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2 **Validity of the stress inversion of orientation data from a dike swarm**
3 **with a radial-parallel pattern transition**
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18 Running title: Paleostress analyses of radial and parallel dike swarm
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20 **ABSTRACT**
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22 The fitting of a stress trajectory pattern predicted by a 2D elastic model to an
23 observed radial-parallel transitional dike pattern determines the orientation of far-
24 field stress and pressure in the central magma chamber. We found such a transitional
25 pattern in the Amakusa region, western Japan. On the other hand, the 3D dike
26 orientations can be inverted using Bingham distributions to determine stress axes,
27 stress ratios, and driving pressure ratios. From the Amakusa data, the 2D analysis
28 indicated roughly N-S extensional far-field stress. The 3D analyses conducted in the
29 parts of the dike swarm yielded horizontal maximum stress axes roughly parallel to
30 the local stress orientations predicted by the 2D model. Subsets obtained in distant

areas from the central magma chamber indicated a normal faulting stress regime with roughly N-S extension. So, both the 2D and 3D approaches led to consistent results. The 3D analyses indicated the declining frequency of the injection of highly overpressured magmas with distance from the chamber. Thus, we conclude the 3D analysis is useful to study dike swarms with transitional patterns. The paleostress determined from the Amakusa dikes suggests that the Shikoku basin subducted under Kyushu including the Amakusa region in the early middle Miocene. (200 words)

Keywords: dike, sill, elastic model, tectonic stress, magma pressure

Highlights

A dike swarm with a radial-parallel pattern transition was found in western Japan

Stress trajectory predicted by a 2D elastic model was fitted very well to the pattern

Stresses inverted from 3D dike orientations consistent with the trajectories

Magma overpressure declined with distance from the central magma chamber

The 3D stress inversion applied to the parts of radial dikes indicate local stresses

1. Introduction

Subsurface stress affects the spatial and orientation distribution of dilational fractures (Stevens, 1911; Iddings, 1914, p. 222), magma transport (Takada, 1994a), and eventually volcanic activities (Nakamura, 1977; Takada, 1994b). Conversely, the general trend of dikes indicates the maximum horizontal principal stress orientation at the time of magmatism (Evans, 1925; Anderson, 1942, p. 22). This approach has been so well developed that it is now possible to determine not only stress axes but also stress and driving pressure ratios from the 3D orientations of planar intrusions,

i.e., dikes and sills (Delaney et al., 1986; Bear et al., 1994; Jolly and Sanderson, 1997; Yamaji et al., 2010). In terms of principal stresses, the ratios are defined as $\Phi = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ and $p = (p_f - \sigma_3)/(\sigma_1 - \sigma_3)$, respectively, where p_f is local magma pressure and $\sigma_1 \geq \sigma_2 \geq \sigma_3$ (Table 1). The method is widely used to infer paleostress fields in the world (e.g., Martínez-Poza et al., 2014; Bhatt et al., 2019; Umino et al., 2020; Roquer et al., 2021).

On the other hand, dikes show characteristic, map-scale, radial and parallel patterns (Geikie, 1882, p. 555). Knopf (1936) mapped about 300 dikes and found a transition from radial to parallel patterns in the Spanish Peaks region, Colorado. Assuming an elastic plate with a circular hole, Odé (1957) explained the transition by the superposition of far-field stress and pressure from the central magma chamber. That is, dikes were thought to be formed along the trajectories of maximum horizontal stress (Fig. 1): Dike trends in the radial and parallel parts are thought to be determined mainly by pushing from the central magma reservoir and by far-field stress, respectively. Later, such transitional patterns have been found in other areas in the world and on other planets (e.g., Ernst et al., 2001). The 2D elastic model developed initially by Odé (1957) has been often used to determine paleostress orientations from dike patterns (e.g., Muller and Pollard, 1977; Baer and Reches, 1991; McKenzie et al., 1992; Koenig and Pollard, 1998).

Then, questions arise. What if 3D dike orientations in a swarm showing the radial-parallel transition are inverted? Are the solutions of such stress inversion useful? Are the solutions determined from dikes in the radial part meaningless or consistent with the far-field stress inferred from the parallel part? This paper aims to answer these questions. To this end, we mapped 250 planar intrusions in the Amakusa region, western Japan (Fig. 2).

The dike swarm in this region is useful to answer the questions. First, the theoretical stress trajectory pattern (Jaeger et al., 2007, p. 220) has two isotropic points (Fig. 1) where horizontal differential stress vanishes. The trajectories approach such a point from three directions (Fig. 1). No such isotropic point has been described in dike patterns as long as the present authors know, but we found it

in the Amakusa region. Second, the 2D model expects upright or steeply dipping dikes. However, the dikes in our study area had various dips and strikes even in a small portion of the swarm, having allowed us to invert 3D dike orientations to determine paleostresses and driving pressure ratios. So, the dikes in the region provided us with a rare opportunity to answer the questions.

In this paper, a 2D theoretical stress trajectory pattern was, first, fitted to the trends and spatial distributions of dikes in the Amakusa region to determine far-field stress. We devised the objective function for the fitting to take into account not only the trends but dip angles of dikes because planar intrusions had various dip angles in the region. Second, the 3D orientation data from dikes in the region were partitioned into subsets according to the locations where they were obtained in the field. And the stress inversion technique by Yamaji and Sato (2011) was applied to the subsets. As a result, the technique yielded the maximum horizontal stress orientations roughly parallel to the stress trajectories predicted by the 2D elastic model.

2. Data

2.1. Geological setting

We observed felsic planar intrusions in the Amakusa region at the junction of the SW Japan and Ryukyu arcs (Fig. 2). The intrusions were hosted by Cretaceous and Eocene, non-volcanic formations with a total thickness of ~7000 m which covered Cretaceous metamorphic rocks in the region (e.g., Saito et al., 2010) (Fig. 2). The deposits are composed mainly of shallow marine and non-marine alternating sandstone and siltstone, and folded with NNE-trending axes (e.g., Takai et al., 1997), and have dip angles usually at 20–30°. After the folding (Takahashi, 1965; Sakai, 1993; Takai et al., 1997; Ushimaru and Yamaji, 2020), the volcanic front migrated oceanward in the early Middle Miocene (e.g., Shinjoe et al., 2019), and felsic dikes, sills, and stocks intruded into the formations (e.g., Nagao 1922; Nōtomi, 1930; Toyohara and Hase, 1991; Hamasaki, 1997; Takai et al., 1997; Iwasaki et al., 1998).

2.2. *Felsic planar intrusions*

We measured the attitudes of 250 felsic planar intrusions in the Amakusa region, and found that they showed a lateral transition from radial to parallel pattern (Fig. 2). The intrusions were observed mainly along the coasts except for ~100 ones in the northwestern part of the region. The attitudes and locations of the planar intrusions where we observed them are listed in Table A1 (Appendix A). The planar intrusions had various dip angles. There were sills as well. Here, dikes are referred to as planar intrusions oblique to local bedding in the host rocks irrespective of the dips of the intrusions; and planar intrusions parallel to local ones are called sills. The composition of the planar intrusions were rhyolite, dacite, andesite, and quartz porphyry. Their thicknesses ranged from 0.03 to 12 m and the average thickness was 2.2 m.

A dozen of radiometric ages around 14 Ma have been reported from the felsic intrusions (Nagao et al., 1992; Hamasaki, 1996; Yamamoto et al., 1997; Miura and Ohira, 2012; Ohira et al., 2012). Shibata and Togashi (1975) obtained a K-Ar age of ~19 Ma, but this age is unreliable because they collected biotite from somewhat weathered granodiorite. There is no systematic age difference between felsic dikes, sills, and stocks. This suggests that all the felsic intrusions resulted from geologically contemporaneous magmatism at ~14 Ma. This is supported by the field observation that sills are often connected with dikes in the region (Figs. 3a, 3d).

We followed the criteria of Yamaji et al. (2010, Fig. 7) for data acquisition with a threshold length of 2 m. That is, the attitude of an intrusion was measured if the planar part of the intrusion in an outcrop was longer than this threshold. We measured the attitude of a planar part of a curved intrusion if the part was longer than 2 m on an outcrop. The formations hosting the intrusions in our study area had variously oriented fractures (Fig. 3b). The dilation of pre-existing fractures that intersected each other resulted in zig-zag dike walls (Figs. 3c, d). The planar part of such a wall gave an orientation datum as well if the part cropped out with a length greater than 2 m.

Field observations support that the felsic intrusions in the Amakusa region were

emplaced in pre-existing fractures. As an example, the orientations of fractures around the felsic intrusions in Fig. 3d are shown in Fig. 3e. It is a weak point of fracture analysis that fractures have uncertain ages. Thus, we measured fracture orientations in the intrusions and in their host rocks separately to infer the orientations of fractures older than the intrusions. The intrusions encompassed orthogonal systematic joints with ENE- and NNW trends. The host rocks had not only the systematic joints but N-S to NE-trending fractures (Fig. 3e). In addition, the cluster made by the orientations of latter fractures coincides with the one made by the planar intrusions. It means that they and the bedding planes were pre-existing fractures that were dilated to form the intrusions.

