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OPTIMAL DESIGN AND SCHEDULING OF BATCH PROCESSES
WITH INTERMEDIATE STORAGE TANKS

SHINJI HASEBE

1984

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Chapter 1

THE PRESENT STATUS OF DESIGN AND SCHEDULING FOR BATCH PROCESSES

1. Introduction

In many chemical processes, considerable effort has been devoted to upgrading batch-wise operated systems to continuous systems in order to increase the production capacity, save the capital investment and labour and keep stability of process operations.

As the major drawbacks of batch processes, it is usually pointed out that an item of batch equipment needs much more labor and peripheral equipment for charging, emptying and other operations than continuous equipment, and this is one of the reasons why the capital and labor costs of a batch process are high.

However, the development of control technique for batch processes based on highly capable computers, is just going to overcome the above drawbacks. That is, the size of the individual equipment is able to be increased and as a result, the cost of peripheral equipment is drastically cut. Moreover, the development of control technique has made it possible to operate the process automatically, and as a result manpower can be reduced.

Recently, the growing importance of process developments for producing various products of high added value and low volume has been stressed by people in the industrial sector. The production of such products usually requires complicated synthesis procedures, long reaction time or high conversion rate. Therefore, for the production of such a product, a

batch process or a mixed continuous and batch process may be most appropriate. Furthermore, batch processes still have an important place, specifically in fields such as dyes, drugs, fermentation products and other fine chemicals.

As batch processes are inherently operated in the unsteady state manner, the problems relevant to the design, operation and control of these processes become different from those of continuous processes. In order to solve these problems, we therefore need new solution methods which are different from the methods developed for the continuous process. By taking into account this point, the design and scheduling problems of batch processes are studied in this thesis.

In this chapter, the characteristics of batch processes are made clear and the present status of the research related to this topic is surveyed. And the contents of the thesis is briefly stated.

2. Characteristics of Batch Processes

We can classify the elements in a general batch process as shown in Figure 1-1 into three kinds. The first kind is true batch equipment which is operated batch-wise and specified by volume and not by throughput or processing rate. The second kind is semi-continuous equipment such as pumps or heat exchangers etc. which is operated continuously but intermittently, and specified by throughput or processing rate. The third kind is intermediate storage tanks. One of the prominent characteristics of chemical processes is that fluid materials are mainly handled in a process. Therefore, storage tanks have an important role in the operation of a batch process.

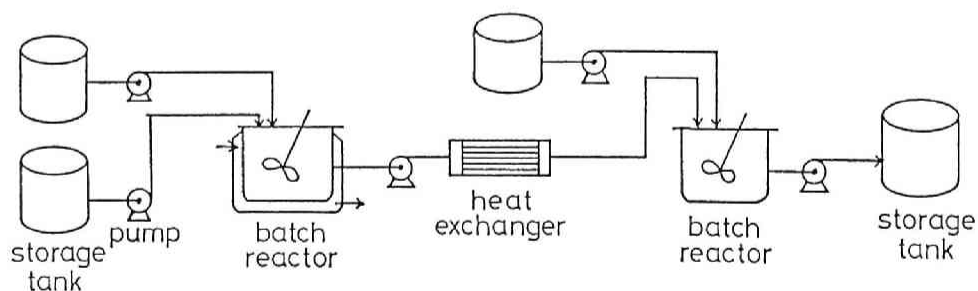


Figure 1-1. An example of a batch process

A batch equipment is generally operated cyclically by repeating the charging, processing, discharging and cleaning steps. Therefore, a batch process has the following characteristics with respect to its operation.

1) Easy to start-up and shut-down

The start-up and shut-down of each batch unit can be done independently compared with an item in a continuous process. Then, a batch process is easy to meet the changing demand on the production rate and to arrange the schedule so as to stop the operation of the process at weekends or at nights. From this point of view, batch processes are used to produce to make products in small quantities or products which are intermittently ordered.

2) Easy to prevent the propagation of the contamination

In a batch process, materials are handled in batch-wise. Then, the material of a batch is not mixed with the material of the different batch. This means that it is easy in a batch process to prevent the propagation of any bad influence due to contamination or miss processing. The amount of the off-specification product is limited to that of a batch. From this point of view, the batch process is used in the fields such as drugs and fermentation products.

Moreover, as the material is not mixed with that of the different batch, many kinds of products are often produced in the same batch equipment by coordinating their production periods. Such a process is called the "multi-product or multi-purpose process."

3) Easy to treat materials uniformly

The material in a batch unit is more uniformly treated compared with that in a continuous unit. For example, usually the residence time distribution need not be considered in a batch process. The duration of the processing can also be arbitrary chosen and it is easy to practice the complicated processing procedures. Therefore, batch equipment is fit for the operation which needs a long reaction time, a sophisticated processing or a high conversion rate.

These three characteristics mentioned above are major advantages of the batch process. However in order to optimally design and effectively operate the batch process, there are many problems to be solved.

For example, if two batch units are directly connected, the outlet flow from a batch unit directly becomes the flow into the successive batch unit. In this case, we cannot arbitrarily choose the amount of material produced in a batch, the cycle time and the starting time of the operation of each batch item. Therefore, to optimally design and operate such a process, it is necessary to design each batch item by taking account of the design problem of the whole process, and it is also necessary to consider the design and the scheduling problems simultaneously. Otherwise, it is often the cases where some batch items have to be operated with fairly long idle time.

At following three sections, the present status of the design and scheduling problems and the problem related with

the flexibility of a batch process is briefly surveyed, and the characteristics of these problems are clarified.

3. Design Problem of Batch Processes

3-1. Single product process

In a single product batch process, only one kind of product is produced by repeating the series of operations periodically. For the simplicity of the explanation, it is assumed that the batch process consists of only two batch stages as shown in Figure 1-2.

