women and their relative wage increased during this period, with WWII (Doepke, Hazan and Maoz, 2015) or the Great Depression (Bellou and Cardia, 2014) as the ultimate cause. When the model captures the increased labor demand for young women by the increased opportunity cost of child-rearing, it can explain these facts.

Finally, as for the period after the mid-1970s, the fertility rate of elder mothers rose, fertility rate of younger mothers fell, and total fertility rate changed little, while average years of education, particularly of tertiary education of women, increased. Empirical works show that the relative earnings of college graduate women to men increased greatly from the late 1960s (Goldin, 2014) and the wage premium of college graduate increased sharply from the late 1970s (Goldin and Katz, 2008), although the premium stopped increasing around 2015 (Autor, Goldin, and Katz, 2020). When the model captures the increased relative earnings of educated, particularly female, workers by the increased productivity of (or increased return to) adulthood education, it can explain these facts except the largely unchanged total fertility rate.

Our work is mainly related to Iyigun (2000) and d'Albisa, Greulich and Ponthiere (2018) that develop models with educational investment and two periods of childbearing. Iyigun (2000) develops an OLG model that assumes that individuals' human capital depends positively on their education and parental human capital. He shows that if the

complementarity between parental human capital and education is relatively large, an increase in the parental human capital raises the opportunity cost of having children while young and induces individuals to delay childbearing. d'Albisa, Greulich and Ponthiere (2018) develop a unified growth model to rationalize the decreasing of total fertility rate and the U-shaped trend of childbearing age. They show that a traditional economy, where individuals do not invest in education and increased income lowers childbearing age, can progressively converge towards a modern economy, where individuals invest in education and increased income raises childbearing age.

Our work differs from Iyigun (2000) and d'Albisa, Greulich and Ponthiere (2018) mainly in three aspects. First, as mentioned above, Iyigun (2000) focuses on the postponement of childbearing, and d'Albisa, Greulich and Ponthiere (2018) focus on the decreasing of total fertility rate and the U-shaped trend of childbearing age. By contrast, we focus on four periods of distinctive fertility trends in countries such as the US and the UK: rapid fertility decline in the early 20th century; the Baby boom; the “baby bust”; and diverging fertility trends of elder and younger mothers after the mid-1970s. Second, in addition to the decision on one's own education in adulthood (late adolescence), we also consider decisions on education of her children. This specification allows us to consider the interactions between adulthood education, education of children and fertility. Finally, we examine how various factors, such as cost of household chores, productivity and cost of adulthood (late adolescence) education and relative cost of early and late childbearing, affect fertility and education, which are not included in these works.

Our work is also related to Momota and Horii (2013) and Cigno and Ermisch (1989) that consider both childbearing age and total fertility rate. Momota and Horii (2013) develop an OLG model with exogenous fertility and show that delayed childbearing causes fluctuations in the age composition of workers, which generates cycles in the capital–labor ratio and lifetime welfare. Instead of examining how fertility and childbearing age change over time, given these variables, they analyze the impact of delayed childbearing on economic growth. Cigno and Ermisch (1989) develop a model with multiple periods of childbearing and show numerically that women with more human capital at the time of marriage hasten childbearing and lower total fertility rate. Although they show how certain

factors affect fertility, the multiple childbearing periods setting is complicated and the changes of total fertility rate and childbearing age over time cannot be theoretically analyzed.

The rest of this work is organized as follows. Section 2 presents the model, and Section 3 analyzes the model. Section 4 examines how the model can explain the observed change in total fertility rate, childbearing age and education of the actual economy. Finally, Section 5 concludes the work. Appendix A analyzes the case when several parameters affecting fertility and education change over time. Appendix B includes proofs of propositions in the main text. Appendix C includes proofs of propositions in Appendix A.

3.2 Model

Consider an economy in which individuals live for three periods, the first period as a child and the remaining two periods as an adult, and thus three generations coexist in every period. In childhood, individuals do not make any decisions and just receive nurturing and education. In each period of adulthood, they decide how many children they have, how much time to spend on rearing and education of the children, and how much time to work. Further, in the first period of adulthood, which includes late adolescence in real life, they decide how much time to spend on their own education, which corresponds to tertiary education in real life and raises earnings in the next period.

The utility function of an adult depends on consumption in two periods and total human capital of their children:

$$U = \ln c_1 + \ln c_2 + \ln(n_1 h_{c1} + n_2 h_{c2}), \quad (3.1)$$

where c_i , n_i , and h_{ci} ($i = 1, 2$) are respectively consumption, and the number and the

human capital of children born in period i of adulthood. Parents care about both the quantity and the quality, but not the timing of birth, of their children.

First, consider the first period of adulthood. The production function of consumption goods is linear in effective labor (human capital times labor supply) of workers. Let the total factor productivity be A . Then, the wage rate per unit time of an individual in the first period of adulthood equals

$$w_1 = Ah_1, \quad (3.2)$$

where h_1 is her human capital in this period.

She has total time of 1 to allocate to household chores (such as cleaning, cooking, and washing), her education, rearing and education of their children, and work. Thus, her consumption in this period equals

$$c_1 = w_1[1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1] = Ah_1[1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1], \quad (3.3)$$

where $\delta, \eta, \theta_q, \theta_e \in (0,1)$. In the equation, δ is the fixed time spent on household chores, e is the amount of education she takes in this period, η is the time cost of the education, θ_q is the per child time cost of rearing, e_{c1} is the amount of education received by her children born in this period, and θ_e is the per child time cost of the education. The costs of education

of herself and of her children reflect not only the opportunity cost but also the cost of paying for teachers' time.¹⁸ She allocates time to the various activities to maximize her utility.

Education she receives in this period raises her human capital in the next period, the second period of adulthood. The human capital production function is given by

$$h_2 = (1 + \rho e)^\alpha h_1, \alpha \in (0,1), \quad (3.4)$$

where ρ is the effectiveness or productivity of adulthood (late adolescence) education, which corresponds to higher education in real life. In the real economy, ρ could increase not only from the advancement of education technology but also from the increased economic value of workers with the advanced education. When $e = 0$, human capital in the next period is the same as this period.

Next, consider the second period of her adulthood. The wage rate per unit time in this period equals

$$w_2 = gAh_2, \quad (3.5)$$

where g is the growth rate of total factor productivity.

In this period too, she has total time of 1 to allocate to various activities. The difference from the first period of adulthood is that she no longer spends time on her own education. It is assumed that time costs of rearing and education of children in the second

¹⁸ This is because the cost of hiring teachers is proportional to the wage rate. Other components of the cost of school education, such as study materials and school facilities, are abstracted from the model for analytical tractability.

period of adulthood are weakly greater than the costs in the first period by the same proportion, where the last part of the assumption is made to simplify the analysis greatly:

$$\theta_{q2} = \lambda \theta_q, \lambda \geq 1, \quad (3.6)$$

$$\theta_{e2} = \lambda \theta_e. \quad (3.7)$$

The assumption of the greater costs reflects the fact that older parents have less energy and feel more tiredness from taking care of children, which implies less effective time left for other activities, than younger parents. In a poor health environment, the assumption also reflects the maternal health has negative health effect on mothers.

Hence, her consumption in this period equals

$$c_2 = w_2[1 - \delta - (\theta_{q2} + \theta_{e2}e_{c2})n_2] = gA(1 + \rho e)^\alpha h_1[1 - \delta - \lambda(\theta_q + \theta_e e_{c2})n_2]. \quad (3.8)$$

Education received by children raises their human capital in the next period, the first period of their adulthood. Thus, their human capital production function is given by

$$h_{ci} = (1 + \rho_c e_{ci})^{\alpha_c}, i = 1, 2, \alpha_c \in (0, 1), \quad (3.9)$$

where ρ_c is the effectiveness or productivity of childhood education, which corresponds to primary and secondary education, in raising human capital. Human capital in the next period is positive even without childhood education.

3.3 Analysis

Values of endogenous variables are obtained by solving the utility maximization problem backward, starting from the maximization problem in the second period of

adulthood. Appendix 3.2 provides detailed analysis, and the following proposition summarizes the result, in particular, values of the number of children and the amount of education children receive in each period of adulthood of their parents, and the amount of education adults take in the first period of adulthood.

We assume the following condition.

$$\rho_c > \frac{\theta_e}{\alpha_c \theta_q}. \quad (3.10)$$

With this assumption, we can eliminate the case $e_{c1} = e_{c2} = 0$, which is not realistic because it means that individuals do not receive any education in their childhood.

The solutions of n_i ($i = 1, 2$) and e are classified into four cases depending on whether n_2 and e are positive or zero, as presented in the next proposition.

Proposition 3.1 Under the condition of $\rho_c > \frac{\theta_e}{\alpha_c \theta_q}$, $e_{c1} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c)\theta_e \rho_c}$, $e_{c2} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c)\theta_e \rho_c}$

if $n_2 > 0$, otherwise, $e_{c2} = 0$.

(1) When $\rho \in \left(0, \frac{2\eta}{(1-\delta)\alpha}\right]$ and $\lambda \geq 2$, $n_1 = \frac{(1-\alpha_c)(1-\delta)\rho_c}{2(\rho_c \theta_q - \theta_e)}$, $n_2 = 0$, and $e = 0$.

(2) When $\rho \in \left[\frac{2\eta}{(1-\delta)\alpha}, \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)}\right]$ and $\lambda \geq 2$, $n_1 = \frac{(1-\alpha_c)\rho_c[(1-\delta)\rho+\eta]}{(2+\alpha)\rho(\rho_c \theta_q - \theta_e)}$, $n_2 = 0$,

and $e = \frac{(1-\delta)\alpha\rho-2\eta}{(2+\alpha)\rho\eta}$.

(3) When $\rho \in \left(0, \frac{3\lambda\eta}{(1-\delta)\alpha(1+\lambda)}\right]$ and $\lambda \leq 2$, $n_1 = \frac{(1-\delta)\rho_c(1-\alpha_c)(2\lambda-1)}{3\lambda(\rho_c \theta_q - \theta_e)}$, $n_2 =$

$\frac{(1-\delta)\rho_c(1-\alpha_c)(2-\lambda)}{3\lambda(\rho_c \theta_q - \theta_e)}$, and $e = 0$.

(4) When $\rho > \max \left[\frac{3\lambda\eta}{(1-\delta)\alpha(1+\lambda)}, \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)} \right]$, $n_1 = \frac{(1-\alpha_c)\rho_c\{(1-\delta)\rho[2\lambda-(1+\alpha)]+2\lambda\eta\}}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)}$,
 $n_2 = \frac{(1-\alpha_c)\rho_c[(1-\delta)\rho(2+\alpha-\lambda)-\lambda\eta]}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)}$, and $e = \frac{(1-\delta)\rho\alpha(1+\lambda)-3\lambda\eta}{(3+\alpha)\lambda\rho\eta}$.

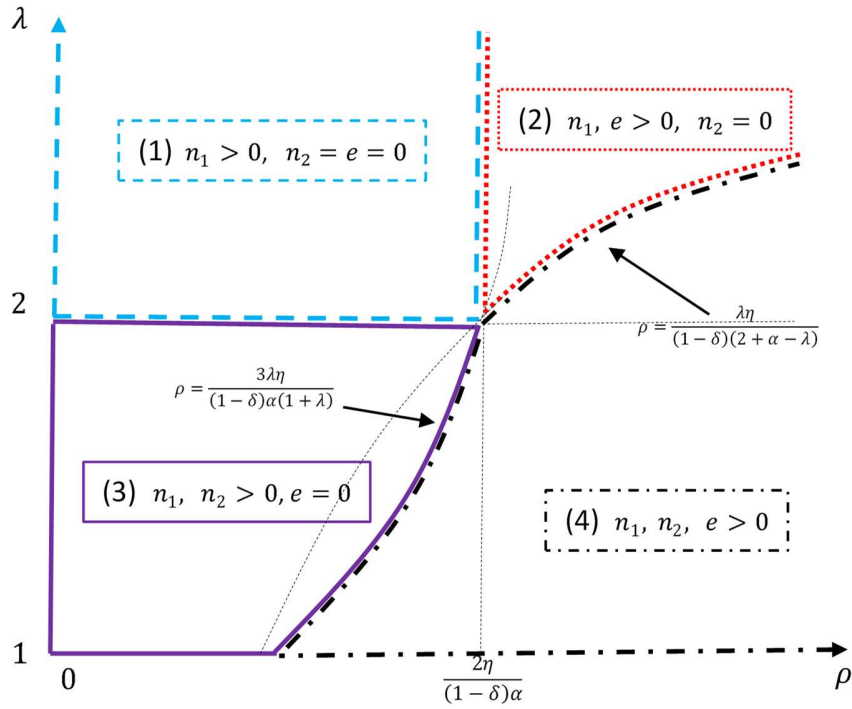


Figure 3.1 – Fertility and education (Proposition 3.1)

Based on the proposition, Figure 3.1 illustrates the region of each of the four cases on the (ρ, λ) plane. When the productivity of adulthood (late adolescence) education ρ is low and the relative cost of child-rearing and education older parents bear λ is high (case (1)), the benefit of adulthood education is small and the cost of late childbearing is high. Thus, adults do not take education, i.e., $e = 0$, and do not have babies in the second period

of adulthood, i.e., $n_2 = 0$. Equations in the proposition show how parameters affect fertility and education. Increases in the productivity of childhood education ρ_c and per child cost of rearing θ_q (an increase in per child cost of the education θ_e) lower (raises) the number of babies parents have in the first period of adulthood, n_1 , and raise (lowers) the amount of childhood education their children receive, e_{c1} . An increase in the fixed time spent on household chores δ lowers individuals' disposable time and thus n_1 .

When ρ and λ are large (case (2)), i.e., the productivity of adulthood education and the cost of late child-bearing are high, $e > 0$ and $n_2 = 0$. An increase in ρ (an increase in the cost of the education η) raises (lowers) the amount of education adults take, e , and thus lowers (raises) n_1 . An increase in δ lowers disposable time and thus e . As in case (1), increases in ρ_c and θ_q (an increase in θ_e) raise (lowers) e_{c1} and lower (raises) n_1 , and an increase in δ lowers n_1 .

When ρ and λ are small (case (3)), $e = 0$ and $n_2 > 0$. An increase in λ raises the relative cost of late childbearing and thus raises n_1 and lowers n_2 . As with e_{c1} and n_1 , increases in ρ_c and θ_q (an increase in θ_e) raise (lowers) e_{c2} and lower (raises) n_2 , and an increase in δ lowers n_2 . The remaining results are the same as in case (1) and case (2).

Finally, when ρ is large and λ is small (case (4)), $e > 0$ and $n_2 > 0$. As in case (2), an increase in ρ (an increase in η) raises (lowers) e and lowers (raises) n_1 . By contrast, the increase raises (lowers) n_2 because of intertemporal substitution of childbearing. As in case (3), an increase in λ raises n_1 and lowers n_2 . An increase in λ lowers e because it raises n_1 and thus lowers disposable time in the first period of adulthood. The remaining results are the same as in other cases.

The next corollary presents the solutions of total fertility rate $n_1 + n_2$. Since $n_2 = 0$ in case (1) and case (2), we only consider case (3) and case (4).

Corollary 3.1 Under the condition of $\rho_c > \frac{\theta_e}{\alpha_c \theta_q}$,

(1) In case (3), when $\rho \in (0, \frac{3\lambda\eta}{(1-\delta)\alpha(1+\lambda)}]$ and $\lambda \leq 2$, $n_1 + n_2 = \frac{(1-\delta)\rho_c(1-\alpha_c)(\lambda+1)}{3\lambda(\rho_c\theta_q-\theta_e)}$.

(2) In case (4), when $\rho > \max\left[\frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)}, \frac{3\lambda\eta}{(1-\delta)\alpha(1+\lambda)}\right]$, $n_1 + n_2 = \frac{(1-\alpha_c)\rho_c[(1-\delta)\rho(\lambda+1)+\lambda\eta]}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)}$.

While an increase in λ raises n_1 and lowers n_2 , it lowers $n_1 + n_2$. That is, an increased cost of late childbearing lowers total fertility rate. As with n_1 and n_2 , increases in ρ_c , θ_q and δ (an increase in θ_e) lower (raises) $n_1 + n_2$. While an increase in ρ (an increase in η) lowers (raises) n_1 and raises (lowers) n_2 , it lowers (raises) $n_1 + n_2$. As with n_1 and n_2 , the remaining results are the same as in case (3).

3.4 Explaining evolutions of fertility and education

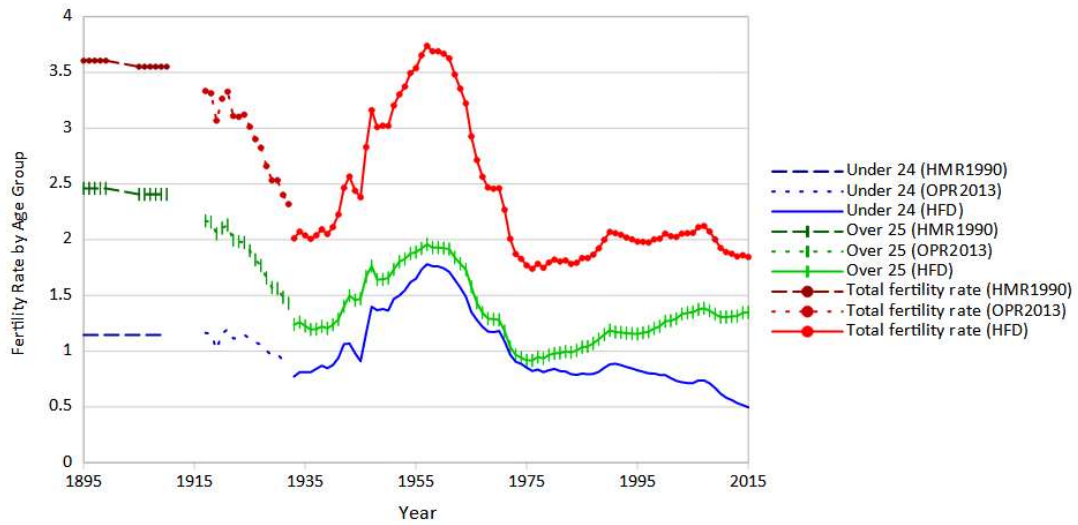


Figure 3.2 – Fertility rate by age group (United States)

Figure 3.2 illustrates the total fertility rate, the fertility rate of women under age 24, and that of women over age 25 of the United States for the years 1895-2015.¹⁹ After the relatively stable period of the early 20th century, the fertility rates decreased sharply between the early 1920s and the early 1930s. From the early 1940s, particularly after the World War II, they rose sharply and peaked at the end of the 1950s (the baby boom). Then, they fell sharply. After the mid-1970s, the three series showed different trends. Whereas the fertility rate of women under age 24 continued to fall except the late 1980s, the fertility rate of women over age 25 increased consistently. The total fertility rate increased moderately until the mid-2000s after which it decreased slightly.

¹⁹ Data sources of Figure 3.2 are as follows. 1895-1899 and 1905-1910: Haines, M.R. (1990); 1917-1932: Office of Population Research at Princeton University (2013); 1933-2015: Human Fertility Database (2018).

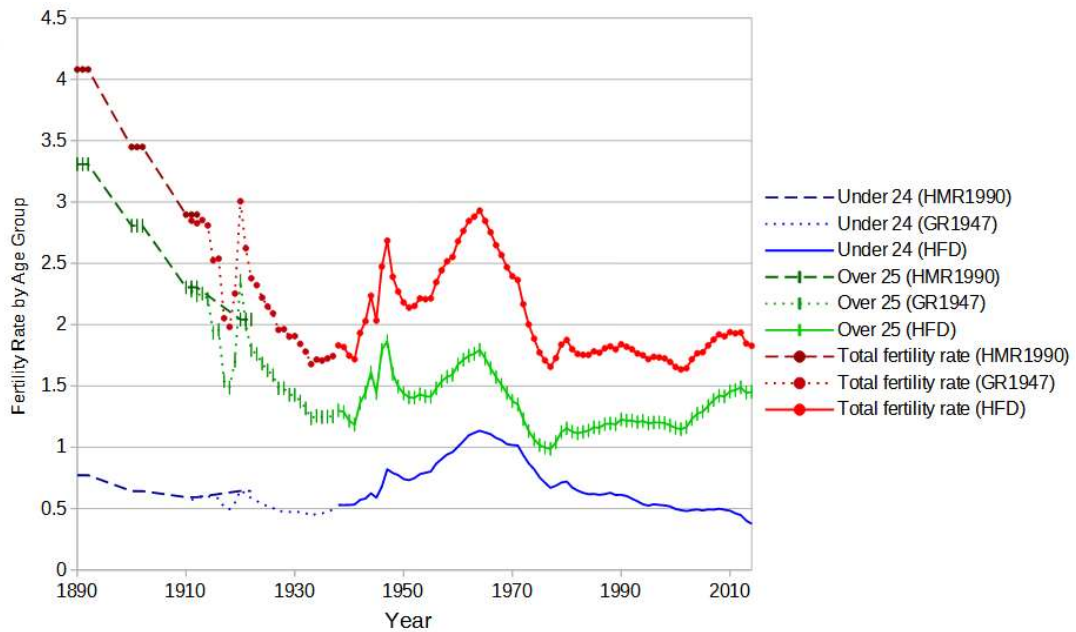


Figure 3.3 – Fertility rate by age group (United Kingdom)

Figure 3.3 shows the corresponding series for the United Kingdom for the years 1890-2014.²⁰ The fertility rates showed similar transitions as those of the US, although there were slight differences: the decreases of the series started earlier (at least since 1890); they fluctuated greatly associated with the WWI; there were two peaks for the baby boom, the first just after the WWII, and the second in the mid-1960s.

In sum, in both countries (and in countries such as France and Sweden), there exist four periods of distinctive fertility trends: the period of the rapid fertility decline from no later than the early 1920s until the early 1930s; the period of baby boom from the early

²⁰ Data sources of Figure 3.3 are as follows. 1891, 1901, 1911, 1921: Haines, M.R. (1990); 1911-1937: General Register Office (1947); 1938-2014: Human Fertility Database (2018).

1940s until the early 1960s; the period of “baby bust” until the mid-1970s; and the period of rising fertility of elder mothers and falling fertility of younger mothers afterwards.

As for education, according to the Lee and Lee dataset (Lee and Lee, 2016), average years of education of 15-24 year olds increased rapidly in the 1920s and the 1930s (from 7.00 in 1920 to 9.68 in 1940 in the US, and from 5.48 to 8.07 in the UK). During this period, average years of higher education of the young generation were very low (the number for women was almost 0 in the UK) and changed little. During the period of baby boom, average years of overall education and of higher education of 15-24 year olds increased moderately: in the US, the former increased from 9.68 in 1940 to 10.24 in 1960 and the latter increased from 0.2 to 0.35. In the period of the “baby bust” too, these series increased moderately: in the US, the former increased from 10.24 in 1960 to 11.59 in 1975 and the latter increased from 0.35 to 0.69. After the mid-1970s, the increase of average years of overall education was small in the US, but the increase of average years of tertiary education of women was relatively large (from 0.69 in 1975 to 1.02 in 2010 in the US, and from 0.24 in 1975 to 0.56 in 2005 in the UK).

For each of the four periods, possible factors contributing to the development of fertility rates have been proposed in the literature or can be discerned from empirical facts. In the following, these factors are briefly described and how they can be interpreted within the model is explained. Then, it is examined whether the model can capture the development of fertility and education of the actual economy.

Rapid fertility decline in the early 20th century: Becker, Murphy, and Tamura (1990) show theoretically that increased returns to human capital investment lead to

substitution of child quality for quantity, and Galor and Weil (2000), based on a theoretical analysis, argue that increased demand for human capital leads to fertility decline and increased education in the latter stage of economic development. Goldin and Katz (1998), based on data of U.S. manufacturing for the years 1909-40, show that the introduction of mass production methods during this period raised the relative demand for skilled workers, who corresponded to high school graduates at the time. In the model, the increased demand for high school graduates may be captured by an increase in the productivity of childhood education ρ_c of the human capital production function, because increased ρ_c implies increased return to childhood education.

Baby boom: Greenwood, Seshadri, and Vandenbroucke (2005), based on quantitative analysis of an OLG model, argue that increased productivity of the household sector from the diffusion of home appliances, such as automated washing machines and refrigerators, in the 1940s and 1950s (in the US) is a main driver of the baby boom. In the model, the increased productivity of the sector can be captured by a decrease in the fixed time spent on household chores, δ , and to a lesser extent, a decrease in the per child time cost of rearing, θ_q .

Albanesi and Olivetti (2014) find empirically that a large drop in maternal mortality, which is a proxy for the overall maternal cost of having children, including pregnancy-related diseases, during the mid-1930s and the 1950s in the US is strongly associated with a surge in fertility during the period. In the model, the decreased cost of having children can be represented by a decrease in θ_q and in the relative cost of late childbearing, λ . This is because pregnancy-related diseases often had long-term negative effects on maternal

health and thus labor supply and the large drop in morbidity benefited particularly older mothers who had higher risks of the pregnancy-related troubles.

Tamura and Simon (2017), based on quantitative analysis of a model calibrated to many developed economies, argue that a sudden fall in per space housing costs largely due to suburbanization that started after World War II and ended by 1970 is a main contributor to the baby boom. In the model, the falling price for space can be captured by decreasing θ_q .

“Baby bust”: Doepke, Hazan and Maoz (2015), using a quantitative model calibrated to the US economy, provide a unified explanation of the baby boom and the "baby bust". They argue that the entry of a large number of women into the labor force during WWII, who continued to work after the war, lowered demand for inexperienced young women and their relative wage in the 1940s and the 1950s and led them to have more children, whereas the retirement of these old women after the 1960s stimulated the entry of young women into the labor market and contributed to the baby bust. Bellou and Cardia (2014), based on econometric analysis, propose a similar mechanism with the Great Depression rather than WWII as the ultimate cause.²¹ In the model, the increased labor demand for young women and their relative wage during the period of baby bust could be captured by an increase in θ_q , because the increased demand raises the opportunity cost of child-bearing.

²¹ Another possibly important factor contributing to the baby bust is examined by Bailey (2010), who shows empirically that the birth control pill accelerated the fall in fertility rate after 1960. This factor, however, cannot be considered in the present model.

Diverging fertility trends of elder and younger mothers after the mid-1970s:

Because women bear large burdens of having and rearing children, it is mainly women who face the trade-offs between their tertiary education, fertility and child-rearing. Thus, it would be appropriate to consider e as the level of tertiary education of women and ρ as the effectiveness of (or return to) adulthood (adolescence) education they receive. During this period, ρ seems to have increased greatly. First, earnings of college graduate women relative to earnings of college graduate men increased substantially. Most of the increase occurred for cohorts who were born between 1938 and 1953 and the rise in the relative earnings is noticeable after age 30 and is largest in early 40s, that is, between the late 1970s and the mid-1990s (Goldin, 2014). Second, since the late 1970s or the early 1980s, the wage premium of college graduates has increased greatly (Goldin and Katz, 2008), although the premium stopped increasing around 2015 (Autor, Goldin, and Katz, 2020).

From Figures 3.2 and 3.3, since the fertility rate of women over age 25 (late childbearing) was consistently well above 0 (the same result holds even when the fertility rate of women over age 30 is used), it is realistic to analyze case (3) and case (4), where $n_2 > 0$ holds. Hence, it is assumed that λ is smaller than 2 (see Figure 3.1).

As mentioned above, it would be appropriate to interpret e as the level of tertiary education of women. At the beginning of the 20th century, average years of higher education of young women were very low (almost 0 in the UK), so we suppose that the model is in case (3) initially.

When the model is applied to explain changes of fertility and education of the actual economy, it must assume that the above-mentioned parameters change over time. This

implies that values of the fixed time spent on household chores δ and of the productivity of childhood education ρ_c individuals face in their first and second periods of adulthood are different. Let δ and ρ_c be values of these variables in the first period and $g_\delta\delta$ and $g_{\rho_c}\rho_c$, where $g_\delta \leq 1$ and $g_{\rho_c} \geq 1$, be values in the second period of adulthood. Detailed analysis of the time-dependent case is presented in Appendix 3.1 (Propositions A3.1, A3.2 and A3.3). Proofs of the propositions are presented in Appendix 3.3.

Proposition 3.2 summarizes the effects of the above-mentioned changes of the parameters in each period of the fertility trends on fertility and education when the economy is in case (3) or case (4).

Proposition 3.2 *Consider the time-dependent case. Assume that $\lambda < 2$ and $g_{\rho_c} \geq 1$ are not large and $g_\delta \leq 1$ is not small.*

(1) *An increase in ρ_c raises e_{ci} ($i = 1, 2$) and lowers n_1 , n_2 and $n_1 + n_2$, and in case (4), it raises e .*

(2) (a) *A decrease in δ raises n_1 , n_2 and $n_1 + n_2$, and in case (4), it raises e .*

(b) *A decrease in λ lowers n_1 and raises n_2 and $n_1 + n_2$, and in case (4), it raises e .*

(c) *A decrease in θ_q lowers e_{ci} ($i = 1, 2$) and raises n_1 , n_2 and $n_1 + n_2$ and in case (4), it lowers e .*

(3) *An increase in θ_q raises e_{ci} ($i = 1, 2$) and lowers n_1 , n_2 and $n_1 + n_2$, and in case (4), it raises e .*

(4) In case (4), an increase in ρ lowers n_1 and $n_1 + n_2$ and raises n_2 and e .

Under the assumptions in Proposition 3.2, the results of the time-dependent case are mostly same as the results of the time-invariant case, which can be obtained from Proposition 3.1 and Corollary 3.1. The exception is that, in case (4), an increase in ρ_c raises e and a decrease in θ_q lowers e in the time-dependent case, while they have no effects on e in the time-invariant case. These assumptions appear to be not very restrictive because it seems unlikely that ρ_c and δ change substantially within 10 or 20 years (between the first and the second periods of adulthood) and, as long as the changes of these parameters are not large, the assumption on λ is only a little stronger than $\lambda < 2$ of the time-invariant case.

As for the period roughly from the early 1920s to the early 1930s, data show that age-specific and total fertility rates fell sharply, average years of education of young generations rose rapidly, while their average years of tertiary education (especially of women) changed little. Evidence suggests that the demand for skilled workers, who corresponded to high school graduates at the time, increased greatly (Goldin and Katz, 1998). In the model, this can be captured by the increased productivity of (or increased return to) childhood education ρ_c . Because average years of higher education of young women were very low (almost 0 in the UK) during this period, it would be reasonable to suppose that the economy is in case (3) and thus $e = 0$. Proposition 3.2 (1) shows that an increase in ρ_c raises e_{ci} ($i = 1, 2$) and lowers n_1 , n_2 and $n_1 + n_2$ in case (3). Hence, the model can explain the above-mentioned facts qualitatively.

As for the period of baby boom roughly from the early 1940s to the early 1960s, age-specific and total fertility rates rose rapidly, and average years of secondary education and

of tertiary education of young generations rose moderately. Researchers argue that the baby boom was caused by the increased productivity of the household sector from the diffusion of home appliances (Greenwood, Seshadri, and Vandenbroucke, 2005), a large fall in maternal mortality, which is a proxy for the overall maternal cost of having children (Albanesi and Olivetti, 2014), and a decrease in per space housing costs due to suburbanization (Tamura and Simon, 2017). In the model, the increased productivity of the household sector can be captured by the decreased fixed time spent on household chores δ and to a lesser extent, by the decreased cost of child-rearing θ_q ; the decreased cost of having children can be captured by the decreased relative cost of late childbearing λ and by the decreased θ_q ; the decreased price for space can be captured by the decreased θ_q .