3. Fitting of stress trajectory pattern

The S_H -orientation of far-field stress was determined by fitting a stress trajectory pattern to the observed spatial and orientation distributions of planar intrusions in the Amakusa region.

3.1. Stress trajectory pattern

The theoretical stress trajectory pattern is given as the solution of a 2D elastic problem. That is, a horizontally extensive elastic plate with an internally pressured circular hole is assumed to be subjected to far-field stress with the principal values, $M \pm S$ (Fig. 4). M is the mean horizontal stress, and $2S$ is the horizontal differential stress. The hole is assumed to be filled with inviscid magma with the pressure, P . Let us use the polar coordinates (r, θ) whose origin is placed at the center of the magma chamber, and r is radial distance normalized by the radius of the hole, R . It means that the elastic body is subjected to the pressure, P , at $r = 1$. Local, horizontal, principal stresses are denoted by σ_H and σ_h . Kirsch (1898) first gave the analytical solutions for the problem. That is, the local stress tensor has the components (Mériaux and Lister, 2002; Jaeger et al., 2007, p. 220),

$$\sigma_{rr} = M + \frac{kS}{r^2} + S \left(1 - \frac{4}{r^2} + \frac{3}{r^4} \right) \cos 2\theta \quad (1)$$

$$\sigma_{\theta\theta} = M - \frac{kS}{r^2} - S \left(1 + \frac{3}{r^4}\right) \cos 2\theta \quad (2)$$

$$\sigma_{r\theta} = -S \left(1 + \frac{2}{r^2} - \frac{3}{r^4}\right) \sin 2\theta, \quad (3)$$

where non-dimensionalized magma overpressure is denoted as

$$k = (P - M)/S. \quad (4)$$

The σ_H -axis is perpendicular to the circular wall at $r = 1$, because inviscid magma is assumed to fill the hole. Let α be the angle made by the local σ_H -orientation and the radial coordinate (Fig. 4). Then, it is related to the stress components as

$$\alpha = \frac{1}{2} \tan^{-1} \left(\frac{2\sigma_{r\theta}}{\sigma_{rr} - \sigma_{\theta\theta}} \right). \quad (5)$$

Substituting Eqs. (1)–(3) into (5), we have

$$\alpha = \frac{1}{2} \tan^{-1} \left[\frac{-(r^4 + 2r^2 - 3) \sin 2\theta}{kr^2 + (r^4 - 2r^2 + 3) \cos 2\theta} \right]. \quad (6)$$

The angle is expressed in this equation only in terms of non-dimensional quantities. This equation also says that the stress trajectory pattern is determined only by k and that the pattern does not depend on elastic modulus.

3.2. Pattern fitting

Since the planar intrusions have various dip angles in the Amakusa region, we devised an inversion scheme to fit the stress trajectory pattern predicted by Eq. (6) to the observed one for determining the S_H -orientation and k . This orientation is denoted by the angle, ψ , which is measured counterclockwise from due east (Fig. 4). The pattern generally has paired isotropic points on both sides of the central magma chamber, where horizontal differential stress vanishes, i.e., $\sigma_H = \sigma_h$. The locations of the points are described in Appendix B.

Let Δ_i be the misfit angle of the trend of the i th planar intrusion from the theoretical orientation, and d_i be the dip angle of the intrusion. Then, the pattern is said to be fitted to the data if the misfit angles are small. We used the dip angles as the weights for the fitting. That is, we minimized the objective function for the fitting,

$$F(k, \psi) = \sum_{i=1}^N \Delta_i \sin d_i, \quad (7)$$

where N is the number of data and $\sin d_i$ is the weight (Appendix C).

The following three lines of evidence made us propose the location of the central magma chamber at the Tomioka Peninsula in the northwestern part of our study area (Fig. 5). First, felsic stocks cluster in and around the peninsula (e.g., Takai et al., 1997). Second, the number density of planar intrusions decreases with distance from the peninsula. Third, the variation in the attitudes of such intrusions also decreases with the distance (Fig. 2): Such a diminishing trend suggests decreasing magma pressure (e.g., Jolly and Sanderson, 1997). Planar intrusions form a parallel dike swarm away from the peninsula. Taking into account the spatial extent of the onshore and offshore outcrops of the stocks (Otsuka, 1970), we assumed that $R = 2.5$ km, and placed the center of the magma chamber at the tombolo ($32^\circ 31' 15''\text{N}$, $130^\circ 02' 05''\text{E}$) (Fig. 5).

The optimal stress trajectory pattern was determined by exhaustive search (Fig. 5). As a result, we obtained the optimal values $k = 24.5$ and $\psi = 3.3^\circ$, the latter of which indicates an E-W trending S_H -axis ($\text{N}86.7^\circ\text{E}$). Since the basin of the function value is elongated subparallel to the k -axis (Fig. 6), the uncertainty of the optimal k is greater than that of the optimal ψ . The contours for $k \lesssim 20$ near the optimal point in Fig. 6 indicate a steep slope. The bootstrap solutions in Fig. 6 suggest the error ranges from ca. 20 to 140 for k and from ca. 0° to 25° for ψ . That is, the optimal S_H -orientation was $\text{N}86.7^\circ\text{E}$, but was possibly between $\text{N}65^\circ\text{E}$ and $\text{N}90^\circ\text{E}$, i.e., N-S to NNW-SSE extension.

As far as we know, few previous studies described dike orientations around isotropic points, especially those on the far side of the point, but we found them in the Amakusa region. The pattern made by the trends of planar intrusions shows a ‘whirl’ around the point to the south of the Tomioka Peninsula (Fig. 5). The trends of planar intrusions make a sharp turn around the point. We also found some intrusions roughly parallel to S_H on the far side of the point. The whirl is located ~ 15 km south of the central reservoir, where is close to the isotropic point of the stress

trajectory pattern predicted by the 2D elastic model.

3.3. Determination of k from isotropic point

The fact is that a k value consistent with the optimal one is obtained from the position of the isotropic point of the stress trajectory pattern. The whirl is located ~15 km south of the central reservoir. Combining $r_1 = (15 \text{ km})/R$ and Eq. (B1) we obtain $k = 34.1$, which is close to the optimal k compared to the error range of the optimal one.

4. Stress analyses using 3D Bingham distributions

4.1. Method

The 3D orientations of planar intrusions in the Amakusa region were inverted to determine paleostresses by fitting Bingham distributions using the method of Yamaji and Sato (2011). We used the software by Yamaji (2016) for the inversion. If observed dikes were formed under a few different stress conditions, the software separates the stresses by fitting a mixed Bingham distribution to their orientations. The optimal numbers of stresses were determined by minimizing the Bayesian information criterion (BIC) (Yamaji and Sato, 2011). The inversion results did not necessarily have horizontal stress axes. The σ_H -orientations were computed using the technique of Lund and Townend (2007) to objectively determine the σ_H -orientation from the inversion results.

The inverse method can constrain driving pressure ratios as well (Yamaji et al, 2010). That is, assuming that magma pressure had temporal fluctuations, the method determines a driving pressure index (DPI) which is the 95 percentile of driving pressure ratios during the fluctuations (Faye et al., 2018). Note that the pressure, in this case, does not refer to P but to the local pressure, p_f , of magma that injected fractures to form the planar intrusions in the area where dike orientations were observed. A DPI value close to zero indicates that p_f barely exceeded σ_3 . If the index is close to 1, p_f often approached σ_1 .

4.2. Results

The 3D orientation data from the Amakusa region were divided into eight subsets from A through H to investigate the spatial variation of the stress condition during the magmatism (Fig. 7). We also refer A through H to the areas where the subsets were obtained. As a result, a total of fourteen stresses were obtained from eight subsets (Figs. 7, A1). BIC values indicate that all the subsets yielded two stresses except for the subsets, C and E, each of which was explained by a stress. The poles to dikes and sills in the areas A, F, and H had distinctive clusters, but the orientations of dikes and sills in other areas made clusters indistinguishable from each other.

We also inverted the subsets A through H excluding sill orientations. As a result, it was found that each of the subsets excluding the sill orientations was explained by a stress except for the subset G which yielded two stresses (Figs. 8, A2).

4.3. Interpretations

4.3.1. Sills

The following lines of evidence suggest that sill orientations are merely the indicators of the mechanical weakness of bedding planes rather than of the stress condition at the time of their formation. First, we observed dikes and sills connected in the Amakusa region (Fig. 3a, d). The connection is explained by the tendency of dikes to become sills when they meet a layer boundary with a sharp contrast in the material toughness of the layers (e.g., Gudmundsson et al., 2011). Second, dike orientations in each of the subsets can be explained by a stress with the σ_H -orientation concordant with the stress trajectories determined by the pattern fitting (Fig. 9c). Although the subset G yielded two stresses, they had similar σ_H -orientations. Therefore, we do not address the stresses obtained from both dikes and sills (Fig. 7, 9a) but address those from only dikes (Fig. 8, 9c) for comparing the results of the 2D and 3D analyses.