We first consider a batch process which is operated in "non-overlapping manner", that is, there can be only one batch of material in the process at any one time.

Figure 1-3 shows the Gantt chart of the operation for the case that the process is operated in non-overlapping manner. In this case, the outlet flow from the batch item in batch stage 1 (batch item 1) directly becomes the inlet flow to the batch item in batch stage 2 (batch item 2).

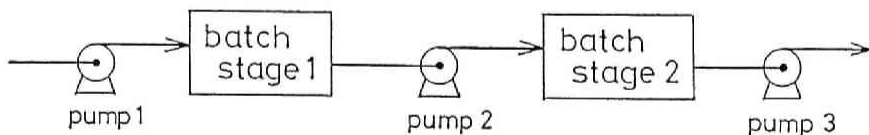


Figure 1-2. Process consisting of two batch stages

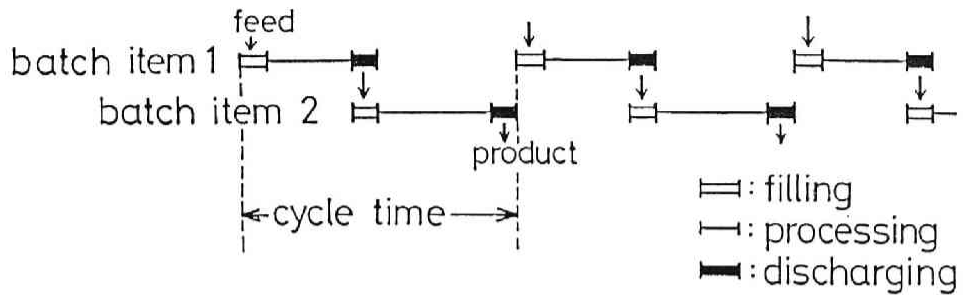


Figure 1-3. Gantt chart of the operation
for non-overlapping case

Therefore, the amount of material produced in a batch, that is, the batch size of each batch item has to be identical each other. The cycle time of the process, W , is given as the sum of the processing times and the filling and discharging times of both batch items, i.e.

$$W = S/U_1 + T_1 + S/U_2 + T_2 + S/U_3 \quad (1-1)$$

where

S = the batch size of each batch item,

U_i = the capacity of pump i ,

T_i = the processing time of batch item i .

Then, the design problem of this process is stated as follows:

"When the production requirement per unit time, P , and processing time of each batch item, T_i , are given, find the optimal batch size, S , and processing rates of each semi-continuous equipment, U_j , so as to minimize the following performance index".

$$P.I. = \sum_{i=1}^2 p_i(S) + \sum_{j=1}^3 r_j(U_j) \quad (1-2)$$

subject to

$$S/W = P \quad (1-3)$$

$$W = \sum_{i=1}^2 T_i + \sum_{j=1}^3 (S/U_j) \quad (1-1)$$

where

p_i = the cost function of a batch equipment in batch stage i ,

r_j = the cost function of a semi-continuous equipment succeeded by batch stage j .

This is a typical nonlinear programming problem with equality constraints. Ketner^[1] dealt with this type of problem for the case that the performance index is given as a linear function of the batch size, S , and processing rates of feeding and discharging pumps, U_j . Loonkar & Robinson^[2] solved the same problem for the case that the performance index is given as a linear function of S^α and $U_j^{\beta_j}$; where α and β_j are positive numbers.

Figure 1-4 shows the case where overlapping of the operation is allowed. That is, a new batch can begin on a

stage as soon as the discharging for the previous batch is completed.

In a real batch process, some item of equipment must be cleaned after the discharging step. However, in order to simplify the explanation, it is assumed in this section that the law material can be charged as soon as the discharging step is finished. Under the above assumption, minimal cycle time of batch item i is given by

$$S/U_i + T_i + S/U_{i+1} .$$

Then, the design problem of this process can be stated as follows:

"Find the optimal batch size, S , and processing rate of each semi-continuous equipment, U_j , so as to minimize the performance index given by Eq.(1-2) subject to the following constraints:

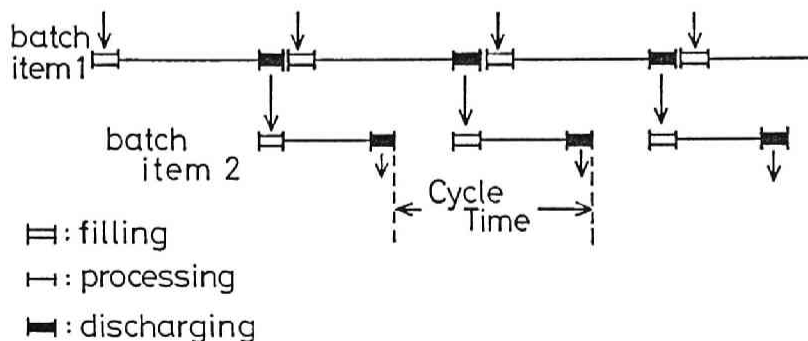


Figure 1-4. Gantt chart of the operation for overlapping case

$$P = S/W \quad (1-3)$$

$$W \geq S/U_i + T_i + S/U_{i+1} \quad (i = 1, 2) \quad (1-4)$$

This problem is also a typical nonlinear programming problem with inequality constraints.

When the processing times of two batch items are different each other in the large scale as shown in Figure 1-5a, the batch item which has shorter processing time (i.e. batch item in stage 2) has to be operated with a large idle time for waiting. In order to avoid this kind of inefficiency in the operation of the process, batch items are installed in parallel for batch stage 1. If two batch items are installed in parallel, the cycle time of batch stage 1 becomes one half of the previous cycle time as shown in Figure 1-5b. This means that the given production requirement can be achieved by using batch items of smaller size.