During the period of baby boom, average years of tertiary education of young women rose moderately.²² Thus, it would be plausible that the model economy transits from case (3) to case (4) at some point of this period. In Figure 3.1 of the previous section, decreases in λ and δ shift the boundary line between case (3) and case (4) to the left, and a decrease in λ shifts the model economy downward. Hence, the model economy transits to case (4) when δ and λ become sufficiently small. Proposition 3.2 (2) shows that a decrease in δ raises n_1 , n_2 , $n_1 + n_2$ and in case (4), e ; a decrease in λ lowers n_1 and raises n_2 , $n_1 + n_2$ and in case (4), e ; and a decrease in θ_q raises n_1 , n_2 and $n_1 + n_2$ and lowers e_{ci} ($i = 1, 2$) and in case (4), e . A decrease in δ can explain the facts except the increased years of secondary education; a decrease in λ can explain the facts except the increased fertility rate

²² Average years of higher education of 15-24 old women increased from 0.21 in 1940 to 0.39 in 1965 in the US and from 0.01 to 0.13 in the UK.

of young mothers and years of secondary education; and a decrease in θ_q can explain the facts except the increased years of secondary and tertiary education. Hence, the model can explain the facts except the increased years of secondary education, as long as the effects of the changes of λ and θ_q are not very large.

As for the period of "baby bust" roughly from the early or middle 1960s to the mid-1970s, age-specific and total fertility rates fell rapidly, while average years of education, particularly of tertiary education, of young generations rose moderately. Researchers argue that the labor demand for young women and their relative wage increased during this period, with WWII (Doepke, Hazan and Maoz, 2015) or the Great Depression (Bellou and Cardia, 2014) as the ultimate cause. In the model, the increased labor demand could be captured by the increased opportunity cost of child-rearing θ_q .²³ Proposition 3.2 (3) shows that an increase in θ_q lowers n_1 , n_2 and $n_1 + n_2$ and raises e_{ci} ($i = 1, 2$) and e in case (4). Hence, the model can explain the facts qualitatively.

Finally, as for the period after the mid-1970s, the fertility rate of elder mothers rose, fertility rate of younger mothers fell, and total fertility rate changed little, while average years of education, particularly of tertiary education of women, increased. Empirical works show that the relative earnings of college graduate women to men increased greatly from the late 1960s (Goldin, 2014) and the wage premium of college graduate increased sharply

²³ To be more accurate, the increased labor demand for young women raises not only their opportunity cost of child-rearing but also that of adulthood education and the maximum effective time available for work, which is $1 - \delta$ in the present setting. Taking into account these factors complicates the analysis, but the result is qualitatively unchanged, as long as the importance of the opportunity cost in the total cost of adulthood education, which includes the cost of paying for teachers' time, is not too large.

from the late 1970s (Goldin and Katz, 2008) except after around 2015 (Autor, Goldin and Katz, 2020). In the model, these can be captured by the increased productivity of (or increased return to) adulthood education ρ . Proposition 3.2 (4) shows that an increase in ρ lowers n_1 and $n_1 + n_2$ and raises n_2 and e in case (4). Hence, it can explain these facts except the largely unchanged total fertility rate.

In summary, changes of fertility and education in the period roughly from the early 1920s to the early 1930s can be explained by the increased ρ_c of the model; the changes in the period of baby boom can be explained by the decreased δ , λ and θ_q except the increased years of secondary education; the changes in the period of "baby bust" can be explained by the increased θ_q ; and the changes in the period after the mid-1970s can be explained by the increased ρ except the largely unchanged total fertility rate.

3.5 Conclusions

In this work, we develop a three-period model with two periods of childbearing to consider two aspects of fertility, that is, total fertility rate and childbearing age (age-specific fertility rate) at the same time. Also, we incorporate decisions on one's own education in adulthood (corresponds to tertiary education) and education of her children (corresponds to primary and secondary education) into the model to study the interactions between fertility and education.

We examine how well the model can explain the long-run changes in fertility and education in countries such as the US and the UK qualitatively. First, as for the period roughly from the early 1920s to the early 1930s, data show that age-specific and total fertility rates fell sharply, average years of education of young generations rose rapidly,

while their average years of tertiary education (especially of women) changed little. Evidence suggests that the demand for skilled workers (corresponded to high school graduates) increased greatly. When the model captures the increased skill demand by the increased productivity of (or increased return to) childhood education, it can explain these facts.

Second, as for the period of baby boom roughly from the early 1940s to the early 1960s, observed facts are that age-specific and total fertility rates rose rapidly, and average years of secondary education and of tertiary education of young generations rose moderately. Researchers argue that the baby boom was caused by the increased productivity of the household sector, a large fall in maternal mortality, and a decrease in housing costs. When the model captures these changes by the decreased fixed time spent on household chores, the decreased relative cost of late childbearing, and the decreased cost of child-rearing, it can explain the facts except the increased years of secondary education, as long as the effects of the changes of the relative cost of late childbearing and the cost of child-rearing are not very large.

Third, as for the period of "baby bust" roughly from the early or middle 1960s to the mid-1970s, age-specific and total fertility rates fell rapidly, while average years of education, particularly of tertiary education, of young generations rose moderately. Researchers argue that the labor demand for young women and their relative wage increased, with WWII or the Great Depression as the ultimate cause. When the model captures the increased labor demand by the increased opportunity cost of child-rearing, it can explain these facts.

Finally, as for the period after the mid-1970s, the fertility rate of elder mothers rose, fertility rate of younger mothers fell, and total fertility rate changed little, while average years of education, particularly of tertiary education of women, increased. Empirical works show that the relative earnings of educated, particularly female, workers, increased greatly. When the model captures it by the increased productivity of (or increased return to) adulthood education, it can explain these facts except the largely unchanged total fertility rate.

The interactions between education and fertility have been considered by Iyigun (2000) and d'Albisa, Greulich and Ponthiere (2018). Our work differs from their works mainly in three aspects. First, Iyigun (2000) focuses on the postponement of childbearing, and d'Albisa, Greulich and Ponthiere (2018) focus on the decreasing of total fertility rate and the U-shaped trend of childbearing age. By contrast, as mentioned above, we focus on four periods of distinctive fertility trends. Second, in addition to the decision on one's own adulthood education, we also consider decisions on education of her children. This specification allows us to consider the interactions between adulthood education, education of children and fertility. Finally, we examine how various factors, such as the cost of household chores, the productivity of adulthood education and the relative cost of late childbearing, affect fertility and education, which are not included in these works.

Appendix The time-dependent case and Proofs of Propositions in chapter 3

A3.1 The time-dependent case

Since the dynamic analysis is focused on case (3) and case (4), we consider these cases when the fixed time spent on household chores δ and the productivity of childhood education ρ_c are time-dependent. Let δ and ρ_c be the variables in period 1 of adulthood and $g_\delta\delta$ and $g_{\rho_c}\rho_c$ be the variables in period 2, where $g_\delta \leq 1$ and $g_{\rho_c} \geq 1$.

The values and properties of e_{ci} and n_i ($i = 1, 2$) in case (3) are summarized in the next proposition. Proofs of the propositions in this Appendix are presented in Appendix 3.2.

Proposition A3.1 Assume $\rho_c > \frac{\theta_e}{\alpha_c\theta_q}$, $g_\delta \leq 1$ and $g_{\rho_c} \geq 1$. In Case (3) of the time-dependent case, the following holds. $e = 0$, $e_{c1} = \frac{\alpha_c\rho_c\theta_q - \theta_e}{(1-\alpha_c)\theta_e\rho_c}$, $e_{c2} = \frac{\alpha_cg_{\rho_c}\rho_c\theta_q - \theta_e}{(1-\alpha_c)\theta_eg_{\rho_c}\rho_c}$.

(1) $n_1 = \frac{(1-\alpha_c)\rho_c[(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}2\lambda - (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c}]}{3\lambda(\rho_c\theta_q - \theta_e)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}$. It increases (decreases) with λ , g_δ and θ_e (with ρ_c , g_{ρ_c} , δ , and θ_q).

(2) $n_2 = \frac{(1-\alpha_c)\rho_c[2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} - (1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda]}{3\lambda(\rho_c\theta_q - \theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q - \theta_e)}$. It decreases with λ and g_δ . It decreases (increases) with ρ_c when $\lambda < (\geq) \frac{2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}[\alpha_c(\rho_c\theta_q - \theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q - \theta_e)]}$, with g_{ρ_c} when $\lambda < (\geq) \frac{2\theta_e(1-g_\delta\delta)(\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{(1-\delta)\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}$, with δ when $\lambda < (\geq) \frac{2g_{\rho_c}g_\delta(\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}$, with θ_q when $\lambda < (\geq) \frac{2(1-g_\delta\delta)g_{\rho_c}^2(\rho_c\theta_q - \theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}[\alpha_cg_{\rho_c}(\rho_c\theta_q - \theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q - \theta_e)]}$, and increases (decreases) with θ_e when $\lambda < (\geq) \frac{2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}[\alpha_c(\rho_c\theta_q - \theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q - \theta_e)]}$.

The values and properties of e_{ci} , n_i ($i = 1, 2$) and e in case (4) are summarized in the next proposition.

Proposition A3.2 Assume $\rho_c > \frac{\theta_e}{\alpha_c \theta_q}$, $g_\delta \leq 1$ and $g_{\rho_c} \geq 1$. In Case (4) of the time-dependent case, the following holds. $e_{c1} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c) \theta_e \rho_c}$, $e_{c2} = \frac{\alpha_c g_{\rho_c} \rho_c \theta_q - \theta_e}{(1-\alpha_c) \theta_e g_{\rho_c} \rho_c}$.

$$(1) \quad n_1 = \frac{(1-\alpha_c) \rho_c \{ 2[(1-\delta) \rho + \eta] (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c} \lambda - (1+\alpha) \rho (1-g_\delta \delta) g_{\rho_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c} \}}{(3+\alpha) \lambda \rho (\rho_c \theta_q - \theta_e) (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c}}. \quad \text{It}$$

increases (decreases) with λ , η , g_δ and θ_e (with ρ , ρ_c , g_{ρ_c} , and θ_q). It decreases with δ when g_{ρ_c} is not large and g_δ is not small.

$$(2) \quad n_2 = \frac{(1-\alpha_c) \rho_c [(2+\alpha) \rho (1-g_\delta \delta) g_{\rho_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c} - [(1-\delta) \rho + \eta] (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c} \lambda]}{(3+\alpha) \lambda \rho (\rho_c \theta_q - \theta_e)^{1-\alpha_c} (g_{\rho_c} \rho_c \theta_q - \theta_e)}. \quad \text{It}$$

increases (decreases) with ρ (with λ , η and g_δ). It decreases (increases) with ρ_c when $\lambda <$

$$(\geq) \frac{(2+\alpha) \rho (1-g_\delta \delta) g_{\rho_c} (\rho_c \theta_q - \theta_e)^{2-\alpha_c}}{[(1-\delta) \rho + \eta] (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c} [\alpha_c (\rho_c \theta_q - \theta_e) + (1-\alpha_c) (g_{\rho_c} \rho_c \theta_q - \theta_e)]}, \quad \text{with } g_{\rho_c} \text{ when } \lambda <$$

$$(\geq) \frac{(2+\alpha) \rho \theta_e (1-g_\delta \delta) (\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{[(1-\delta) \rho + \eta] \alpha_c \rho_c \theta_q (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c}}, \quad \text{with } \delta \text{ when } \lambda < (\geq) \frac{(2+\alpha) g_{\rho_c} g_\delta (\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{(g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c}}, \quad \text{with}$$

$$\theta_q \text{ when } \lambda < (\geq) \frac{(2+\alpha) \rho (1-g_\delta \delta) g_{\rho_c}^2 (\rho_c \theta_q - \theta_e)^{2-\alpha_c}}{[(1-\delta) \rho + \eta] (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c} [\alpha_c g_{\rho_c} (\rho_c \theta_q - \theta_e) + (1-\alpha_c) (g_{\rho_c} \rho_c \theta_q - \theta_e)]}, \quad \text{and increases}$$

$$(\text{decreases}) \text{ with } \theta_e \text{ when } \lambda < (\geq) \frac{(2+\alpha) \rho (1-g_\delta \delta) g_{\rho_c} (\rho_c \theta_q - \theta_e)^{2-\alpha_c}}{[(1-\delta) \rho + \eta] (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c} [\alpha_c (\rho_c \theta_q - \theta_e) + (1-\alpha_c) (g_{\rho_c} \rho_c \theta_q - \theta_e)]}.$$

$$(3) \quad e = \frac{\alpha \rho [(1-\delta) (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c} \lambda + (1-g_\delta \delta) g_{\rho_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}] - 3 \lambda \eta (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c}}{(3+\alpha) \lambda \rho \eta (g_{\rho_c} \rho_c \theta_q - \theta_e)^{1-\alpha_c}}. \quad \text{It}$$

increases (decreases) with ρ , ρ_c , g_{ρ_c} and θ_q (with λ , η , δ , g_δ and θ_e).

The values and properties of $n_1 + n_2$ in case (3) and case (4) are summarized in the next proposition.

Proposition A3.3 Assume $\rho_c > \frac{\theta_e}{\alpha_c \theta_q}$, $g_\delta \leq 1$ and $g_{\rho_c} \geq 1$. In Case (3) and case (4) of the time-dependent case, the following holds.

(1) In case (3),

$n_1 + n_2 = \frac{(1-\alpha_c)\rho_c}{3\lambda(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)} \left\{ (1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} \left[2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} - (\rho_c\theta_q-\theta_e)^{\alpha_c} \right] \lambda + (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} \left[2(\rho_c\theta_q-\theta_e)^{\alpha_c} - (g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} \right] \right\}$. It increases (decreases) with θ_e (with λ , ρ_c , δ , g_δ and θ_q) when g_{ρ_c} is not large. It decreases (increases) with g_{ρ_c} when

$$\lambda < (\geq) \frac{(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-2\alpha_c} [(\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} + 2\theta_e(\rho_c\theta_q-\theta_e)^{\alpha_c}]}{(1-\delta)\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}.$$

(2) In case (4),

$n_1 + n_2 = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)} \left\{ [(1-\delta)\rho + \eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} \left[2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} - (\rho_c\theta_q-\theta_e)^{\alpha_c} \right] \lambda + \rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} \left[(2+\alpha)(\rho_c\theta_q-\theta_e)^{\alpha_c} - (1+\alpha)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} \right] \right\}$. It increases (decreases) with η (with ρ). It increases (decreases) with θ_e (with λ , ρ_c , δ , g_δ and θ_q) when g_{ρ_c} is not large. It decreases (increases) with g_{ρ_c} when $\lambda < (\geq) \frac{(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-2\alpha_c} \rho [(1+\alpha)(\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} + (2+\alpha)\theta_e(\rho_c\theta_q-\theta_e)^{\alpha_c}]}{[(1-\delta)\rho + \eta]\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$.

As in the time-invariant case, e_{ci} increases (decreases) with ρ_c and θ_q (θ_e); n_1 increases (decreases) with λ and θ_e (with ρ_c , δ and θ_q) and in case (4), it increases (decreases) with η (with ρ);²⁴ n_2 decreases with λ and in case (4), it increases (decreases) with ρ (with η); e increases (decreases) with ρ (with λ , η and δ); and in case (4), $n_1 + n_2$ increases (decreases) with η (with ρ).

By contrast, n_2 increases (decreases) with θ_e (with ρ_c , δ and θ_q) when λ is small and decreases (increases) with θ_e (with ρ_c , δ and θ_q) when λ is large in the time-dependent case, while n_2 always increases (decreases) with θ_e (with ρ_c , δ and θ_q) in the time-invariant case. $n_1 + n_2$ increases (decreases) with θ_e (with λ , δ , ρ_c and θ_q) when g_{ρ_c} is not large, while it always holds in the time-invariant case. e increases (decreases) with ρ_c

²⁴ In case (4), the result on δ holds when g_{ρ_c} is not large and g_δ is not small.

and θ_q (with θ_e), while these parameters do not affect e in the time-invariant case. As for new parameters, e_{c2} increases with g_{ρ_c} ; n_1 increases (decreases) with g_δ (with g_{ρ_c}); n_2 decreases with g_δ and decreases (increases) with g_{ρ_c} when λ is small (large); $n_1 + n_2$ decreases with g_δ when g_{ρ_c} is not large and decreases (increases) with g_{ρ_c} when λ is small (large); e increases (decreases) with g_{ρ_c} (with g_δ).

However, the results of the time-dependent case are mostly same as the time-invariant case when λ and g_{ρ_c} are not large and g_δ is not small. When $g_{\rho_c} = g_\delta = 1$, the inequalities on λ (except those associated with g_{ρ_c}) of the above propositions become $\lambda < (\geq) 2$ in case (3) and $\lambda < (\geq) \frac{(2+\alpha)\rho}{(1-\delta)\rho+\eta}$ (the one associated with δ becomes $\lambda < (\geq) 2 + \alpha$) in case (4). Hence, in the area $\lambda < 2$ where both case (3) and case (4) hold in the time-invariant case, the results of the time-dependent case are mostly same as the time-invariant case for the parameters present in both cases. In case (4), ρ_c , θ_q and θ_e have no effect on e in the time-invariant case. Further, the direction of the effect of g_δ is the same as the effect of δ .²⁵ The same results hold for $g_{\rho_c} > 1$ and $g_\delta < 1$ even when λ is not very small, as long as g_{ρ_c} is not large and g_δ is not small.

A3.2 Proofs of Propositions 3.1 and 3.2

Proof of Proposition 3.1

Necessary conditions of the utility maximization problem

Results of the proposition are obtained by solving the utility maximization problem.

The maximization problem is solved backward. The objective function of the period 2 maximization problem is

²⁵ The inequalities associated with g_{ρ_c} becomes $\lambda < (\geq) \frac{2\theta_e}{\alpha_c \rho_c \theta_q}$ in case (3) and $\lambda < (\geq) \frac{(2+\alpha)\rho \theta_e}{[(1-\delta)\rho+\eta]\alpha_c \rho_c \theta_q}$ in case (4) when $g_{\rho_c} = g_\delta =$

1. Thus, the direction of the effect of g_{ρ_c} is the same as the effect of ρ_c when λ is not large.

$$\ln\{gA(1+\rho e)^\alpha h_1[1-\delta-\lambda(\theta_q+\theta_e e_{c2})n_2]\} + \ln[n_1(1+\rho_c e_{c1})^{\alpha_c} + n_2(1+\rho_c e_{c2})^{\alpha_c}]. \quad (A3.1)$$

First order conditions are

$$\frac{\partial U}{\partial e_{c2}} = n_2 \left[\frac{-\lambda\theta_e}{1-\delta-\lambda(\theta_q+\theta_e e_{c2})n_2} + \frac{\alpha_c \rho_c (1+\rho_c e_{c2})^{\alpha_c-1}}{n_1(1+\rho_c e_{c1})^{\alpha_c} + n_2(1+\rho_c e_{c2})^{\alpha_c}} \right] \leq 0, \quad (A3.2)$$

$$\frac{\partial U}{\partial n_2} = \frac{-\lambda(\theta_q+\theta_e e_{c2})}{1-\delta-\lambda(\theta_q+\theta_e e_{c2})n_2} + \frac{(1+\rho_c e_{c2})^{\alpha_c}}{n_1(1+\rho_c e_{c1})^{\alpha_c} + n_2(1+\rho_c e_{c2})^{\alpha_c}} \leq 0. \quad (A3.3)$$

When $n_2 > 0$, from $\frac{\partial U}{\partial n_2} = 0$,

$$n_1(1+\rho_c e_{c1})^{\alpha_c} + n_2(1+\rho_c e_{c2})^{\alpha_c} = \frac{(1+\rho_c e_{c2})^{\alpha_c}}{\lambda(\theta_q+\theta_e e_{c2})} [1-\delta-\lambda(\theta_q+\theta_e e_{c2})n_2]. \quad (A3.4)$$

By substituting this equation into (A3.2),

$$\begin{aligned} \frac{\partial U}{\partial e_{c2}} &= n_2 \left[\frac{-\lambda\theta_e}{1-\delta-\lambda(\theta_q+\theta_e e_{c2})n_2} + \frac{\alpha_c \rho_c (1+\rho_c e_{c2})^{\alpha_c-1}}{1-\delta-\lambda(\theta_q+\theta_e e_{c2})n_2} \frac{\lambda(\theta_q+\theta_e e_{c2})}{(1+\rho_c e_{c2})^{\alpha_c}} \right] \\ &= \frac{\lambda n_2}{1-\delta-\lambda(\theta_q+\theta_e e_{c2})n_2} \left[-\theta_e + \frac{\alpha_c \rho_c (\theta_q+\theta_e e_{c2})}{1+\rho_c e_{c2}} \right]. \end{aligned} \quad (A3.5)$$

From $\rho_c > \frac{\theta_e}{\alpha_c \theta_q}$ (equation (3.10)), $e_{c2} > 0$ if $n_2 > 0$, otherwise $e_{c2} = 0$.

When $n_2 > 0$ from $\frac{\partial U}{\partial e_{c2}} = 0$,

$$e_{c2} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c)\theta_e \rho_c}. \quad (A3.6)$$

When $n_2 = (>)0$, from (A3.3),

$$\frac{\partial U}{\partial n_2} \Big|_{n_2=0} = \frac{-\lambda(\theta_q+\theta_e e_{c2})}{1-\delta} + \frac{(1+\rho_c e_{c2})^{\alpha_c}}{n_1(1+\rho_c e_{c1})^{\alpha_c}} \leq (>)0. \quad (A3.7)$$

Depending on whether n_2 is positive or zero, there are two cases to examine.

[1] When $n_2 = 0$

In this case, the objective function of the period 1 maximization problem becomes

$$U = \ln\{Ah_1[1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1]\} + \ln[gA(1 + \rho e)^\alpha h_1(1 - \delta)] \\ + \ln[n_1(1 + \rho_c e_{c1})^{\alpha_c}]. \quad (A3.8)$$

First order conditions are

$$\frac{\partial U}{\partial n_1} = \frac{-(\theta_q + \theta_e e_{c1})}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} + \frac{1}{n_1} \leq 0, \quad (A3.9)$$

$$\frac{\partial U}{\partial e_{c1}} = \frac{-\theta_e n_1}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} + \frac{\alpha_c \rho_c}{1 + \rho_c e_{c1}} \leq 0, \quad (A3.10)$$

$$\frac{\partial U}{\partial e} = \frac{-\eta}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} + \frac{\alpha \rho}{1 + \rho e} \leq 0. \quad (A3.11)$$

In this case, $n_1 > 0$ always holds because $\frac{\partial U}{\partial n_1} = \infty$ when $n_1 = 0$. Thus from $\frac{\partial U}{\partial n_1} = 0$,

$$1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1 = (\theta_q + \theta_e e_{c1})n_1.$$

By substituting this equation into (A3.10),

$$\frac{\partial U}{\partial e_{c1}} = \frac{-\theta_e}{\theta_q + \theta_e e_{c1}} + \frac{\alpha_c \rho_c}{1 + \rho_c e_{c1}}. \quad (A3.12)$$

Hence, $e_{c1} > 0$ from $\rho_c > \frac{\theta_e}{\alpha_c \theta_q}$ (equation (3.10)). From $\frac{\partial U}{\partial e_{c1}} = 0$,

$$e_{c1} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1 - \alpha_c) \theta_e \rho_c}. \quad (A3.13)$$

When $e = (>)0$, from (A3.12),

$$\frac{\partial U}{\partial e} \Big|_{e=0} = \frac{-\eta}{1 - \delta - (\theta_q + \theta_e e_{c1})n_1} + \alpha \rho \leq (>)0. \quad (A3.14)$$

Depending on whether e is positive or zero, there are two cases to consider.

① When $n_1 > 0$ and $e = 0$

In this case, values of variables are $n_2 = e = e_{c2} = 0$, $e_{c1} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c)\theta_e \rho_c}$, and

$$n_1 = \frac{(1-\alpha_c)(1-\delta)\rho_c}{2(\rho_c \theta_q - \theta_e)}. \quad (A3.15)$$

By substituting these values of n_1 , e_{c1} , and e_{c2} into $\frac{\partial U}{\partial n_2} \leq 0$ when $n_2 = 0$, (A3.7),

$$\frac{-\lambda \theta_q}{1-\delta} + \frac{1}{\frac{(1-\alpha_c)(1-\delta)\rho_c}{2(\rho_c \theta_q - \theta_e)} \left[\frac{\alpha_c(\rho_c \theta_q - \theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}} \leq 0.$$

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} \leq 0$ when $e = 0$, (A3.14),

$$\frac{-\eta}{1-\delta - \frac{(\rho_c \theta_q - \theta_e)(1-\alpha_c)(1-\delta)\rho_c}{(1-\alpha_c)\rho_c}} + \alpha \rho \leq 0.$$

Hence, the necessary conditions for this case to be realized are

$$\lambda \geq \frac{2\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}, \quad (A3.16)$$

$$\rho \leq \frac{2\eta}{\alpha(1-\delta)}. \quad (A3.17)$$

The right-hand side of (A3.16) is less than 2, because,

$$\begin{aligned} \frac{2\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} &= \frac{2\theta_e^{\alpha_c}}{\theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} \frac{(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c} \\ &< \frac{2\theta_e^{\alpha_c}}{\theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} \frac{\left(\frac{\theta_e}{\alpha_c \theta_q} \theta_q - \theta_e \right)^{1-\alpha_c}}{\frac{\theta_e}{\alpha_c \theta_q}} = 2. \end{aligned} \quad (A3.18)$$

where the inequality holds because, from $\rho_c > \frac{\theta_e}{\alpha_c \theta_q}$ (equation (3.10)),

$$\frac{\partial \left(\frac{\rho_c \theta_q - \theta_e}{\rho_c} \right)^{1-\alpha_c}}{\partial \rho_c} = \frac{(\rho_c \theta_q - \theta_e)^{-\alpha_c} (\theta_e - \alpha_c \rho_c \theta_q)}{\rho_c^2} < 0.$$

② When $n_1 > 0$ and $e > 0$

In this case, values of variables are $n_2 = e_{c2} = 0$, $e_{c1} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c) \theta_e \rho_c}$,

$$n_1 = \frac{(1-\alpha_c) \rho_c [(1-\delta) \rho + \eta]}{(2+\alpha) \rho (\rho_c \theta_q - \theta_e)}, \quad (A3.19)$$

and

$$e = \frac{(1-\delta) \alpha \rho - 2\eta}{(2+\alpha) \rho \eta}. \quad (A3.20)$$

By substituting these values of n_1 , e_{c1} , and e_{c2} into $\frac{\partial U}{\partial n_2} \leq 0$ when $n_2 = 0$, (A3.7),

$$\frac{-\lambda \theta_q}{1-\delta} + \frac{(2+\alpha) \rho (\rho_c \theta_q - \theta_e)}{(1-\alpha_c) \rho_c [(1-\delta) \rho + \eta] \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{(1-\alpha_c) \theta_e} \right]^{\alpha_c}} \leq 0.$$

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} > 0$ when $e = 0$, (A3.14),

$$\frac{-\eta}{1-\delta - \frac{\rho_c \theta_q - \theta_e}{(1-\alpha_c) \rho_c} \frac{(1-\alpha_c) \rho_c [(1-\delta) \rho + \eta]}{(2+\alpha) \rho (\rho_c \theta_q - \theta_e)}} + \alpha \rho > 0$$

Hence, the necessary conditions for this case to be realized are

$$\begin{aligned} \lambda &\geq \frac{(2+\alpha)(1-\delta) \rho \theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q [(1-\delta) \rho + \eta] \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} \\ &\Leftrightarrow \rho \leq \frac{\lambda \eta}{(1-\delta) \left[\frac{(2+\alpha) \theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} - \lambda \right]}, \end{aligned} \quad (A3.21)$$

$$\rho > \frac{2\eta}{\alpha(1-\delta)}. \quad (A3.22)$$

[2] When $n_2 > 0$

Since $n_2 > 0$, $e_{c2} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c)\theta_e \rho_c}$ from (A3.6). Thus,

$$\theta_q + \theta_e e_{c2} = \frac{\rho_c \theta_q - \theta_e}{(1-\alpha_c)\rho_c},$$

$$(1 + \rho_c e_{c2})^{\alpha_c} = \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{\theta_e (1-\alpha_c)} \right]^{\alpha_c}.$$

By substituting these equations into (A3.4) and solving for n_2 ,

$$n_2 = \frac{(1-\delta)\rho_c(1-\alpha_c) \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{\theta_e (1-\alpha_c)} \right]^{\alpha_c} - \lambda(\rho_c \theta_q - \theta_e)(1 + \rho_c e_{c1})^{\alpha_c} n_1}{2\lambda(\rho_c \theta_q - \theta_e) \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{\theta_e (1-\alpha_c)} \right]^{\alpha_c}}. \quad (\text{A3.23})$$

Hence,

$$1 - \delta - \lambda(\theta_q + \theta_e e_{c2})n_2 = \frac{(1-\delta)\rho_c(1-\alpha_c) \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{\theta_e (1-\alpha_c)} \right]^{\alpha_c} + \lambda(\rho_c \theta_q - \theta_e)(1 + \rho_c e_{c1})^{\alpha_c} n_1}{2\rho_c(1-\alpha_c) \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{\theta_e (1-\alpha_c)} \right]^{\alpha_c}},$$

$$n_1(1 + \rho_c e_{c1})^{\alpha_c} + n_2(1 + \rho_c e_{c2})^{\alpha_c} = \frac{(1-\delta)\rho_c(1-\alpha_c) \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{\theta_e (1-\alpha_c)} \right]^{\alpha_c} + \lambda(\rho_c \theta_q - \theta_e)(1 + \rho_c e_{c1})^{\alpha_c} n_1}{2\lambda(\rho_c \theta_q - \theta_e)}.$$

By substituting these equations into (A3.1), the objective function of the period 1 maximization problem becomes

$$U = \ln\{Ah_1[1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1]\} \\ + \ln\left\{gA(1 + \rho e)^{\alpha} h_1 \frac{(1-\delta)\rho_c(1-\alpha_c) \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{\theta_e (1-\alpha_c)} \right]^{\alpha_c} + \lambda(\rho_c \theta_q - \theta_e)(1 + \rho_c e_{c1})^{\alpha_c} n_1}{2\rho_c(1-\alpha_c) \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{\theta_e (1-\alpha_c)} \right]^{\alpha_c}}\right\}$$

$$+ \ln \left\{ \frac{(1-\delta)\rho_c(1-\alpha_c) \left[\frac{\alpha_c(\rho_c\theta_q - \theta_e)}{\theta_e(1-\alpha_c)} \right]^{\alpha_c} + \lambda(\rho_c\theta_q - \theta_e)(1+\rho_ce_{c1})^{\alpha_c}n_1}{2\lambda(\rho_c\theta_q - \theta_e)} \right\} \quad (A3.24)$$

First order conditions are

$$\begin{aligned} \frac{\partial U}{\partial n_1} &= \frac{-(\theta_q + \theta_e e_{c1})}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} \\ &+ \frac{2\lambda(\rho_c\theta_q - \theta_e)(1 + \rho_ce_{c1})^{\alpha_c}}{(1 - \delta)\rho_c(1 - \alpha_c) \left[\frac{\alpha_c(\rho_c\theta_q - \theta_e)}{\theta_e(1 - \alpha_c)} \right]^{\alpha_c} + \lambda(\rho_c\theta_q - \theta_e)(1 + \rho_ce_{c1})^{\alpha_c}n_1} \leq 0 \quad (A3.25) \end{aligned}$$

$$\begin{aligned} \frac{\partial U}{\partial e_{c1}} &= n_1 \left\{ \frac{-\theta_e}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} \right. \\ &+ \left. \frac{2\lambda(\rho_c\theta_q - \theta_e)\alpha_c\rho_c(1 + \rho_ce_{c1})^{\alpha_c-1}}{(1 - \delta)\rho_c(1 - \alpha_c) \left[\frac{\alpha_c(\rho_c\theta_q - \theta_e)}{\theta_e(1 - \alpha_c)} \right]^{\alpha_c} + \lambda(\rho_c\theta_q - \theta_e)(1 + \rho_ce_{c1})^{\alpha_c}n_1} \right\} \leq 0, \quad (A3.26) \end{aligned}$$

$$\frac{\partial U}{\partial e} = \frac{-\eta}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} + \frac{\alpha\rho}{1 + \rho e} \leq 0. \quad (A3.27)$$

When $n_1 > 0$, from $\frac{\partial U}{\partial n_1} = 0$,

$$\begin{aligned} &(1 - \delta)\rho_c(1 - \alpha_c) \left[\frac{\alpha_c(\rho_c\theta_q - \theta_e)}{\theta_e(1 - \alpha_c)} \right]^{\alpha_c} + \lambda(\rho_c\theta_q - \theta_e)(1 + \rho_ce_{c1})^{\alpha_c}n_1 \\ &= \frac{2\lambda(\rho_c\theta_q - \theta_e)(1 + \rho_ce_{c1})^{\alpha_c}}{\theta_q + \theta_e e_{c1}} [1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1]. \end{aligned}$$

By substituting this equation into (A3.26),

$$\frac{\partial U}{\partial e_{c1}} = \frac{n_1}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} \left[-\theta_e + \frac{\alpha_c\rho_c(\theta_q + \theta_e e_{c1})}{1 + \rho_ce_{c1}} \right]. \quad (A3.28)$$

Thus, $e_{c1} > 0$ if $n_1 > 0$, otherwise $e_{c1} = 0$ from $\rho_c > \frac{\theta_e}{\alpha_c \theta_q}$ (equation (3.10)).