4.3.2. Far-field stress

Since the area H is distant from the central magma chamber, the stress determined from dikes in the area is expected to approximate far-field stress. Dike orientations in the area showed a normal faulting stress regime with N-S extension and an intermediate stress ratio ($\Phi = 0.40$) (Fig. 8). The σ_H -orientation of this stress is in the error range of ψ (§3.2). In addition, the σ_H -orientations determined by the 2D analyses of other data subsets were approximately parallel also to the local σ_H -orientations predicted by the optimal stress trajectory pattern (Fig. 9c).

4.3.3. DPI

DPI determined from the 3D orientations of planar intrusions reflects the spread of the orientations and indicates how frequently overpressured magmas were supplied to the areas where the intrusions were observed (Faye et al., 2018). However, the high DPIs determined from the subset E in Figs. 9b and 9d do not mean that the planar intrusions in the area E were formed from extremely overpressured magmas. Stress trajectories had sharp turns around the isotropic point. So, the involvement of the point in the area and of sill orientations in the subset inevitably resulted in the large spread of the attitudes of the intrusions. A spatially uniform stress condition assumed in the 3D analyses did not hold there. Thus, we are not concerned with the DPIs from the subset anymore.

The DPI values except for those from the subset E showed a negative correlation with the distance of the areas from the magmatic center at Tomioka (Figs. 9b, 9d). The correlation coefficients between the DPI values and the radial distances calculated from the orientations of dikes and sills (Fig. 9b) and those of only dikes (Fig. 9d) were -0.56 and -0.85 , respectively. The exclusion of the sill orientations improved the correlation. This result indicates that the frequency of highly overpressured magmas decreased with the distance from the central magma reservoir, being consistent with the assumption of the 2D analysis.

5. Discussion

5.1. Validity of the 3D analyses

The validity of the 3D stress analysis for radial dike swarms has been an open question. The results of this study demonstrate the validity. First, the local σ_H -orientations determined by the 3D stresses were consistent with the stress trajectory inferred from the 2D model even not only in the radial part but in the parallel part of the swarm (Fig. 9c). Second, the declining frequency of the supply of the highly overpressured magmas with the distance from the central magma chamber (Fig. 9d) was consistent with the physical picture prerequisite for the 2D analysis. In addition, the 3D analyses in the parallel part of the dike swarm could determine a normal faulting regime and the stress ratio of 0.40. The 2D analysis can determine neither stress regime nor stress ratio.

It is a matter of course that many planar intrusions should be observed in narrow areas in the radial part of a dike swarm to understand the tectonic or volcanological implications of the clusters made by the dike orientations. The preliminary study by Ushimaru and Yamaji (2020) dealt with a part of the radial dikes (subset B), but the significance of the inversion result was uncertain.

5.2. Validity of the 2D elastic model

The 2D analysis is based on a simplistic model in which uniform far-field stress is disturbed only by the magmatic pressure acting on the wall of a cylindrical pit (Fig. 1). An actual rock mass has mechanical heterogeneity. In addition, the rock mass hosting a radial dike swarm may have been loaded by a volcanic edifice at the surface to disturb the stress field in the mass (Galgana et al., 2011). However, there is no way to infer the lateral distribution of the topographic load of the volcano, because erosion has removed the superstructure of the magma system. The pressure in the central magma chamber may have temporal variations, though the model assumes a constant pressure during all dikes were formed around the chamber. So, the applicability of the model is a matter of question.

The consistency of the results of the 2D and 3D analyses (Fig. 9) suggests the validity of the 2D model. In addition, the validity is supported by the concordant k values obtained from the fitting of the overall pattern made by stress trajectories (§3.2) and from the location of the isotropic point (§3.3). This coincidence is not trivial because the subset E was only a small part of the entire subsets with which the fitting was done.

However, the P value determined by the 2D analysis was abnormally large as follows. The far-field stress was estimated to be of normal-faulting stress regime (§4). Let σ_V be the overburden pressure which is approximately equal to σ_1 . Combining the definitions of stress ratio and $M = (\sigma_2 + \sigma_3)/2$ and $S = (\sigma_2 - \sigma_3)/2$, we have $\Phi = 2S/(\sigma_V - M + S)$ (Fig. 10a). It follows from Eq. (4) that $P = Sk + M$. Combining these equations, we obtain

$$S = (\sigma_V - M)\Phi/(2 - \Phi) \quad (8)$$

and

$$P = Sk + M = \left(\frac{\sigma_V - M}{2 - \Phi}\right)\Phi k + M \approx -5.1M + 6.1\sigma_V, \quad (9)$$

where the approximation is made using the optimal values, $k = 24.5$ and $\Phi = 0.40$. Since $\sigma_V \approx \sigma_1$, the mean horizontal stress, M , must be smaller than σ_V , i.e., $0 < M/\sigma_V < 1$. It follows from Eq. (9) that this range of the M/σ_V ratio corresponds to the P/σ_V ratio of 1.0–6.1. That is, magma pressure is greater than overburden pressure. This argument applies to the case with the lower bound of the error range of k at ~ 20 as well (Fig. 6). A P/σ_V ratio greater than 1 is geologically unsound because the magma chamber with pressure greater than the overburden cannot be stable for a geologic time scale during which a dike swarm is formed except for occasional volcanic eruptions. Accordingly, the 2D elastic model overestimated P in the Amakusa case. There are, indeed, subhorizontal planar intrusions which suggest the occasional approach of magma pressure to overburden pressure: Such intrusions are common in the area B in the vicinity of the magmatic center (Fig. 7). However, it seems unsound that pressure in the center was kept several times the overburden.

The following argument also explains the overestimation. Since the 2D model assumes the linear elasticity for the rock mass around the central magma chamber, let us assume the biaxial state of stress (Zang and Stephansson, 2010, §5.2), i.e., $\sigma_V/3 = (\sigma_H + \sigma_h)/2$ which is equivalent with

$$\sigma_1/3 = M \quad (10)$$

in the present case. It follows from Eq. (4) that $P/\sigma_1 = (Sk + M)/\sigma_1$. Eliminating M and S using this equation and Eqs. (8) and (10) we obtain

$$\frac{P}{\sigma_1} = \frac{2\Phi k - \Phi + 2}{3(2 - \Phi)}. \quad (11)$$

The right-hand side of this equation as a function of k and Φ is shown in Fig. 10b. Substituting the optimal values, $k = 24.5$ and $\Phi = 0.40$, into Eq. (11), we get $P/\sigma_1 = 4.4$.

The model of Mériaux and Lister (2002) attempted to solve the overestimation of P . They took fracture mechanics into account to predict dike propagation oblique to the stress trajectories of the 2D model. As a result, they succeeded in reducing k and P by a factor of $\sim 1/5$ to $1/2$. Indeed, their model makes P down to a reasonable range for the Amakusa case.

However, their model seems unlikely for the Amakusa case, because the model assumes an intact rock mass in which dikes are formed. Actual rocks contain a number of variously oriented fractures. Such variation is evidenced by the observed spread of dike orientations even in a small area (Fig. 2) and by the pre-existing fractures (Fig. 3e). When fluid is injected into such a fractured medium, fluid travels in a zigzag pattern along intersecting pre-existing fractures. However, the traveling route is roughly and preferentially along a stress trajectory (Li et al., 2020). This physical picture explains the existence of planar intrusions on the far side of the isotropic point from the central magma chamber (Fig. 11). Magma plumbing only along stress trajectories cannot explain those intrusions, because the trajectories on this side were not connected with the chamber.

Is the 2D elastic model applicable to such a fractured rock mass? The following arguments suggest an answer to this question. It is known that the overall mechanical

property of a rock mass involving variously oriented fractures is approximated effectively by linear elasticity (e.g., Grechka and Kachanov, 2006). The effective elastic moduli of such a fractured body may be different from those of an intact body, but the equations of stress trajectory pattern (Eqs. 1–3) do not involve the moduli. It means that the 2D model is applicable to a fractured rock mass. The lithological heterogeneity of host rocks can disturb the stress field in the host, but dike trends in the Amakusa region do not show refraction at the boundaries between Cretaceous and Eocene formations (Fig. 5). In conclusion, it is an open problem why P was significantly overestimated, though the 2D elastic model was so successfully applied to the Amakusa dikes that the k value determined from the overall stress trajectory pattern was consistent with the value from the location of the isotropic point.

5.3. Implications for regional tectonics

The dike pattern with a radial-parallel transition in the Amakusa region is described for the first time in this paper. Felsic dikes in the Amakusa region have been regarded as an NNE-trending parallel dike swarm (Yamamoto, 1991) because several thick altered dikes with the trends to the south of Tomioka (Fig. 2) were famous for yielding ceramic clay (Ueno, 1952; Togashi, 1975). Yamamoto (1991) argued that the formation of the NNE-trending dikes in early middle Miocene was accompanied by rift initiation in the Okinawa Trough—an active back-arc rift of the Ryukyu arc off Amakusa (Fig. 12). However, Ushimaru and Yamaji (2020) and the present study deny this hypothesis.