The minimum cycle time of batch stage i with N_i batch items in parallel is $1/N_i$ as long as that of batch stage i with only one batch item. Then, the problem is stated as the one of how to determine the optimal number of batch items, N_i , the optimal batch size of batch items, S , and the optimal processing rate of each semi-continuous equipment, U_j , so as to minimize the performance index given by the following equation.

$$P.I. = \sum_{i=1}^2 N_i \cdot p_i(S) + \sum_{j=1}^3 r_j(U_j) \quad (1-5)$$

subject to

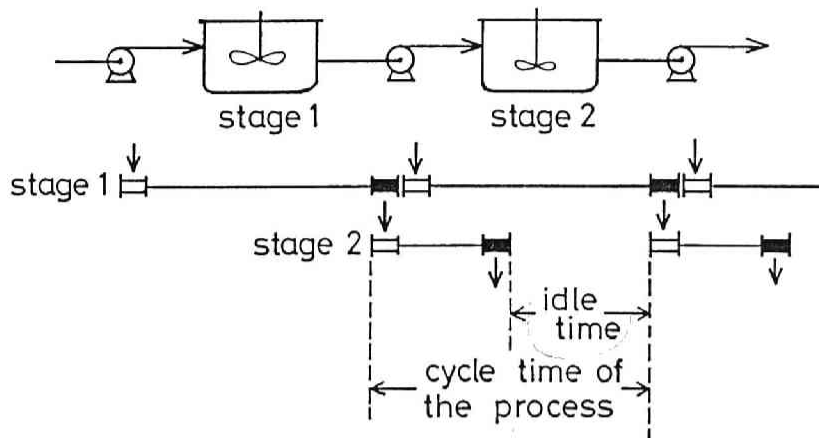


Figure 1-5a. Operation schedule with a large idle time

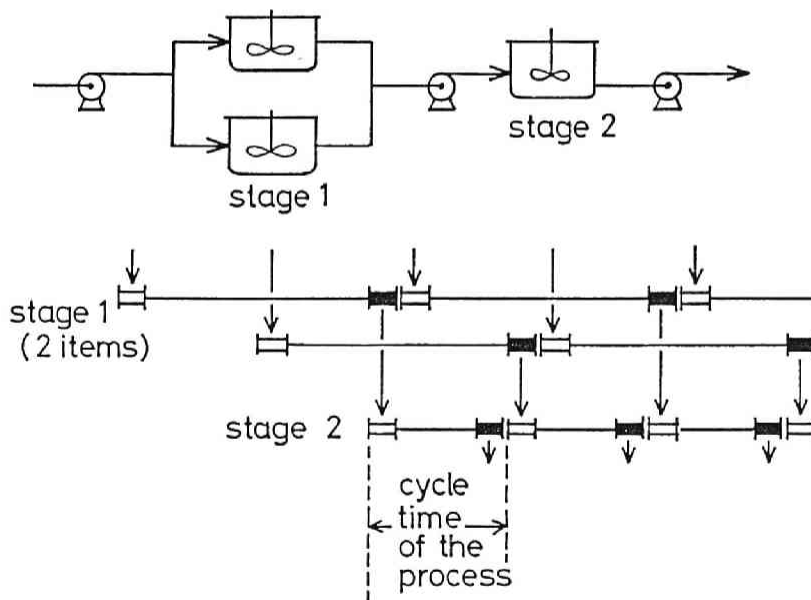


Figure 1-5b. Operation schedule for the case where two batch items are installed in parallel at stage 1

$$W \geq \{(S/U_i) + T_i + (S/U_{i+1})\}/N_i \quad (i = 1, 2) \quad (1-6)$$

$$P = S/W \quad (1-3)$$

The number of parallel batch items must be a natural number. Therefore, the problem stated above is a mixed integer nonlinear programming problem.

Another countermeasure to reduce the idle time of the operation is to install an intermediate storage tank between two batch stages as shown in Figure 1-6. By installing the storage tank, the outlet flow from batch stage 1 is stored in the tank and does not immediately become the inlet flow of batch stage 2. Therefore, the cycle times and batch sizes can be chosen arbitrarily in both batch stages so as to satisfy the given production requirement. The design problem of this kind of process is discussed in Chapter 2 and Chapter 3.

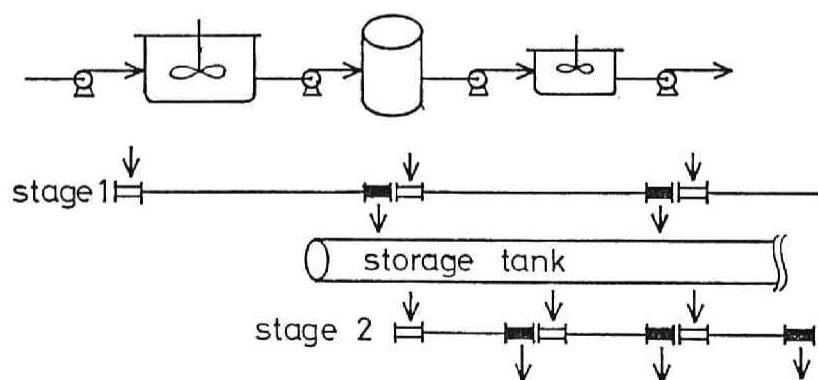


Figure 1-6. Operation schedule of a process with a storage tank

3-2. Multi-product process

In a batch process there are many cases where the process is used to produce many kinds of products. If all products are produced by the same path through the process and only one product is produced at a time, such a process is called "a multi-product batch process".