When $n_1 > 0$, $e_{c1} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c) \theta_e \rho_c}$ from $\frac{\partial U}{\partial e_{c1}} = 0$.

When $n_1 = (>)0$, from (A3.25),

$$\frac{\partial U}{\partial n_1} \Big|_{n_1=0} = \frac{-(\theta_q + \theta_e e_{c1})}{1 - \delta - \eta e} + \frac{2\lambda(\rho_c \theta_q - \theta_e)(1 + \rho_c e_{c1})^{\alpha_c}}{(1 - \delta)\rho_c(1 - \alpha_c) \left[\frac{\alpha_c(\rho_c \theta_q - \theta_e)}{\theta_e(1 - \alpha_c)} \right]^{\alpha_c}} \leq (>)0. \quad (\text{A3.29})$$

When $e = (>)0$, from (A3.27),

$$\frac{\partial U}{\partial e} \Big|_{e=0} = \frac{-\eta}{1 - \delta - (\theta_q + \theta_e e_{c1})n_1} + \alpha \rho \leq (>)0. \quad (\text{A3.30})$$

Depending on whether n_1 and e are positive or zero, there are four cases to consider.

③ When $n_1 > 0$ and $e = 0$

In this case, values of variables are $e = 0$, $e_{c1} = e_{c2} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c) \theta_e \rho_c}$,

$$n_1 = \frac{(1 - \delta)\rho_c(1 - \alpha_c)(2\lambda - 1)}{3\lambda(\rho_c \theta_q - \theta_e)}, \quad (\text{A3.31})$$

and

$$n_2 = \frac{(1 - \delta)\rho_c(1 - \alpha_c)(2 - \lambda)}{3\lambda(\rho_c \theta_q - \theta_e)}. \quad (\text{A3.32})$$

By substituting these values of n_1 , e_{c1} , and e_{c2} into $\frac{\partial U}{\partial n_2} > 0$ when $n_2 = 0$, (A3.7),

$$\frac{-\lambda \frac{\rho_c \theta_q - \theta_e}{\rho_c(1 - \alpha_c)}}{1 - \delta} + \frac{3\lambda(\rho_c \theta_q - \theta_e)}{(1 - \delta)\rho_c(1 - \alpha_c)(2\lambda - 1)} > 0.$$

By substituting the values of e and e_{c1} into $\frac{\partial U}{\partial n_1} > 0$ when $n_1 = 0$, (A3.29),

$$\frac{\frac{\rho_c \theta_q - \theta_e}{\rho_c(1-\alpha_c)}}{1-\delta} + \frac{2\lambda(\rho_c \theta_q - \theta_e)}{(1-\delta)\rho_c(1-\alpha_c)} > 0 \Leftrightarrow \lambda > \frac{1}{2},$$

which always holds because of the assumption of $\lambda \geq 1$.

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} \leq 0$ when $e = 0$, (A3.30),

$$\frac{-\eta}{1-\delta - \frac{\rho_c \theta_q - \theta_e}{\rho_c(1-\alpha_c)} \frac{(1-\delta)\rho_c(1-\alpha_c)(2\lambda-1)}{3\lambda(\rho_c \theta_q - \theta_e)}} + \alpha\rho \leq 0.$$

Hence, the necessary conditions for this case to be realized are

$$\lambda < 2, \quad (A3.33)$$

$$\rho \leq \frac{3\lambda\eta}{(1-\delta)\alpha(1+\lambda)}. \quad (A3.34)$$

④ When $n_1 > 0$ and $e > 0$

In this case, values of variables are $e_{c1} = e_{c2} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c)\theta_e \rho_c}$,

$$n_1 = \frac{(1-\alpha_c)\rho_c\{(1-\delta)\rho[2\lambda - (1+\alpha)] + 2\lambda\eta\}}{(3+\alpha)\lambda\rho(\rho_c \theta_q - \theta_e)}, \quad (A3.35)$$

$$n_2 = \frac{(1-\alpha_c)\rho_c[(1-\delta)\rho(2+\alpha-\lambda) - \lambda\eta]}{(3+\alpha)\lambda\rho(\rho_c \theta_q - \theta_e)}, \quad (A3.36)$$

and

$$e = \frac{(1-\delta)\rho\alpha(1+\lambda) - 3\lambda\eta}{(3+\alpha)\lambda\rho\eta}. \quad (A3.37)$$

By substituting these values of n_1 , e_{c1} , and e_{c2} into $\frac{\partial U}{\partial n_2} > 0$ when $n_2 = 0$, (A3.7),

$$\frac{-\lambda \frac{\rho_c \theta_q - \theta_e}{\rho_c(1-\alpha_c)}}{1-\delta} + \frac{(3+\alpha)\lambda\rho(\rho_c \theta_q - \theta_e)}{(1-\alpha_c)\rho_c\{(1-\delta)\rho[2\lambda - (1+\alpha)] + 2\lambda\eta\}} > 0.$$

By substituting the values of e and e_{c1} into $\frac{\partial U}{\partial n_1} > 0$ when $n_1 = 0$, (A3.29),

$$\frac{-\frac{\rho_c \theta_q - \theta_e}{\rho_c(1-\alpha_c)}}{1-\delta-\eta\frac{(1-\delta)\rho\alpha(1+\lambda)-3\lambda\eta}{(3+\alpha)\lambda\rho\eta}} + \frac{2\lambda(\rho_c \theta_q - \theta_e)}{(1-\delta)(1-\alpha_c)\rho_c} > 0 \Leftrightarrow \lambda > \frac{(1+\alpha)(1-\delta)\rho}{2[(1-\delta)\rho+\eta]},$$

which always holds because of the assumptions of $0 < \alpha < 1$ and $\lambda \geq 1$.

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} > 0$ when $e = 0$, (A3.30),

$$\frac{-\eta}{1-\delta-\frac{\rho_c \theta_q - \theta_e}{\rho_c(1-\alpha_c)}\frac{(1-\alpha_c)\rho_c\{(1-\delta)\rho[2\lambda-(1+\alpha)]+2\lambda\eta\}}{(3+\alpha)\lambda\rho(\rho_c \theta_q - \theta_e)}} + \alpha\rho > 0.$$

Hence, the necessary conditions for this case to be realized are

$$\lambda < \frac{(1-\delta)\rho(2+\alpha)}{(1-\delta)\rho+\eta} \Leftrightarrow \rho > \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)}, \quad (\text{A3.38})$$

$$\lambda < \frac{(1-\delta)\rho\alpha}{3\eta-(1-\delta)\rho\alpha} \Leftrightarrow \rho > \frac{3\lambda\eta}{(1-\delta)\alpha(1+\lambda)}. \quad (\text{A3.39})$$

The right-hand side of (A3.38) is smaller than that of (A3.21),

$$\frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)} < \frac{\lambda\eta}{(1-\delta)\left[(2+\alpha)\frac{\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c(1-\alpha_c)^{1-\alpha_c}}} - \lambda\right]},$$

because $\frac{\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c(1-\alpha_c)^{1-\alpha_c}}} < 1$ from (3.10) and (A3.18).

⑤ When $n_1 = 0$ and $e = 0$

In this case, values of variables are $n_1 = e_{c1} = e = 0$, $e_{c2} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c)\theta_e \rho_c}$, and

$$n_2 = \frac{(1-\delta)(1-\alpha_c)\rho_c}{2\lambda(\rho_c \theta_q - \theta_e)}. \quad (\text{A3.40})$$

By substituting these values of n_1 , e_{c1} , and e_{c2} into $\frac{\partial U}{\partial n_2} > 0$ when $n_2 = 0$, (A3.7),

$$\frac{-\lambda \frac{\rho_c \theta_q - \theta_e}{\rho_c(1-\alpha_c)}}{1-\delta} + \frac{1}{0} = \infty > 0.$$

By substituting the values of e and e_{c1} into $\frac{\partial U}{\partial n_1} \leq 0$ when $n_1 = 0$, (A3.29),

$$\frac{-\theta_q}{1-\delta} + \frac{2\lambda(\rho_c \theta_q - \theta_e)}{(1-\delta)(1-\alpha_c)\rho_c \left[\frac{\alpha_c(\rho_c \theta_q - \theta_e)}{\theta_e(1-\alpha_c)} \right]^{\alpha_c}} \leq 0.$$

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} \leq 0$ when $e = 0$, (A3.30),

$$\frac{-\eta}{1-\delta} + \alpha\rho \leq 0.$$

Hence, the necessary conditions for this case to be realized are

$$\lambda \leq \left[\frac{2\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} \right]^{-1}, \quad (A3.41)$$

$$\rho \leq \frac{\eta}{(1-\delta)\alpha}. \quad (A3.42)$$

When ρ_c is small, the right-hand side of (A3.41) becomes smaller than 1, which means that this case does not exist. Especially when ρ_c converges to $\frac{\theta_e}{\alpha_c \theta_q}$, the right-hand side of (A3.41) converges to $\frac{1}{2}$.

⑥ When $n_1 = 0$ and $e > 0$

In this case, values of variables are, $n_1 = e_{c1} = 0$, $e_{c2} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c)\theta_e \rho_c}$

$$n_2 = \frac{(1-\delta)(1-\alpha_c)\rho_c}{2\lambda(\rho_c \theta_q - \theta_e)}, \quad (A3.43)$$

and

$$e = \frac{(1-\delta)\rho\alpha-\eta}{(1+\alpha)\rho\eta}. \quad (\text{A3.44})$$

By substituting these values of n_1 , e_{c1} , and e_{c2} into $\frac{\partial U}{\partial n_2} > 0$ when $n_2 = 0$, (A3.7),

$$\frac{-\lambda \frac{\rho_c \theta_q - \theta_e}{\rho_c(1-\alpha_c)}}{1-\delta} + \frac{1}{0} = \infty > 0.$$

By substituting the values of e and e_{c1} into $\frac{\partial U}{\partial n_1} \leq 0$ when $n_1 = 0$, (A3.29),

$$\frac{-\theta_q}{1-\delta-\eta \frac{(1-\delta)\rho\alpha-\eta}{(1+\alpha)\rho\eta}} + \frac{2\lambda(\rho_c \theta_q - \theta_e)}{(1-\delta)(1-\alpha_c)\rho_c \left[\frac{\alpha_c(\rho_c \theta_q - \theta_e)}{\theta_e(1-\alpha_c)} \right]^{\alpha_c}} \leq 0.$$

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} > 0$ when $e = 0$, (A3.30),

$$\frac{-\eta}{1-\delta} + \alpha\rho > 0.$$

Hence, the necessary conditions for this case to be realized are

$$\lambda \leq \frac{\rho_c \theta_q}{2} \frac{(1+\alpha)(1-\delta)\rho}{[(1-\delta)\rho+\eta]} \left(\frac{\alpha_c}{\theta_e} \right)^{\alpha_c} \left(\frac{1-\alpha_c}{\rho_c \theta_q - \theta_e} \right)^{1-\alpha_c}$$

$$\Leftrightarrow \rho \geq \frac{\lambda\eta}{(1-\delta) \left[\frac{(1+\alpha)\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{2\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} - \lambda \right]}, \quad (\text{A3.45})$$

$$\rho > \frac{\eta}{(1-\delta)\alpha}. \quad (\text{A3.46})$$

When $\frac{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{2\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} > (<) \frac{2+\alpha}{1+\alpha}$, the right-hand side of (A3.45) is smaller (greater)

than that of (A3.38), i.e., $\frac{\lambda\eta}{(1-\delta) \left[\frac{(1+\alpha)\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{2\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} - \lambda \right]} < (>) \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)}.$

When $\frac{\rho_c \theta_q \alpha_c^{\alpha_c (1-\alpha_c)^{1-\alpha_c}}}{2 \theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} > (<) \frac{3+2\alpha}{3(1+\alpha)}$, the right-hand side of (A3.45) is smaller (greater) than that of (A3.39) at $\lambda = 1$, i.e., $\frac{\lambda \eta}{(1-\delta) \left[\frac{(1+\alpha) \rho_c \theta_q \alpha_c^{\alpha_c (1-\alpha_c)^{1-\alpha_c}}}{2 \theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} - \lambda \right]} < (>) \frac{3\lambda \eta}{(1-\delta) \alpha (1+\lambda)}$.

Solutions of the utility maximization problem

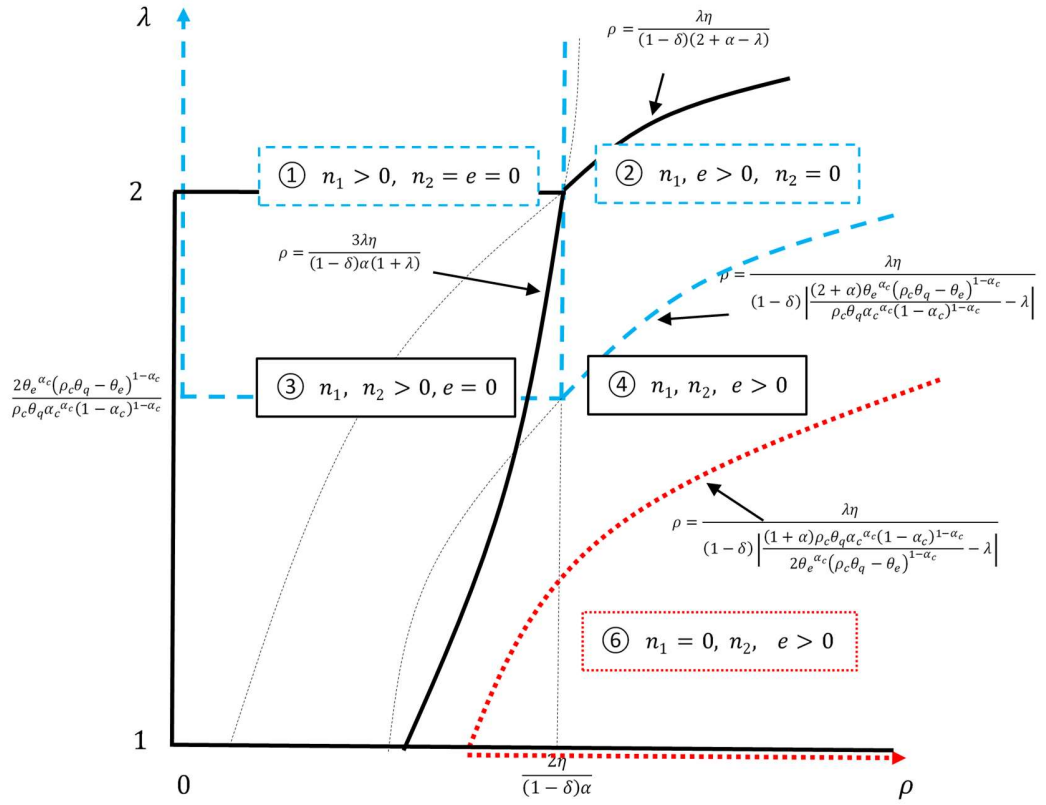


Figure A3.1 – Necessary conditions for the existence of the six cases when

$$\frac{2 \theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c (1-\alpha_c)^{1-\alpha_c}}} > 1$$

$e_{ci} > 0$ if $n_i > 0$, otherwise $e_{ci} = 0$, $i = 1, 2$

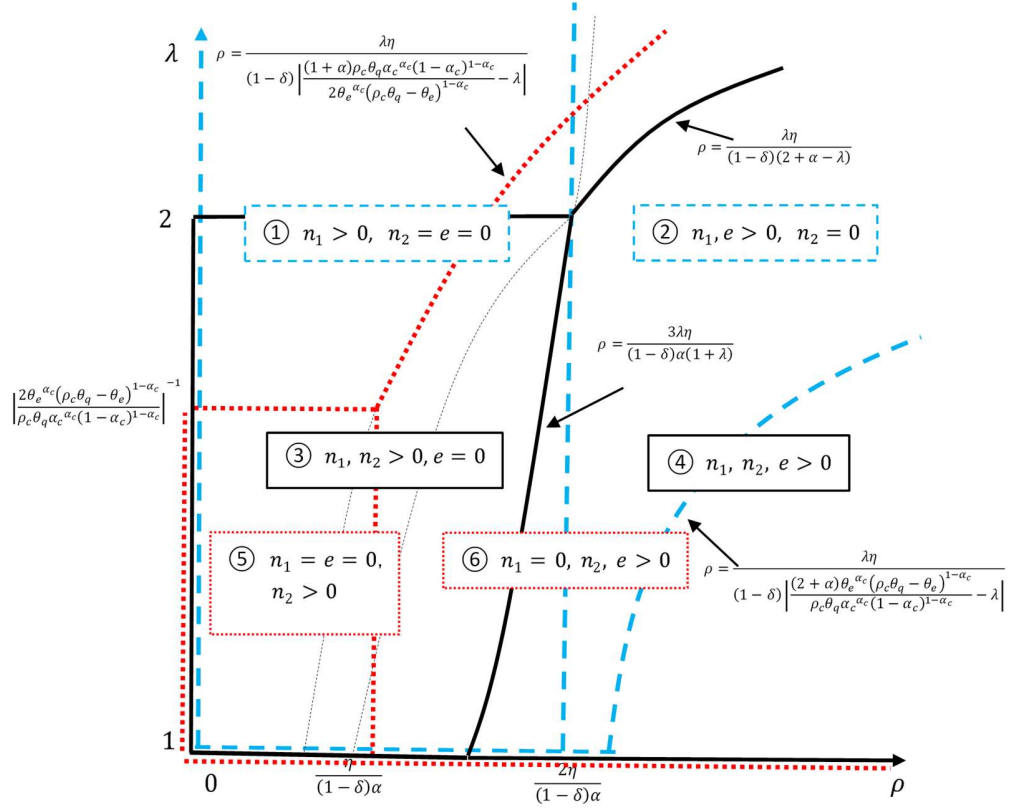


Figure A3.2 – Necessary conditions for the existence of the six cases when $\frac{1}{2} <$

$$\frac{2\theta_e^{a_c}(\rho_c\theta_q-\theta_e)^{1-a_c}}{\rho_c\theta_q\alpha_c^{a_c}(1-\alpha_c)^{1-a_c}} < 1$$

Figure A3.1²⁶ and Figure A3.2²⁷ illustrate necessary conditions for the existence of the six cases on the (ρ, λ) plane. From these figures, we can see that some cases overlap. It does not mean multiple solutions because the utilities obtained from these cases are different as shown later. Therefore, we need to compare the utilities of the overlapping areas to find the correct solutions.

Since $e_{ci} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1-\alpha_c) \theta_e \rho_c}$ if $n_i > 0$, otherwise $e_{ci} = 0$, $i = 1, 2$, from (3.9), we have

$$h_{ci} = \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{(1-\alpha_c) \theta_e} \right]^{\alpha_c} \text{ if } n_i > 0, \text{ otherwise } h_{ci} = 1.$$

① When $n_1 > 0$ and $n_2 = e = 0$

$n_1 = \frac{(1-\alpha_c)(1-\delta)\rho_c}{2(\rho_c \theta_q - \theta_e)}$ from (A3.15). Thus, from (3.3), (3.4) and (3.8), $c_1 = Ah_1 \frac{1-\delta}{2}$, $c_2 = gAh_1(1-\delta)$, and $n_1 h_{c1} + n_2 h_{c2} = \frac{(1-\alpha_c)(1-\delta)\rho_c}{2(\rho_c \theta_q - \theta_e)} \left[\frac{\alpha_c (\rho_c \theta_q - \theta_e)}{(1-\alpha_c) \theta_e} \right]^{\alpha_c}$. Hence, from (3.1), the utility of case ① is

$$U_1 = \ln \left[\frac{gA^2 h_1^2 \rho_c (1-\delta) \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} \right] + 2 \ln \left(\frac{1-\delta}{2} \right). \quad (\text{A3.47})$$

② When $n_1 > 0$, $n_2 = 0$ and $e > 0$

²⁶ Figure A3.1 shows the case when $\frac{2\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} > \frac{3(1+\alpha)}{3+2\alpha}$. When $\frac{2\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} < \frac{3(1+\alpha)}{3+2\alpha}$, the two boundary lines $\rho = \frac{\lambda\eta}{(1-\delta) \left[\frac{(1+\alpha)\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{2\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} - \lambda \right]}$ and $\rho = \frac{3\lambda\eta}{\alpha(1-\delta)(1+\lambda)}$ intersect.

²⁷ Figure A3.2 shows the case when $\frac{2\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} < \frac{1+\alpha}{2+\alpha}$. When $\frac{2\theta_e^{\alpha_c}(\rho_c \theta_q - \theta_e)^{1-\alpha_c}}{\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}} > \frac{1+\alpha}{2+\alpha}$, the positions of the two boundary lines $\rho = \frac{\lambda\eta}{(1-\delta) \left[\frac{(1+\alpha)\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{2\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} - \lambda \right]}$ and $\rho = \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)}$ are reversed, so $\rho = \frac{\lambda\eta}{(1-\delta) \left[\frac{(1+\alpha)\rho_c \theta_q \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{2\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} - \lambda \right]}$ and $\rho = \frac{3\lambda\eta}{\alpha(1-\delta)(1+\lambda)}$ intersect.

$n_1 = \frac{(1-\alpha_c)\rho_c[(1-\delta)\rho+\eta]}{(2+\alpha)\rho(\rho_c\theta_q-\theta_e)}$ and $e = \frac{(1-\delta)\alpha\rho-2\eta}{(2+\alpha)\rho\eta}$ from (A3.19) and (A3.20). Thus, from (3.3), (3.4) and (3.8), $c_1 = Ah_1 \frac{(1-\delta)\rho+\eta}{(2+\alpha)\rho}$, $c_2 = gAh_1(1-\delta) \left\{ \frac{\alpha[(1-\delta)\rho+\eta]}{(2+\alpha)\eta} \right\}^\alpha$, and $n_1 h_{c1} + n_2 h_{c2} = \frac{(1-\alpha_c)\rho_c[(1-\delta)\rho+\eta]}{(2+\alpha)\rho(\rho_c\theta_q-\theta_e)} \left[\frac{\alpha_c(\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}$. Hence, the utility of case ② is

$$U_2 = \ln \left[\frac{gA^2 h_1^2 \rho_c (1-\delta) \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} \right] + 2 \ln \left[\frac{(1-\delta)\rho + \eta}{(2+\alpha)\rho} \right] + \alpha \ln \left\{ \frac{\alpha[(1-\delta)\rho + \eta]}{(2+\alpha)\eta} \right\}. \quad (A3.48)$$

③ When $n_1 > 0$, $n_2 > 0$ and $e = 0$

$n_1 = \frac{(1-\delta)\rho_c(1-\alpha_c)(2\lambda-1)}{3\lambda(\rho_c\theta_q-\theta_e)}$ and $n_2 = \frac{(1-\delta)\rho_c(1-\alpha_c)(2-\lambda)}{3\lambda(\rho_c\theta_q-\theta_e)}$ from (A3.31) and (A3.32). Thus, from (3.3), (3.4) and (3.8), $c_1 = Ah_1(1-\delta) \frac{1+\lambda}{3\lambda}$, $c_2 = gAh_1(1-\delta) \frac{1+\lambda}{3}$, and $n_1 h_{c1} + n_2 h_{c2} = \frac{(1-\delta)\rho_c(1-\alpha_c)(\lambda+1)}{3\lambda(\rho_c\theta_q-\theta_e)} \left[\frac{\alpha_c(\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}$. Hence, the utility of case ③ is

$$U_3 = \ln \left[\frac{gA^2 h_1^2 \rho_c (1-\delta) \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} \right] + 2 \ln \left[\frac{(1-\delta)(1+\lambda)}{3\lambda} \right] + \ln \left(\frac{1+\lambda}{3} \right). \quad (A3.49)$$

④ When $n_1 > 0$, $n_2 > 0$ and $e > 0$

$n_1 = \frac{(1-\alpha_c)\rho_c\{(1-\delta)\rho[2\lambda-(1+\alpha)]+2\lambda\eta\}}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)}$, $n_2 = \frac{(1-\alpha_c)\rho_c[(1-\delta)\rho(2+\alpha-\lambda)-\lambda\eta]}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)}$ and $e = \frac{(1-\delta)\rho\alpha(1+\lambda)-3\lambda\eta}{(3+\alpha)\lambda\rho\eta}$ from (A3.35), (A3.36) and (A3.37). Thus, from (3.3), (3.4) and (3.8), $c_1 = Ah_1 \frac{(1-\delta)\rho(1+\lambda)+\eta\lambda}{(3+\alpha)\lambda\rho}$, $c_2 = gAh_1 \left\{ \frac{\alpha[(1-\delta)\rho(1+\lambda)+\eta\lambda]}{(3+\alpha)\eta\lambda} \right\}^\alpha \frac{(1-\delta)\rho(1+\lambda)+\eta\lambda}{(3+\alpha)\rho}$, and $n_1 h_{c1} + n_2 h_{c2} = \frac{(1-\alpha_c)\rho_c[(1-\delta)\rho(\lambda+1)+\lambda\eta]}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)} \left[\frac{\alpha_c(\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}$. Hence, the utility of case ④ is

$$\begin{aligned}
U_4 = & \ln \left[\frac{gA^2 h_1^2 \rho_c (1-\delta) \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} \right] + 2 \ln \left[\frac{(1-\delta) \rho (1+\lambda) + \eta \lambda}{(3+\alpha) \lambda \rho} \right] \\
& + \ln \left[\frac{(1-\delta) \rho (1+\lambda) + \eta \lambda}{(1-\delta)(3+\alpha) \rho} \right] + \alpha \ln \left\{ \frac{\alpha [(1-\delta) \rho (1+\lambda) + \eta \lambda]}{(3+\alpha) \eta \lambda} \right\}. \quad (A3.50)
\end{aligned}$$

⑤ When $n_1 = 0$, $n_2 > 0$ and $e = 0$

$n_2 = \frac{(1-\delta)(1-\alpha_c)\rho_c}{2\lambda(\rho_c\theta_q-\theta_e)}$ from (A3.40). Thus, from (3.3), (3.4) and (3.8), $c_1 = Ah_1(1-\delta)$, $c_2 = gAh_1 \frac{2(1-\delta)}{3}$, and $n_1 h_{c1} + n_2 h_{c2} = \frac{(1-\delta)(1-\alpha_c)\rho_c}{2\lambda(\rho_c\theta_q-\theta_e)} \left[\frac{\alpha_c(\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}$. Hence, the utility of case ⑤ is

$$U_5 = \ln \left[\frac{gA^2 h_1^2 \rho_c (1-\delta) \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} \right] + 2 \ln \left(\frac{1-\delta}{2} \right) + \ln \left(\frac{1}{\lambda} \right). \quad (A3.51)$$

⑥ When $n_1 = 0$, $n_2 > 0$ and $e > 0$

$n_2 = \frac{(1-\delta)(1-\alpha_c)\rho_c}{2\lambda(\rho_c\theta_q-\theta_e)}$ and $e = \frac{(1-\delta)\rho\alpha-\eta}{(1+\alpha)\rho\eta}$ from (A3.43) and (A3.44). Thus, from (3.3), (3.4) and (3.8), $c_1 = Ah_1 \frac{(1-\delta)\rho+\eta}{(1+\alpha)\rho}$, $c_2 = gAh_1 \left\{ \frac{\alpha[(1-\delta)\rho+\eta]}{(1+\alpha)\eta} \right\}^{\alpha} \frac{1-\delta}{2}$, and $n_1 h_{c1} + n_2 h_{c2} = \frac{(1-\delta)(1-\alpha_c)\rho_c}{2\lambda(\rho_c\theta_q-\theta_e)} \left[\frac{\alpha_c(\rho_c\theta_q-\theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}$. Hence, the utility of case ⑥ is

$$\begin{aligned}
U_6 = & \ln \left[\frac{gA^2 h_1^2 \rho_c (1-\delta) \alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c}}{\theta_e^{\alpha_c} (\rho_c \theta_q - \theta_e)^{1-\alpha_c}} \right] + 2 \ln \left(\frac{1}{2} \right) \\
& + \ln \left[\frac{(1-\delta)[(1-\delta)\rho + \eta]}{(1+\alpha)\lambda\rho} \right] + \alpha \ln \left\{ \frac{\alpha[(1-\delta)\rho + \eta]}{(1+\alpha)\eta} \right\}. \quad (A3.52)
\end{aligned}$$

The case of Figure A3.1

We need to compare the utilities of case ① and case ③, case ① and case ④, case ② and case ④, case ③ and case ⑥, case ④ and case ⑥.

[1] case ① and case ③

From (A3.47) and (A3.49), we have

$$U_3 - U_1 = 2\ln\left[\frac{2(1+\lambda)}{3\lambda}\right] + \ln\left(\frac{1+\lambda}{3}\right) = \ln\left[\frac{4(1+\lambda)^3}{27\lambda^2}\right]. \quad (\text{A3.53})$$

Since

$$\frac{d\left[\frac{4(1+\lambda)^3}{27\lambda^2}\right]}{d\lambda} = \frac{4}{27} \frac{(1+\lambda)^2}{\lambda^3} (\lambda - 2) < 0 \text{ from } \lambda < 2 (\text{equation (A3.33)}),$$

$$U_3 - U_1 > \ln\left[\frac{4(1+2)^3}{27(2)^2}\right] = 0 \text{ and thus } U_3 > U_1 \text{ holds.}$$

[2] case ① and case ④

From (A3.47) and (A3.50), we have

$$\begin{aligned} U_4 - U_1 &= 2\ln\left\{\frac{2[(1-\delta)\rho(1+\lambda)+\eta\lambda]}{(3+\alpha)(1-\delta)\lambda\rho}\right\} + \ln\left[\frac{(1-\delta)\rho(1+\lambda)+\eta\lambda}{(3+\alpha)(1-\delta)\rho}\right] + \alpha\ln\left\{\frac{\alpha[(1-\delta)\rho(1+\lambda)+\eta\lambda]}{(3+\alpha)\eta\lambda}\right\} \\ &= \ln\left\{\frac{2^2\alpha^\alpha[(1-\delta)\rho(1+\lambda)+\eta\lambda]^{3+\alpha}}{(3+\alpha)^{3+\alpha}\eta^\alpha(1-\delta)^3\lambda^{2+\alpha}\rho^3}\right\}. \end{aligned}$$

Because

$$\begin{aligned} \frac{d\left\{\frac{2^2\alpha^\alpha[(1-\delta)\rho(1+\lambda)+\eta\lambda]^{3+\alpha}}{(3+\alpha)^{3+\alpha}\eta^\alpha(1-\delta)^3\lambda^{2+\alpha}\rho^3}\right\}}{d\rho} &= \frac{2^2\alpha^\alpha}{(3+\alpha)^{3+\alpha}\eta^\alpha(1-\delta)^3\lambda^{2+\alpha}} \frac{[(1-\delta)\rho(1+\lambda)+\eta\lambda]^{2+\alpha}}{\rho^4} [\alpha(1-\delta)(1+\lambda)\rho - \\ &3\eta\lambda] > 0 \text{ from } \rho > \frac{3\eta\lambda}{(1-\delta)\alpha(1+\lambda)} \text{ (equation (A3.39))}, \end{aligned}$$

$$U_4 - U_1 > \ln\left\{\frac{2^2\alpha^\alpha\left[(1-\delta)\frac{3\eta\lambda}{(1-\delta)\alpha(1+\lambda)}(1+\lambda)+\eta\lambda\right]^{3+\alpha}}{(3+\alpha)^{3+\alpha}\eta^\alpha(1-\delta)^3\lambda^{2+\alpha}\left[\frac{3\eta\lambda}{(1-\delta)\alpha(1+\lambda)}\right]^3}\right\} = \ln\left[\frac{4(1+\lambda)^3}{27}\right].$$

Since $\lambda < 2$ from (A3.17) and (A3.39), $\frac{d\left[\frac{4(1+\lambda)^3}{27\lambda^2}\right]}{d\lambda} = \frac{4}{27} \frac{(1+\lambda)^2}{\lambda^3} (\lambda - 2) < 0$. Hence,
 $U_4 - U_1 > \ln \left[\frac{4(1+2)^3}{27(2)^2} \right] = 0$ and thus $U_4 > U_1$ holds.