Our results suggest that the Amakusa region belonged not to the Ryukyu arc but to the SW Japan arc during the early middle Miocene magmatism. We found that the felsic dikes were formed under regional N-S extensional stress. This is roughly perpendicular to the SW Japan arc, which was subjected to arc-perpendicular extension around the Median Tectonic Line (MTL) and to arc-parallel extension in the fore- and backarc regions. Such a triple-zoned stress field in the arc is evidenced by the trends of early middle Miocene dike swarms and ore veins (Yora, 1967; Ishii, 1973; Takada, 1987; Umehara et al., 1991; Tsunakawa, 1986; Yamamoto, 1991;

Miyazaki, 2007; Aoya, 2009; Haji and Yamaji, 2020, 2021). The Amakusa region is just on the northwestern side of the extension of MTL, and was sandwiched by the zones of arc-perpendicular dike swarms in southern and northwestern Kyushu (Fig. 12) (Sawada and Kitagawa, 1961; Teshima and Yamamoto, 1972; Matsumoto and Takahashi, 1987; Yamamoto and Ushijima, 1994; Yamamoto and Yamamoto, 1995; Tonai et al., 2008). In the Yakushima Island to the south of Kyushu, most quartz veins contemporaneous with the intrusion of middle Miocene granite have arc-perpendicular trends (Anma, et al., 2014). Thus, the Amakusa region belonged to the SW Japan arc in the early middle Miocene.

The belonging of the Amakusa region implies that the Shikoku basin—the northeastern part of the Philippine Sea plate—is subducted under Kyushu including the Amakusa region in the early middle Miocene. This plate configuration is consistent with the occurrence of high-magnesium andesite along the Median Tectonic Line in the SW Japan arc (e.g., Tatsumi et al., 2001) and in the Amakusa region (Nagao et al., 1992) (Fig. 12). The melting of a very young subducting slab is thought to bring about such andesites (e.g., Kimura et al., 2005). That is, ocean floor spreading in the Shikoku basin started in the Late Oligocene and stopped simultaneously in the early middle Miocene with the felsic magmatism in the Amakusa region (Okino et al., 1994). Several researchers inferred that the Shikoku basin was subducting under not only the SW Japan arc but the northern Ryukyu arc in early middle Miocene (e.g., Underwood and Pickering, 2018; Tatsumi et al., 2020).

6. Conclusions

Both the 2D and 3D stress analyses of dike orientations are questionable. The former is based on a simplistic model, and the latter assumes a large variation in the orientations of pre-existing planes of weakness in the host rocks. The present paper has documented an example for their validity using the orientation data from early middle Miocene felsic dikes in the Amakusa region at the junction of the SW Japan and Ryukyu arcs. The consistent results from the analyses demonstrate the validity,

though the 2D model predicted abnormally high pressure in the central magma chamber. The consistency includes not only stress orientations but the negative correlation of the distance from the chamber and the DPIs estimated through the 3D analyses. The validity of the 2D model is supported by the coincidence of the k values estimated by the fitting of the stress trajectory pattern and by the distance of the isotropic point from the chamber.

Now, we can answer the questions raised in the introduction. That is, the 3D analyses of dike orientations in the parts of a swarm showing the radial-parallel transition results in local stress conditions in the swarm. The result from the parallel part of the swarm is consistent with the far-field stress inferred by the 2D analysis.

The dikes in the Amakusa region indicated regional arc-perpendicular extensional stress, which suggests that the region belonged to the SW Japan arc rather than the Ryukyu arc in the early middle Miocene.

CRedit authorship contribution statement

Kentaro Ushimaru: Data curation, Formal analysis, Funding acquisition, Investigation, Writing draft. **Atsushi Yamaji:** Conceptualization, Investigation, Methodology, Supervision, Writing-review.

Acknowledgments

The first author acknowledges the Amakusa Marine Biological Laboratory, Kyushu University for kind facilities in the field. The field surveys by the first and second authors were supported partly by a grant-in-aid from the Fukada Geological Institute and by JSPS KAKENHI Grant Number 15H02141, respectively.

Appendix A. Supplementary data

Supplementary data to this article can be found online at

<https://doi.org/10.1016/j.jsg.2021.104485>.

Appendix B. Isotropic points

The locations of the isotropic points of a stress trajectory pattern (Fig. 1) can be computed as follows. The points are located on the line defined by $\theta = \pm 90^\circ$ (Fig. 4). Since the locations are symmetric for the coordinate origin, we consider the points only on the ray, $\theta = 90^\circ$. Since the off-diagonal component in Eq. (3) vanishes along this ray; and Eqs. (1) and (2) indicate the principal stresses, $[(M - S)r^4 + S(k + 4)r^2 - 3S]/r^4$ and $[(M - S)r^4 - Skr^2 + 3S]/r^4$. Therefore, we have $\sigma_H - \sigma_h = |2S[-r^4 + (k + 2)r^2 - 3]|/r^4$. This difference vanishes at the isotropic points, the radial coordinates of which is given by solving $\sigma_H - \sigma_h = 0$. As a result, we obtain

$$r_1, r_2 = \frac{1}{2}\sqrt{2k + 4 \pm 2\sqrt{k^2 + 4k - 8}}, \quad (\text{B1})$$

where $r_1 \geq r_2$. When $k < 2(\sqrt{3} - 1) \approx 1.46$, both the solutions, r_1 and r_2 , are complex numbers. Then, the stress trajectory pattern has no isotropic point. When $k = 2(\sqrt{3} - 1)$, we have $r_1 = r_2 \approx 1.32$. When $2 \leq k < 2(\sqrt{3} - 1)$, both the solutions are real (Fig. B1). In this case, the stress condition along the line segment, $r_2 > r > 1$ and $\theta = 90^\circ$, is extensional. r_2 decreases with increasing k , and reach 1 when $k = 2$. Since the isotropic point cannot be in the central magma chamber, r_2 must be greater than 1. When $k \geq 2$, r_2 is not a geologically significant solution.

Appendix C. Weighting function

The object function (Eq. 7) involves $\sin d$ as the weighting function for the pattern fitting, where d is the dip angle of a planar intrusion. This function was chosen to take into account not only upright dikes but planar intrusions with various dip angles not to count the latter ones out. The trend of a gently dipping dike has a measurement error larger than that of a steeply dipping one. And, the uncertainty of

the trend of a horizontal one is 360° . It means that such a horizontal one is useless for the pattern fitting. So, the value of the weighting function must vanish when $d = 0^\circ$, and be a monotonously increasing function of d .

It is reasonable that the weighting function is inversely proportional to ϕ , which is the error of a dike trend (Fig. C1a). The following argument guarantees that $1/\phi$ is approximately proportional to $\sin d$ (Fig. C1b). Suppose that the pole to a dike has a representative angular measurement error of a , which is assumed to be a constant. This angle is as small as a few degrees, and the uncertainty of the pole position is depicted by the circular area centered by P on the lower hemisphere in Fig. C1a. The spherical triangle APQ in the figure has a right angle at Q because the great circle through A is tangent to the circle at Q. The sine rule of this spherical triangle is written as $\sin d / \sin 90^\circ = \sin a / \sin \phi$. It follows that $\phi = \sin^{-1}(\sin a / \sin d)$. Now, we define the function,

$$f(d) \equiv \frac{a}{\phi} = \frac{a}{\sin^{-1}(\sin a / \sin d)} \quad (\text{C1})$$

which is inversely proportional to ϕ , and monotonously increases to $f(90^\circ) = 1$. This function is approximately equal to $\sin d$ except for the interval, $0 \leq d < a$, where $f(d)$ has complex values (Fig. C1b). The width of this interval, a , is as small as a few degrees. Therefore, $\sin d$ is useful for the weighting function.

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Table 1: List of symbols.

k	non-dimensional magma pressure
M	mean horizontal stress
P	magma pressure in the central magma chamber
p	driving pressure ratio
p_f	local magma pressure
r	radial distance from magmatic center normalized by R
R	radius of the central magma chamber
S	half horizontal differential stress
S_H, S_h	maximum and minimum horizontal principal stresses of far-field stress
α	the counterclockwise angle of σ_h -axis measured from the radial direction
θ	angular coordinate measured from the S_H -axis
$\sigma_1, \sigma_2, \sigma_3$	principal stresses ($\sigma_1 \geq \sigma_2 \geq \sigma_3$)
σ_H, σ_h	maximum and minimum horizontal principal stress of local stress
σ_v	overburden pressure
Φ	stress ratio
ψ	the angle indicating the S_H -orientation

Figure Captions

Fig. 1. Schematic illustration of the stress trajectory pattern around a cylindrical magma chamber with pressure, P . The horizontal maximum and minimum principal stresses of far-field stress are denoted by S_H and S_h . Stars indicate the singular points of the stress field, termed the isotropic points, where horizontal differential stress vanishes. The points are the triple junctions of the trajectories.

Fig. 2. Lower-hemisphere, equal-area projections of the poles to planar intrusions in

the areas enclosed by polygons in geological map of the Amakusa region. Open and solid circles in the projections indicate the poles to sills and dikes, respectively. The frequency distribution of bedding poles is shown by contours in each diagram. N indicates the number of measurements. Geological map after Saito et al. (2010), in which thin planar intrusions are omitted.