In order to show how the design problem of the multi-product process is formulated, a simple process shown in Figure 1-2 is taken up again. And it is also assumed that two kinds of products, product A and product B, are produced in the process.

In a multi-product process, the size of equipment required at each stage to produce unit mass of product differs with products. Therefore, the "size factor" is introduced to explain the above characteristic. That is, the size factor C_{ik} is defined as the characteristic size of equipment required at stage i to produce unit mass of product k in a batch leaving the process.

By introducing the size factor, the relationship between the equipment size and the batch size is given as follows:

$$E_i \geq C_{ik} \cdot S_k \quad (i = 1, 2 ; k = A, B) \quad (1-7)$$

where

E_i = the size of batch equipment in stage i ,

S_k = the batch size of product k .

The cycle time of the process for product k , W_k , must satisfy the following inequality:

$$W_k \geq \{(S_k/U_i) + T_{ik} + (S_k/U_{i+1})\}/N_i \quad (i = 1, 2) \quad (1-8)$$

where

T_{ik} = the processing time of the batch item in stage i
for product k .

Let the total production requirement of product k be P_k , then the time which is necessary to produce product k is given by $W_k \cdot P_k / S_k$. Therefore, if the changeover time between two products can be neglected, W_k and S_k must satisfy the following inequality:

$$H \geq \sum_{k=A,B} W_k \cdot P_k / S_k \quad (1-9)$$

where

H = the total production period.

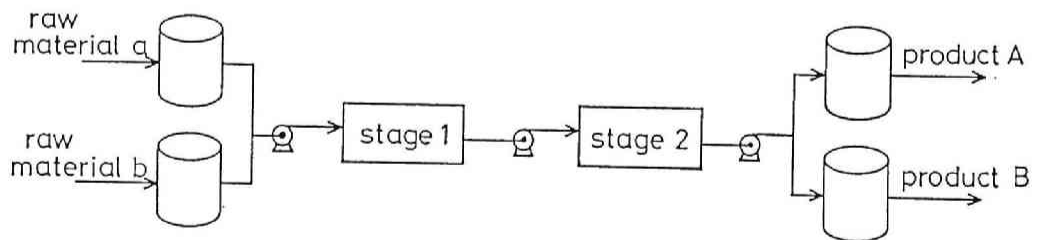


Figure 1-7. Multi-product batch process

After all the design problem of a multi-product batch process shown in Figure 1-7 is formulated as follows:

"Find the optimal number of batch items, N_i , the optimal equipment size of each batch stage, E_i , and the optimal processing rate of each semi-continuous equipment, U_j , which satisfy Eqs.(1-7), (1-8) and (1-9), so as to minimize the performance index given by the following equation".

$$P.I. = \sum_{i=1}^2 N_i \cdot p(E_i) + \sum_{j=1}^3 r_j(U_j) \quad (1-10)$$

In a design problem of a multi-product batch process, the problem of how to allocate the total production period for the production of each kind of product arises. Therefore, the design of a multi-product process becomes more difficult problem compared with that of a single product process.

In a single product batch process, by assuming the number of parallel batch items in each batch stage and the processing rate of each semi-continuous equipment, the minimum batch size which satisfies the production requirement is uniquely determined. However, in a multi-product batch process, many kinds of combinations in batch sizes of products satisfy the same set of production requirements by adjusting the production period of each product.

For example, the cycle time of the process for each product is assumed to take a constant value such that $W_A = W_B = 1.0$ [day], and it is also assumed that the size factors and other data take the following values:

$$C_{1A} = C_{2B} = 1.0, \quad C_{1B} = C_{2A} = 2.0 \quad [\text{m}^3/\text{ton}] \quad ;$$

$$P_A = P_B = 30 \quad [\text{ton}] \quad ; \quad H = 30 \quad [\text{day}].$$

In this example, a combination in batch sizes of product A and B, such that $S_A = 3$, $S_B = 1.5$, satisfies the production requirement. In this case, the minimum equipment sizes of both batch stages are 3 m^3 and 6 m^3 , respectively. But other combinations in batch sizes of both products as shown in Table 1-1 also satisfy the production requirement. This means that even if the processing rate of each semi-continuous equipment is already determined, the equipment sizes in both stages are not uniquely determined.

Table 1-1. Equipment sizes which satisfy the production requirement

	Equipment size		batch size		production period	
	stage 1	stage 2	product A	product B	product A	product B
case 1	$3[\text{m}^3]$	$6[\text{m}^3]$	$3[\text{m}^3]$	$1.5[\text{m}^3]$	10[day]	20[day]
case 2	4	4	2	2	15	15
case 3	6	3	1.5	3	20	10

Robinson and Loonkar^[3] considered the design problem of a multi-product batch process, and proposed the algorithm which can derived the minimum capital cost by using a direct search procedure. However, they did not consider the problem of a process which has equipment in parallel.

Sparrow et al.^{[4],[5],[6]} developed the computer package "MULTI-BATCH" for the design and evaluation of multi-product batch processes. The proposed package can carry out not only sizing equipment to achieve the production requirement but also calculating a heat and material balance for each product and estimating project economics. In this design package, two methods of sizing batch equipment are presented. The first method is based upon defining a new hypothetical product and the equipment is sizes to achieve the required production of the hypothetical product. The second method uses the technique of branch and bound to obtain the optimal batch size for the case that only the discrete values are available as the size of each batch item.

Grossmann and Sargent^[7] also solved the design problem of a multi-product batch process. They formulated the problem as a mixed integer nonlinear programming problem and solved by using a branch and bound technique coupled with the nonlinear programming algorithm.