[3] case ② and case ④

From (A3.48) and (A3.50), we have

$$\begin{aligned} U_4 - U_2 &= (2 + \alpha) \ln \left\{ \frac{(2+\alpha)[(1-\delta)\rho(1+\lambda)+\eta]}{(3+\alpha)[(1-\delta)\rho+\eta]\lambda} \right\} + \ln \left[\frac{(1-\delta)\rho(1+\lambda)+\eta\lambda}{(3+\alpha)(1-\delta)\rho} \right] \\ &= \ln \left\{ \frac{(2+\alpha)^{2+\alpha}[(1-\delta)\rho(1+\lambda)+\eta]^{3+\alpha}}{(3+\alpha)^{3+\alpha}(1-\delta)\lambda^{2+\alpha}\rho[(1-\delta)\rho+\eta]^{2+\alpha}} \right\}. \end{aligned}$$

Because

$$\begin{aligned} \frac{d\left\{ \frac{(2+\alpha)^{2+\alpha}[(1-\delta)\rho(1+\lambda)+\eta]^{3+\alpha}}{(3+\alpha)^{3+\alpha}(1-\delta)\lambda^{2+\alpha}\rho[(1-\delta)\rho+\eta]^{2+\alpha}} \right\}}{d\rho} &= \frac{(2+\alpha)^{2+\alpha}}{(3+\alpha)^{3+\alpha}(1-\delta)\lambda^{2+\alpha}} \frac{\eta[(1-\delta)\rho(1+\lambda)+\eta\lambda]^{2+\alpha}}{\rho^2[(1-\delta)\rho+\eta]^{3+\alpha}} [(1-\delta)\rho(2 + \\ \alpha - \lambda) - \eta\lambda] &> 0 \text{ from } \rho > \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)} \text{ (equation (A3.38))}, \end{aligned}$$

$$U_4 - U_2 > \ln \left\{ \frac{(2+\alpha)^{2+\alpha} \left[(1-\delta) \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)} (1+\lambda) + \eta\lambda \right]^{3+\alpha}}{(3+\alpha)^{3+\alpha}(1-\delta)\lambda^{2+\alpha} \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)} \left[(1-\delta) \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)} + \eta \right]^{2+\alpha}} \right\} = 0 \text{ and thus } U_4 > U_2$$

holds.

[4] case ③ and case ⑥

From (A3.49) and (A3.52), we have

$$\begin{aligned} U_6 - U_3 &= 2 \ln \left[\frac{3\lambda}{2(1-\delta)(1+\lambda)} \right] + \ln \left[\frac{3(1-\delta)[(1-\delta)\rho+\eta]}{(1+\alpha)\lambda(1+\lambda)\rho} \right] + \alpha \ln \left\{ \frac{\alpha[(1-\delta)\rho+\eta]}{(1+\alpha)\eta} \right\} \\ &= \ln \left\{ \frac{3^3 \alpha^\alpha [(1-\delta)\rho+\eta]^{1+\alpha} \lambda}{2^2 (1+\alpha)^{1+\alpha} \eta^\alpha (1-\delta) \rho (1+\lambda)^3} \right\}. \end{aligned}$$

Because

$$\frac{d\left\{\frac{3^3\alpha^\alpha[(1-\delta)\rho+\eta]^{1+\alpha}\lambda}{2^2(1+\alpha)^{1+\alpha}\eta^\alpha(1-\delta)\rho(1+\lambda)^3}\right\}}{d\rho} = \frac{3^3\alpha^\alpha\lambda}{2^2(1+\alpha)^{1+\alpha}\eta^\alpha(1-\delta)(1+\lambda)^3} \frac{[(1-\delta)\rho+\eta]^\alpha}{\rho^2} [\alpha(1-\delta)\rho - \eta] > 0 \quad \text{from}$$

$$\rho > \frac{\eta}{(1-\delta)\alpha} \text{ (equation (A3.55)), from (A3.34),}$$

$$U_6 - U_3 \leq \ln \left\{ \frac{3^3\alpha^\alpha \left[(1-\delta) \frac{3\eta}{(1-\delta)\alpha(1+\lambda)} + \eta \right]^{1+\alpha} \lambda}{2^2(1+\alpha)^{1+\alpha}\eta^\alpha(1-\delta) \frac{3\eta\lambda}{(1-\delta)\alpha(1+\lambda)} (1+\lambda)^3} \right\} = \ln \left[\frac{3^2(3\lambda+\alpha\lambda+\alpha)^{1+\alpha}}{2^2(1+\alpha)^{1+\alpha}(1+\lambda)^{3+\alpha}} \right].$$

$$\text{Further, since } \frac{d\left[\frac{3^2(3\lambda+\alpha\lambda+\alpha)^{1+\alpha}}{2^2(1+\alpha)^{1+\alpha}(1+\lambda)^{3+\alpha}}\right]}{d\lambda} = \frac{3^2(3+\alpha)}{2^2(1+\alpha)^{1+\alpha}} \frac{(3\lambda+\alpha\lambda+\alpha)^\alpha}{(1+\lambda)^{4+\alpha}} (1-2\lambda) < 0 \text{ from } \lambda > 1,$$

$$U_6 - U_3 < \ln \left[\frac{3^2(3+\alpha)^{1+\alpha}}{2^2(1+\alpha)^{1+\alpha}(1+1)^{3+\alpha}} \right] = \ln \left[\left(\frac{3}{4} \right)^2 \left(\frac{3+2\alpha}{2+2\alpha} \right)^{1+\alpha} \right].$$

Because

$$\frac{d\left(\frac{3+2\alpha}{2+2\alpha}\right)^{1+\alpha}}{d\alpha} = \left(\frac{3+2\alpha}{2+2\alpha}\right)^{1+\alpha} \left[\ln \left(\frac{3+2\alpha}{2+2\alpha} \right) - \frac{1}{3+2\alpha} \right] > \left(\frac{3+2\alpha}{2+2\alpha}\right)^{1+\alpha} \left[\ln \left(\frac{5}{4} \right) - \frac{1}{5} \right] >$$

$$\left(\frac{3+2\alpha}{2+2\alpha}\right)^{1+\alpha} [0.22 - 0.2] > 0 \text{ from } \alpha < 1 \text{ and } \frac{d\left[\ln\left(\frac{3+2\alpha}{2+2\alpha}\right) - \frac{1}{3+2\alpha}\right]}{d\alpha} = \frac{2}{3+2\alpha} \left(\frac{1}{3+2\alpha} - \frac{1}{2+2\alpha} \right) < 0,$$

$$U_6 - U_3 < \ln \left[\left(\frac{3}{4} \right)^2 \left(\frac{5}{4} \right)^2 \right] < 0 \text{ and thus } U_6 < U_3 \text{ holds.}$$

[5] case ④ and case ⑥

From (A3.50) and (A3.52), we have

$$U_4 - U_6 = 2\ln \left\{ \frac{2[(1-\delta)\rho(1+\lambda)+\eta\lambda]}{(3+\alpha)(1-\delta)\rho} \right\} + (1+\alpha)\ln \left\{ \frac{(1+\alpha)[(1-\delta)\rho(1+\lambda)+\eta\lambda]}{(3+\alpha)\lambda[(1-\delta)\rho+\eta]} \right\}$$

$$= \ln \left\{ \frac{2^2(1+\alpha)^{1+\alpha}[(1-\delta)\rho(1+\lambda)+\eta\lambda]^{3+\alpha}}{(3+\alpha)^{3+\alpha}(1-\delta)^2\rho^2[(1-\delta)\rho+\eta]^{1+\alpha}\lambda^{1+\alpha}} \right\}.$$

$$\frac{d\left\{\frac{2^2(1+\alpha)^{1+\alpha}[(1-\delta)\rho(1+\lambda)+\eta\lambda]^{3+\alpha}}{(3+\alpha)^{2+\alpha}(1-\delta)^2\rho^2[(1-\delta)\rho+\eta]^{1+\alpha}\lambda^{1+\alpha}}\right\}}{d\rho} = \frac{2^2(1+\alpha)^{1+\alpha}}{(3+\alpha)^{2+\alpha}(1-\delta)^2\lambda^{1+\alpha}} \frac{\eta[(1-\delta)\rho(1+\lambda)+\eta\lambda]^{2+\alpha}}{\rho^3[(1-\delta)\rho+\eta]^{2+\alpha}} [(1-\delta)\rho(1+$$

$$\alpha - 2\lambda) - 2\eta\lambda] < 0 \text{ and}$$

$$\frac{d\left\{\frac{2^2(1+\alpha)^{1+\alpha}[(1-\delta)\rho(1+\lambda)+\eta]^{3+\alpha}}{(3+\alpha)^{2+\alpha}(1-\delta)^2\rho^2[(1-\delta)\rho+\eta]^{1+\alpha}\lambda^{1+\alpha}}\right\}}{d\lambda} = \frac{2^2(1+\alpha)^{1+\alpha}}{(3+\alpha)^{2+\alpha}(1-\delta)^2\rho^2[(1-\delta)\rho+\eta]^{1+\alpha}} \frac{[(1-\delta)\rho(1+\lambda)+\eta\lambda]^{2+\alpha}}{\lambda^{2+\alpha}} \{2[(1-\delta)\rho+\eta]\lambda - (1+\alpha)(1-\delta)\rho\} > 0 \text{ from } \lambda > 1.$$

Hence, $U_4 - U_6 > (U_4 - U_6)|_{\lambda=1, \rho=\infty} = \ln \left[\frac{4^2(2+2\alpha)^{1+\alpha}}{(3+\alpha)^{3+\alpha}} \right] > \ln \left[\frac{4^2 4^2}{4^4} \right] = 0$ from $\alpha < 1$ and $\frac{d\left[\frac{(2+2\alpha)^{1+\alpha}}{(3+\alpha)^{3+\alpha}}\right]}{d\alpha} = \frac{(2+2\alpha)^{1+\alpha}}{(3+\alpha)^{3+\alpha}} \ln \left(\frac{2+2\alpha}{3+\alpha} \right) < 0$ and thus $U_4 > U_6$ holds.

Based on these results, solutions of the maximization problem and the conditions for the existence of these cases can be classified as presented in Proposition 3.1 and as illustrated in Figure 3.1.

The case of Figure A3.2

Besides the comparisons above, we need to compare the utilities of case ① and case ⑤, case ① and case ⑥, case ② and case ⑥, case ③ and case ⑤.

[6] case ① and case ⑤

From (A3.47) and (A3.51), we have

$$U_5 - U_1 = \ln \left(\frac{1}{\lambda} \right) < 0 \text{ from } \lambda > 1 \text{ and thus } U_5 < U_1 \text{ holds.}$$

[7] case ① and case ⑥

From (A3.47) and (A3.52), we have

$$U_6 - U_1 = \ln \left(\frac{(1-\delta)\rho+\eta}{(1+\alpha)(1-\delta)\rho\lambda} \right) + \alpha \ln \left\{ \frac{\alpha[(1-\delta)\rho+\eta]}{(1+\alpha)\eta} \right\} = \ln \left\{ \frac{\alpha^\alpha[(1-\delta)\rho+\eta]^{1+\alpha}}{(1+\alpha)^{1+\alpha}(1-\delta)\eta^\alpha\rho\lambda} \right\}.$$

From Figure A3.2, case ① and case ⑥ overlap in the area where $\rho \in \left[\frac{\eta}{(1-\delta)\alpha}, \frac{2\eta}{(1-\delta)\alpha} \right]$ and $\lambda \in \left[1, \frac{(1-\delta)\rho}{(1-\delta)\rho+\eta} \frac{(1+\alpha)\rho_c\theta_q\alpha_c^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}}{2\theta_e^{\alpha_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}} \right]$ hold. We have already shown that $U_3 > U_1$ and $U_3 > U_6$ in the sub-area where $\rho \leq \frac{3\lambda\eta}{(1-\delta)\alpha(1+\lambda)}$ and $\lambda < 2$ hold, and $U_4 > U_1$ and $U_4 > U_6$ in the sub-area where $\rho > \frac{3\lambda\eta}{(1-\delta)\alpha(1+\lambda)}$ holds. Hence, there is only the area where $\rho < \frac{2\eta}{(1-\delta)\alpha}$ and $\lambda \geq 2$ hold left to consider (see Figure A3.2).

Because

$$\frac{d\left\{ \frac{\alpha^\alpha[(1-\delta)\rho+\eta]^{1+\alpha}}{(1+\alpha)^{1+\alpha}(1-\delta)\eta^\alpha\rho\lambda} \right\}}{d\rho} = \frac{\alpha^\alpha}{(1+\alpha)^{1+\alpha}(1-\delta)\eta^\alpha\lambda} \frac{[(1-\delta)\rho+\eta]^\alpha}{\rho^2} [\alpha(1-\delta)\rho - \eta] > 0 \quad \text{from } \rho > \frac{\eta}{(1-\delta)\alpha}$$

(equation (A3.46)) and $\frac{\alpha^\alpha[(1-\delta)\rho+\eta]^{1+\alpha}}{(1+\alpha)^{1+\alpha}(1-\delta)\eta^\alpha\rho\lambda}$ decreases with λ , $U_6 - U_1 < (U_6 - U_1)|_{\lambda=2, \rho=\frac{2\eta}{(1-\delta)\alpha}} = \ln \frac{(2+\alpha)^{1+\alpha}}{4(1+\alpha)^{1+\alpha}} < 0$ and thus $U_6 < U_1$ holds.

[8] case ② and case ⑥

From (A3.48) and (A3.52), we have

$$U_6 - U_2 = \ln \left\{ \frac{(2+\alpha)(1-\delta)\rho}{4\lambda[(1-\delta)\rho+\eta]} \right\} + (1+\alpha) \ln \left(\frac{2+\alpha}{1+\alpha} \right).$$

Case ② and case ⑥ overlap in the area where

$$\rho \in \left[\frac{2\eta}{(1-\delta)\alpha}, \frac{\lambda\eta}{(1-\delta)\left[\frac{(2+\alpha)\theta_e^{\alpha_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{\rho_c\theta_q\alpha_c^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}} - \lambda \right]} \right] \quad \text{and} \quad \lambda \in \left[1, \frac{(1-\delta)\rho}{(1-\delta)\rho+\eta} \frac{(1+\alpha)\rho_c\theta_q\alpha_c^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}}{2\theta_e^{\alpha_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}} \right]$$

hold from Figure A3.2. We have already shown that $U_4 > U_6$ and $U_4 > U_2$ in the sub-area where $\rho > \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)}$ holds. Hence, there is only the area $\rho < \frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)}$ left to consider.

Because $\frac{(2+\alpha)(1-\delta)\rho}{4\lambda[(1-\delta)\rho+\eta]}$ increases with ρ and decreases with λ ,

$$U_6 - U_2 < (U_6 - U_2) \Big|_{\lambda=1, \rho=\frac{\lambda\eta}{(1-\delta)(2+\alpha-\lambda)}} = \ln \frac{(2+\alpha)^{1+\alpha}}{4(1+\alpha)^{1+\alpha}} < 0 \text{ and thus } U_6 < U_2 \text{ holds.}$$

[9] case ③ and case ⑤

We have already shown that $U_3 > U_1$ and $U_1 > U_5$ and thus $U_3 > U_5$ holds.

Based on these results, solutions of the maximization problem and the conditions for the existence of these cases can be classified as presented in Proposition 3.1 and as illustrated in Figure 3.1. ■

Proof of Proposition 3.2

When $g_{\rho_c} = g_\delta = 1$, the inequalities on λ (those associated with ρ_c , δ and θ_q) of Propositions A3.1, A3.2 and A3.3 become $\lambda < (\geq) 2$ in case (3) and $\lambda < (\geq) \frac{(2+\alpha)\rho}{(1-\delta)\rho+\eta}$ (the one associated with δ becomes $\lambda < (\geq) 2 + \alpha$) in case (4). Hence, the inequalities in case (3) and the one associated with δ in case (4) hold with "<" for any $\lambda < 2$. The inequalities associated with ρ_c and δ too hold with "<", as long as $\lambda < 2$ is not large. These inequalities hold with "<" when $g_{\rho_c} > 1$ is not large and $g_\delta < 1$ is not small, as long as $\lambda < 2$ is not large, because the expressions of the right-hand sides of the inequalities are not very different from the ones when $g_{\rho_c} = g_\delta = 1$. Hence, the results of the time-dependent case are the same as the time-invariant case for ρ_c , δ and θ_q . The remaining results are straightforward from Propositions A3.1, A3.2 and A3.3 and the assumption that g_{ρ_c} is not large and g_δ is not small of Proposition 3.2.

A3.3 Proofs of Propositions A3.1, A3.2 and A3.3

Solutions and conditions of the utility maximization problem

The objective function of the period 2 maximization problem now becomes

$$\ln\{gA(1 + \rho e)^\alpha h_1[1 - g_\delta \delta - \lambda(\theta_q + \theta_e e_{c2})n_2]\}$$

$$+ \ln[n_1(1 + \rho_c e_{c1})^{\alpha_c} + n_2(1 + g_{\rho_c} \rho_c e_{c2})^{\alpha_c}]. \quad (A3.54)$$

First order conditions are

$$\begin{aligned} \frac{\partial U}{\partial e_{c2}} = n_2 & \left[\frac{-\lambda \theta_e}{1 - g_\delta \delta - \lambda(\theta_q + \theta_e e_{c2})n_2} \right. \\ & \left. + \frac{\alpha_c g_{\rho_c} \rho_c (1 + g_{\rho_c} \rho_c e_{c2})^{\alpha_c - 1}}{n_1(1 + \rho_c e_{c1})^{\alpha_c} + n_2(1 + g_{\rho_c} \rho_c e_{c2})^{\alpha_c}} \right] \leq 0, \end{aligned} \quad (A3.55)$$

$$\begin{aligned} \frac{\partial U}{\partial n_2} = & \frac{-\lambda(\theta_q + \theta_e e_{c2})}{1 - g_\delta \delta - \lambda(\theta_q + \theta_e e_{c2})n_2} \\ & + \frac{(1 + g_{\rho_c} \rho_c e_{c2})^{\alpha_c}}{n_1(1 + \rho_c e_{c1})^{\alpha_c} + n_2(1 + g_{\rho_c} \rho_c e_{c2})^{\alpha_c}} \leq 0. \end{aligned} \quad (A3.56)$$

From $g_\delta \leq 1$, $\frac{\partial U}{\partial n_2} = 0$ could hold. Thus, when $n_2 > 0$, from $\frac{\partial U}{\partial n_2} = 0$,

$$\begin{aligned} & n_1(1 + \rho_c e_{c1})^{\alpha_c} + n_2(1 + g_{\rho_c} \rho_c e_{c2})^{\alpha_c} \\ & = \frac{(1 + g_{\rho_c} \rho_c e_{c2})^{\alpha_c}}{\lambda(\theta_q + \theta_e e_{c2})} [1 - g_\delta \delta - \lambda(\theta_q + \theta_e e_{c2})n_2]. \end{aligned} \quad (A3.57)$$

By substituting this equation into (A3.55),

$$\frac{\partial U}{\partial e_{c2}} = \frac{\lambda n_2}{1 - g_\delta \delta - \lambda(\theta_q + \theta_e e_{c2})n_2} \left[-\theta_e + \frac{\alpha_c g_{\rho_c} \rho_c (\theta_q + \theta_e e_{c2})}{1 + g_{\rho_c} \rho_c e_{c2}} \right]. \quad (A3.58)$$

When $n_2 > 0$, $\frac{\partial U}{\partial e_{c2}} = 0$ from $g_{\rho_c} \geq 1$. Thus,

$$e_{c2} = \frac{\alpha_c g_{\rho_c} \rho_c \theta_q - \theta_e}{(1 - \alpha_c) \theta_e g_{\rho_c} \rho_c}. \quad (A3.59)$$

When $n_2 > 0$, from (A3.56),

$$\frac{\partial U}{\partial n_2} \Big|_{n_2=0} = \frac{-\lambda(\theta_q + \theta_e e_{c2})}{1 - g_\delta \delta} + \frac{(1 + g_{\rho_c} \rho_c e_{c2})^{\alpha_c}}{n_1 (1 + \rho_c e_{c1})^{\alpha_c}} > 0. \quad (A3.60)$$

From (A3.57) and (A3.59), the equation corresponding to (A3.23) of the time-invariant case is

$$n_2 = \frac{(1 - g_\delta \delta) g_{\rho_c} \rho_c (1 - \alpha_c) \left[\frac{\alpha_c (g_{\rho_c} \rho_c \theta_q - \theta_e)}{\theta_e (1 - \alpha_c)} \right]^{\alpha_c} - \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e) (1 + \rho_c e_{c1})^{\alpha_c} n_1}{2 \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e) \left[\frac{\alpha_c (g_{\rho_c} \rho_c \theta_q - \theta_e)}{\theta_e (1 - \alpha_c)} \right]^{\alpha_c}}. \quad (A3.61)$$

Thus, the objective function of the period 1 maximization problem becomes

$$\begin{aligned} U = & \ln \{ A h_1 [1 - \delta - \eta e - (\theta_q + \theta_e e_{c1}) n_1] \} \\ & + \ln \left\{ g A (1 + \rho e)^\alpha h_1 \frac{(1 - g_\delta \delta) g_{\rho_c} \rho_c (1 - \alpha_c) \left[\frac{\alpha_c (g_{\rho_c} \rho_c \theta_q - \theta_e)}{\theta_e (1 - \alpha_c)} \right]^{\alpha_c} + \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e) (1 + \rho_c e_{c1})^{\alpha_c} n_1}{2 g_{\rho_c} \rho_c (1 - \alpha_c) \left[\frac{\alpha_c (g_{\rho_c} \rho_c \theta_q - \theta_e)}{\theta_e (1 - \alpha_c)} \right]^{\alpha_c}} \right\} \\ & + \ln \left\{ \frac{(1 - g_\delta \delta) g_{\rho_c} \rho_c (1 - \alpha_c) \left[\frac{\alpha_c (g_{\rho_c} \rho_c \theta_q - \theta_e)}{\theta_e (1 - \alpha_c)} \right]^{\alpha_c} + \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e) (1 + \rho_c e_{c1})^{\alpha_c} n_1}{2 \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e)} \right\}. \end{aligned} \quad (A3.62)$$

First order conditions are

$$\begin{aligned} \frac{\partial U}{\partial n_1} = & \frac{2 \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e) (1 + \rho_c e_{c1})^{\alpha_c}}{(1 - g_\delta \delta) g_{\rho_c} \rho_c (1 - \alpha_c) \left[\frac{\alpha_c (g_{\rho_c} \rho_c \theta_q - \theta_e)}{\theta_e (1 - \alpha_c)} \right]^{\alpha_c} + \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e) (1 + \rho_c e_{c1})^{\alpha_c} n_1} \\ & + \frac{-(\theta_q + \theta_e e_{c1})}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1}) n_1} \leq 0, \end{aligned} \quad (A3.63)$$

$$\begin{aligned} \frac{\partial U}{\partial e_{c1}} = & n_1 \left\{ \frac{2 \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e) \alpha_c \rho_c (1 + \rho_c e_{c1})^{\alpha_c - 1}}{(1 - g_\delta \delta) g_{\rho_c} \rho_c (1 - \alpha_c) \left[\frac{\alpha_c (g_{\rho_c} \rho_c \theta_q - \theta_e)}{\theta_e (1 - \alpha_c)} \right]^{\alpha_c} + \lambda (g_{\rho_c} \rho_c \theta_q - \theta_e) (1 + \rho_c e_{c1})^{\alpha_c} n_1} \right. \\ & \left. - \frac{\theta_e}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1}) n_1} \right\} \leq 0, \end{aligned} \quad (A3.64)$$

$$\frac{\partial U}{\partial e} = \frac{-\eta}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} + \frac{\alpha\rho}{1 + \rho e} \leq 0. \quad (A3.65)$$

When $n_1 > 0$, from $\frac{\partial U}{\partial n_1} = 0$,

$$\begin{aligned} (1 - g_\delta \delta)g_{\rho_c}\rho_c(1 - \alpha_c) \left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q - \theta_e)}{\theta_e(1 - \alpha_c)} \right]^{\alpha_c} + \lambda(g_{\rho_c}\rho_c\theta_q - \theta_e)(1 + \rho_c e_{c1})^{\alpha_c} n_1 \\ = \frac{2\lambda(g_{\rho_c}\rho_c\theta_q - \theta_e)(1 + \rho_c e_{c1})^{\alpha_c}}{\theta_q + \theta_e e_{c1}} [1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1]. \end{aligned}$$

By substituting this equation into (A3.64),

$$\frac{\partial U}{\partial e_{c1}} = \frac{n_1}{1 - \delta - \eta e - (\theta_q + \theta_e e_{c1})n_1} \left[-\theta_e + \frac{\alpha_c \rho_c (\theta_q + \theta_e e_{c1})}{1 + \rho_c e_{c1}} \right]. \quad (A3.66)$$

Thus, when $n_1 > 0$, $e_{c1} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1 - \alpha_c) \theta_e \rho_c}$ from $\frac{\partial U}{\partial e_{c1}} = 0$.

When $n_1 = (>)0$, from (A3.63),

$$\begin{aligned} \frac{\partial U}{\partial n_1} \Big|_{n_1=0} &= \frac{2\lambda(g_{\rho_c}\rho_c\theta_q - \theta_e)(1 + \rho_c e_{c1})^{\alpha_c}}{(1 - g_\delta \delta)g_{\rho_c}\rho_c(1 - \alpha_c) \left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q - \theta_e)}{\theta_e(1 - \alpha_c)} \right]^{\alpha_c}} \\ &\quad - \frac{\theta_q + \theta_e e_{c1}}{1 - \delta - \eta e} \leq (>)0. \end{aligned} \quad (A3.67)$$

When $e = (>)0$, from (A3.65),

$$\frac{\partial U}{\partial e} \Big|_{e=0} = \frac{-\eta}{1 - \delta - (\theta_q + \theta_e e_{c1})n_1} + \alpha\rho \leq (>)0. \quad (A3.68)$$

Case (3): When $n_1 > 0$ and $e = 0$

In this case, values of variables are $e = 0$, $e_{c1} = \frac{\alpha_c \rho_c \theta_q - \theta_e}{(1 - \alpha_c) \theta_e \rho_c}$ and $e_{c2} = \frac{\alpha_c g_{\rho_c} \rho_c \theta_q - \theta_e}{(1 - \alpha_c) \theta_e g_{\rho_c} \rho_c}$,

$$n_1 = \frac{(1 - \alpha_c)\rho_c \left[(1 - \delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} 2\lambda - (1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right]}{3\lambda(\rho_c\theta_q - \theta_e)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}, \quad (\text{A3.69})$$

and

$$n_2 = \frac{(1 - \alpha_c)\rho_c \left[2(1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} - (1 - \delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} \lambda \right]}{3\lambda(\rho_c\theta_q - \theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q - \theta_e)}. \quad (\text{A3.70})$$

By substituting these values of n_1 , e_{c1} , and e_{c2} into $\frac{\partial U}{\partial n_2} > 0$ when $n_2 = 0$, (A3.60),

$$\begin{aligned} & \frac{-\lambda \frac{g_{\rho_c}\rho_c\theta_q - \theta_e}{g_{\rho_c}\rho_c(1 - \alpha_c)}}{1 - g_\delta\delta} + \\ & \frac{\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q - \theta_e)}{(1 - \alpha_c)\theta_e} \right]^{\alpha_c}}{\left[\frac{\alpha_c(\rho_c\theta_q - \theta_e)}{(1 - \alpha_c)\theta_e} \right]^{\alpha_c}} \frac{3\lambda(\rho_c\theta_q - \theta_e)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{(1 - \alpha_c) \left[(1 - \delta)\rho_c(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} 2\lambda - (1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right]} > 0. \end{aligned}$$

By substituting the values of e and e_{c1} into $\frac{\partial U}{\partial n_1} > 0$ when $n_1 = 0$, (A3.67),

$$\begin{aligned} & \frac{\frac{\rho_c\theta_q - \theta_e}{\rho_c(1 - \alpha_c)}}{1 - \delta} + \frac{2\lambda(g_{\rho_c}\rho_c\theta_q - \theta_e) \left[\frac{\alpha_c(\rho_c\theta_q - \theta_e)}{(1 - \alpha_c)\theta_e} \right]^{\alpha_c}}{(1 - g_\delta\delta)g_{\rho_c}\rho_c(1 - \alpha_c) \left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q - \theta_e)}{(1 - \alpha_c)\theta_e} \right]^{\alpha_c}} > 0 \\ & \Leftrightarrow \lambda > \frac{(1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{2(1 - \delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}. \quad (\text{A3.71}) \end{aligned}$$

Because g_{ρ_c} is not large and g_δ is not small, (A3.71) always holds.

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} \leq 0$ when $e = 0$, (A3.68),

$$\frac{-\eta}{1 - \delta - \frac{\rho_c\theta_q - \theta_e}{\rho_c(1 - \alpha_c)} \frac{(1 - \alpha_c) \left[(1 - \delta)\rho_c(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} 2\lambda - (1 - g_\delta\delta)g_{\rho_c}\rho_c(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right]}{3\lambda(\rho_c\theta_q - \theta_e)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}} + \alpha\rho \leq 0.$$

Hence, the conditions for this case to be realized are

$$\lambda < \frac{2(1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{(1 - \delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}, \quad (\text{A3.72})$$

$$\rho \leq \frac{3\lambda\eta(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{\alpha \left[(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda + (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right]}. \quad (A3.73)$$

Case (4): When $n_1 > 0$ and $e > 0$

In this case, values of variables are $e_{c1} = \frac{\alpha_c\rho_c\theta_q - \theta_e}{(1-\alpha_c)\theta_e\rho_c}$, $e_{c2} = \frac{\alpha_cg_{\rho_c}\rho_c\theta_q - \theta_e}{(1-\alpha_c)\theta_eg_{\rho_c}\rho_c}$,

$$n_1 = \frac{(1-\alpha_c)\rho_c \left\{ 2[(1-\delta)\rho + \eta](g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda - (1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right\}}{(3+\alpha)\lambda\rho(\rho_c\theta_q - \theta_e)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}, \quad (A3.74)$$

$$n_2 = \frac{(1-\alpha_c)\rho_c \left[(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} - [(1-\delta)\rho + \eta](g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda \right]}{(3+\alpha)\lambda\rho(\rho_c\theta_q - \theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q - \theta_e)}, \quad (A3.75)$$

and

$$e = \frac{\alpha\rho \left[(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda + (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right] - 3\lambda\eta(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{(3+\alpha)\lambda\rho\eta(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}. \quad (A3.76)$$

By substituting these values of n_1 , e_{c1} , and e_{c2} into $\frac{\partial U}{\partial n_2} > 0$ when $n_2 = 0$, (A3.60),

$$\begin{aligned} & -\lambda \frac{g_{\rho_c}\rho_c\theta_q - \theta_e}{g_{\rho_c}\rho_c(1-\alpha_c)} + \\ & \frac{\left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q - \theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}}{\left[\frac{\alpha_c(\rho_c\theta_q - \theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}} \frac{(3+\alpha)\lambda\rho(\rho_c\theta_q - \theta_e)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{(1-\alpha_c) \left[2[(1-\delta)\rho + \eta]\rho_c(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda - (1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right]} > 0. \end{aligned}$$

By substituting the values of e and e_{c1} into $\frac{\partial U}{\partial n_1} > 0$ when $n_1 = 0$, (A3.67),

$$\begin{aligned} & -\frac{\rho_c\theta_q - \theta_e}{\rho_c(1-\alpha_c)} \\ & \frac{1-\delta-\eta \frac{\alpha\rho \left[(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda + (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right] - 3\lambda\eta\rho_c(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{(3+\alpha)\lambda\rho\eta\rho_c(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}}{1-\delta-\eta \frac{\alpha\rho \left[(1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda + (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right] - 3\lambda\eta\rho_c(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{(3+\alpha)\lambda\rho\eta\rho_c(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}} \\ & + \frac{2\lambda(g_{\rho_c}\rho_c\theta_q - \theta_e) \left[\frac{\alpha_c(\rho_c\theta_q - \theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}}{(1-g_\delta\delta)(1-\alpha_c)g_{\rho_c}\rho_c \left[\frac{\alpha_c(g_{\rho_c}\rho_c\theta_q - \theta_e)}{(1-\alpha_c)\theta_e} \right]^{\alpha_c}} > 0 \end{aligned}$$

$$\Leftrightarrow \lambda > \frac{(1 + \alpha)\rho(1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{2[(1 - \delta)\rho + \eta](g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}. \quad (\text{A3.77})$$

Because g_{ρ_c} is not large and g_δ is not small, (A3.77) always holds.