Fig. 3. Photographs of felsic planar intrusions and fractures in the Amakusa region. (a) A dike branching from a sill. (b) A representative outcrop of the formations that host the intrusions. Joints with various orientations on an Eocene siltstone bedding plane. The dike width is 0.9 m. (c) A dike with a zig-zag wall in Cretaceous siltstone at the hammer. The planar parts of the wall were parallel to joints. (d) A drone image of dikes and sills deflected at bedding planes and fractures in Eocene alternating sandstone and siltstone. Black arrows indicate dike-parallel fractures. (e) Lower hemisphere, equal-area projections of poles to fractures, bedding planes, and the walls of the intrusions observed at and in the vicinity of the outcrop of (d). The fractures in the host rock and the felsic intrusions were measured separately to investigate the presence of pre-existing fractures. Here we measured the planar part of the walls of dikes and sills with a threshold length of 50 cm. Fracture sets with similar orientations are distinguished by blue, green and red ovals. N indicates the number of measurements.

Fig. 4. Schematic illustration of the 2D elastic model for dike trends. An elastic plate with infinite horizontal extension is subjected to the far-field stress, $M \pm S$, and magma pressure, P , the latter of which is exerted at the circumference of a circular hole with a radius of R . The origin of the polar coordinates (r, θ) is placed at the center of the circular magma chamber. The orientation of the S_H -axis is denoted by ψ . Local principal stresses are denoted by σ_H and σ_h , i.e., the maximum and minimum horizontal compression. The orientation of the former is indicated by α , and is measured counterclockwise from the radial direction. Stars indicate the isotropic points.

Fig. 5. Map showing the stress trajectory pattern (smooth curves) in the Amakusa region that best fits the observed trends of planar intrusions (bars). Their dip angles are indicated by the lengths of the bars. A circle centered by a diamond depicts the assumed central magma chamber. The isotropic point is indicated by a star. See Fig. 2 for the legend of geological map patterns.

Fig. 6. Contours of $F(k, \psi)$. The optimal and 1,000 bootstrap solutions are indicated by a star and circles, respectively.

Fig. 7. Paleostresses (Lower-hemisphere, equal-area projections) determined by fitting Bingham distributions to the 3D orientation data from planar intrusions in the areas A through H, each of which is enclosed by a polygon. When two stresses were determined, the memberships of the planar intrusions to the stresses are indicated by the varying colors of the poles to the intrusions. Red, green, and blue lines enclosing stress axes in the lower-hemisphere, equal-area projections depict the 95% confidence ellipses of the principal orientations. N indicates the number of measurements. See Fig. 2 for the legend of geological map patterns.

Fig. 8. Paleostresses (Lower-hemisphere, equal-area projections) determined by fitting Bingham distributions to the 3D orientation data only from dikes in the areas A through H. N indicates the number of measurements. See Fig. 2 for the legend of geological map patterns and Fig 7 for details about the lower-hemisphere, equal-area projections.

Fig. 9. (a, c) Lower-hemisphere, equal-area projections showing the 95% confidence ellipses of the stress axes determined from the 3D orientations of (a) dikes and sills and (c) only dikes. Paired solid arrows in (a) and (c) indicate the orientation of the

maximum horizontal stress determined from the subsets in Fig. 7 and 8, respectively. The stress trajectory pattern fitted to their trends is shown by gray lines. See Fig. 2 for the legend of geological map patterns. Dashed lines indicate the error range of the S_H -orientation determined by the pattern fitting: The extensions of the lines and the bold line meet at the center of the magma chamber depicted by a diamond. The isotropic point is represented by a star. (b, d) The driving pressure indices (DPI) evaluated from the subsets in Fig. 7 and 8. If two stresses were detected from a data subset, the corresponding two DPIs are plotted for the subset. The length of a horizontal bar depicts the range of distances from the magmatic center. Thick grey lines are the regression lines determined from DPIs excluding the data of subset E.

Fig. 10. (a) Schematic Mohr diagram for the normal faulting regime of stress. (b) Contours of P/σ_1 evaluated by Eq. (11). The domain in the range, $0 < P/\sigma_1 < 1$, is hatched. The optimal k and Φ values determined from the Amakusa data are indicated by an open circle.

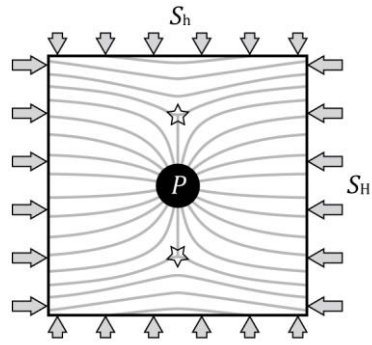
Fig. 11. Schematic illustration of the magma plumbing system through pre-existing fractures in a radial dike swarm. Since the stress field predicted by Eqs. (1)–(3) has orthorhombic symmetry, only a quadrant is shown here. Short bars indicate σ_H -orientations. Dikes parallel or subparallel to those orientations are dominant, but there are dikes at high angles to the orientations especially around the central magma chamber due to the frequent supply of overpressured magmas. The isotropic point is indicated by a star.

Fig. 12. The maximum horizontal stress orientations inferred from middle Miocene dikes (solid bars) and mineral veins (open bars) in the SW Japan and northern Ryukyu arcs. Reddish ovals indicate regions where middle Miocene high-magnesium andesites occur (Ishizaka and Carlson, 1983; Nagao et al., 1992; Shiraki et al., 1995). MTL: Median Tectonic Line. KPR: Kyushu-Palau ridge which defines the western border of the Shikoku basin. OT: Okinawa Trough. The references for the data of

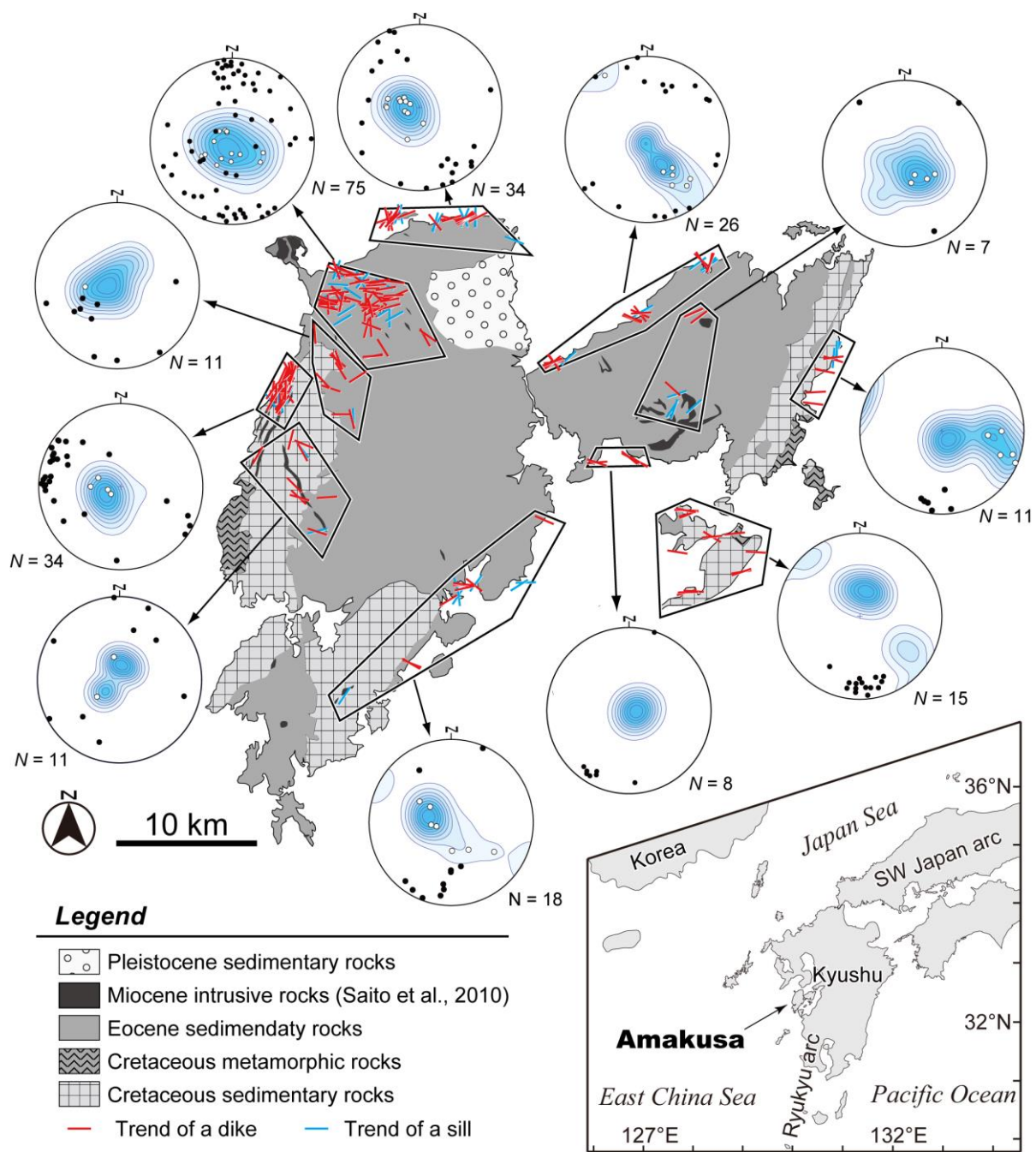
dikes and veins are 1: Sawada and Kitagawa (1961); Matsumoto and Takahashi (1987); 2: Matsui et al. (1989); 3: Teshima and Yamamoto (1972); 4: Yamamoto and Ushijima (1994); 5: Yamamoto and Yamamoto (1995); 6: Tonai et al. (2008); 7: Anma et al. (2014); 8: Sato and Haji, (2021); 9: Yamaji and Sato (2011); 10: Umehara et al. (1991); Miyazaki (2007); Aoya (2009); 11; Murakami et al. (1989); 12: Tatsumi (1983); 13: Haji and Yamaji (2021); 14: Sawai and Sato (1989); 15: Ishii (1973); 16: Yora (1967); 17: Haji and Yamaji (2020); 18: Takada (1987). The bathymetric contours are based on the map of Japan and its surroundings 1:5,000,000 published by the Geospatial Information Authority of Japan.