Flatz^[8] proposed the procedure for determining equipment sizes of a multi-product batch process by manual calculations. In his algorithm, most expensive item of equipment is first designed so as to be fully utilized in the production of all

of the products. Then, the number of batch items in parallel and the equipment sizes of other batch stages are calculated.

Knopf et al.^[9] solved the design problem of an actual batch/semicontinuous process, and insisted on the significance of the inclusion of operation costs such as energy costs in design optimization.

As stated above, many kind of procedures which derive the optimal or sub-optimal solutions have been proposed for designing of a multi-product batch process. However, the design problem of a multi-product batch process with intermediate storage tanks has not been studied yet.

3-3. Multi-purpose process

The process which produces multiple products and does not come under the heading of the "multi-product process" is called the "multi-purpose process". In such a process, two or more products may be produced simultaneously by sharing process equipment or products may be produced one by one by taking different routes. Furthermore, batches of the same product may take different routes through the process.

Suhami and Mah^{[10], [11]} proposed the solution procedure for the design problem of a multi-purpose batch process under the condition that the production route of each product has already been known. They proposed a heuristic procedure in which randomly generated configurations of products produced simultaneously are scanned as a result of which the best configuration is identified. After the configuration

is determined, then the design problem is formulated as a mixed integer nonlinear programming problem.

4. Scheduling Problem of Batch Processes

4-1. Introduction

In the previous section, the problem of designing the batch process so as to satisfy the long term production demand was considered. Batch items in a batch process is operated by repeating some processing steps. Moreover, in a batch process many kinds of products are often produced by sharing the available production time between products. Therefore, it becomes a very important problem to determine the production order of products and the starting moment of the production for each product so as to optimize some performance index, even though the size of each batch equipment is already determined.

The scheduling problem is mainly studied in the field of the operations research. However, almost all of these works are for non-chemical processes such as mechanical processes. So, in this section the differences between the scheduling for batch processes and that for non-chemical processes are clarified and the present status of the scheduling for batch processes is briefly surveyed. More extensive review of the scheduling problems of batch chemical processes has been provided by Reklaitis^[12].

4-2. Flowshop scheduling problem

The flowshop problem is one of the most general production scheduling problems in the field of the operations

research. There are some survey papers for the flowshop problem^{[13],[14]}. So we avoid explaining the flowshop problem in detail. But it is noted that in order to derive the exact optimal solution for the flowshop problem, the solution time grows exponentially with the characteristic size of the problem^[15]. This fact holds not only in the flowshop problem but also in most of scheduling problems, that is, most of scheduling problems are considered to be NP-complete^[12]. So, the size of the problem for which the optimal solution can be derived is limited even if the implicit enumeration technique such as the branch and bound procedure is used to solve the problem.

Therefore, in addition to the solution procedures to obtain the optimal solution, a lot of procedures based on the rule of thumb are proposed to derive the sub-optimal solutions^{[16],[17]}.

In the flowshop problem, all kinds of products are assumed to be processed on the same set of facilities with an identical precedence ordering of the processing steps. The scheduling problems of a multi-product batch process can be regarded as a kind of flowshop problem. However, in order to formulate the scheduling problem of a multi-product batch process, modifications of the normal flowshop problem are required in some points.

The materials processed in a batch process are liquids, gases or fine powders. To store these materials, the special facility for storage, that is, the storage tank must be

installed between two processing stages. Therefore, the operation of a batch process must be scheduled by taking into account the places where intermediate storage tanks are installed and the capacities of these tanks, though most of the operations research work has been done with the assumption that unlimited intermediate storage is available between two stages. Moreover, intermediate products of a batch process are not always stable enough to be stored.

By taking account of the status of intermediate products, the flowshop problems of batch processes can be classified into four types of problems^[12]:

- (a) unlimited intermediate storage between stages (UIS),
- (b) finite intermediate storage between stages (FIS),
- (c) zero wait processing (ZW),
- (d) no intermediate storage between stages (NIS).

As mentioned before, in most of the operations research work, unlimited storage is assumed to be available between processing stages, but almost all of the flowshop problems of batch processes cannot be considered as UIS problems. The flowshop problem of a batch process with storage tanks can be regarded as an FIS problem, but there are few studies on this type of problem^[18].

In a batch process, there are some cases where raw materials must be processed nonstop on all of the batch units once its processing is started on the first unit. In this case, the flowshop problem can be regarded as a ZW problem. The ZW problem which minimizes the total time

required to produce all products, that is, "the make span", can be formulated as a traveling salesman problem and has been solved by many kinds of procedures^{[19],[20],[21]}.

If intermediate storage is not allowed between two successive stages, the flowshop problem of a non-chemical process can be regarded as the ZW problem. However in a batch chemical process, there are some cases where materials can be held in a batch unit after the processing is completed. In this NIS case, the batch unit can not commence processing for the next product until the material in the batch unit is completely discharged to the batch item in the next stage. The NIS problem has received very little attention compared with other problems, but solution procedures for minimizing the make span of the NIS problem have been developed recently^{[11],[22],[23]}.

In order to show the difference among these flowshop problems, an example of schedules of a three-unit three-product flowshop is shown by using Gantt charts. In this example, it is assumed that processing times are given in Table 1-2. Schedules under UIS, ZW and NIS mode are depicted in Figures 1-8a, 1-8b and 1-8c, respectively. In NIS mode, raw material c has completed processing in unit 1 at time 32, but must be held for 4 time increments until the material in unit 2 is discharged.