By substituting the values of n_1 and e_{c1} into $\frac{\partial U}{\partial e} > 0$ when $e = 0$, (A3.68),

$$\frac{-\eta}{1-\delta_1-\frac{\rho_c\theta_q-\theta_e}{\rho_c(1-\alpha_c)}\frac{(1-\alpha_c)[2[(1-\delta)\rho+\eta]\rho_c(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda-(1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}\rho_c(\rho_c\theta_q-\theta_e)^{1-\alpha_c}]}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}} + \alpha\rho > 0.$$

Hence, the conditions for this case to be realized are

$$\lambda < \frac{(2 + \alpha)\rho(1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{[(1 - \delta)\rho + \eta](g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}, \quad (\text{A3.78})$$

$$\lambda < \frac{\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{[3\eta-(1-\delta)\rho\alpha](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}$$

$$\Leftrightarrow \rho > \frac{3\lambda\eta(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}{\alpha[(1 - \delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}\lambda + (1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c}]} \quad (\text{A3.79})$$

Proof of Proposition A3.1

$$(1) n_1 = \frac{(1-\alpha_c)\rho_c[(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}2\lambda-(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}]}{3\lambda(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}} \text{ from (A3.69).}$$

$$\frac{dn_1}{d\rho_c} = \frac{(1-\alpha_c)\theta_e}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}} \left\{ \frac{1}{\lambda} g_{\rho_c}(1 - g_\delta\delta)(\rho_c\theta_q - \theta_e)^{1-\alpha_c} [\alpha_c\rho_c\theta_q(g_{\rho_c} - 1) + (\rho_c\theta_q - \theta_e)] - 2(1 - \delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{2-\alpha_c} \right\} < \frac{2(1-\alpha_c)^2(1-\delta)\rho_c\theta_q\theta_e}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)} (1 - g_{\rho_c}) \leq 0 \text{ from (A3.71) and } g_{\rho_c} \geq 1.$$

$$\frac{dn_1}{dg_{\rho_c}} = \frac{(1-\alpha_c)\rho_c(1-g_\delta\delta)}{3\lambda(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}} (\theta_e - \alpha_c g_{\rho_c}\rho_c\theta_q) < 0 \quad \text{from } \rho_c > \frac{\theta_e}{\alpha_c\theta_q} \quad \text{and} \\ g_{\rho_c} \geq 1.$$

$$\frac{dn_1}{d\delta} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}} \left[g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c} - 2\lambda(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right] < \frac{(1-\alpha_c)\rho_c g_{\rho_c}}{3(1-\delta)\lambda(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}} (g_\delta-1) \leq 0 \text{ from (A3.71) and } g_\delta \leq 1.$$

$$\frac{dn_1}{d\theta_q} = \frac{(1-\alpha_c)\rho_c^2}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}} \left\{ -2(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c} + \frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)g_{\rho_c}(\rho_c\theta_q-\theta_e)] \right\} < \frac{2(1-\alpha_c)^2(1-\delta)\rho_c^2\theta_e}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)} (1-g_{\rho_c}) \leq 0 \text{ from (A3.71) and } g_{\rho_c} \geq 1.$$

$$\frac{dn_1}{d\theta_e} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}} \left\{ 2(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c} - \frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)(\rho_c\theta_q-\theta_e)] \right\} > \frac{2(1-\alpha_c)^2(1-\delta)\rho_c^2\theta_q}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)} (g_{\rho_c}-1) \geq 0 \text{ from (A3.71) and } g_{\rho_c} \geq 1.$$

$$(2) \ n_2 = \frac{(1-\alpha_c)\rho_c [2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} - (1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda]}{3\lambda(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)} \text{ from (A3.70).}$$

Since

$$\frac{dn_2}{d\rho_c} = \frac{(1-\alpha_c)\theta_e}{3(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left\{ (1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} [\alpha_c(\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)] - 2\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c} \right\},$$

$$\frac{dn_2}{d\rho_c} < (\geq) 0 \text{ when } \lambda < (\geq) \frac{2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} [\alpha_c(\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}.$$

Since

$$\frac{dn_2}{dg_{\rho_c}} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left[(1-\delta)\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} - 2\frac{1}{\lambda}\theta_e(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right], \frac{dn_2}{dg_{\rho_c}} < (\geq) 0 \text{ when } \lambda < (\geq) \frac{2\theta_e(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(1-\delta)\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}.$$

Since

$$\frac{dn_2}{d\delta} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)} \left[(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} - 2\frac{1}{\lambda}g_{\rho_c}g_\delta(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right],$$

$$\frac{dn_2}{d\delta} < (\geq) 0 \text{ when } \lambda < (\geq) \frac{2g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}.$$

Since

$$\begin{aligned} \frac{dn_2}{d\theta_q} &= \frac{(1-\alpha_c)\rho_c^2}{3(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left\{ (1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} [\alpha_c g_{\rho_c}(\rho_c\theta_q - \theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q - \theta_e)] - 2\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}^2(\rho_c\theta_q - \theta_e)^{2-\alpha_c} \right\}, \\ \frac{dn_2}{d\theta_q} &< (\geq) 0 \text{ when } \lambda < (\geq) \frac{2(1-g_\delta\delta)g_{\rho_c}^2(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}[\alpha_c g_{\rho_c}(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}. \end{aligned}$$

Since

$$\begin{aligned} \frac{dn_2}{d\theta_e} &= \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left\{ 2\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{2-\alpha_c} - (1-\delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} [\alpha_c(\rho_c\theta_q - \theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q - \theta_e)] \right\}, \\ \frac{dn_2}{d\theta_e} &> (\leq) 0 \text{ when } \lambda < (\geq) \frac{2(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}. \end{aligned}$$

Proof of Proposition A3.2

$$(1) \quad n_1 = \frac{(1-\alpha_c)\rho_c \{ 2[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} \lambda - (1+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} \}}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}} \text{ from}$$

(A3.74).

$$\begin{aligned} \frac{dn_1}{d\rho_c} &= \frac{(1-\alpha_c)\theta_e}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}} \left\{ \frac{1}{\lambda}(1+\alpha)(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right. \\ &\quad \left[(g_{\rho_c}\rho_c\theta_q - \theta_e) - (1-\alpha_c)\rho_c\theta_q(g_{\rho_c} - 1) \right] - 2 \left[(1-\delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q - \theta_e)^{2-\alpha_c} \right\} \\ &< \frac{2(1-\alpha_c)^2\rho_c\theta_q\theta_e \left[(1-\delta) + \frac{\eta}{\rho} \right]}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)} (1-g_{\rho_c}) \leq 0 \text{ from (A3.77) and } g_{\rho_c} \geq 1. \end{aligned}$$

$$\frac{dn_1}{dg_{\rho_c}} = \frac{(1+\alpha)(1-\alpha_c)\rho_c(1-g_\delta\delta)}{(3+\alpha)\lambda(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}} (\theta_e - \alpha_c g_{\rho_c}\rho_c\theta_q) < 0 \text{ from } \rho_c > \frac{\theta_e}{\alpha_c\theta_q} \text{ and}$$

$$g_{\rho_c} \geq 1.$$

$$\frac{dn_1}{d\delta} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}} \left[\frac{1}{\lambda}(1+\alpha)g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c} - \right.$$

$$2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} \left. \right] \leq \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}} \left[(1+\alpha)g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c} - 2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right] < 0$$
 from $\lambda \geq 1$ and the assumption that g_{ρ_c} is not large and g_δ is not small.

$$\frac{dn_1}{d\theta_q} = \frac{(1-\alpha_c)\rho_c^2}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}} \left\{ -2 \left[(1-\delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c} + \right.$$

$$\frac{1}{\lambda}(1+\alpha)(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)g_{\rho_c}(\rho_c\theta_q-\theta_e)] \left. \right\} < \frac{2(1-\alpha_c)^2 \left[(1-\delta) + \frac{\eta}{\rho} \right] \rho_c^2 \theta_e}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)} (1-g_{\rho_c}) \leq 0$$
 from (A3.77) and $g_{\rho_c} \geq 1$.

$$\frac{dn_1}{d\theta_e} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}} \left\{ 2 \left[(1-\delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c} - \frac{1}{\lambda}(1+\alpha)(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)(\rho_c\theta_q-\theta_e)] \right\} >$$

$$\frac{2(1-\alpha_c)^2 \left[(1-\delta) + \frac{\eta}{\rho} \right] \rho_c^2 \theta_q}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)} (g_{\rho_c} - 1) \geq 0$$
 from (A3.77) and $g_{\rho_c} \geq 1$.

$$(2) \quad n_2 = \frac{(1-\alpha_c)\rho_c \left[(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} - [(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda \right]}{(3+\alpha)\lambda\rho(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)} \quad \text{from (A3.75).}$$

Since

$$\frac{dn_2}{d\rho_c} = \frac{(1-\alpha_c)\theta_e}{(3+\alpha)(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left\{ \left[(1-\delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} [\alpha_c(\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)] - (2+\alpha)\frac{1}{\lambda}(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c} \right\},$$

$$\frac{dn_2}{d\rho_c} < (\geq) 0 \text{ when } \lambda < (\geq) \frac{(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}[\alpha_c(\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}.$$

Since

$$\frac{dn_2}{dg_{\rho_c}} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left[\left[(1-\delta) + \frac{\eta}{\rho} \right] \alpha_c \rho_c \theta_q (g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} - (2 + \alpha) \frac{1}{\lambda} \theta_e (1 - g_\delta \delta) (\rho_c \theta_q - \theta_e)^{1-\alpha_c} \right],$$

$$\frac{dn_2}{dg_{\rho_c}} < (\geq) 0 \text{ when } \lambda < (\geq) \frac{(2+\alpha)\rho\theta_e(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{[(1-\delta)\rho+\eta]\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}.$$

Since

$$\frac{dn_2}{d\delta} = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^{1-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)} \left[(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} - (2 + \alpha) \frac{1}{\lambda} g_{\rho_c} g_\delta (\rho_c \theta_q - \theta_e)^{1-\alpha_c} \right], \frac{dn_2}{d\delta} < (\geq) 0 \text{ when } \lambda < (\geq) \frac{(2+\alpha)g_{\rho_c}g_\delta(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}.$$

Since

$$\frac{dn_2}{d\theta_q} = \frac{(1-\alpha_c)\rho_c^2}{(3+\alpha)(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left\{ -(2 + \alpha) \frac{1}{\lambda} (1 - g_\delta \delta) g_{\rho_c}^2 (\rho_c \theta_q - \theta_e)^{2-\alpha_c} + \left[(1 - \delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} [\alpha_c g_{\rho_c} (\rho_c \theta_q - \theta_e) + (1 - \alpha_c) (g_{\rho_c}\rho_c\theta_q - \theta_e)] \right\},$$

$$\frac{dn_2}{d\theta_q} < (\geq) 0 \text{ when } \lambda < (\geq) \frac{(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}^2(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}[\alpha_c g_{\rho_c}(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}.$$

Since

$$\frac{dn_2}{d\theta_e} = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)^{2-\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left\{ (2 + \alpha) \frac{1}{\lambda} (1 - g_\delta \delta) g_{\rho_c} (\rho_c \theta_q - \theta_e)^{2-\alpha_c} - \left[(1 - \delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} [\alpha_c (\rho_c \theta_q - \theta_e) + (1 - \alpha_c) (g_{\rho_c}\rho_c\theta_q - \theta_e)] \right\},$$

$$\frac{dn_2}{d\theta_e} > (\leq) 0 \text{ when } \lambda < (\geq) \frac{(2+\alpha)\rho(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{2-\alpha_c}}{[(1-\delta)\rho+\eta](g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}[\alpha_c(\rho_c\theta_q-\theta_e)+(1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)]}.$$

$$(3) \quad e = \frac{\alpha\rho[(1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}\lambda+(1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c}]-3\lambda\eta(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(3+\alpha)\lambda\rho\eta(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}} \text{ from}$$

(A3.76).

$$\frac{de}{d\rho_c} = \frac{\alpha(1-g_\delta\delta)g_{\rho_c}(1-\alpha_c)\theta_q\theta_e}{(3+\alpha)\lambda\eta(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(g_{\rho_c}-1) \geq 0 \text{ from } g_{\rho_c} \geq 1.$$

$$\frac{de}{dg_{\rho_c}} = \frac{\alpha(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c}}{(3+\alpha)\lambda\eta(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(\alpha_c g_{\rho_c}\rho_c\theta_q - \theta_e) > 0 \text{ from } \rho_c > \frac{\theta_e}{\alpha_c\theta_q} \text{ and } g_{\rho_c} \geq 1.$$

$$\frac{de}{d\theta_q} = \frac{\alpha(1-\alpha_c)(1-g_\delta\delta)g_{\rho_c}\theta_e}{(3+\alpha)\lambda\eta(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(g_{\rho_c}-1) \geq 0 \text{ from } g_{\rho_c} \geq 1.$$

$$\frac{de}{d\theta_e} = \frac{\alpha(1-\alpha_c)(1-g_\delta\delta)g_{\rho_c}\rho_c\theta_q}{(3+\alpha)\lambda\eta(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{2-\alpha_c}}(1-g_{\rho_c}) \leq 0 \text{ from } g_{\rho_c} \geq 1.$$

Proof of Proposition A3.3

(1) $n_1 + n_2 = \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)} \{ [2(g_{\rho_c}\rho_c\theta_q - \theta_e)^{\alpha_c} - (\rho_c\theta_q - \theta_e)^{\alpha_c}] (1 - \delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} + [2(\rho_c\theta_q - \theta_e)^{\alpha_c} - (g_{\rho_c}\rho_c\theta_q - \theta_e)^{\alpha_c}] \frac{1}{\lambda} (1 - g_\delta\delta)g_{\rho_c}(\rho_c\theta_q - \theta_e)^{1-\alpha_c} \}$ from (A3.69) and (A3.70). $n_1 + n_2$ decreases with λ , δ and g_δ from the assumption that g_{ρ_c} is not large.

$$\begin{aligned} \frac{d(n_1+n_2)}{d\rho_c} &= \frac{(1-\alpha_c)\theta_e}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left(\{ (g_{\rho_c}\rho_c\theta_q - \theta_e)^{\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q - \theta_e) + (1 - \alpha_c)(\rho_c\theta_q - \theta_e)] - 2(\rho_c\theta_q - \theta_e)^{1+\alpha_c} \} \frac{1}{\lambda} g_{\rho_c}(1 - g_\delta\delta)(\rho_c\theta_q - \theta_e)^{1-\alpha_c} + \{ (\rho_c\theta_q - \theta_e)^{\alpha_c} [\alpha_c(\rho_c\theta_q - \theta_e) + (1 - \alpha_c)(g_{\rho_c}\rho_c\theta_q - \theta_e)] - 2(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1+\alpha_c} \} (1 - \delta)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c} \right) < 0 \text{ from the assumption that } g_{\rho_c} \text{ is not large.} \end{aligned}$$

Since

$$\begin{aligned} \frac{d(n_1+n_2)}{dg_{\rho_c}} &= \frac{(1-\alpha_c)\rho_c}{3\lambda(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left\{ \lambda(1-\delta)\alpha_c\rho_c\theta_q(\rho_c\theta_q-\theta_e)^{\alpha_c}(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} - (1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c} [(\alpha_c g_{\rho_c}\rho_c\theta_q - \theta_e)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{\alpha_c} + 2\theta_e(\rho_c\theta_q - \theta_e)^{\alpha_c}] \right\}, \frac{d(n_1+n_2)}{dg_{\rho_c}} < (\geq) 0 \text{ when} \end{aligned}$$

$$\lambda < (\geq) \frac{(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-2\alpha_c} [(\alpha_c g_{\rho_c}\rho_c\theta_q - \theta_e)(g_{\rho_c}\rho_c\theta_q - \theta_e)^{\alpha_c} + 2\theta_e(\rho_c\theta_q - \theta_e)^{\alpha_c}]}{(1-\delta)\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q - \theta_e)^{1-\alpha_c}}.$$

$$\begin{aligned} \frac{d(n_1+n_2)}{d\theta_q} &= \frac{(1-\alpha_c)\rho_c^2}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left(\left\{ (\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-g_{\rho_c}\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)] - 2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c} \right\} (1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} + \right. \\ &\left. \left\{ (g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-g_{\rho_c}\theta_e)] - 2g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1+\alpha_c} \right\} \frac{1}{\lambda} (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right) < 0 \text{ from the assumption that } g_{\rho_c} \text{ is not large.} \end{aligned}$$

$$\begin{aligned} \frac{d(n_1+n_2)}{d\theta_e} &= \frac{(1-\alpha_c)\rho_c}{3(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left(\left\{ 2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c} - (\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)] \right\} (1-\delta)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} + \right. \\ &\left. \left\{ 2(\rho_c\theta_q-\theta_e)^{1+\alpha_c} - (g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)(\rho_c\theta_q-\theta_e)] \right\} \frac{1}{\lambda} (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right) > 0 \text{ from the assumption that } g_{\rho_c} \text{ is not large.} \end{aligned}$$

$$(2) \quad n_1 + n_2 = \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)} \left\{ [2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} - (\rho_c\theta_q-\theta_e)^{\alpha_c}] \left[(1-\delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} + [(2+\alpha)(\rho_c\theta_q-\theta_e)^{\alpha_c} - (1+\alpha)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}] \frac{1}{\lambda} (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right\} \text{ from (A3.74) and (A3.75). } n_1 + n_2 \text{ decreases with } \lambda, \delta \text{ and } g_\delta \text{ from the assumption that } g_{\rho_c} \text{ is not large.}$$

$$\begin{aligned} \frac{d(n_1+n_2)}{d\rho_c} &= \frac{(1-\alpha_c)\theta_e}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left(\left\{ (1+\alpha)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)(\rho_c\theta_q-\theta_e)] - (2+\alpha)(\rho_c\theta_q-\theta_e)^{1+\alpha_c} \right\} \frac{1}{\lambda} g_{\rho_c}(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c} + \right. \\ &\left. \left\{ (\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)] - 2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c} \right\} \left[(1-\delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right) < 0 \text{ from the assumption that } g_{\rho_c} \text{ is not large.} \end{aligned}$$

Since

$$\begin{aligned} \frac{d(n_1+n_2)}{dg_{\rho_c}} &= \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left\{ \left[(1-\delta) + \frac{\eta}{\rho} \right] \alpha_c \rho_c \theta_q (\rho_c\theta_q-\theta_e)^{\alpha_c} (g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} - \frac{1}{\lambda} (1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-\alpha_c} [(1+\alpha)(\alpha_c g_{\rho_c}\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} + \right. \right. \\ &\left. \left. (2+\alpha)\theta_e(\rho_c\theta_q-\theta_e)^{\alpha_c}] \right\}, \frac{d(n_1+n_2)}{dg_{\rho_c}} < (\geq) 0 \text{ when} \end{aligned}$$

$$\lambda < (\geq) \frac{(1-g_\delta\delta)(\rho_c\theta_q-\theta_e)^{1-2\alpha_c}\rho[(1+\alpha)(\alpha_cg_{\rho_c}\rho_c\theta_q-\theta_e)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c}+(2+\alpha)\theta_e(\rho_c\theta_q-\theta_e)^{\alpha_c}]}{[(1-\delta)\rho+\eta]\alpha_c\rho_c\theta_q(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c}}.$$

$$\begin{aligned} \frac{d(n_1+n_2)}{d\theta_q} &= \frac{(1-\alpha_c)\rho_c^2}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left(\left\{ (\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-g_{\rho_c}\theta_e) + \right. \right. \\ &\quad \left. \left. (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)] - 2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c} \right\} \left[(1-\delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} + \right. \\ &\quad \left. \left\{ (1+\alpha)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-g_{\rho_c}\theta_e)] - (2+\right. \right. \\ &\quad \left. \left. \alpha)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1+\alpha_c} \right\} \frac{1}{\lambda} (1-g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right) < 0 \text{ from the assumption that } \\ &g_{\rho_c} \text{ is not large.} \end{aligned}$$

$$\begin{aligned} \frac{d(n_1+n_2)}{d\theta_e} &= \frac{(1-\alpha_c)\rho_c}{(3+\alpha)(\rho_c\theta_q-\theta_e)^2(g_{\rho_c}\rho_c\theta_q-\theta_e)^2} \left(\left\{ (2+\alpha)(\rho_c\theta_q-\theta_e)^{1+\alpha_c} - (1+\right. \right. \\ &\quad \left. \left. \alpha)(g_{\rho_c}\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(g_{\rho_c}\rho_c\theta_q-\theta_e) + (1-\alpha_c)(\rho_c\theta_q-\theta_e)] \right\} \frac{1}{\lambda} (1-\right. \\ &\quad \left. g_\delta\delta)g_{\rho_c}(\rho_c\theta_q-\theta_e)^{1-\alpha_c} + \left\{ 2(g_{\rho_c}\rho_c\theta_q-\theta_e)^{1+\alpha_c} - (\rho_c\theta_q-\theta_e)^{\alpha_c} [\alpha_c(\rho_c\theta_q-\theta_e) + \right. \right. \\ &\quad \left. \left. (1-\alpha_c)(g_{\rho_c}\rho_c\theta_q-\theta_e)] \right\} \left[(1-\delta) + \frac{\eta}{\rho} \right] (g_{\rho_c}\rho_c\theta_q-\theta_e)^{1-\alpha_c} \right) > 0 \text{ from the assumption} \\ &\text{that } g_{\rho_c} \text{ is not large.} \end{aligned}$$

■

CHAPTER 4. Structural change, elasticity of substitution, and growth

4.1 Introduction

Structural change in economic growth encompasses a shift or change in the relative importance of sectors and locations of economic activities, demographic transition, change of income distribution and among others. In particular, since Clark (1940) and Kuznets (1966), many works have documented the continuous decrease in the employment share and value-added share of the agricultural sector in both developed and developing countries, accompanied by growth in income per capita.

As a theoretical explanation for the declining shares, a preference-driven mechanism on the demand side and technology-driven mechanisms on the supply side are proposed. In the former mechanism (for example, Boppart (2014)), structural change results from non-homothetic preferences (the difference in the income elasticity of demand among consumer goods). In the latter ones, two mechanisms, that is, differences in productivity growth rate and the capital elasticity of output between sectors (for example, Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) respectively), which relate to the sectoral difference in production technology, are proposed.

In this chapter, we consider an additional source of technology-driven structural change, that is, the sectoral difference in the elasticity of substitution between capital and labor (ES). The main objective of this chapter is to analyze how sectoral labor, capital and

value-added shares, the factor income ratio, and the ES of the macro economy change over time by using a model with the sectoral difference in the ES. There are two reasons for introducing this difference into structural change analysis.

First, it has been empirically shown that the ES varies between sectors and the ES of a macro economy changes over time. Herrendorf et al. (2015) estimate sectoral ESs using postwar data of the United States. They find that the ESs of services and manufacturing sectors are below 1, and the ES of the agricultural sector is well above 1. Pereira (2003) shows that during the period 1890-1991 of Japan, 1870-1991 of the United Kingdom and 1890-1992 of the United States, the macro-ES increases over time.

Second, existing two-sector models used for the structural change analysis are unable to explain a part of observed structural changes. The main reason is that most models assume that sectoral production functions are Cobb-Douglas type (the ES is 1). Herrendorf et al. (2015) show that capital-labor ratios grow at a higher rate in the agricultural sector than other sectors using postwar data of the United States.²⁸ When sectoral production functions are Cobb-Douglas, this result cannot arise under free factor mobility. Piketty (2014) shows that during the period 1770-2010 of UK, the capital income share shows an

²⁸ The yearly average growth rate of the capital-labor ratio in the agricultural sector reaches 1.9% since 1960, compared to 1.4% for the whole economy. The higher growth rate of the capital labor ratio in the agricultural sector is implicitly shown in our model.

inverted U-shaped trend except recent periods.²⁹ This observation is also inconsistent with the Cobb–Douglas specification.

Alvarez-Cuadrado, Long and Poschke (2017, 2018) use Constant Elasticity of Substitution (CES) production functions to introduce sectoral differences in the ES. Although they show interesting results using numerical simulations, the dynamics cannot be theoretically analyzed under the CES setting.

Therefore, in this work, in order to enable the theoretical analysis, we use a linear production function for the agricultural sector and a Cobb-Douglas production function for the non-agricultural sector, considering the fact that the ES of the former sector is much bigger than the latter sector. Also, we consider sectoral differences in productivity growth rate and the capital elasticity of output. As for the utility function, we consider non-homothetic preferences in which the income elasticity of the non-agricultural good (the agricultural good) is greater (smaller) than 1. This simple framework allows us not only to analyze the new mechanism that we are proposing, but also to capture those preference-driven and technology-driven mechanisms stressed in the previous literature.

Based on the model, we examine how the income ratio of capital to labor, the capital, labor share and value-added share of the non-agricultural sector, and the ES of the economy change over time. Main results are summarized as follows.

²⁹ The capital share of income increases from 0.35 to 0.43 in year 1870, and decreases to 0.2 in year 1970, then increases to 0.28 eventually.

First, the income ratio of capital to labor shows an inverted U-shaped trend, that is, it increases in the early stage of development and decreases in the late stage of development. This result is consistent with the empirical works of Piketty (2014) based on 1770-2010 data of the United Kingdom, Fukao, Makino and Settsu (2019) based on 1885-1970 data of Japan, Piketty and Zucman (2014) based on 1820-2010 data of France and Mallick (2017) based on 1980-2010 data of India, except recent periods in developed countries in which the income ratio is rising.

Second, the capital share of the non-agricultural sector shows a U-shaped trend, that is, it decreases in the early stage of development and increases in the late stage of development. Anchorena and Modalsli (2009) show that during the period 1600-1860 of the United Kingdom, the capital share of the non-agricultural sector shows a U-shaped trend. By contrast, unlike the theoretical result, the capital share of the sector increases over time (see, e.g., Fukao, Makino and Settsu (2019) and Mallick (2017)). A likely reason for the difference between our result and these findings is that in our model, capital stands for equipment such as machines and power products, while in these works, capital includes structure as well as equipment. As explained later, if capital represents structure as well in the model, there could be no decrease in the capital share of the non-agricultural sector in the early stage of development. It would be important to empirically examine how the equipment share of the sector changes with development.

Third, the ES decreases in the early stage of development and increases in the late stage of development. The result of our model is consistent with the works of Jalava et al. (2006) based on 1902-2003 data of Finland and Singh and Sharma (2009) based on 1961-1990 data of India. By contrast, Pereira (2003) shows that during the period 1890-1991 of

Japan, 1870-1991 of United Kingdom and 1890-1992 of United States, ES increases over time. A main reason for the discrepancy between the theoretical result and this finding could be that capital stands for equipment in our model, while Pereira (2003) includes structure as well as equipment. As explained later, if capital stands for structure as well, there could be no decrease in the ES in the early stage of development.

Finally, the employment share and value-added share of the non-agricultural sector increase over time as with standard structural change models and empirical findings (see, e.g., Dennis and İşcan (2009) and Timmer, de Vries and de Vries (2014)).

This chapter is related to Alvarez-Cuadrado, Long and Poschke (2017, 2018) that use general CES production functions. Alvarez-Cuadrado, Long and Poschke (2017) show that during the shift from the agricultural sector to the non-agricultural sector, a factor rebalancing effect, which tends to make production in the more flexible sector (agricultural sector) more intensive in the input that becomes more abundant (capital), is operative. By quantitative analysis, they show that in certain cases, this effect dominates and shares of capital and labor in a given sector can move in different directions. Alvarez-Cuadrado, Long and Poschke (2018) develop a model to analyze the shift from the manufacturing sector to the service sector. They calibrate the model to the US economy and show that the change of sectoral labor income shares depends partially on the sectoral difference in the ES.

This chapter differs from Alvarez-Cuadrado, Long and Poschke (2017, 2018) mainly in two aspects. First, instead of general CES production functions, we use linear and Cobb-Douglas functions to introduce the sectoral difference in the ES. This simple specification

allows theoretical analysis of the dynamics. Second, unlike their papers, we examine how the factor income ratio and the macro-ES change over time. As for the capital share of the non-agricultural sector, Alvarez-Cuadrado, Long and Poschke (2017) conduct relatively short-term analysis, while this chapter examines the long run dynamics.

This chapter is also related to Palivos and Karagiannis (2010) and Gomez (2018). Palivos and Karagiannis (2010) use a convex growth model with non-reproducible labor and a general production function, and show that the ES that eventually becomes greater than one can counteract the role of diminishing returns to capital, which in turn can lead to unbounded endogenous growth. Gomez (2018) uses a Ramsey model with elastic labor supply and a CES production function and shows that if the initial value of capital per unit of effective labor is below its steady-state value, an increase in the ES generates higher steady state values of income, capital and consumption per capita. These two works use one-sector models and structural change is not incorporated. Instead of examining how the sectoral difference in the ES affects structural change, they analyze the impact of the change in the ES on economic growth.

The rest of this chapter is organized as follows. Section 4.2 presents the model, and Section 4.3 analyzes the statics and dynamics. Section 4.4 presents the results. Finally, Section 4.5 concludes the work. Appendix includes proofs of lemmas and propositions.

4.2 Model

We consider a closed economy in discrete time. There are two kinds of consumption goods, the agricultural good and non-agricultural good, where the latter one is also used as investment good. All markets are perfectly competitive.

Production

There are two production sectors: sector 1 produces the non-agricultural good using labor and capital with a Cobb–Douglas production technology and sector 2 produces the agricultural good with a linear production technology. Capital stands for equipment such as machines and power generators. There are two reasons for choosing the linear production function in the agricultural sector. First, it has been empirically shown that the elasticity of substitution between capital and labor (ES) of the agricultural sector is much larger than the non-agricultural sectors (for example, Herrendorf et al. (2015) and Koesler and Schymura (2015)).³⁰ Second, unlike other models that assume different ESs between sectors, the model is analytically tractable.

Production function of the non-agricultural good is,

$$Y_1(t) = (A_1(t)L_1(t))^{1-\alpha}K_1(t)^\alpha, 0 < \alpha < 1, \quad (4.1)$$

where $Y_1(t)$, $A_1(t)$, $L_1(t)$ and $K_1(t)$ are, respectively, levels of output, technology, labor and capital of sector 1. The non-agricultural good is also used as the investment good.

Production function of the agricultural good is,

$$Y_2(t) = (1 - \beta)A_2(t)L_2(t) + \beta K_2(t), 0 < \beta < 1, \quad (4.2)$$

³⁰ Herrendorf et al. (2015) use postwar data of the United States to estimate the sectoral ESs. They find that the ESs of the manufacturing and service sectors are 0.8 and 0.75 respectively, and the ES of the agricultural sector is 1.58. Koesler and Schymura (2015) use the WIOD database (the dataset covering 27 EU countries and 13 other major regions) to estimate the ESs of 35 sectors. They find that the ES of the agricultural sector is 1.07, and the ESs of the manufacturing and service sectors are 0.23 and 0.17 respectively.

where $Y_2(t)$, $A_2(t)$, $L_2(t)$ and $K_2(t)$ are, respectively, levels of output, technology, labor and capital of sector 2.