Fig. B1. The graphs of r_1 and r_2 (Eq. B1) are tangent to the dotted line through the point indicated by the open circle.

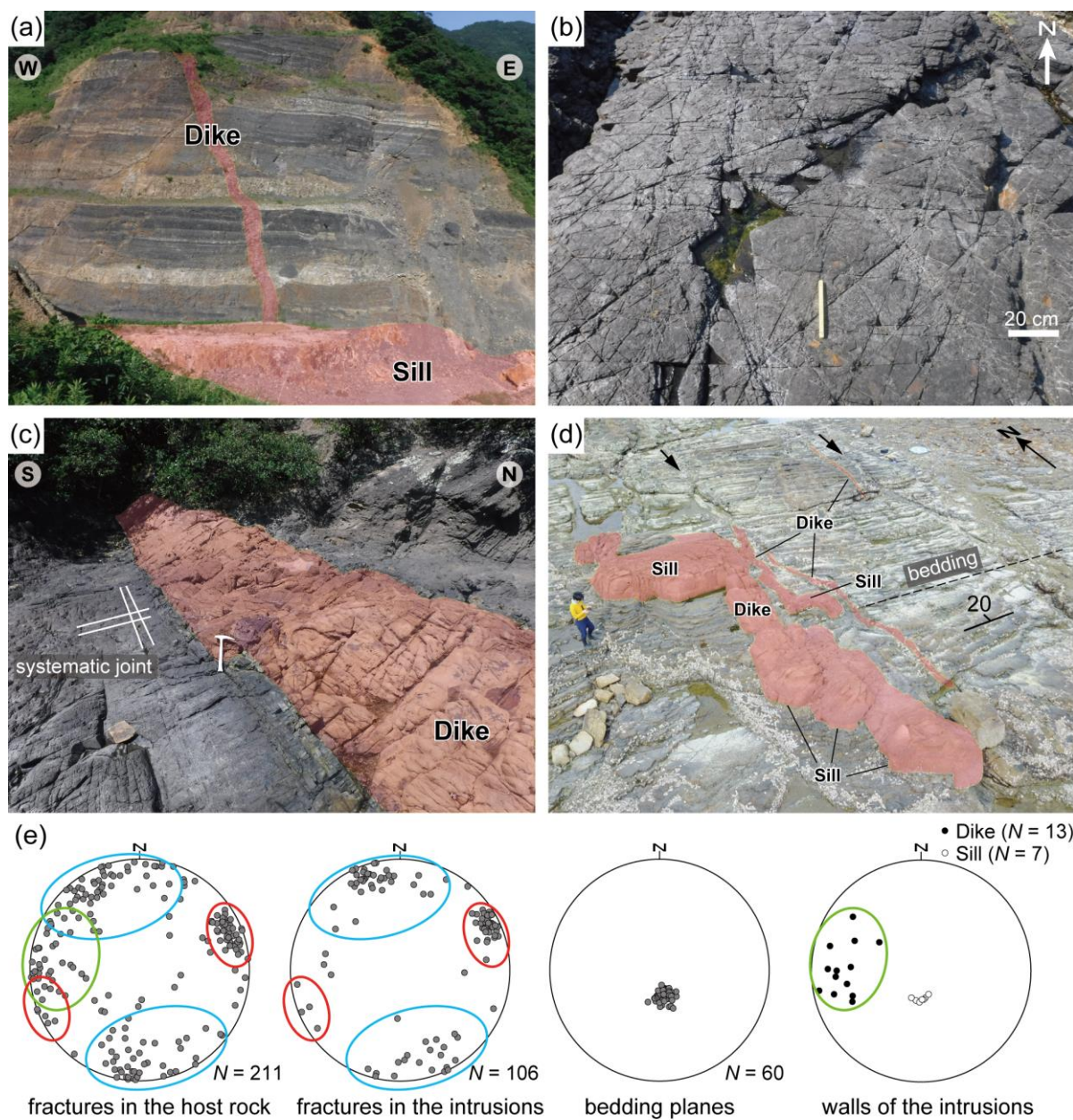
Fig. C1. (a) Schematic illustration of the measurement error of the trend, ϕ , of an inclined dike. Dashed lines indicate the uncertainty of the trend. The pole to the dike is indicated by the point, P, on the lower hemisphere centered by O. The measurement error of this pole is depicted by a circular area on the hemisphere, the lowest point of which is indicated by A. The great circle through A is tangent to the circle at the point, Q. The angular distance between P and Q is denoted by a , which is the radius of the circle and is as small as a few degrees. (b) The graphs of $f(d)$ (Eq. C1) and $\sin d$. The value, $a = 3^\circ$, is used here. $f(d)$ has complex values for the interval, $0 \leq d < a$, because the circular area includes the point, A.