If the production demand of a product exceeds the production capacity of a batch, the process has to be operated in a certain number of batches to meet the given production

Table 1-2. Processing time at each stage

stage	1	2	3
product A	12[hr]	12[hr]	12[hr]
product B	4	4	4
product C	8	8	8

demand. In a non-chemical process, materials processed at a stage are not directly transferred to the next stage, but stored to formulate a definite amount, i.e. a "lot". Materials are transferred lot by lot from one stage to another. On the other hand, in a batch chemical process, a batch of material processed at a stage is transferred to the next stage as one batch. So the operation schedule for this type of batch processes differs from that of non-chemical processes even if the production order and time required for each processing are equal in both types of processes.

Figure 1-9 shows an example of schedules for both cases, where four pieces/batches of product A and three pieces/batches of product B are produced at a three-unit process. As shown in Figure 1-9a, the scheduling problem of a non-chemical process can be reduced to the UIS problem if the lot size of each product is determined. However, in a

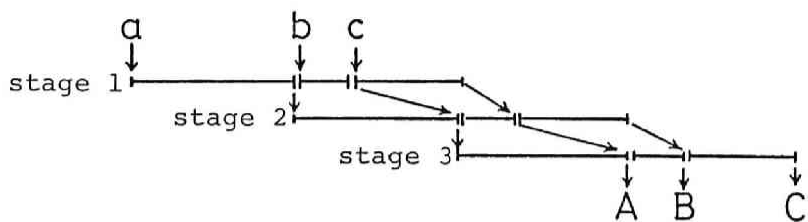


Figure 1-8a. Schedule for UIS case

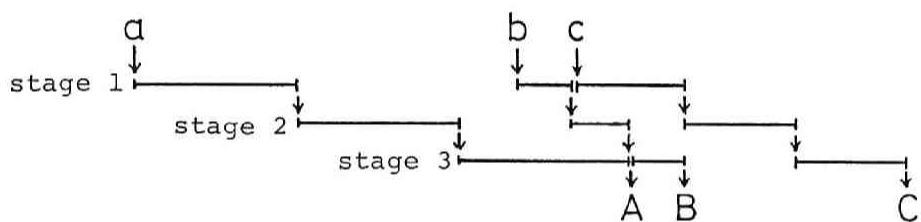


Figure 1-8b. Schedule for ZW case

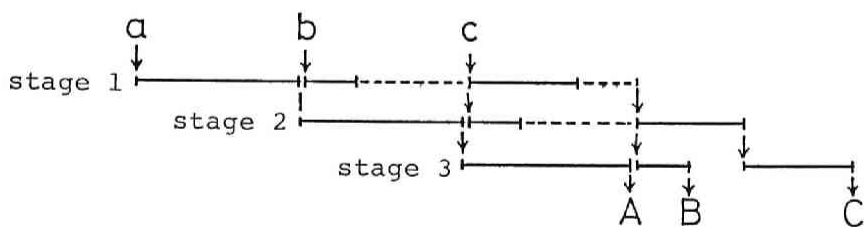


Figure 1-8c. Schedule for NIS case

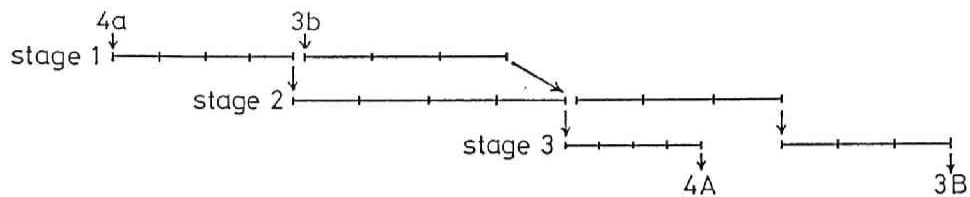


Figure 1-9a. Schedule for the lot production system

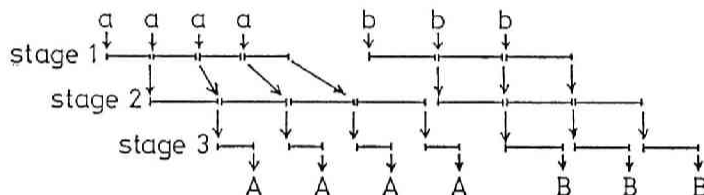


Figure 1-9b. Schedule of a batch chemical process

batch process we need the new formulation for the scheduling problem as shown in Figure 1-9b, and this type of problem has not been studied yet.

4-3. Jobshop scheduling problem

The jobshop scheduling problem is also one of the most general production scheduling problems. In a jobshop problem, each product has a fixed processing order but this order varies with products. The scheduling problem of a multi-purpose process can be regarded as a kind of jobshop

problem. However as mentioned in the previous section, the treatment of intermediate products affects the formulation of the problem. Moreover, in order to effectively operate the process, production order and batch units which are used for the production of each product must be determined by taking account not only of the processing time at each batch stage but also of the amount of product produced in a batch. On the other hand, in a non-chemical process, parts are processed one by one at each machine. Therefore, the schedule can be determined by taking account only of the processing time at each machine.

Mauderli et al.^[24] proposed the procedure to solve a general production scheduling problem of a multi-purpose batch process. In their procedure, alternative production strategies are first generated, and are screened by identifying and rejecting non-dominant strategies. Then, the optimal production schedule is obtained by solving an optimization problem in which the production time is allocated to the various dominant production strategies. They incorporated the procedure outlined above into a computer program "BATCHMAN"^{[25], [26]}.

Swanson^[27] modeled the scheduling problem of a multi-purpose batch process using linear production and inventory costs and linear constraints a subset of which gives a feasible schedule. Then, the problem was transformed into the dual problem and solved by using the revised simplex method coupled with the branch and bound technique.