Technological progress

The technological progress rates in both sectors are exogenous and constant:

$$\frac{\dot{A}_1(t)}{A_1(t)} = g_1, \quad (4.3)$$

$$\frac{\dot{A}_2(t)}{A_2(t)} = g_2. \quad (4.4)$$

Because the speed of technological progress is faster in the non-agricultural sector than in the agricultural sector in developing countries (Szirmai, 2012), we assume that $g_1 > g_2$.³¹

We define $R(t)$, $W(t)$, $P_1(t)$ and $P_2(t)$ as the rental rate of capital, wage rate and the prices of the non-agricultural and agricultural good respectively. We assume that factors are perfectly mobile between two sectors. Then, from the first order conditions of the profit maximization of competitive firm,

$$R(t) = P_1(t)\alpha \frac{Y_1(t)}{K_1(t)} = P_2(t)\beta, \quad (4.5)$$

³¹ Szirmai (2012) shows that in 16 developing countries, the non-agricultural sector is more dynamic than the agricultural sector in terms of productivity growth, especially in the period 1950–1973. Therefore, we assume that the agricultural productivity increases at a smaller rate than the non-agricultural productivity.

where the second equality holds when sector 2 uses capital,

$$W(t) = P_1(t)(1 - \alpha) \frac{Y_1(t)}{L_1(t)} = P_2(t)(1 - \beta)A_2(t) \quad (4.6)$$

where the second equality holds when sector 2 uses labor.

Household activities

Each household spends labor and capital incomes from sectors 1 and 2 on investment and the consumption of the agricultural and non-agricultural goods. We assume that each household invests a certain proportion (v) of their income from sector 1, and consumes the rest.³²

A household maximizes the utility function

$$u(t) = \gamma \ln(c_1(t) + \underline{c}_1) + (1 - \gamma) \ln(c_2(t) - \underline{c}_2), 0 < \gamma < 1,$$

subject to

$$P_1(t)c_1(t) + P_1(t)i(t) + P_2(t)c_2(t) = W(t) + R(t) \frac{K(t)}{L(t)}, \quad (4.7)$$

$$i(t) = vy_1(t). \quad (4.8)$$

³² We could assume that investment is determined through intertemporal utility maximization, but the analysis of the model becomes very difficult. To make the model analytically tractable, we assume that fixed fraction of sector 1 income is invested.

In the above equations, $c_1(t)$ and $c_2(t)$ are consumptions of the non-agricultural and agricultural goods; $i(t)$ is the investment of the non-agricultural good; $K(t)$ and $L(t)$ are respectively the total capital and labor endowment of the economy; $y_1(t)$ is real labor and capital income from sector 1. $\underline{c}_1$ and $\underline{c}_2$ are constants. $\underline{c}_1$ could be interpreted as domestic production, such as household services, and $\underline{c}_2$ could be interpreted as the subsistence level of food consumption.

From the first order conditions of the utility maximization problem, the marginal rate of substitution between the two consumption goods equals their relative price

$$\frac{\frac{\partial u(t)}{\partial c_1(t)}}{\frac{\partial u(t)}{\partial c_2(t)}} = \frac{\frac{\gamma}{c_1(t) + \underline{c}_1}}{\frac{1 - \gamma}{c_2(t) - \underline{c}_2}} = \frac{P_1(t)}{P_2(t)},$$

which can be expressed as

$$\frac{\gamma}{1 - \gamma} \frac{\frac{Y_2(t)}{L(t)} - \underline{c}_2}{(1 - \gamma) \frac{Y_1(t)}{L(t)} + \underline{c}_1} = \frac{P_1(t)}{P_2(t)} \quad (4.9)$$

from the market equilibrium conditions below.

Market equilibrium conditions

Market equilibrium conditions of factors and goods are

$$L(t) = L_1(t) + L_2(t), \quad (4.10)$$

$$K(t) = K_1(t) + K_2(t), \quad (4.11)$$

$$Y_1(t) = (c_1(t) + i(t))L(t), \quad (4.12)$$

$$Y_2(t) = c_2(t)L(t). \quad (4.13)$$

Labor and capital change according to

$$\frac{\dot{L}(t)}{L(t)} = n, \quad (4.14)$$

$$\dot{K}(t) = i(t)L(t) - \delta K(t) = vY_1(t) - \delta K(t), \quad (4.15)$$

where n is constant and δ stands for the depreciation rate of capital.

4.3 Analysis

The Depending on the relation between labor cost ($\frac{1}{(1-\beta)A_2(t)}W(t)$) and capital cost ($\frac{1}{\beta}R(t)$) per output of the agricultural production, there are three cases. When $\frac{1}{(1-\beta)A_2(t)}W(t) < \frac{1}{\beta}R(t)$, the agricultural production uses only labor (case 1); when $\frac{1}{(1-\beta)A_2(t)}W(t) = \frac{1}{\beta}R(t)$, the agricultural production uses both labor and capital (case 2); when $\frac{1}{(1-\beta)A_2(t)}W(t) > \frac{1}{\beta}R(t)$, the agricultural production uses only capital (case 3).

In the following analysis, we use the following symbols.

$$k(t) = \frac{K(t)}{L(t)} : \text{capital per worker}, \quad (4.16)$$

$\kappa(t) = \frac{K_1(t)}{K(t)}$, $0 \leq \kappa(t) \leq 1$ and $\lambda(t) = \frac{L_1(t)}{L(t)}$, $0 \leq \lambda(t) \leq 1$: shares of capital and labor allocated to sector 1,

$$\chi(t) = \frac{K(t)}{A_1(t)L(t)} = \frac{k(t)}{A_1(t)} : \text{aggregate capital per unit of effective labor.}^{33} \quad (4.17)$$

We drop the time indicator when there is no risk of ambiguity.

In case 1, $Y_2 = (1 - \beta)A_2L_2$, $\kappa = 1$, and $W = P_2(1 - \beta)A_2 = P_1(1 - \alpha)\frac{Y_1}{L_1}$. From the last equation, we have

$$\frac{P_2}{P_1} = \frac{1 - \alpha}{1 - \beta} \frac{Y_1}{A_2L_1}. \quad (4.18)$$

From (4.9) and (4.18),

$$\frac{\gamma}{1 - \gamma} \frac{\frac{Y_2}{L} - \underline{c_2}}{(1 - v)\frac{Y_1}{L} + \underline{c_1}} \frac{1 - \alpha}{1 - \beta} \frac{Y_1}{A_2L_1} = 1. \quad (4.19)$$

By substituting (4.1) and (4.2) into (4.19) and using the definitions of λ and χ , we have

$$\frac{\gamma}{1 - \gamma} \frac{(1 - \beta)A_2(1 - \lambda) - \underline{c_2}}{(1 - v)A_1(\lambda)^{1 - \alpha}(\chi)^\alpha + \underline{c_1}} \frac{1 - \alpha}{1 - \beta} \frac{A_1(\lambda)^{1 - \alpha}(\chi)^\alpha}{A_2\lambda} = 1.$$

³³ The reason for using $A_1(t)L(t)$ as effective labor instead of $A_1(t)L_1(t) + A_2(t)L_2(t)$ is that, with technological progress, effective labor gradually approaches $A_1(t)L(t)$ due to the faster technology progress in sector 1 ($g_1 \geq g_2$).

Thus, given χ , λ is obtained as the solution to

$$\chi = \lambda \left\{ \frac{\frac{c_1}{\bar{A}_1}}{\frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{\bar{A}_2} \right] - \left[(1-\nu) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \lambda} \right\}^{\frac{1}{\alpha}}. \quad (4.20)$$

In case 2, $Y_2 = (1-\beta)A_2L_2 + \beta K_2$, $R = P_1\alpha \frac{Y_1}{K_1} = P_2\beta$ and $W = P_1(1-\alpha) \frac{Y_1}{L_1} = P_2(1-\beta)A_2$. From the last equation, we have

$$\frac{P_2}{P_1} = \frac{1-\alpha}{1-\beta} \frac{Y_1}{A_2L_1}. \quad (4.21)$$

By substituting (4.5) and (4.6) into the condition for case 2, we have

$$A_2\lambda = \kappa k \frac{\beta}{1-\beta} \frac{1-\alpha}{\alpha}. \quad (4.22)$$

From (4.9) and (4.21), we have

$$\frac{\gamma}{1-\gamma} \frac{\frac{Y_2}{L} - \frac{c_2}{\bar{A}_2}}{(1-\nu) \frac{Y_1}{L} + \frac{c_1}{\bar{A}_1}} \frac{1-\alpha}{1-\beta} \frac{Y_1}{A_2L_1} = 1. \quad (4.23)$$

By substituting (4.1) and (4.2) into (4.23) and using the definitions of κ , λ and χ , we have

$$\frac{\gamma}{1-\gamma} \frac{(1-\beta)A_2(1-\lambda) + \beta(1-\kappa)A_1\chi - \frac{c_2}{\bar{A}_2}}{(1-\nu)A_1(\lambda)^{1-\alpha}(\kappa\chi)^\alpha + \frac{c_1}{\bar{A}_1}} \frac{1-\alpha}{1-\beta} \frac{A_1(\lambda)^{1-\alpha}(\kappa\chi)^\alpha}{A_2\lambda} = 1.$$

By substituting (4.22) into the above equation and solving for κ , we have

$$\kappa = \frac{\alpha \left[(1 - \beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1 - \gamma}{\gamma} \left(\frac{1 - \beta}{1 - \alpha} \right)^{1 - \alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1 - \alpha} + \beta \chi \right]}{\beta \left[1 + (1 - v) \frac{1 - \gamma}{\gamma} \right] \chi}, \quad (4.24)$$

From (4.22) and (4.24), we have

$$\lambda = \frac{(1 - \alpha) \left[(1 - \beta) - \frac{c_2}{A_2} - \frac{c_1}{A_2} \frac{1 - \gamma}{\gamma} \left(\frac{1 - \beta}{1 - \alpha} \right)^{1 - \alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1 - \alpha} + \beta \frac{A_1}{A_2} \chi \right]}{(1 - \beta) \left[1 + (1 - v) \frac{1 - \gamma}{\gamma} \right]}. \quad (4.25)$$

In case 3, $Y_2 = \beta K_2$, $\lambda = 1$, and $R = P_1 \alpha \frac{Y_1}{K_1} = P_2 \beta$. From the last equation, we have

$$\frac{P_2}{P_1} = \frac{\alpha Y_1}{\beta K_1}. \quad (4.26)$$

From (4.9) and (4.26), we have

$$\frac{\gamma}{1 - \gamma} \frac{\frac{Y_2}{L} - \frac{c_2}{L}}{(1 - v) \frac{Y_1}{L} + \frac{c_1}{L}} \frac{\alpha Y_1}{\beta K_1} = 1. \quad (4.27)$$

By substituting (4.1) and (4.2) into (4.27) and using the definitions of κ and χ , we have

$$\frac{\gamma}{1 - \gamma} \frac{\beta(1 - \kappa)\chi - \frac{c_2}{A_1}}{(1 - v)(\kappa\chi)^\alpha + \frac{c_1}{A_1}\beta} \frac{\alpha (\kappa\chi)^\alpha}{\kappa\chi} = 1.$$

Thus, given χ , κ is obtained as the solution to

$$\chi = \left[1 + \frac{1}{\alpha} (1 - v) \frac{1 - \gamma}{\gamma} \right] \kappa \chi + \frac{1}{\beta} \frac{c_2}{A_1} + \frac{1}{\alpha} \frac{1 - \gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{1 - \alpha}. \quad (4.28)$$

The following lemma summarizes the range of χ in each case. Proofs of all lemmas and propositions are presented in Appendix.

Lemma 4.1 *The economy is in case 1 for $0 < \chi < \chi_1$, in case 2 for $\chi_1 \leq \chi \leq \chi_2$ and*

in case 3 for $\chi > \chi_2$, where $\chi_1 \equiv \frac{\alpha \left[(1 - \beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1 - \gamma}{\gamma} \left(\frac{1 - \beta}{1 - \alpha} \right)^{1 - \alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1 - \alpha} \right]}{\beta \left[(1 - \alpha) + (1 - v) \frac{1 - \gamma}{\gamma} \right]}$ and

$$\chi_2 \equiv \frac{\left[\alpha + (1 - v) \frac{1 - \gamma}{\gamma} \right] \frac{A_2}{A_1} (1 - \beta) + (1 - \alpha) \left[\frac{c_2}{A_1} + \frac{c_1}{A_1} \frac{1 - \gamma}{\gamma} \left(\frac{1 - \beta}{1 - \alpha} \right)^{1 - \alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1 - \alpha} \right]}{(1 - \alpha) \beta}.$$

In case 1, aggregate capital per unit of effective labor is small ($\chi < \chi_1$), thus capital is scarce compared to labor. Hence, capital cost per output of the agricultural good exceeds labor cost, and the agricultural production uses only labor. In case 2, χ is high enough that capital cost equals labor cost ($\chi_1 \leq \chi \leq \chi_2$), thus the agricultural production uses both labor and capital. In case 3, χ is large ($\chi > \chi_2$), thus labor is scarce compared to capital. Hence, labor cost exceeds capital cost, and the agricultural production uses only capital. By contrast, the non-agricultural production always uses both factors.

The result that the agricultural sector uses only labor in case 1 would not be implausible considering that capital stands for equipment such as machines and energy generators. More generally, these results are consistent with the observations that labor (capital) is the dominate factor of production in the agricultural sector of developing (developed) countries, while in the non-agricultural sector, the difference between developing and developed countries in the use of labor and capital is not as stark as in the

agricultural sector. Chen (2020), using data of 62 countries (covering both rich and poor ones) for 1967-1992, shows that the richer countries have higher capital-labor ratio in both sectors, but the difference between rich and poor countries is much larger in the agricultural sector.³⁴

Dynamic analysis

We consider the economy with $0 < \chi(0) < \chi_1$ (that is, the economy is in case 1 initially). The dynamic equations of χ are derived as follows.

From (4.14)-(4.17),

$$\frac{\dot{\chi}}{\chi} = \frac{\dot{k}}{k} - \frac{\dot{A}_1}{A_1} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} - g_1 = \frac{vY_1 - \delta K}{K} - n - g_1.$$

By substituting (4.1), (4.17) and the definition of κ and λ into the above equation, the dynamic equation of χ in case 1 becomes

$$\frac{\dot{\chi}}{\chi} = v \left(\frac{\lambda}{\chi} \right)^{1-\alpha} \kappa^\alpha - (\delta + n + g_1) = v \left(\frac{\lambda}{\chi} \right)^{1-\alpha} - (\delta + n + g_1), \quad (4.29)$$

where λ satisfies (4.20).

From (4.24), (4.25) and (4.29), the equation in case 2 becomes

³⁴ The non-agricultural capital-labor ratio differs by around 10-fold between the United States and the 20% poorest countries in the sample, while the agricultural capital-labor ratio differs by 165-fold.

$$\begin{aligned} \frac{\dot{\chi}}{\chi} = & v \left(\frac{1-\alpha}{1-\beta} \right)^{1-\alpha} \left(\frac{\alpha}{\beta} \right)^{\alpha} \frac{(1-\beta) \left(\frac{A_2}{A_1} \right)^{\alpha} - \frac{c_2}{A_1^{\alpha} A_2^{1-\alpha}} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^{\alpha} + \beta \left(\frac{A_1}{A_2} \right)^{1-\alpha}}{\left[1 + (1-\gamma) \frac{1-\gamma}{\gamma} \right] \chi} \\ & - (\delta + n + g_1). \end{aligned} \quad (4.30)$$

Since $\lambda = 1$, from (4.29), the equation in case 3 becomes

$$\frac{\dot{\chi}}{\chi} = v \frac{(\kappa \chi)^{\alpha}}{\chi} - (\delta + n + g_1), \quad (4.31)$$

where κ satisfies (4.28).

The following lemma examines the properties of $\frac{\dot{\chi}}{\chi}$.

Lemma 4.2 $\frac{d\dot{\chi}}{d\chi} < 0$ in all the three cases.³⁵

³⁵ We can also prove that $\frac{d\dot{\chi}}{dA_1} \frac{dA_1}{dt} + \frac{d\dot{\chi}}{dA_2} \frac{dA_2}{dt} > 0$ in cases 1 and 3. In case 2, when χ is close to χ_2 , $\frac{d\dot{\chi}}{dA_1} \frac{dA_1}{dt} + \frac{d\dot{\chi}}{dA_2} \frac{dA_2}{dt} > 0$, but when χ

is close to χ_1 , its sign is ambiguous.

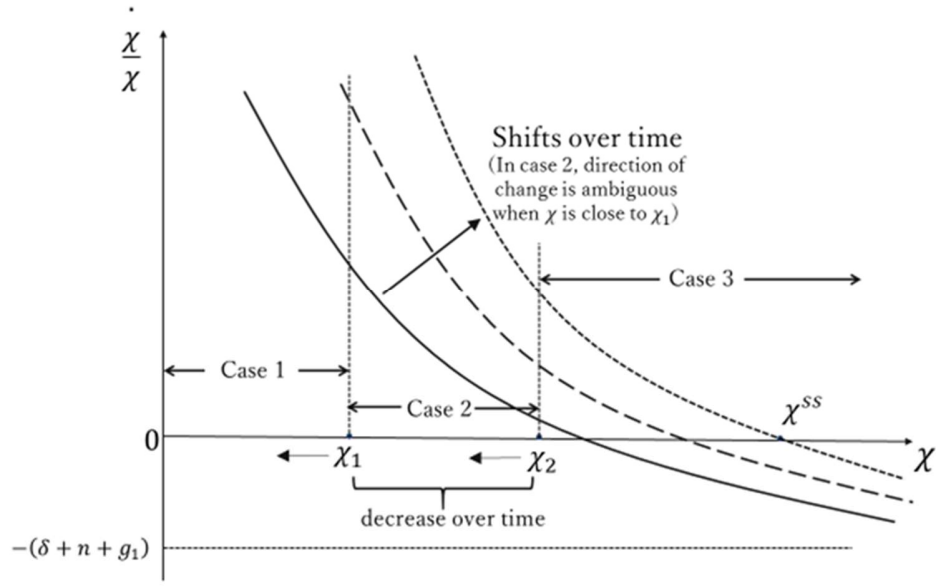


Figure 4.1 – Dynamics of aggregate capital per unit of effective labor χ

χ has direct and indirect effects on $\frac{\dot{\chi}}{\chi}$. The direct effect is negative in all the three cases because an increase in aggregate capital per unit of effective labor χ lowers the growth rate because of diminishing returns with respect to capital. At the same time, the indirect effect is positive, because increasing χ raises λ in case 1, $\lambda^{1-\alpha}\kappa^\alpha$ in case 2, and κ in case 3 (see (4.29)), which means that an increase in χ raises the output share of the non-agricultural sector, and thus capital investment. It turns out that the direct effect dominates in all the three cases and $\frac{\dot{\chi}}{\chi}$ decreases with χ .

Based on Lemma 4.2, Proposition 4.1 examines the dynamics of χ .

Proposition 4.1 *The economy transits from case 1 to case 2 then to case 3 over time,*

and converges to the steady state $\chi^{ss} = \left(\frac{v}{\delta+n+g_1}\right)^{\frac{1}{1-\alpha}} \left[\frac{\alpha+(1-v)\frac{1-\gamma}{\gamma}}{\alpha}\right]^{\frac{\alpha}{1-\alpha}}$ in case 3.

Figure 4.1 illustrates the relationship between χ and $\frac{\dot{\chi}}{\chi}$ for given technological levels.

The growth rate of aggregate capital per unit of effective labor χ decreases with the increase of χ and thus the slope of the curve is negative. Also, Figure 4.1 shows that technological progress shifts the curve upwardly and thus raises χ except when χ is close to χ_1 in case 2 (from footnote 35). Eventually, χ converges to the steady state value χ^{ss} .

4.4 Results

We now examine how the shares of capital and labor allocated to sector 1 κ and λ , the value ratio of the non-agricultural production to the agricultural production $\frac{P_1 Y_1}{P_2 Y_2}$, the income ratio of the two production factors $\frac{RK}{WL}$, and the elasticity of substitution between labor and capital σ change over time.

We assume that

$$1 - \beta > \beta \frac{1 - \alpha}{\alpha} \frac{v}{1 - v} \frac{1}{g_2} \frac{g_1 - g_2}{\delta + n + g_1} \frac{c_1}{A_2(0)}. \quad (4.32)$$

This assumption holds when the initial values of A_2 is sufficiently large.

Proposition 4.2 analyses the dynamics of κ and λ during the structural change.

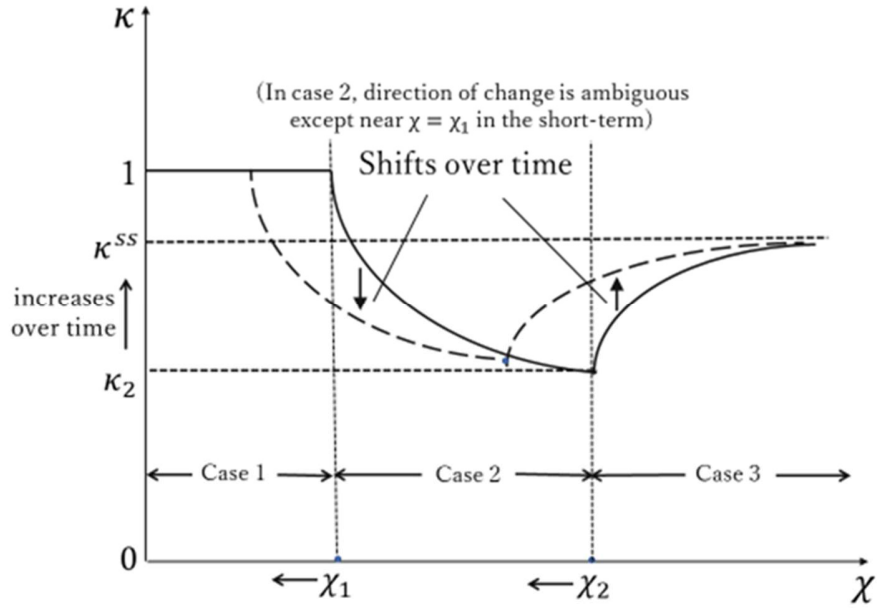


Figure 4.2 – Dynamics of the share of capital allocated to non-agricultural sector κ

Proposition 4.2

(1) In case 1, $\kappa = 1$. In case 2, κ decreases over time in the long run (the direction of change of κ is ambiguous except near $\chi = \chi_1$ in the short term) and reaches $\kappa_2 \in (0,1)$ at $\chi = \chi_2$. In case 3, κ increases over time and converges to $\kappa^{ss} = \frac{\alpha}{\alpha + (1-\nu)^{\frac{1-\gamma}{\gamma}}} < 1$.

(2) In cases 1 and 2, λ increases over time. In case 3, $\lambda = 1$.

Figure 4.2 illustrates the relationship between χ and κ for given technological levels and the effect of technological progress on the relationship. In case 1, the share of capital allocated to sector 1 equals 1. In case 2, κ decreases with the increase of χ and technological progress in the long run (the direction of change of κ is ambiguous except near $\chi = \chi_1$ in the short term), and reaches $\kappa_2 \in (0,1)$ at $\chi = \chi_2$. In case 3, κ increases

with the increase of χ and technological progress and converges to the steady state value

$$\kappa^{ss} = \frac{\alpha}{\alpha + (1-\nu)^{\frac{1-\gamma}{\gamma}}}.$$

The dynamics of λ are simple. In cases 1 and 2, the share of labor allocated to sector 1 increases with the increase of χ and technological progress, and in case 3, λ equals 1.

In case 1, sector 2 uses only labor (thus $\kappa = 1$) because capital is scarce and thus capital cost exceeds labor cost. Increases of $\chi = \frac{K}{A_1 L}$ and A_1 (an increase of A_2) raise the output in sector 1 (sector 2) and lower the relative price of goods produced in the sector. Because the income and price elasticities of good 1 (good 2) are greater (smaller) than 1, the share of production of sector 1 (sector 2) increases (decreases). Hence, increased production in sector 1 pulls labor from sector 2, increased production in sector 2 pushes labor to sector 1, and λ increases.

In case 3, sector 2 uses only capital (thus $\lambda = 1$) because labor is scarce and thus labor cost exceeds capital cost. Increases of χ and A_1 (an increase of $k = A_1 \chi$) raise the output in sector 1 (sector 2) and lower the relative price of the sector's goods. Because the elasticities of good 1 (good 2) are greater (smaller) than 1, the share of production of sector 1 (sector 2) increases (decreases). Hence, increased production in sector 1 pulls capital from sector 2, increased production in sector 2 pushes capital to sector 1, and κ increases.

In case 2, increased production in sector 1 (sector 2) from increases of χ and A_1 (an increase of A_2) pulls capital and labor from sector 2 (pushes capital and labor to sector 1), which raises κ and λ . Besides the push and pull effects, effects associated with factor price exist. An increase of $k = A_1 \chi$ (A_2) lowers (raises) the marginal productivity of capital

(labor) in sector 1 (sector 2) and thus increases the sector's $\frac{W}{R}$. For the factor price ratios of the two sectors to remain equal, the labor-capital ratio must increase in sector 1 (in sector 2), which lowers (raises) κ and raises (lowers) λ . Because the effect of $k = A_1\chi$ is greater than A_2 , the aggregate factor price effect lowers κ and raises λ . Further, this effect dominates the push and pull effects, hence κ decreases and λ increases in case 2.

Proposition 4.3 examines the dynamics of $\frac{P_1Y_1}{P_2Y_2}$ and $\frac{RK}{WL}$ during the structural change.

Proposition 4.3

(1) $\frac{P_1Y_1}{P_2Y_2}$ increases over time in all the three cases (in case 2 under the assumption of

(32)) and converges to $\left(\frac{P_1Y_1}{P_2Y_2}\right)^{ss} = \frac{1}{1-\nu} \frac{\gamma}{1-\gamma}$ in case 3.

(2) In cases 1 and 2, $\frac{RK}{WL}$ increases over time. In case 3, $\frac{RK}{WL}$ decreases over time and

converges to $\left(\frac{RK}{WL}\right)^{ss} = \frac{\alpha+(1-\nu)\frac{1-\gamma}{\gamma}}{1-\alpha}$.

The dynamics of $\frac{P_1Y_1}{P_2Y_2}$ is simple. $\frac{P_1Y_1}{P_2Y_2}$ increases with the increase of χ and technological progress in all the cases (in case 2 under the assumption of (4.32)).

Eventually, $\frac{P_1Y_1}{P_2Y_2}$ converges to the steady state value $\left(\frac{P_1Y_1}{P_2Y_2}\right)^{ss} = \frac{1}{1-\nu} \frac{\gamma}{1-\gamma}$.

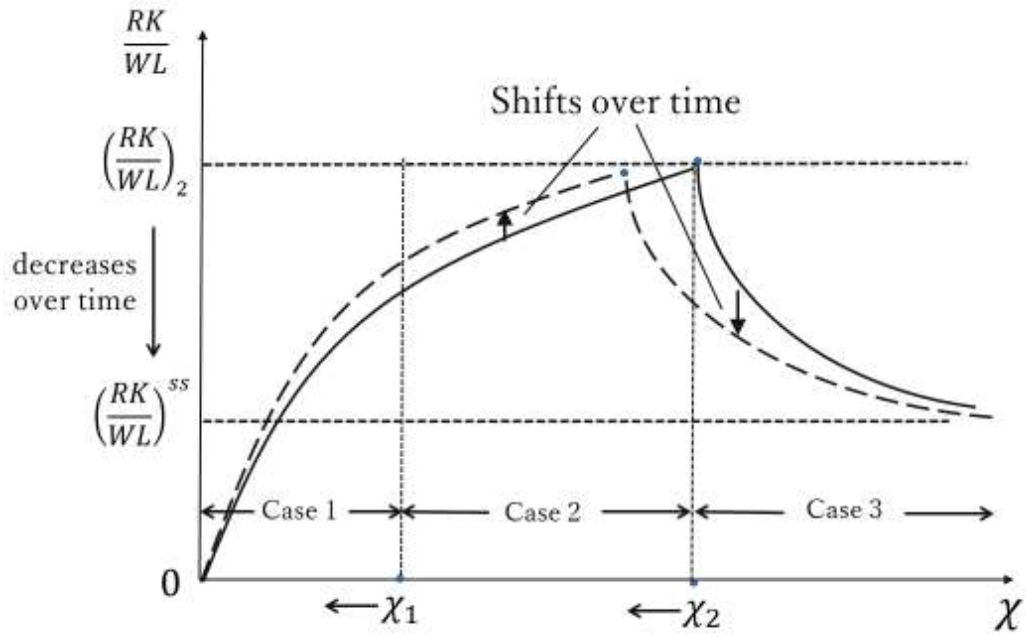


Figure 4.3 – Dynamics of the income ratio of capital and labor $\frac{RK}{WL}$

Increases of χ and A_1 in all three cases raise the output in sector 1 and lower the relative price of goods produced in the sector. By contrast, an increase of A_2 in case 1, increases of A_2 and $k = A_1\chi$ in case 2 and an increase of $k = A_1\chi$ in case 3 raise the output in sector 2 and lower the relative price of goods produced in the sector. Since the income and price elasticities of good 1 (good 2) are greater (smaller) than 1, the output share of sector 1 increases and thus the output ratio $\frac{P_1Y_1}{P_2Y_2}$ increases.

Figure 4.3 illustrates the relationship between χ and $\frac{RK}{WL}$ for given technological levels and the effect of technological progress on the relationship. In cases 1 and 2, $\frac{RK}{WL}$ increases with the increase of χ and technological progress. In case 3, $\frac{RK}{WL}$ decreases with

the increase of χ and technological progress. Eventually, $\frac{RK}{WL}$ converges to the steady state

$$\text{value} \left(\frac{RK}{WL} \right)^{ss} = \frac{\alpha + (1-\alpha)^{\frac{1-\gamma}{\gamma}}}{1-\alpha}.$$

Sector 1 always uses both factors, which means that the factor price ratio $\frac{R}{W}$ is proportional to the ratio of labor and capital allocated to sector 1, $\frac{L_1}{K_1}$. Therefore, $\frac{RK}{WL}$ is proportional to $\frac{L_1}{K_1} \frac{K}{L} = \frac{\lambda}{\kappa}$. From Proposition 4.2, $\frac{RK}{WL}$ increases from the increase of λ in case 1 and decreases from the increase of κ in case 3. In case 2, $\frac{RK}{WL}$ increases from the increase of $\frac{\lambda}{\kappa}$.

Proposition 4.4 analyses the dynamics of the elasticity of substitution between labor and capital (ES), σ .

Proposition 4.4 *In case 1, σ decreases over time and reaches $\sigma_1 > 1$ at $\chi = \chi_1$. In case 2, $\sigma = \infty$. In case 3, starting from $\sigma_2 < 1$ at $\chi = \chi_2$, σ increases over time and converges to 1.*

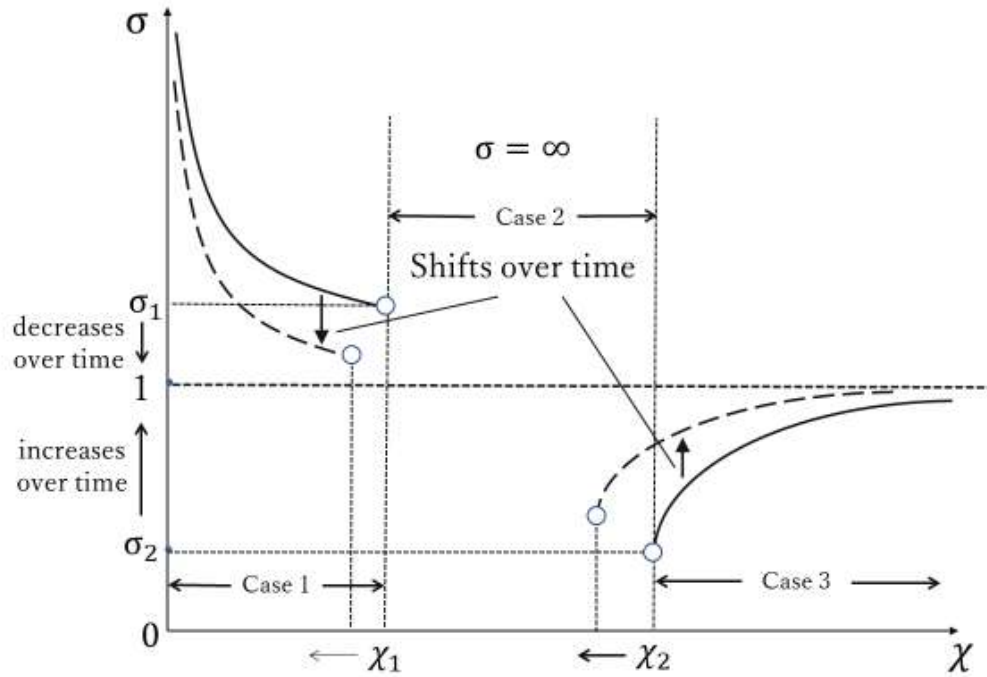


Figure 4.4 – Dynamics of the elasticity of substitution between labor and capital σ

Figure 4.4 illustrates the relationship between χ and σ for given technological levels and the effect of technological progress on the relationship. In case 1, the ES, σ , decreases with the increase of χ and technological progress, and reaches $\sigma_1 > 1$. In case 2, $\sigma = \infty$. In case 3, starting from $\sigma_2 < 1$, σ increases with the increase of χ and technological progress, and converges to the steady state value 1.