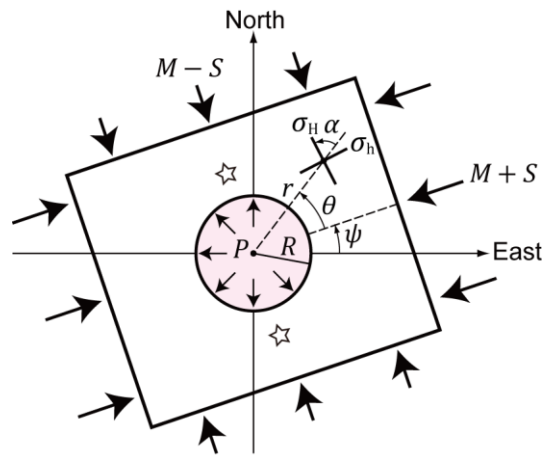
Ushimaru and Yamaji (2022), Fig. 1



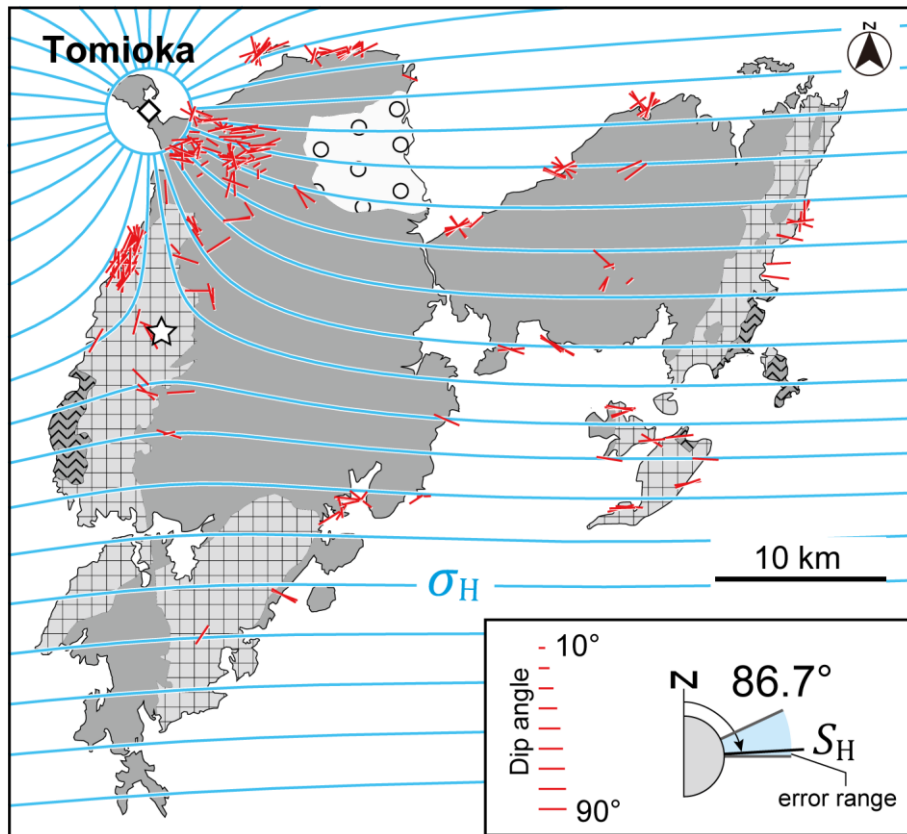
Ushimaru and Yamaji (2022), Fig. 2



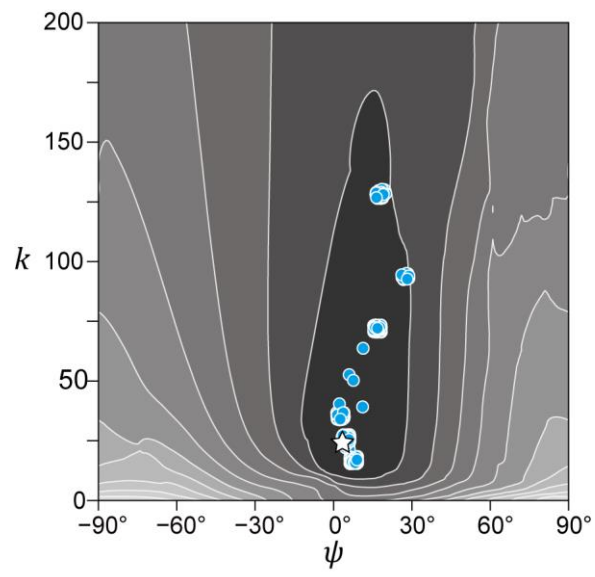
Ushimaru and Yamaji (2022), Fig. 3



Ushimaru and Yamaji (2022), Fig. 4



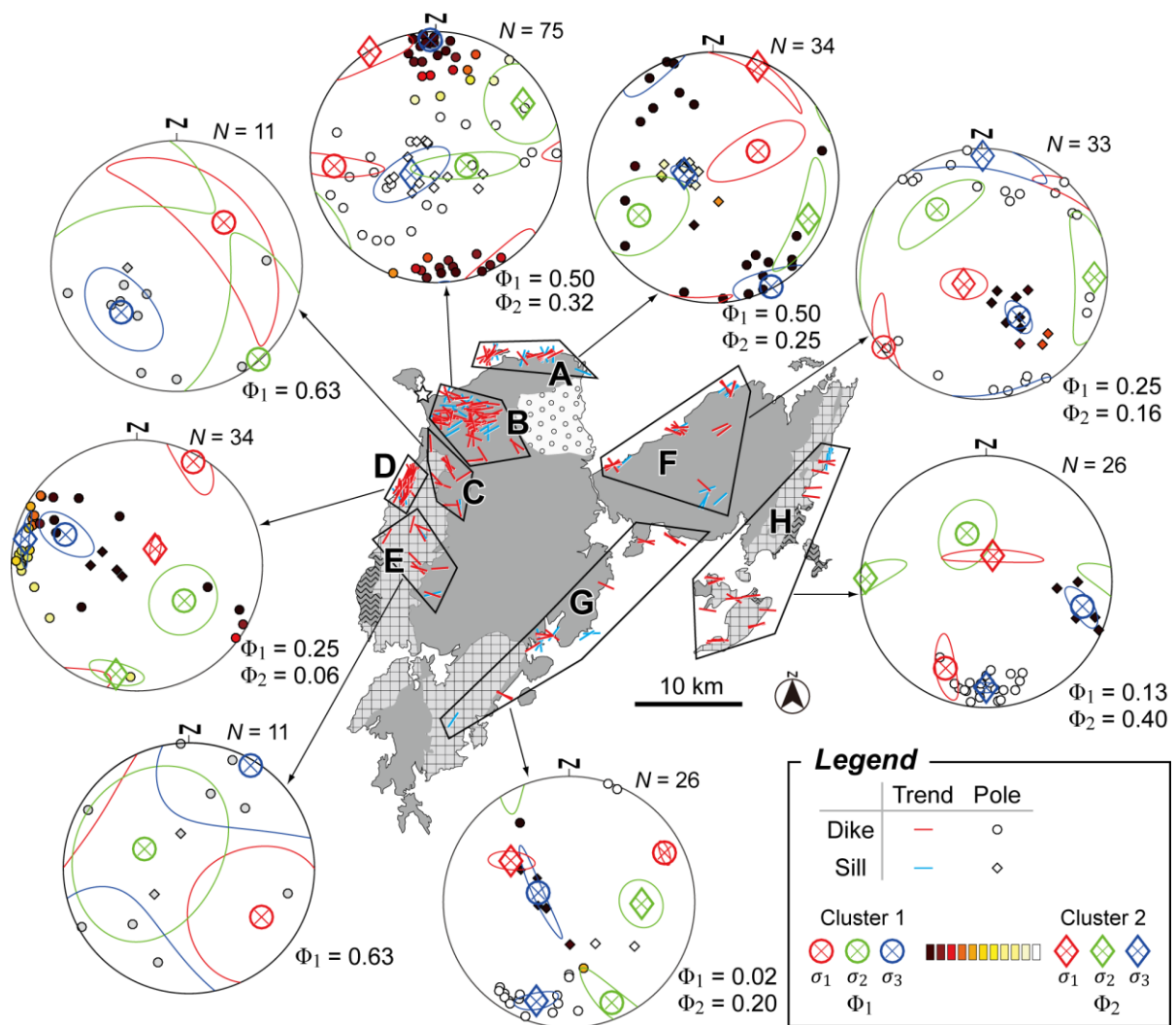
Ushimaru and Yamaji (2022), Fig. 5



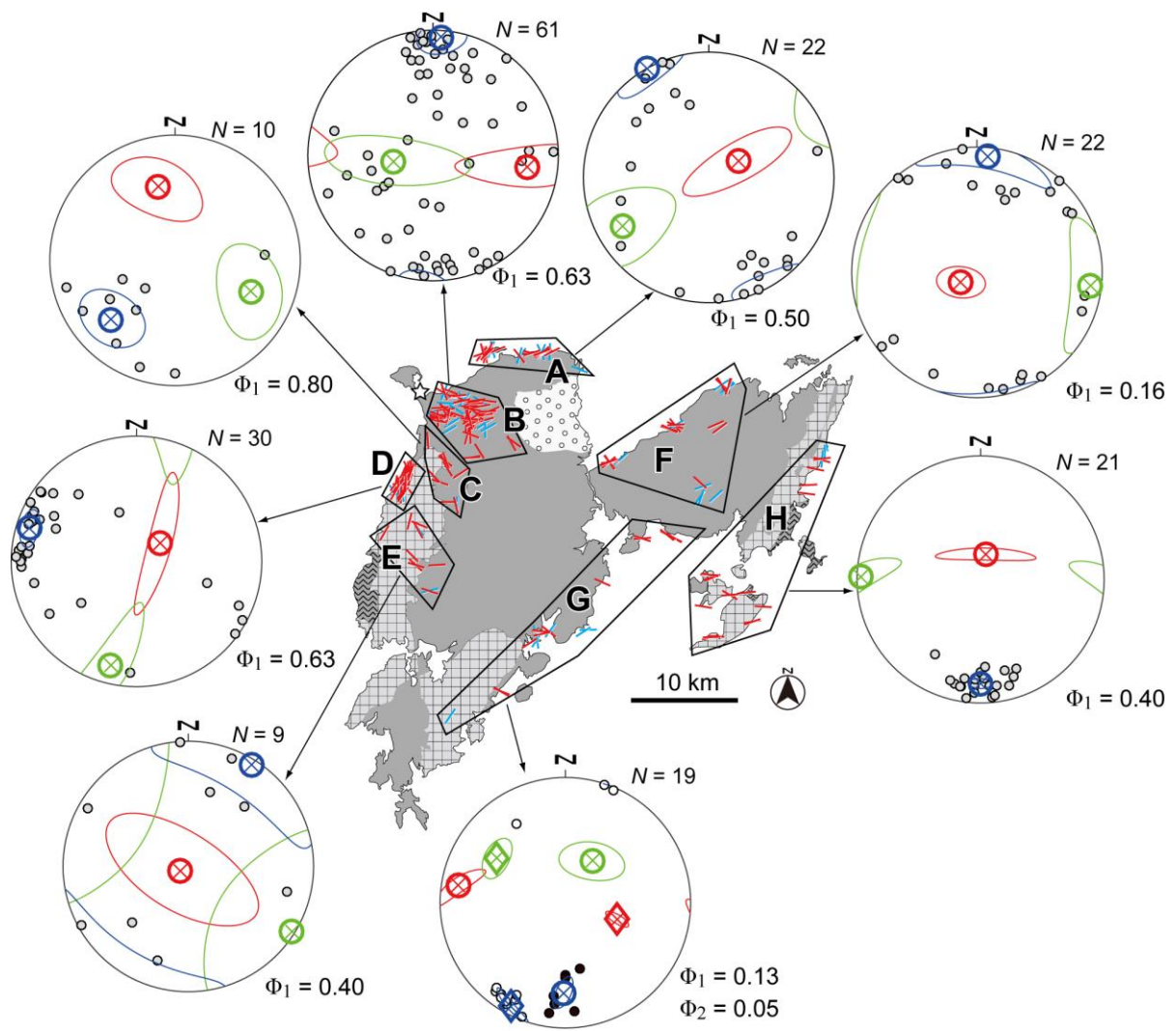
Ushimaru and Yamaji (2022), Fig. 6

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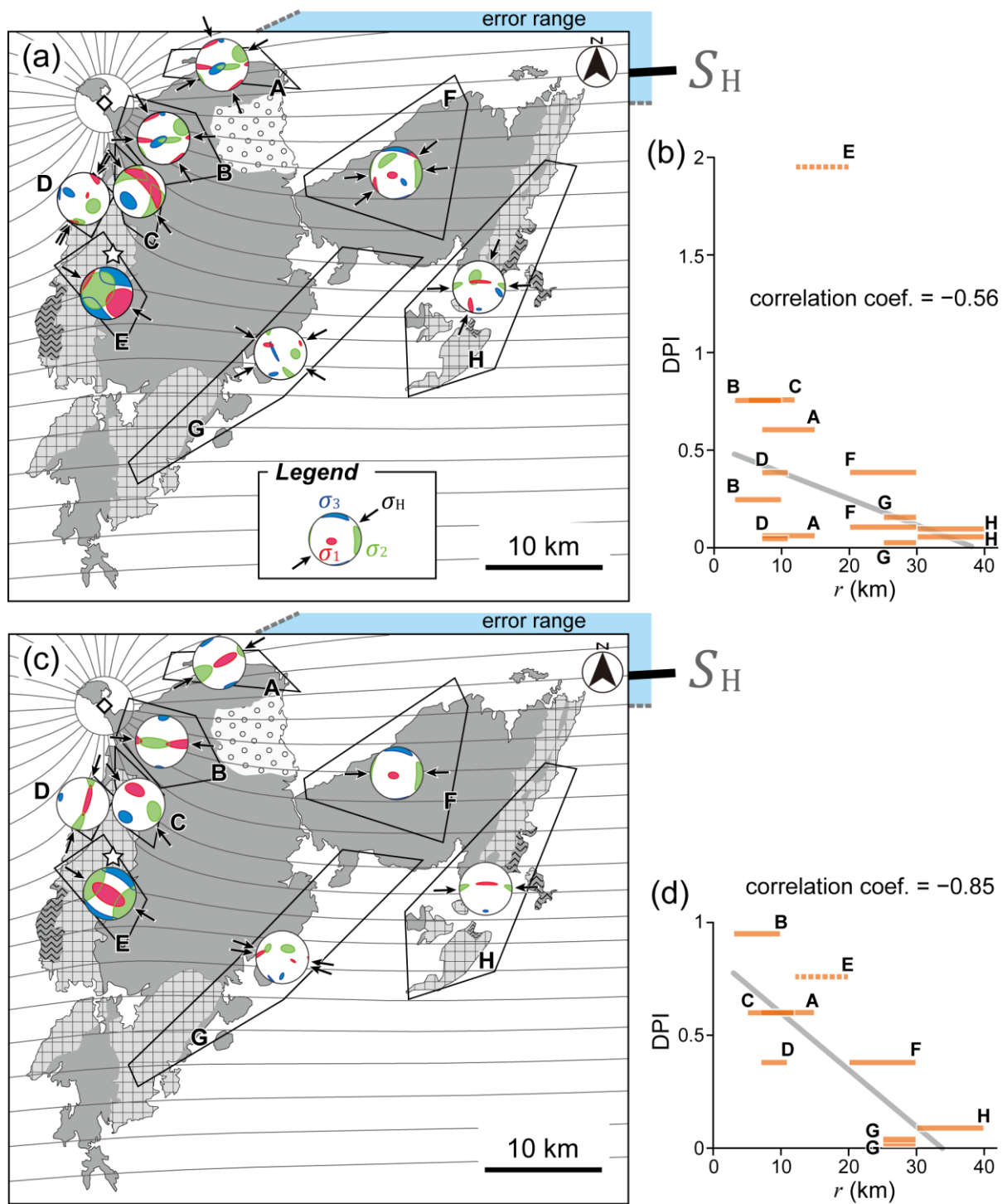
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Ushimaru and Yamaji (2022), Fig. 7