4-4. Scheduling with utility constraints

In a batch process, many processing steps are simultaneously executed. And each processing step needs a particular kind of equipment and different amounts of utilities such as manpower, electricity and steam etc., for its execution. There are some cases where the maximum level of utilization is limited for some utilities and/or available units are restricted. In these cases, the operation of the batch process must be scheduled under the restrictions on utility usage.

Scheduling problems with utility/resource constraints have been studied in the field of the project scheduling. A lot of techniques have been proposed to solve the project scheduling problems, and there have been some papers which made comparisons of the effectiveness of various heuristic rules relative to an optimal solution^{[28],[29]}. Davis^{[30],[31]} provided excellent reviews of the history and research on the project scheduling under resource constraints. However, it needs some modification to apply the technique developed in the field of the project scheduling to the scheduling problem of batch processes.

For example, in a batch process each material must be processed to reach a stable state without any interruption. That is, most of processing step can not be divided into sub-steps. Furthermore, there are some cases where two successive processing steps must be executed without any idle time. Therefore, if the operation of the process must

be stoped at night or over week-ends, we must schedule the operation of the process so that no material which partially completed the processing step remains in the process during the night or week-end.

On the other hand, if each processing step can be stoped at any time and can be resumed after some time period, the operation of the process can be scheduled without considering the times when it is stoped such as those during the night or week-end.

By taking account of the effect of stability of the material, Egri and Rippin^[32] proposed the scheduling program for multi-product batch processes.

As mentioned in the previous section, in most of batch processes products are produced by cyclically operating the process. So, the methods developed for solving the project scheduling problem are not applicable to solve the scheduling problem of a batch process which is operated cyclically. Because most of the solution procedures in the project scheduling problem use the arrow diagram to indicate the precedence order of processing steps. If the precedence order of processing steps for a cyclically operated batch process is expressed by using an arrow diagram, we must distinguish between the first batch of a processing step and the second batch of the same processing step. In this case the arrow diagram becomes a very complicated one.

For example, it is assumed that the precedence order of three processing steps, A, B and C are expressed by using an

arrow diagram as shown in Figure 1-10a. If these three steps are performed more than once, the arrow diagram which shows the precedence order becomes as shown in Figure 1-10b.

Very little attention has been paid to the scheduling problem for cyclically operated batch processes^{[33],[34],[35]}. This problem is discussed more precisely in Chapter 5.



Figure 1-10a. Precedence order of three steps

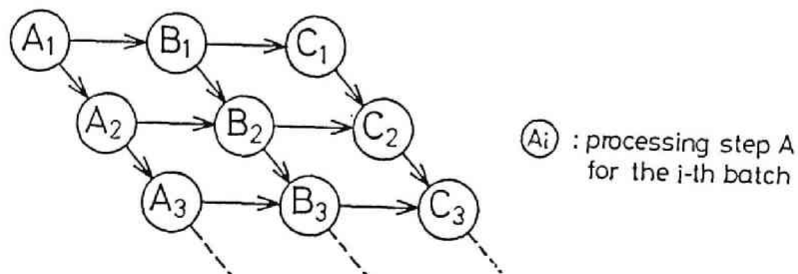


Figure 1-10b. Precedence order of three steps which are performed more than once

5. Flexibility of Batch Processes

In previous sections, we briefly surveyed the present status of the design and scheduling problems of batch processes, where the production requirement of each product and parameters related to the design and operation are regarded as constant values. However, in a real process, the amount of material to be produced and its due date may be changed owing to the variations of the production demand, and moreover the processing time and/or cleaning time of some batch unit may become shorter or longer than the preassigned nominal values. Therefore, it is very important to design a process so as to ensure the operation of the process for a range of process parameter variations and/or production demand variations.

In a batch process, materials are handled in batch-wise. So, it is easy to start up and shut down each unit compared with that in a continuous process. This characteristic shows that in a batch process it is easy to operate the process so as to follow the variation of production demand as well as to ensure the operation of the process for a range of parameter variations. From this point of view, we can say that batch processes have large flexibility.

Recently the importance of designing the process flexible is stressed, and the problems such as how to define the flexibility of chemical processes, how to measure the magnitude of flexibility and how to design a process with a fixed

degree of flexibility, etc. are vigorously studied^{[36],[37],[38]}. However, almost all of these works are for continuous processes.

In order to design a flexible batch process, two different approaches have been proposed so far. One is the "stochastic approach" and the other is the "deterministic approach".

By taking a stochastic approach, Smith and Rudd^[39] showed that the variation in processing times affects the performance of a batch process. They solved simple problems by using queuing theory. Simulation was then presented to solve the more complicated problems. Ross^[40] also studied the problem of how to determine the optimal tank volume so as to assure the smooth operation of the whole process by using the Monte Carlo simulation technique. Overturf et al.^[41] have also taken a stochastic approach. By using the simulation package, they solved the complex combined scheduling and design problem that typically occurs in batch processes.

However, there are some drawbacks in a stochastic approach. The first is that it is difficult to determine the probability distribution of each stochastic variable before the process is operated. The second is that the tremendous amount of calculation is required to derive the result. Because a large number of simulations must be done once the design and operation variables are fixed at certain values, and then the optimal design and operation variables

must be searched by changing these fixed values.

Oi et al. [42], [43] took a deterministic approach and analyzed the flexibility of a process consisting of parallel batch units, a storage tank and a continuous section. In their study, the allowable range of the fluctuations of processing times of batch units was derived so that the operation of the continuous section following the tank was not affected by the variation in the operation of any batch unit before the tank, in so far as these variations remain in the allowable range obtained.

It is impossible to design a process which can cope with the all kinds of variations with arbitrary magnitude. Therefore in a deterministic approach, a process is designed to ensure its feasible operation for a limited number of variations the ranges of which are bounded. In Chapter 4, we deal with the problem of how to design a batch process with a fixed degree of flexibility.