The ES of the macro economy σ equals the rate of change of the capital-labor ratio divided by the rate of the change of the factor price ratio in absolute value, i.e., $\sigma = \left| \frac{d(\frac{K}{L})}{\frac{K}{L}} / \frac{d(\frac{R}{W})}{\frac{R}{W}} \right|$. The ES of the non-agricultural sector is 1, which means that the rate of change of the capital-labor ratio of the sector $\frac{K_1}{L_1}$ equals the rate of change of $\frac{R}{W}$ in absolute

value, i.e., $\left| \frac{d\left(\frac{K_1}{L_1}\right)}{\frac{K_1}{L_1}} \right| = \left| \frac{d\left(\frac{R}{W}\right)}{\frac{R}{W}} \right|$. In case 1 (case 3), since $\frac{K}{L} = \frac{K_1}{L_1} \lambda$ and λ increase ($\frac{K}{L} = \frac{K_1}{L_1} \frac{1}{\kappa}$ and κ increase and thus $\frac{1}{\kappa}$ decreases), the rate of change of $\frac{K}{L}$ is larger (smaller) than that of $\frac{K_1}{L_1}$ and thus $\sigma > 1$ ($\sigma < 1$).³⁶ σ decreases (increases) over time largely because the rate of change of λ decreases with the increase in λ (of κ decreases and thus of $\frac{1}{\kappa}$ increases with the increase in κ) in case 1 (case 3). In case 2, both sectors use both factors, which means that $\frac{R}{W}$ is constant and $\sigma = \infty$.

Implications

These results have several interesting implications.

First, our model shows that the employment share λ and the value-added share of the non-agricultural sector $\frac{P_1 Y_1}{P_1 Y_1 + P_2 Y_2} = 1 / \left(1 + \frac{P_2 Y_2}{P_1 Y_1} \right)$ increases over time. All developed countries have undergone rises in these shares during structural change. For example, Dennis and İşcan (2009) show that during the period 1840-2000 of the United States, the employment share of the non-agricultural sector increases from 0.32 to under 0.97, and the value-added share of the sector increases from 0.54 to slightly below 0.99. This change is also observed in developing countries. Timmer, de Vries and de Vries (2014) show that during the period 1960-2010 of developing countries, the employment and value-added

36 $\sigma = \left| \frac{d\left(\frac{K}{L}\right)}{\frac{K}{L}} / \frac{d\left(\frac{R}{W}\right)}{\frac{R}{W}} \right| = \left| \frac{d\left(\frac{K}{L}\right)}{\frac{K}{L}} / \frac{d\left(\frac{K_1}{L_1}\right)}{\frac{K_1}{L_1}} \right|$ and thus $\sigma = \left| \left[\frac{d\left(\frac{K_1}{L_1}\right)}{\frac{K_1}{L_1}} + \frac{d\lambda}{\lambda} \right] / \frac{d\left(\frac{K_1}{L_1}\right)}{\frac{K_1}{L_1}} \right|$ in case 1 and $\sigma = \left| \left[\frac{d\left(\frac{K_1}{L_1}\right)}{\frac{K_1}{L_1}} + \frac{d\frac{1}{\kappa}}{\frac{1}{\kappa}} \right] / \frac{d\left(\frac{K_1}{L_1}\right)}{\frac{K_1}{L_1}} \right| = \left| \left[\frac{d\left(\frac{K_1}{L_1}\right)}{\frac{K_1}{L_1}} - \frac{d\kappa}{\kappa} \right] / \frac{d\left(\frac{K_1}{L_1}\right)}{\frac{K_1}{L_1}} \right|$ in case 3.

shares of the non-agricultural sector increase from 0.52 to 0.79 and from 0.74 to 0.92 respectively in Asian countries, from 0.27 to 0.49 and from 0.62 to 0.78 respectively in African countries, and from 0.53 to 0.86 and from 0.81 to 0.93 respectively in Latin American countries.

Second, according to our model, the income ratio of capital to labor $\frac{RK}{WL}$ shows an inverted U-shaped trend, that is, it increases in the early stage of development and decreases in the late stage of development. Piketty (2014) show that during the period 1770-2010 of the United Kingdom, the capital share of income increases from 0.35 to 0.43 in year 1870, and decreases to 0.2 in year 1970, then increases to 0.28 eventually. Fukao, Makino and Settsu (2019) show that during the period 1885-1970 of Japan, the capital share of income increases from 0.13 to 0.45 in year 1944, then decreases to 0.31 eventually. Piketty and Zucman (2014) show that during the period 1820-2010 of France, the capital share of income increases from 0.3 to 0.45 in year 1860, then decreases to 0.15 in year 1940, and eventually increases to 0.23. Mallick (2017) shows that during the period 1980-2010 of India, the capital income share increases from 0.29 to 0.54 in year 2006 then slightly decreases to 0.50. Our result is consistent with these findings, except recent periods in developed countries in which the capital income share is rising.

Third, the capital share of the non-agricultural sector κ show a U-shaped trend, that is, it decreases in the early stage of development and increases in the late stage of development. Anchorena and Modalsli (2009) show that during the period 1600-1860 of the United Kingdom, the capital share of the non-agricultural sector decreases from 0.6 to 0.4 in year 1700, then increases to 0.8 eventually. Fukao, Makino and Settsu (2019) show

that during the period 1885-1970 of Japan, the capital share of the non-agricultural sector increases from 0.75 to 0.89. Mallick (2017) shows that during the period 1980-2010 of India, the capital share of the non-agricultural sector increases from 0.77 to 0.89. These findings are different from our result except Anchorena and Modalsli (2009). A likely reason is that in our model, capital stands for equipment such as machines and power generators, while in these works, capital includes structure as well as equipment. Intuitively, if agricultural capital also includes structure such as storage facilities and livestock shelters, the capital share of the agricultural sector would be larger than that of the non-agricultural sector in the early stage of development, where the output share of the latter sector is small. Therefore, there could be no decrease in the capital share of the non-agricultural sector during structural change. In order to assess the plausibility of the result, it would be important to empirically examine how the equipment share of the sector changes with development.

Finally, in our model, the elasticity of substitution between labor and capital (ES) σ decreases over time in the early stage of development (case 1) and increases over time in the late stage of development (case 3). Jalava et al. (2006) estimate the ES of Finland, and shows that the value is 0.78 during 1902-1939, 0.08 during 1945-1974 and 0.66 during 1975-2003. Singh and Sharma (2009) estimate the ES of India, and shows that the value decreases from 13 to 4.62 during 1961-1990, and increases from 0.48 to 0.66 during 1991-2004. These findings are consistent with our model. By contrast, Pereira (2003) shows that during the period 1890-1991 of Japan, 1870-1991 of United Kingdom and 1890-1992 of United States, the ES increases over time. The absence of falling ES in the early stage of development might be due to the following factors. First, capital stands for equipment in

our model, while Pereira (2003) includes structure as well as equipment. If agricultural capital includes structure, the agricultural sector also uses both capital and labor in the early stage of development. Then, case 1 would not exist and thus there could be no decrease in the ES in the early stage of development. Second, there is a possibility that case 1 is already completed before the study period.

4.5 Conclusions

In this chapter, we develop a two-sector model where a preference-driven mechanism on the demand side (non-homothetic preference) and technology-driven mechanisms on the supply side (sectoral difference in sectoral technological progress rate, the capital elasticity of output and the elasticity of substitution between capital and labor (ES)) lead to a process of structural change. The ES mechanism is new.

There are two reasons for introducing this mechanism into structural change analysis. First, it has been empirically shown that the ES varies between sectors and the ES of a macro economy changes over time. Second, existing two-sector models used for the structural change analysis are unable to explain a part of observed facts, such as changes in the factor income share and sectoral capital–labor ratio.

Based on the model, we examine how the income ratio of capital to labor, the capital, labor and value-added shares of the non-agricultural sector, and the ES of macro economy change over time. Main results are summarized as follows.

First, the income ratio of capital to labor shows an inverted U-shaped trend, that is, it increases in the early stage of development and decreases in the late stage of development.

This result is consistent with empirical observations except recent periods in developed countries in which the capital income ratio is rising.

Second, the capital share of the non-agricultural sector shows a U-shaped trend, that is, it decreases in the early stage of development and increases in the late stage of development. By contrast, many empirical works find that the capital share increases over time. A likely reason for the discrepancy is that in our model, capital stands for equipment such as machines and power products, while in these works, capital includes structure as well as equipment.

Third, the ES decreases in the early stage of development and increases in the late stage of development, which is consistent with some empirical works. By contrast, other studies find that the ES increases over time, which maybe due to the same reason as above.

Finally, the employment share and value-added share of the non-agricultural sector increases over time as with standard structural change models and empirical findings.

The new ES mechanism of structural change has recently been considered by Alvarez-Cuadrado et al. (2017, 2018). Our model differs from their work mainly in two aspects. First, instead of general CES production functions, we use linear and Cobb-Douglas functions to introduce the sectoral difference in the ES. This simple specification allows theoretical analysis of the dynamics, while Alvarez-Cuadrado, Long and Poschke (2017, 2018) rely on numerical simulations for the analysis. Second, unlike their papers, we examine how the factor income ratio and the macro-ES change over time. Also, instead of relatively short-term analysis, this work examines the long run dynamics.

Appendix Proofs of lemmas and Propositions in chapter 4

A4.1 Proof of Lemma 4.1

[Case 1] By substituting (4.5) and (4.6) into the condition for case 1, we have

$$\chi < \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta} \frac{A_2}{A_1} \lambda. \quad (A4.1)$$

From (4.20) and (A4.1), we have

$$\lambda < \lambda_1 \equiv \frac{(1-\alpha) \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{\left[(1-\alpha) + (1-\nu) \frac{1-\gamma}{\gamma} \right] (1-\beta) \frac{A_2}{A_1}}. \quad (A4.2)$$

From (4.20) and (A4.2), the condition for case 1 is represented as

$$0 < \chi < \chi_1 \equiv \chi(\lambda = \lambda_1) = \frac{\alpha \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{\beta \left[(1-\alpha) + (1-\nu) \frac{1-\gamma}{\gamma} \right]}.$$

[Case 2]

Since $\kappa \leq 1$ and $\lambda \leq 1$, from (4.24) and (4.25), the condition for case 2 is

$$\begin{aligned} \chi_1 &= \frac{\alpha \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{\beta \left[(1-\alpha) + (1-\nu) \frac{1-\gamma}{\gamma} \right]} \leq \chi \leq \chi_2 \\ &\equiv \frac{\left[\alpha + (1-\nu) \frac{1-\gamma}{\gamma} \right] \frac{A_2}{A_1} (1-\beta) + (1-\alpha) \left[\frac{c_2}{A_1} + \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{(1-\alpha)\beta}. \end{aligned}$$

[Case 3]

By substituting (4.5) and (4.6) into the condition for case 3, we have

$$\kappa\chi > \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta} \frac{A_2}{A_1}. \quad (\text{A4.3})$$

From (4.28) and (A4.3), the condition for case 3 is

$$\chi > \chi_2 \equiv \frac{\left[\alpha + (1-v) \frac{1-\gamma}{\gamma} \frac{A_2}{A_1} (1-\beta) + (1-\alpha) \left[\frac{c_2}{A_1} + \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right] \right]}{(1-\alpha)\beta}.$$

■

A4.2 Proof of Lemma 4.2

[Case 1] Totally differentiating (4.20) with respect to λ and χ , we have

$$\frac{d\lambda}{d\chi} = \left\{ \frac{\frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] - \left[(1-v) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \lambda}{\frac{c_1}{A_1}} \right\}^{\frac{1}{\alpha}} \frac{\alpha \left\{ \frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] - \left[(1-v) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \lambda \right\}}{\alpha \frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] + (1-\alpha) \left[(1-v) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \lambda},$$

where $\frac{d\lambda}{d\chi} > 0$ because $\frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] - \left[(1-v) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \lambda > 0$ from $\lambda < \lambda_1$.

From (4.29), we have

$$\frac{\partial \chi}{\partial \lambda} = -v(1-\alpha) \left(\frac{\lambda}{\chi} \right)^{1-\alpha} \frac{1}{\chi} < 0,$$

$$\frac{\partial \chi}{\partial \lambda} = v(1-\alpha) \left(\frac{\lambda}{\chi} \right)^{1-\alpha} \frac{1}{\lambda} > 0.$$

Hence,

$$\frac{d\dot{\chi}}{d\chi} = \frac{\partial \dot{\chi}}{\partial \chi} + \frac{\partial \dot{\chi}}{\partial \lambda} \frac{d\lambda}{d\chi} = v(1-\alpha) \left(\frac{\lambda}{\chi}\right)^{1-\alpha} \frac{1}{\chi} \frac{-[(1-v)+(1-\alpha)\frac{\gamma}{1-\gamma}]\lambda}{\alpha \frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} [(1-\beta) - \frac{c_2}{A_2}] + (1-\alpha)[(1-v)+(1-\alpha)\frac{\gamma}{1-\gamma}]\lambda} < 0.$$

[Case 2] From (4.30), we have

$$\frac{d\dot{\chi}}{d\chi} = -v \left(\frac{1-\alpha}{1-\beta}\right)^{1-\alpha} \left(\frac{\alpha}{\beta}\right)^{\alpha} \left(\frac{A_2}{A_1}\right)^{\alpha} \frac{(1-\beta)^{-\frac{c_2}{A_2} - \frac{c_1}{A_2} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha}}}{[1+(1-v)\frac{1-\gamma}{\gamma}]} \left(\frac{\beta}{\alpha}\right)^{\alpha} \left(\frac{A_2}{A_1}\right)^{1-\alpha} \frac{1}{\chi^2} < 0 \text{ from}$$

$$\chi_1 = \frac{\alpha \left[(1-\beta)^{\frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha}} \left(\frac{\beta}{\alpha}\right)^{\alpha} \left(\frac{A_2}{A_1}\right)^{1-\alpha} \right]}{\beta [(1-\alpha)+(1-v)\frac{1-\gamma}{\gamma}]} > 0.$$

[Case 3] By totally differentiating (4.28) with respect to χ and κ , we have

$$\frac{d\kappa}{d\chi} = \frac{\alpha \left[\frac{1}{\beta A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa\chi)^{1-\alpha} \right]}{\left\{ \left[\alpha + (1-v)\frac{1-\gamma}{\gamma} \right] + (1-\alpha)\frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa\chi)^{-\alpha} \right\} \chi^2} > 0.$$

From (4.31), we have

$$\frac{\partial \dot{\chi}}{\partial \chi} = -(1-\alpha)v \frac{\kappa^{\alpha}}{\chi^{2-\alpha}} < 0,$$

$$\frac{\partial \dot{\chi}}{\partial \kappa} = \alpha v \frac{1}{(\kappa\chi)^{1-\alpha}} > 0,$$

$$\frac{d\dot{\chi}}{d\chi} = \frac{\partial \dot{\chi}}{\partial \chi} + \frac{\partial \dot{\chi}}{\partial \kappa} \frac{d\kappa}{d\chi} = v \frac{-(1-\alpha) \left[\alpha + (1-v)\frac{1-\gamma}{\gamma} \right] (\kappa\chi)^{\alpha} + (2\alpha-1) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} + \frac{\alpha^2 c_2}{\beta A_1} (\kappa\chi)^{-(1-\alpha)}}{\left\{ \left[\alpha + (1-v)\frac{1-\gamma}{\gamma} \right] + (1-\alpha)\frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa\chi)^{-\alpha} \right\} \chi^2}.$$

From (A4.3) in the proof of Lemma 1, we have

$$-(1-\alpha) \left[\alpha + (1-v)\frac{1-\gamma}{\gamma} \right] (\kappa\chi)^{\alpha} + (2\alpha-1) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} + \frac{\alpha^2 c_2}{\beta A_1} (\kappa\chi)^{-(1-\alpha)}$$

$$\begin{aligned}
&< -(1-\alpha) \left[\alpha + (1-v) \frac{1-\gamma}{\gamma} \right] \left(\frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta} \frac{A_2}{A_1} \right)^\alpha + (2\alpha-1) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} \\
&\quad + \frac{\alpha^2}{\beta} \frac{c_2}{A_1} \left(\frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta} \frac{A_2}{A_1} \right)^{-(1-\alpha)} \\
&= - \left(\frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta} \frac{A_2}{A_1} \right)^{-(1-\alpha)} \left\{ \frac{\alpha^2}{\beta} \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right] \right. \\
&\quad \left. + (1-\alpha)(1-v) \frac{1-\gamma}{\gamma} \frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta} \frac{A_2}{A_1} + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} \left(\frac{\alpha}{1-\alpha} \frac{1-\beta}{\beta} \frac{A_2}{A_1} \right)^{1-\alpha} \right\}.
\end{aligned}$$

Since $\chi_1 = \frac{\alpha \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{\beta \left[(1-\alpha) + (1-v) \frac{1-\gamma}{\gamma} \right]} > 0$, we have

$$(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} > 0.$$

Hence,

$$\frac{d\chi}{d\lambda} < 0,$$

■

A4.3 Proof of Proposition 4.1

From Lemma 4.2, $\frac{d\chi}{d\lambda} < 0$ in all three cases.

$$\text{When } \chi \rightarrow \chi_1 = \frac{\alpha \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{\beta \left[(1-\alpha) + (1-v) \frac{1-\gamma}{\gamma} \right]},$$

$$\lambda \rightarrow \lambda_1 \equiv \frac{(1-\alpha) \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{\left[(1-\alpha) + (1-v) \frac{1-\gamma}{\gamma} \right] (1-\beta) \frac{A_2}{A_1}} \text{ from the proof of Lemma 4.1.}$$

Thus, from (4.20) and (4.29), as $\chi \rightarrow \chi_1$,

$$\begin{aligned} \frac{\dot{\chi}}{\chi} &\rightarrow v \left\{ \frac{\frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} [(1-\beta) - \frac{c_2}{A_2}] - [(1-v) + (1-\alpha) \frac{\gamma}{1-\gamma}] \lambda_1}{\frac{c_1}{A_1}} \right\}^{\frac{1-\alpha}{\alpha}} - (\delta + n + g_1) \\ &= v \left(\frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta} \frac{A_1}{A_2} \right)^{1-\alpha} - (\delta + n + g_1). \end{aligned}$$

Because $\frac{A_1}{A_2}$ increases over time from $g_1 > g_2$, $\frac{\dot{\chi}}{\chi}$ at $\chi = \chi_1$ becomes positive eventually.

Thus, the economy shifts from case 1 to case 2.

$$\text{When } \chi \rightarrow \chi_2 = \frac{[\alpha + (1-v) \frac{1-\gamma}{\gamma} \frac{A_2}{A_1} (1-\beta) + (1-\alpha) \frac{[\frac{c_2}{A_1} + \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} (\frac{1-\beta}{1-\alpha})^{1-\alpha} (\frac{\beta}{\alpha})^\alpha (\frac{A_2}{A_1})^{1-\alpha}]}{(1-\alpha)\beta}],$$

from (4.30), $\frac{\dot{\chi}}{\chi} \rightarrow$

$$\begin{aligned} &v \left(\frac{1-\alpha}{1-\beta} \right)^{1-\alpha} \left(\frac{\alpha}{\beta} \right)^\alpha \frac{(1-\beta) \left(\frac{A_2}{A_1} \right)^\alpha - \frac{c_2}{A_2} \left(\frac{A_2}{A_1} \right)^\alpha - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha + \beta \left(\frac{A_2}{A_1} \right)^{-(1-\alpha)} \chi_2}{[1 + (1-v) \frac{1-\gamma}{\gamma}] \chi_2} - (\delta + n + g_1) \\ &= v \frac{[(1-\alpha)\beta]^{1-\alpha} [\alpha(1-\beta)]^\alpha}{\left[\alpha + (1-v) \frac{1-\gamma}{\gamma} (1-\beta) \left(\frac{A_2}{A_1} \right)^{1-\alpha} + (1-\alpha) \left[\frac{c_2}{A_1^{1-\alpha} A_2^\alpha} + \frac{c_1}{A_1^{1-\alpha} A_2^\alpha} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]} - (\delta + n + g_1). \end{aligned}$$

Because A_1 , A_2 and $\frac{A_1}{A_2}$ increase over time from $g_1 > g_2$, $\frac{\dot{\chi}}{\chi}$ at $\chi = \chi_2$ becomes

positive eventually. Thus, the economy shifts from case 2 to case 3.

When $\chi \rightarrow \infty$, from (4.31), $\frac{\dot{\chi}}{\chi} \rightarrow -(\delta + n + g_1) < 0$. Hence, when A_1 and $A_2 \rightarrow \infty$, $\frac{\dot{\chi}}{\chi}$ converges to the steady state in case 3, where from (4.28) and (4.31), $\chi^{ss} =$

$$\left(\frac{v}{\delta+n+g_1}\right)^{\frac{1}{1-\alpha}} \left[\frac{\alpha+(1-v)\frac{1-\gamma}{\gamma}}{\alpha}\right]^{\frac{\alpha}{1-\alpha}}.$$

■

A4.4 Proof of Proposition 4.2

(1) [Case 2]

Because $\kappa = 1$ when $\chi = \chi_1$ and $\kappa < 1$ when $\chi = \chi_2$, κ decreases over time in the long run. Since $\kappa = 1$ when $\chi \leq \chi_1$, κ non-increases over time when $\chi \rightarrow \chi_1$. The sign of $\frac{d\kappa}{dt}$ is ambiguous except near $\chi = \chi_1$.

When $\chi \rightarrow \chi_2 = \frac{[\alpha+(1-v)\frac{1-\gamma}{\gamma}]^{\frac{A_2}{A_1}}(1-\beta)+(1-\alpha)[\frac{c_2}{A_1}+\frac{c_1}{A_1}\frac{1-\gamma}{\gamma}(\frac{1-\beta}{1-\alpha})^{1-\alpha}(\frac{\beta}{\alpha})^{\alpha}(\frac{A_2}{A_1})^{1-\alpha}]}{(1-\alpha)\beta}$, from (4.24),

$$\begin{aligned} \kappa \rightarrow \kappa_2 &\equiv \frac{\alpha[(1-\beta)^{\frac{A_2}{A_1}}\frac{c_2}{A_1}-\frac{c_1}{A_1}\frac{1-\gamma}{\gamma}(\frac{1-\beta}{1-\alpha})^{1-\alpha}(\frac{\beta}{\alpha})^{\alpha}(\frac{A_2}{A_1})^{1-\alpha}+\beta\chi_2]}{\beta[1+(1-v)\frac{1-\gamma}{\gamma}]\chi_2} \\ &= \frac{\alpha(1-\beta)^{\frac{A_2}{A_1}}}{[\alpha+(1-v)\frac{1-\gamma}{\gamma}](1-\beta)^{\frac{A_2}{A_1}}+(1-\alpha)[\frac{c_2}{A_1}+\frac{c_1}{A_1}\frac{1-\gamma}{\gamma}(\frac{1-\beta}{1-\alpha})^{1-\alpha}(\frac{\beta}{\alpha})^{\alpha}(\frac{A_2}{A_1})^{1-\alpha}]} \in (0,1). \end{aligned}$$

[Case 3] From the proof of Lemma 4.2, we have $\frac{d\kappa}{d\chi} > 0$.

By totally differentiating (4.28) with respect to A_1 and κ , we have

$$\frac{d\kappa}{dA_1} = \frac{\frac{\alpha c_2}{\beta A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa\chi)^{1-\alpha}}{\left\{ \left[\alpha + (1-v) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa\chi)^{-\alpha} \right\} A_1 \chi} > 0.$$

Since $\frac{d\kappa}{dt} = \frac{d\kappa}{d\chi} \frac{d\chi}{dt} + \frac{d\kappa}{dA_1} \frac{dA_1}{dt} > 0$, κ increase over time in case 3.

When $A_1 \rightarrow \infty$ and thus $\chi \rightarrow \chi^{ss}$, from (4.28), $\chi^{ss} = \left[1 + \frac{1}{\alpha} (1-v) \frac{1-\gamma}{\gamma} \right] \kappa^{ss} \chi^{ss}$.

Hence, $\kappa \rightarrow \kappa^{ss} \equiv \frac{\alpha}{\alpha + (1-v) \frac{1-\gamma}{\gamma}}$.

(2) [Case 1] From the proof of Lemma 4.2, we have the $\frac{d\lambda}{d\chi} > 0$.

Totally differentiating (4.20) with respect to λ , A_1 and A_2 , we have

$$\frac{d\lambda}{dA_1} = \frac{\frac{c_1}{A_1^2}}{\left(\frac{\chi}{\lambda} \right)^\alpha \left\{ \frac{\alpha}{\lambda} \frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] + (1-\alpha) \left[(1-v) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \right\}} > 0,$$

$$\frac{d\lambda}{dA_2} = \frac{\frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \frac{c_2}{A_2^2}}{\frac{c_1}{A_1} \left(\frac{\chi}{\lambda} \right)^\alpha + \left[(1-v) + (1-\alpha) \frac{\gamma}{1-\gamma} \right]} > 0.$$

Therefore, $\frac{d\lambda}{dt} = \frac{d\lambda}{d\chi} \frac{d\chi}{dt} + \frac{d\lambda}{dA_1} \frac{dA_1}{dt} + \frac{d\lambda}{dA_2} \frac{dA_2}{dt} > 0$.

[Case 2] From (4.25), we have

$$\frac{d\lambda}{d\chi} = \frac{(1-\alpha) \beta \frac{A_1}{A_2}}{(1-\beta) \left[1 + (1-v) \frac{1-\gamma}{\gamma} \right]} > 0,$$

$$\frac{d\lambda}{dA_1} = \frac{(1-\alpha)}{(1-\beta) \left[1 + (1-v) \frac{1-\gamma}{\gamma} \right]} \left\{ \beta \frac{1}{A_2} \chi + (1-\alpha) \frac{1}{A_1} \frac{c_1}{A_2} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right\},$$

$$\frac{d\lambda}{dA_2} = \frac{(1-\alpha)\beta}{(1-\beta)\left[1+(1-\nu)\frac{1-\gamma}{\gamma}\right]} \frac{A_1}{A_2^2} \left\{ -\chi + \frac{1}{\beta} \frac{c_2}{A_1} + \frac{\alpha}{\beta} \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha} \left(\frac{\beta}{\alpha}\right)^\alpha \left(\frac{A_2}{A_1}\right)^{1-\alpha} \right\},$$

$$\frac{d\lambda}{dA_1} \frac{dA_1}{dt} + \frac{d\lambda}{dA_2} \frac{dA_2}{dt} = \frac{(1-\alpha)\beta}{(1-\beta)\left[1+(1-\nu)\frac{1-\gamma}{\gamma}\right]}$$

$$\times \left\{ \beta \frac{A_1}{A_2} (g_1 - g_2) \chi + g_2 \frac{c_2}{A_2} + [(1-\alpha)g_1 + \alpha g_2] \frac{c_1}{A_2} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha} \left(\frac{\beta}{\alpha}\right)^\alpha \left(\frac{A_2}{A_1}\right)^{1-\alpha} \right\} > 0.$$

$$\text{Therefore, } \frac{d\lambda}{dt} - \frac{d\lambda}{d\chi} \frac{d\chi}{dt} + \frac{d\lambda}{dA_1} \frac{dA_1}{dt} + \frac{d\lambda}{dA_2} \frac{dA_2}{dt} > 0.$$

■

A4.5 Proof of Proposition 4.3

(1) [Case 1]

From (4.1), (4.2), (4.9) and $K_2=0$, we have

$$\begin{aligned} \frac{P_1 Y_1}{P_2 Y_2} &= \frac{\gamma}{1-\gamma} \frac{\frac{(1-\beta)A_2 L_2 + \beta K_2 - \underline{c_2}}{L}}{(1-\nu) \frac{(A_1 L_1)^{1-\alpha} (K_1)^\alpha}{L} + \underline{c_1}} \frac{(A_1 L_1)^{1-\alpha} (K_1)^\alpha}{(1-\beta)A_2 L_2 + \beta K_2} \\ &= \frac{\gamma}{1-\gamma} \frac{\frac{(1-\beta)A_2 L_2 - \underline{c_2}}{L}}{(1-\nu) \frac{(A_1 L_1)^{1-\alpha} (K_1)^\alpha}{L} + \underline{c_1}} \frac{(A_1 L_1)^{1-\alpha} (K_1)^\alpha}{(1-\beta)A_2 L_2}. \end{aligned} \quad (A4.4)$$

By rewriting (A4.4) with $\kappa=1$, λ and χ ,

$$\frac{P_1 Y_1}{P_2 Y_2} = \frac{\gamma}{1-\gamma} \frac{(1-\beta)A_2(1-\lambda) - \underline{c_2}}{(1-\nu)A_1(\lambda)^{1-\alpha}(\chi)^\alpha + \underline{c_1}} \frac{A_1(\lambda)^{1-\alpha}(\chi)^\alpha}{(1-\beta)A_2(1-\lambda)}.$$

By substituting (4.20) into the above equation, we have

$$\frac{P_1 Y_1}{P_2 Y_2} = \frac{\gamma}{1-\gamma} \frac{\left[(1-\beta)A_2(1-\lambda) - \underline{c_2} \right]}{\frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{\underline{c_2}}{A_2} \right] - (1-\alpha) \frac{\gamma}{1-\gamma} \lambda} \frac{\lambda}{(1-\beta)A_2(1-\lambda)}$$

$$= \frac{1 - \frac{c_2}{A_2} \frac{1-\alpha}{1-\beta}}{\frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \frac{1}{\lambda} - (1-\alpha) \right]}. \quad (A4.5)$$

From (A4.5), we have

$$\frac{\partial \frac{P_1 Y_1}{P_2 Y_2}}{\partial \lambda} = \frac{(1-\alpha) \frac{1}{\lambda^2} \frac{1}{(1-\lambda)^2} \left(\lambda - \frac{(1-\beta) - \frac{c_2}{A_2}}{1-\beta} \right)^2}{\left\{ \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \frac{1}{\lambda} - (1-\alpha) \right] \right\}^2} > 0,$$

$$\frac{\partial \frac{P_1 Y_1}{P_2 Y_2}}{\partial A_2} = \frac{1-\alpha}{1-\beta} \frac{\frac{c_2}{A_2^2}}{\left\{ \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \frac{1}{\lambda} - (1-\alpha) \right] \right\}^2} = 0.$$

Since $\frac{d\lambda}{dt} > 0$ from Proposition 4.2 (2), $\frac{d \frac{P_1 Y_1}{P_2 Y_2}}{dt} = \frac{d \frac{P_1 Y_1}{P_2 Y_2}}{d\lambda} \frac{d\lambda}{dt} > 0$.