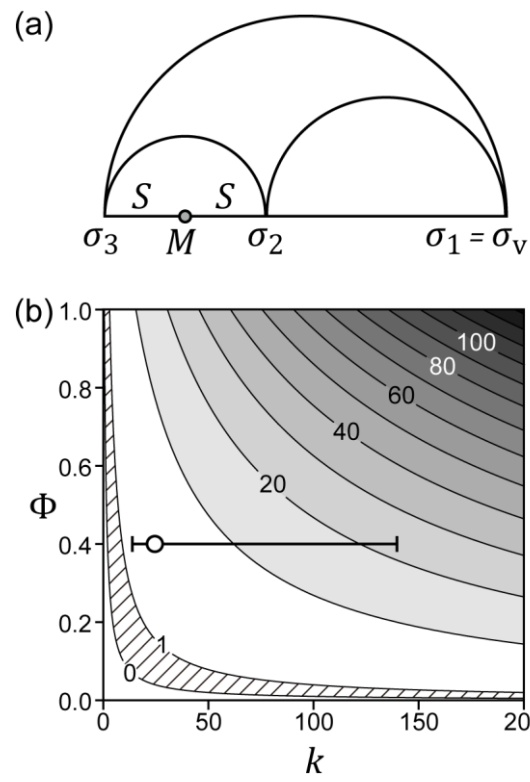
Ushimaru and Yamaji (2022), Fig. 8



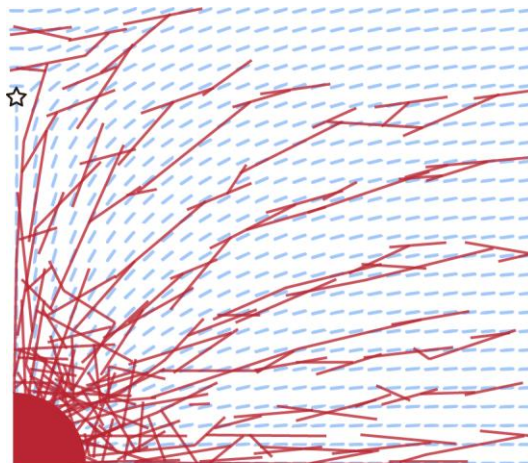
Ushimaru and Yamaji (2022), Fig. 9

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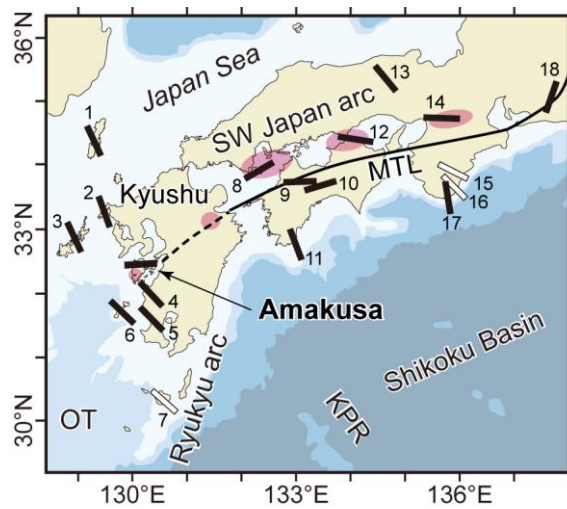
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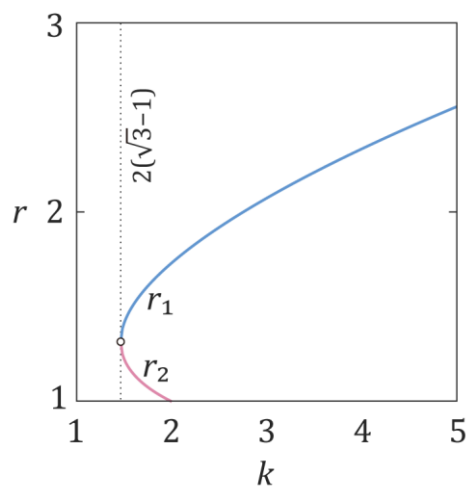
Ushimaru and Yamaji (2022), Fig. 10



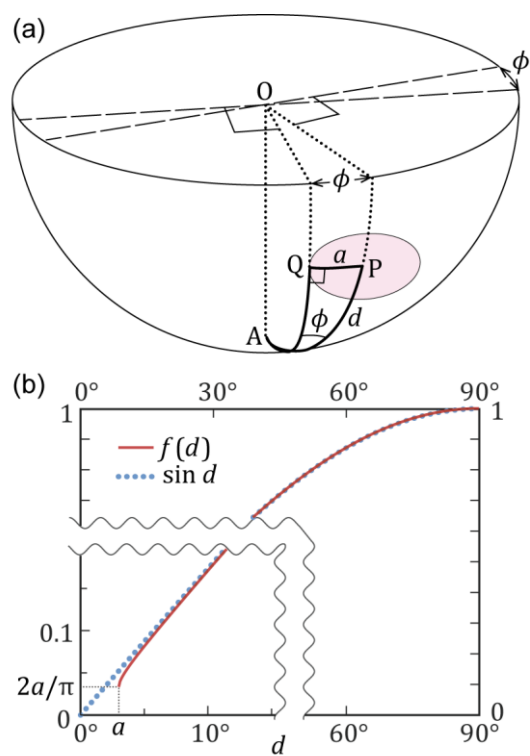
Ushimaru and Yamaji (2022), Fig. 11



Ushimaru and Yamaji (2022), Fig. 12



Ushimaru and Yamaji (2022), Fig. B1



Ushimaru and Yamaji (2022), Fig. C1

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963