[Case 2] By rewriting (A4.5) with κ , λ and χ , we have

$$\begin{aligned} \frac{P_1 Y_1}{P_2 Y_2} &= \frac{\gamma}{1-\gamma} \frac{\frac{(1-\beta)A_2 L_2 + \beta K_2 - c_2}{L}}{(1-v) \frac{(A_1 L_1)^{1-\alpha} (K_1)^\alpha}{L} + \underline{c_1}} \frac{(A_1 L_1)^{1-\alpha} (K_1)^\alpha}{(1-\beta)A_2 L_2 + \beta K_2} = \\ &= \frac{\gamma}{1-\gamma} \frac{(1-\beta)A_2(1-\lambda) + \beta(1-\kappa)A_1\chi - \underline{c_2}}{(1-v)A_1(\lambda)^{1-\alpha}(\kappa\chi)^\alpha + \underline{c_1}} \frac{A_1(\lambda)^{1-\alpha}(\kappa\chi)^\alpha}{(1-\beta)A_2(1-\lambda) + \beta(1-\kappa)A_1\chi}. \end{aligned}$$

By substituting (4.24) and (4.25) into the above equation, we have

$$\frac{P_1 Y_1}{P_2 Y_2} = \frac{\gamma}{1-\gamma} \frac{1 - \frac{c_2}{\Phi}}{(1-v) + \underline{c_1} \Psi}, \quad (A4.6)$$

where

$$\Phi = (1-\beta)A_2 + \frac{1}{1+(1-v)\frac{1-\gamma}{\gamma}}$$

$$\times \left\{ \beta(1-v) \frac{1-\gamma}{\gamma} A_1 \chi - \left[(1-\beta) A_2 - \underline{c}_2 - \underline{c}_1 \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right] \right\},$$

$$\Psi = \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \frac{1+(1-v) \frac{1-\gamma}{\gamma}}{\beta \frac{A_1^{2-\alpha}}{A_2^{1-\alpha}} \chi + (1-\beta) A_1^{1-\alpha} A_2^\alpha - \underline{c}_2 \left(\frac{A_1}{A_2} \right)^{1-\alpha} - \underline{c}_1 \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha}.$$

From (A4.6), we have

$$\frac{d \frac{P_1 Y_1}{P_2 Y_2}}{d \chi} > 0,$$

$$\frac{d \frac{P_1 Y_1}{P_2 Y_2}}{d A_1} = \frac{\gamma}{1-\gamma} \frac{1}{\left[1+(1-v) \frac{1-\gamma}{\gamma} \right] \left[(1-v) + \underline{c}_1 \Psi \right]^2}$$

$$\times \left\{ \frac{\left[\beta(1-v) \frac{1-\gamma}{\gamma} \chi - (1-\alpha) \frac{\underline{c}_1 1-\gamma}{A_1 \gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right] \left[(1-v) + \underline{c}_1 \Psi \right]}{\Phi^2} \underline{c}_2 \right. \\ \left. + \frac{\left[(2-\alpha) \beta \left(\frac{A_1}{A_2} \right)^{1-\alpha} \chi + (1-\alpha) (1-\beta) \left(\frac{A_2}{A_1} \right)^\alpha - (1-\alpha) \underline{c}_2 \frac{1}{A_1^\alpha A_2^{1-\alpha}} \right] \left(1 - \frac{\underline{c}_2}{\Phi} \right) \Psi^2}{\left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha} \underline{c}_1 \right\},$$

$$\frac{d \frac{P_1 Y_1}{P_2 Y_2}}{d A_2} = \frac{\gamma}{1-\gamma} \frac{1}{\left[1+(1-v) \frac{1-\gamma}{\gamma} \right] \left[(1-v) + \underline{c}_1 \Psi \right]^2}$$

$$\times \left\{ \frac{\left[(1-\beta) (1-v) \frac{1-\gamma}{\gamma} + (1-\alpha) \frac{\underline{c}_1 1-\gamma}{A_2 \gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right] \{ (1-v) + \underline{c}_1 \Psi \}}{\Phi^2} \underline{c}_2 \right. \\ \left. + \frac{\left[-(1-\alpha) \beta \left(\frac{A_1}{A_2} \right)^{2-\alpha} \chi + \alpha (1-\beta) \left(\frac{A_1}{A_2} \right)^{1-\alpha} + (1-\alpha) \frac{\underline{c}_2}{A_2} \left(\frac{A_1}{A_2} \right)^{1-\alpha} \right] \left(1 - \frac{\underline{c}_2}{\Phi} \right) \Psi^2}{\left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha} \underline{c}_1 \right\}.$$

Hence,

$$\begin{aligned}
& \frac{d \frac{P_1 Y_1}{P_2 Y_2}}{d A_1} \frac{d A_1}{d t} + \frac{d \frac{P_1 Y_1}{P_2 Y_2}}{d A_2} \frac{d A_2}{d t} = \frac{\gamma}{1-\gamma} \frac{1}{\left[1+(1-\nu) \frac{1-\gamma}{\gamma}\right] \left[\frac{(1-\nu)+\underline{c}_1}{\Phi^2} \Psi\right]^2} \\
& \times \left(\frac{\left[\frac{(1-\nu)+\underline{c}_1}{\Phi^2} \Psi\right]}{\Phi^2} \underline{c}_2 \left\{ \beta(1-\nu) \frac{1-\gamma}{\gamma} g_1 A_1 \chi - (g_1 - g_2)(1-\alpha) \underline{c}_1 \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha} \left(\frac{\beta}{\alpha}\right)^\alpha \left(\frac{A_2}{A_1}\right)^{1-\alpha} \right. \right. \\
& \quad \left. \left. + (1-\beta)(1-\nu) \frac{1-\gamma}{\gamma} g_2 A_2 \right\} \right. \\
& \quad \left. + \left(\frac{A_1}{A_2}\right)^{1-\alpha} \frac{\Psi^2 \left(1-\frac{c_2}{\Phi}\right)}{\left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha} \left(\frac{\beta}{\alpha}\right)^\alpha} \underline{c}_1 \left\{ [(2-\alpha)g_1 - (1-\alpha)g_2] \beta A_1 \chi + [(1-\alpha)g_1 + \alpha g_2](1-\beta) A_2 \right. \right. \\
& \quad \left. \left. - (g_1 - g_2)(1-\alpha) \underline{c}_2 \right\} \right). \tag{A4.7}
\end{aligned}$$

From the proof of Proposition 4.1, we have $v \left(\frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta} \frac{A_1}{A_2}\right)^{1-\alpha} - (\delta + n + g_1) > 0$

$$\Leftrightarrow - \left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha} \left(\frac{\beta}{\alpha}\right)^\alpha \left(\frac{A_2}{A_1}\right)^{1-\alpha} > - \frac{\beta}{\alpha} \frac{v}{\delta+n+g_1}.$$

Therefore,

$$\begin{aligned}
& \beta(1-\nu) \frac{1-\gamma}{\gamma} g_1 A_1 \chi - (g_1 - g_2)(1-\alpha) \underline{c}_1 \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha} \left(\frac{\beta}{\alpha}\right)^\alpha \left(\frac{A_2}{A_1}\right)^{1-\alpha} \\
& + (1-\beta)(1-\nu) \frac{1-\gamma}{\gamma} g_2 A_2 > \\
& \beta(1-\nu) \frac{1-\gamma}{\gamma} g_1 A_1 \chi - (g_1 - g_2)(1-\alpha) \underline{c}_1 \frac{1-\gamma}{\gamma} \frac{\beta}{\alpha} \frac{v}{\delta+n+g_1} \\
& + (1-\beta)(1-\nu) \frac{1-\gamma}{\gamma} g_2 A_2 =
\end{aligned}$$

$$\beta(1-v)\frac{1-\gamma}{\gamma}g_1A_1\chi + (1-v)\frac{1-\gamma}{\gamma}g_2A_2\left[(1-\beta) - \beta\frac{1-\alpha}{\alpha}\frac{v}{1-v}\frac{1}{g_2}\frac{g_1-g_2}{\delta+n+g_1}\frac{\underline{c}_1}{A_2}\right].$$

From the assumption of (4.32), we have

$$1-\beta > \beta\frac{1-\alpha}{\alpha}\frac{v}{1-v}\frac{1}{g_2}\frac{g_1-g_2}{\delta+n+g_1}\frac{\underline{c}_1}{A_2(0)} > \beta\frac{1-\alpha}{\alpha}\frac{v}{1-v}\frac{1}{g_2}\frac{g_1-g_2}{\delta+n+g_1}\frac{\underline{c}_1}{A_2}.$$

Thus,

$$\begin{aligned} & \beta(1-v)\frac{1-\gamma}{\gamma}g_1A_1\chi - (g_1-g_2)(1-\alpha)\underline{c}_1\frac{1-\gamma}{\gamma}\left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha}\left(\frac{\beta}{\alpha}\right)^\alpha\left(\frac{A_2}{A_1}\right)^{1-\alpha} \\ & + (1-\beta)(1-v)\frac{1-\gamma}{\gamma}g_2A_2 \text{ in (A4.7) is positive.} \end{aligned}$$

$$\text{Since } (1-\beta)(A_1)^{1-\alpha}(A_2)^\alpha - \underline{c}_2\left(\frac{A_1}{A_2}\right)^{1-\alpha} = (A_1)^{1-\alpha}(A_2)^\alpha\left[(1-\beta) - \frac{\underline{c}_2}{A_2}\right] > 0$$

from

$$\chi_1 = \frac{\alpha\left[(1-\beta)\frac{A_2}{A_1}\frac{\underline{c}_2}{A_1}\frac{\underline{c}_1}{A_1}\frac{1-\gamma}{\gamma}\left(\frac{1-\beta}{1-\alpha}\right)^{1-\alpha}\left(\frac{\beta}{\alpha}\right)^\alpha\left(\frac{A_2}{A_1}\right)^{1-\alpha}\right]}{\beta\left[(1-\alpha)+(1-v)\frac{1-\gamma}{\gamma}\right]} > 0,$$

$$[(2-\alpha)g_1 - (1-\alpha)g_2]\beta\left(\frac{A_1}{A_2}\right)^{1-\alpha}A_1\chi + [(1-\alpha)g_1 + \alpha g_2](1-\beta)(A_1)^{1-\alpha}(A_2)^\alpha$$

$$-(g_1-g_2)(1-\alpha)\underline{c}_2\left(\frac{A_1}{A_2}\right)^{1-\alpha} \text{ in (A4.7) is positive.}$$

$$\text{Hence, } \frac{d\frac{P_1Y_1}{P_2Y_2}}{dA_1}\frac{dA_1}{dt} + \frac{d\frac{P_1Y_1}{P_2Y_2}}{dA_2}\frac{dA_2}{dt} > 0.$$

$$\text{Therefore, } \frac{d\frac{P_1Y_1}{P_2Y_2}}{dt} - \frac{d\frac{P_1Y_1}{P_2Y_2}}{d\chi}\frac{d\chi}{dt} + \frac{d\frac{P_1Y_1}{P_2Y_2}}{dA_1}\frac{dA_1}{dt} + \frac{d\frac{P_1Y_1}{P_2Y_2}}{dA_2}\frac{dA_2}{dt} > 0.$$

[Case 3] By rewriting (A4.4) with $\kappa, \lambda = 1$ and χ , we have

$$\frac{P_1 Y_1}{P_2 Y_2} = \frac{\gamma}{1-\gamma} \frac{\frac{\beta K_2 - c_2}{L} (A_1 L_1)^{1-\alpha} (K_1)^\alpha}{(1-v) \frac{(A_1 L_1)^{1-\alpha} (K_1)^\alpha}{L} + \underline{c_1}} \frac{1}{\beta K_2} = \frac{\gamma}{1-\gamma} \frac{\beta (1-\kappa) A_1 \chi - \underline{c_2}}{(1-v) A_1 (\kappa \chi)^\alpha + \underline{c_1}} \frac{(\kappa \chi)^\alpha}{\beta (1-\kappa) \chi}.$$

By rewriting the above equation, we have

$$\frac{P_1 Y_1}{P_2 Y_2} = \frac{\gamma}{1-\gamma} \frac{1 - \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi}}{(1-v) + \frac{c_1}{A_1} (\kappa \chi)^{-\alpha}}. \quad (A4.8)$$

From (A4.8), we have

$$\frac{\partial \frac{P_1 Y_1}{P_2 Y_2}}{\partial \chi} = \frac{\gamma}{1-\gamma} \frac{\alpha \frac{c_1}{A_1} \frac{1}{\kappa^\alpha \chi^{1+\alpha}} \left[1 - \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi} \right] + \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi^2} \left[(1-v) + \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right]}{\left[(1-v) + \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right]^2},$$

$$\frac{\partial \frac{P_1 Y_1}{P_2 Y_2}}{\partial \kappa} = \frac{\gamma}{1-\gamma} \frac{\alpha \frac{c_1}{A_1} \frac{1}{\kappa^{1+\alpha} \chi^\alpha} \left[1 - \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi} \right] - \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)^2 \chi^2} \left[(1-v) + \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right]}{\left[(1-v) + \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right]^2},$$

$$\frac{\partial \frac{P_1 Y_1}{P_2 Y_2}}{\partial A_1} = \frac{\gamma}{1-\gamma} \frac{\frac{c_1}{A_1^2} \frac{1}{(\kappa \chi)^\alpha} \left[1 - \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi} \right] + \frac{1}{\beta} \frac{c_2}{A_1^2} \frac{1}{(1-\kappa)\chi} \left[(1-v) + \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right]}{\left[(1-v) + \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right]^2}.$$

From the proofs of Lemma 4.2 and Proposition 4.2 (1) [case 3], we have

$$\frac{d\kappa}{d\chi} = \frac{\alpha \left[\frac{1}{\beta A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{1-\alpha} \right]}{\left\{ \left[\alpha + (1-v) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right\} \chi^2} > 0,$$

$$\frac{d\kappa}{dA_1} = \frac{\frac{\alpha c_2}{\beta A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{1-\alpha}}{\left\{ \left[\alpha + (1-v) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right\} A_1 \chi} > 0.$$

Hence,

$$\begin{aligned}
\frac{d\frac{P_1Y_1}{P_2Y_2}}{d\chi} &= \frac{\partial\frac{P_1Y_1}{P_2Y_2}}{\partial\chi} + \frac{\partial\frac{P_1Y_1}{P_2Y_2}}{\partial\kappa} \frac{d\kappa}{d\chi} = \frac{\gamma}{1-\gamma} \frac{1}{\left[(1-v) + \frac{c_1}{A_1}(\kappa\chi)^{-\alpha}\right]^2} \left\{ \alpha \frac{c_1}{A_1} (\kappa\chi)^{-\alpha} \left[1 - \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi}\right] \left(\frac{1}{\chi} + \frac{1}{\kappa} \frac{d\kappa}{d\chi}\right) \right. \\
&\quad \left. + \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi} \left[(1-v) + \frac{c_1}{A_1}(\kappa\chi)^{-\alpha}\right] \frac{(1-v)^{\frac{1-\gamma}{\gamma}} + (1-\alpha)^{\frac{1-\gamma}{\gamma}} \frac{c_1}{A_1} (\kappa\chi)^{-\alpha}}{\left[\alpha + (1-v)^{\frac{1-\gamma}{\gamma}}\right] + (1-\alpha)^{\frac{1-\gamma}{\gamma}} \frac{c_1}{A_1} (\kappa\chi)^{-\alpha}} (1-\kappa)\chi \right\}, \\
\frac{d\frac{P_1Y_1}{P_2Y_2}}{dA_1} &= \frac{\partial\frac{P_1Y_1}{P_2Y_2}}{\partial A_1} + \frac{\partial\frac{P_1Y_1}{P_2Y_2}}{\partial\kappa} \frac{d\kappa}{dA_1} = \frac{\gamma}{1-\gamma} \frac{1}{\left[(1-v) + \frac{c_1}{A_1}(\kappa\chi)^{-\alpha}\right]^2} \\
&\quad \times \left(\left[1 - \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi}\right] \left(\frac{1}{A_1} + \alpha \frac{1}{\kappa} \frac{d\kappa}{dA_1}\right) (\kappa\chi)^{-\alpha} \frac{c_1}{A_1} \right. \\
&\quad \left. + \left\{ \frac{(1-v)^{\frac{1-\gamma}{\gamma}} \kappa\chi + \frac{\alpha c_2}{\beta A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa\chi)^{1-\alpha}}{\alpha} - \frac{\frac{\alpha c_2}{\beta A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa\chi)^{1-\alpha}}{\left[\alpha + (1-v)^{\frac{1-\gamma}{\gamma}}\right] + (1-\alpha)^{\frac{1-\gamma}{\gamma}} \frac{c_1}{A_1} (\kappa\chi)^{-\alpha}} \right\} \frac{\left[(1-v) + \frac{c_1}{A_1}(\kappa\chi)^{-\alpha}\right] \frac{c_2}{A_1^2}}{\beta(1-\kappa)^2 \chi^2} \right).
\end{aligned}$$

Because $\chi - \kappa\chi > \frac{1}{\beta} \frac{c_2}{A_1} \Leftrightarrow 1 - \frac{1}{\beta} \frac{c_2}{A_1} \frac{1}{(1-\kappa)\chi} > 0$ from (4.28), we have

$$\frac{d\frac{P_1Y_1}{P_2Y_2}}{d\chi} > 0 \text{ and } \frac{d\frac{P_1Y_1}{P_2Y_2}}{dA_1} > 0.$$

$$\text{Therefore, } \frac{d\frac{P_1Y_1}{P_2Y_2}}{dt} - \frac{d\frac{P_1Y_1}{P_2Y_2}}{d\chi} \frac{d\chi}{dt} + \frac{d\frac{P_1Y_1}{P_2Y_2}}{dA_1} \frac{dA_1}{dt} > 0.$$

When $A_1 \rightarrow \infty$ and thus $\chi \rightarrow \chi^{ss}$, from (A4.8),

$$\frac{P_1Y_1}{P_2Y_2} \rightarrow \left(\frac{P_1Y_1}{P_2Y_2}\right)^{ss} \equiv \frac{1}{1-v} \frac{\gamma}{1-\gamma}.$$

(2) [Case 1]

From (4.5) and (4.6), the income ratio of two production factors equals

$$\frac{RK}{WL} = \frac{\alpha}{1-\alpha} \frac{L_1}{L} \frac{K}{K_1} = \frac{\alpha}{1-\alpha} \frac{\lambda}{\kappa}. \quad (A4.9)$$

Since $\kappa=1$ in case 1,

$$\frac{RK}{WL} = \frac{\alpha}{1-\alpha} \lambda \text{ and } \frac{\partial \frac{RK}{WL}}{\partial \lambda} = \frac{\alpha}{1-\alpha}.$$

Because $\frac{d\lambda}{dt} > 0$ from Proposition 4.2 (2), we have $\frac{d\frac{RK}{WL}}{dt} = \frac{d\frac{RK}{WL}}{d\lambda} \frac{d\lambda}{dt} > 0$.

[Case 2] Because $\frac{RK}{WL} = \frac{\beta}{1-\beta} \frac{A_1}{A_2} \chi$ from (4.24), (4.25) and (A4.9), we have

$$\frac{d\frac{RK}{WL}}{d\chi} = \frac{\beta}{1-\beta} \frac{A_1}{A_2} > 0, \frac{d\frac{RK}{WL}}{dA_1} = \frac{\beta}{1-\beta} \frac{1}{A_2} \chi > 0, \frac{d\frac{RK}{WL}}{dA_2} = -\frac{\beta}{1-\beta} \frac{A_1}{A_2^2} \chi < 0.$$

$$\text{Hence, } \frac{d\frac{RK}{WL}}{dA_1} \frac{dA_1}{dt} + \frac{d\frac{RK}{WL}}{dA_2} \frac{dA_2}{dt} = \frac{\beta}{1-\beta} \frac{A_1}{A_2} \chi (g_1 - g_2) \geq 0.$$

$$\text{Therefore, } \frac{d\frac{RK}{WL}}{dt} = \frac{d\frac{RK}{WL}}{d\chi} \frac{d\chi}{dt} + \frac{d\frac{RK}{WL}}{dA_1} \frac{dA_1}{dt} + \frac{d\frac{RK}{WL}}{dA_2} \frac{dA_2}{dt} > 0.$$

[Case 3] From (A4.9), $\frac{RK}{WL} = \frac{\alpha}{1-\alpha} \frac{1}{\kappa}$.

Since $\frac{d\kappa}{dt} > 0$ from Proposition 4.2 (1), we have $\frac{d\frac{RK}{WL}}{dt} = \frac{d\frac{RK}{WL}}{d\kappa} \frac{d\kappa}{dt} < 0$.

When $A_1 \rightarrow \infty$ and thus $\chi \rightarrow \chi^{ss}$, from (4.28),

$$\frac{RK}{WL} \rightarrow \left(\frac{RK}{WL} \right)^{ss} \equiv \frac{\alpha + (1-\nu)^{\frac{1-\gamma}{\gamma}}}{1-\alpha}.$$

■

A4.6 Proof of Proposition 4.4

[Case 1] From (4.5), (4.6) and definitions of κ and λ , we have

$$\sigma = -\frac{\frac{R}{W}}{\frac{K}{L}} \frac{d\left(\frac{K}{L}\right)}{d\left(\frac{R}{W}\right)} = -\frac{\alpha}{1-\alpha} \frac{\lambda}{\kappa k^2} \frac{1}{\frac{d\left(\frac{\alpha}{1-\alpha} \frac{\lambda}{\kappa k}\right)}{dk}}. \quad (A4.10)$$

Since $\kappa = 1$,

$$\sigma = -\frac{\alpha}{1-\alpha} \frac{\lambda}{k^2} \frac{1}{\frac{d\left(\frac{\alpha}{1-\alpha} \frac{\lambda}{k}\right)}{dk}} = \frac{1}{1 - \frac{d\lambda}{dk} \frac{k}{\lambda}}. \quad (A4.11)$$

From (4.20), we have

$$k = A_1 \lambda \left\{ \frac{\frac{c_1}{A_1}}{\frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] - \left[(1-\nu) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \lambda} \right\}^{\frac{1}{\alpha}}.$$

Totally differentiating the above equation with respect to κ and λ , we have

$$\frac{d\lambda}{dk} = \frac{\lambda}{k} \frac{\alpha \left\{ \frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] - \left[(1-\nu) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \lambda \right\}}{\alpha \frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] + (1-\alpha) \left[(1-\nu) + (1-\alpha) \frac{\gamma}{1-\gamma} \right] \lambda}.$$

By substituting the above equation into (A4.11), we have

$$\sigma = \frac{\alpha \frac{1-\alpha}{1-\beta}}{(1-\alpha) + (1-\nu) \frac{1-\gamma}{\gamma}} \frac{(1-\beta) - \frac{c_2}{A_2}}{\lambda} + (1-\alpha). \quad (A4.12)$$

From (A4.12), we have

$$\frac{\partial \sigma}{\partial \lambda} = -\frac{\alpha \frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta}}{(1-\nu)+(1-\alpha) \frac{\gamma}{1-\gamma}} \frac{(1-\beta) - \frac{c_2}{A_2}}{\lambda^2} < 0,$$

$$\frac{\partial \sigma}{\partial A_2} = \frac{\alpha \frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta}}{(1-\nu)+(1-\alpha) \frac{\gamma}{1-\gamma}} \frac{c_2}{\lambda A_2^2} > 0.$$

Because $\frac{d\lambda}{d\chi} > 0$, $\frac{d\lambda}{dA_1} > 0$ and $\frac{d\lambda}{dA_2} = \frac{\frac{\gamma}{1-\gamma} \frac{1-\alpha}{1-\beta} \frac{c_2}{A_2^2}}{\frac{c_1}{A_1} \frac{\alpha}{\lambda} \left(\frac{\lambda}{\chi}\right)^\alpha + [(1-\nu)+(1-\alpha) \frac{\gamma}{1-\gamma}]} > 0$ from the proofs of

Lemma 4.2 and Proposition 4.2 (2) [case 1], $\frac{d\sigma}{d\chi} = \frac{\partial \sigma}{\partial \lambda} \frac{d\lambda}{d\chi} < 0$, $\frac{d\sigma}{dA_1} = \frac{\partial \sigma}{\partial \lambda} \frac{d\lambda}{dA_1} < 0$, and

$$\frac{d\sigma}{dA_2} = \frac{\partial \sigma}{\partial A_2} + \frac{\partial \sigma}{\partial \lambda} \frac{d\lambda}{dA_2} = \frac{\alpha \frac{1-\alpha}{1-\beta}}{(1-\alpha)+(1-\nu) \frac{1-\gamma}{\gamma}} \frac{c_2}{\lambda A_2^2} \frac{(1-\alpha) \left\{ [(1-\alpha)+(1-\nu) \frac{1-\gamma}{\gamma}] \lambda - \frac{1-\alpha}{1-\beta} [(1-\beta) - \frac{c_2}{A_2}] \right\}}{\alpha \frac{1-\alpha}{1-\beta} [(1-\beta) - \frac{c_2}{A_2}] + (1-\alpha) [(1-\alpha)+(1-\nu) \frac{1-\gamma}{\gamma}] \lambda}.$$

Because $\left[(1-\alpha) + (1-\nu) \frac{1-\gamma}{\gamma} \right] \lambda - \frac{1-\alpha}{1-\beta} \left[(1-\beta) - \frac{c_2}{A_2} \right] < 0$ from

$$\lambda < \lambda_1 = \frac{(1-\alpha) \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{\left[(1-\alpha) + (1-\nu) \frac{1-\gamma}{\gamma} \right] (1-\beta) \frac{A_2}{A_1}}, \frac{d\sigma}{dA_2} < 0.$$

Hence, $\frac{d\sigma}{dt} = \frac{d\sigma}{d\chi} \frac{d\chi}{dt} + \frac{d\sigma}{dA_1} \frac{dA_1}{dt} + \frac{d\sigma}{dA_2} \frac{dA_2}{dt} < 0$.

When $\chi \rightarrow 0$ and thus $\lambda \rightarrow 0$ from (4.20), $\sigma \rightarrow \infty$ from (A4.12). When $\chi \rightarrow \chi_1$ and thus

$$\lambda \rightarrow \lambda_1 = \frac{(1-\alpha) \left[(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{\left[(1-\alpha) + (1-\nu) \frac{1-\gamma}{\gamma} \right] (1-\beta) \frac{A_2}{A_1}} \text{ from the proof of lemma 4.1,}$$

$$\sigma \rightarrow \frac{(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - (1-\alpha) \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha}}{(1-\beta) \frac{A_2}{A_1} - \frac{c_2}{A_1} - \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha}} = \sigma_1 > 1 \text{ from (A4.12).}$$

[Case 2] From (4.5) and (4.6),

$$\sigma = \frac{\frac{R}{\bar{W}}}{\frac{K}{L}} \frac{1}{\frac{d(\frac{R}{\bar{W}})}{d(\frac{K}{L})}} = \frac{1}{0} = \infty.$$

[Case 3] From (A4.10) and $\lambda = 1$,

$$\sigma = -\frac{\alpha}{1-\alpha} \frac{1}{\kappa k^2} \frac{1}{\frac{d\left(\frac{\alpha}{1-\alpha} \frac{1}{\kappa k}\right)}{dk}} = \frac{1}{1 + \frac{d\kappa}{dk} \frac{k}{\kappa}}. \quad (A4.13)$$

From (4.28), we have

$$k = \left[1 + \frac{1}{\alpha}(1-v) \frac{1-\gamma}{\gamma}\right] \kappa k + \frac{1}{\beta} c_2 + \frac{1}{\alpha} \frac{1-\gamma}{\gamma} \frac{c_1}{A_1^{1-\alpha}} (\kappa k)^{1-\alpha}.$$

Totally differentiating the above equation with respect to κ and k , we have

$$\frac{dk}{d\kappa} = \frac{1}{k} \left\{ \frac{1}{\left[1 + \frac{1}{\alpha}(1-v) \frac{1-\gamma}{\gamma}\right] + \frac{1-\alpha}{\alpha} \frac{1-\gamma}{\gamma} \frac{c_1}{A_1^{1-\alpha}} (\kappa k)^{-\alpha}} - \kappa \right\}.$$

By substituting the above equation into (A4.13),

$$\sigma = \left[1 + \frac{1}{\alpha}(1-v) \frac{1-\gamma}{\gamma}\right] \kappa + \frac{1-\alpha}{\alpha} \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} \kappa^{1-\alpha} \chi^{-\alpha}. \quad (A4.14)$$

From (A4.14), we have

$$\frac{\partial \sigma}{\partial \chi} = -(1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} \frac{\kappa^{1-\alpha}}{\chi^{1+\alpha}},$$

$$\frac{\partial \sigma}{\partial \kappa} = \left[1 + \frac{1}{\alpha}(1-v) \frac{1-\gamma}{\gamma}\right] + \frac{(1-\alpha)^2}{\alpha} \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa k)^{-\alpha},$$

$$\frac{\partial \sigma}{\partial A_1} = -\frac{1-\alpha}{\alpha} \frac{1-\gamma}{\gamma} \frac{c_1}{A_1^2} \kappa^{1-\alpha} \chi^{-\alpha}.$$

From the proofs of lemma 4.2 and Proposition 4.2 (1) [case 3], we have

$$\frac{d\kappa}{d\chi} = \frac{\alpha \left[\frac{1}{\beta A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{1-\alpha} \right]}{\left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right\} \chi^2} > 0,$$

$$\frac{d\kappa}{dA_1} = \frac{\frac{\alpha c_2}{\beta A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{1-\alpha}}{\left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{-\alpha} \right\} A_1 \chi} > 0.$$

Hence,

$$\begin{aligned} \frac{d\sigma}{d\chi} &= \frac{\partial \sigma}{\partial \chi} + \frac{\partial \sigma}{\partial \kappa} \frac{d\kappa}{d\chi} = \frac{1}{\alpha \left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa k)^{-\alpha} \right\} \chi^2} \\ &\times \left(\left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] \kappa \chi + \frac{\alpha^2}{\beta} \frac{c_2}{A_1} + \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{1-\alpha} \right\} \left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] + (1-\alpha)^2 \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa k)^{-\alpha} \right\} \right. \\ &\quad \left. - \left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa k)^{-\alpha} \right\} \left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] \kappa \chi + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{1-\alpha} \right\} \right) \\ &> \frac{\alpha \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa \chi)^{1-\alpha}}{\left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa k)^{-\alpha} \right\} \chi^2} > 0, \end{aligned}$$

$$\frac{d\sigma}{dA_1} \frac{dA_1}{dt} = \left(\frac{\partial \sigma}{\partial A_1} + \frac{\partial \sigma}{\partial \kappa} \frac{d\kappa}{dA_1} \right) \frac{dA_1}{dt}$$

$$= \frac{\left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] + (1-\alpha)^2 \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa k)^{-\alpha} \right\} \frac{1}{\beta A_1} + \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa k)^{1-\alpha}}{\alpha \left\{ \left[\alpha + (1-\gamma) \frac{1-\gamma}{\gamma} \right] + (1-\alpha) \frac{1-\gamma}{\gamma} \frac{c_1}{A_1} (\kappa k)^{-\alpha} \right\} A_1 \chi} > 0.$$

Therefore, $\frac{d\sigma}{dt} = \frac{d\sigma}{d\chi} \frac{d\chi}{dt} + \frac{d\sigma}{dA_1} \frac{dA_1}{dt} > 0$.

When $\chi \rightarrow \chi_2$, from (A4.14),

$$\sigma \rightarrow \frac{(1-\beta) \left[\alpha + (1-\nu) \frac{1-\gamma}{\gamma} \frac{A_2}{A_1} + (1-\alpha)^2 \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right]}{(1-\beta) \left[\alpha + (1-\nu) \frac{1-\gamma}{\gamma} \frac{A_2}{A_1} + (1-\alpha) \left[\frac{c_2}{A_1} + \frac{c_1}{A_1} \frac{1-\gamma}{\gamma} \left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^\alpha \left(\frac{A_2}{A_1} \right)^{1-\alpha} \right] \right]} \equiv \sigma_2 < 1.$$

When $A_1 \rightarrow \infty$ and thus $\chi \rightarrow \chi^{ss}$, $\sigma \rightarrow 1$.

■

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