We next consider the flexibility of the operation schedule. In most of scheduling procedures taken up in section 4, it is assumed that the demand patterns for many products have been known when the schedule is determined, and it is also assumed that the demand patterns do not change irregularly during the scheduling period.

However, it is more common that the market demands are uncertain and change with time. Moreover, the parameter values related to the operation may vary from the preassigned nominal values. Therefore, the derived schedule is seldom

implemented in its entirety without revision. So, it is very important to develop the scheduling procedure in due consideration of how to modify the previous schedule.

The rolling scheduling technique is one of the scheduling procedures in which the rescheduling is considered^{[44],[45]}. In a rolling scheduling technique, total scheduling period is divided into many smaller periods. Then the scheduling problem is solved for several time periods from the beginning, but only the schedule of the first period is implemented. One period later, the multi-period model is updated and the schedule is reoptimized based on the revised and additional information.

The other procedure in which the rescheduling is considered is the interactive scheduling technique^{[46],[47]}. It is difficult to manually reschedule the complicated operations of a process so as to meet the variable demand. It is also difficult to develop a scheduling procedure fully taking into account a rule of thumb that skillful process schedulers have learned from experience.

Therefore, the symbiotic relationship between human being and electronic computer would be more powerful at certain types of problem-solving activities than either would be alone. However as described in the literature^[46], interactive systems have become widely used in design areas, but little has been done in the area of scheduling. In Chapter 6, an interactive scheduling system developed for a multi-product process is stated.

6. Introduction to the thesis

Recently, the advantages of batch processes have been recognized, and as mentioned in the previous sections, many research works have been done for the design and scheduling problems. However, little attention has been paid to the design and scheduling problems of a batch process with storage tanks.

One of the characteristics of the batch process is that the materials processed are liquids, gases or fine powders. Therefore, storage tanks are indispensable to holding the products or intermediate products in the process. From this point of view, in this thesis the design and scheduling problems of batch processes with intermediate storage tanks are studied. The aim of this thesis is to give an answer for the question of how we should formulate and solve the design and/or scheduling problem of a batch process with storage tanks.

As to the effects on storage tanks to the design and scheduling problems, the following points are stressed in this thesis:

- 1) In order to optimally design a batch process with storage tanks, it is necessary to consider the design problem and the operation schedule of the process simultaneously.
- 2) There are some cases where the total investment cost which is required to satisfy the given production requirement is reduced by installing intermediate storage tanks.

3) The capacities of intermediate storage tanks as well as their installation places affect the flexibility of the operation.

From Chapter 2 to Chapter 4, the design problem of batch processes are studied. In Chapter 2, a simple process consisting of periodically operated parallel batch units and intermediate storage tanks installed before and after the batch section is taken up. And the problem of determining the schedule of the parallel batch operations and the tank capacities is discussed.

By taking into account the result obtained in Chapter 2, the general problem of the optimal design and operation of a single product batch process consisting of many batch stages and intermediate storage tanks is dealt with in Chapter 3.

In Chapter 4, the results derived at the previous chapter are applied to the problem of the design of a flexible batch process. In this chapter, the variations related to the operation schedule and to the batch size of each batch stage are considered as uncertain variations in the process. And we deal with the problem of how to design a batch process in which the storage tanks never overflow nor run out of stored material even though the variations within a fixed range may occur.

In Chapter 5 and Chapter 6, the scheduling problems of batch processes are studied. In Chapter 5, a cyclically operated batch process is taken up. And the problem of

determining the starting moments of batch units so as to smooth the peak consumption of utilities is discussed.

In Chapter 6, the scheduling problem of a multi-product process is studied. In such a process, the storage capacity of fluid materials strongly affects the operation scheduling of the process. Here a process consisting of many production stages and storage tanks is taken up, and an algorithm is developed to derive feasible production schedules which minimize the sum of the operation and the change-over costs.

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Chapter 2

OPTIMAL DESIGN AND SCHEDULING OF A BATCH PROCESS WITH PARALLEL BATCH UNITS AND STORAGE TANKS

1. Introduction

There are many cases where a chemical process system consists of continuous and batch sections. In such a case, intermediate storage tanks are installed before and after the batch section to maintain and assure the continuous operation of the whole system. In this chapter, a simple process consisting of a batch section and two storage tanks shown in Figure 2-1 is taken up. And the problem of determining the size of parallel batch units and their operation schedule as well as the storage tank capacities is discussed.

First, a relationship between minimum storage tank capacities and optimal process timing of each batch unit is analytically derived for the case where the batch section consists of N -parallel identical units. Then we deal with the case for which each batch unit has a different operation scheme. To obtain the optimal solution, it is necessary to carry out tremendous computer calculations. So it is very

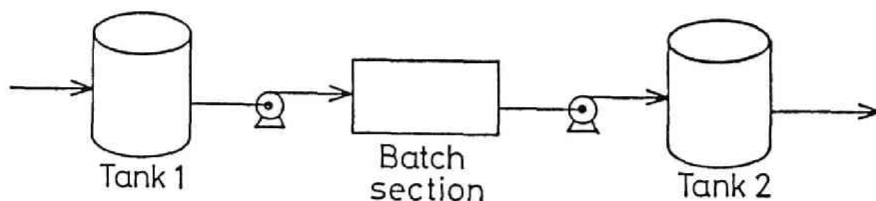


Figure 2-1. Process consisting of a batch section and tanks

important to be able to reduce the searching domain as small as possible. It is discussed here the searching domain can be reduced and how it results in a drastic decrease in computation time.

2. A Process with Only One Batch Unit

In this section the simplest case of a process with one batch operation unit and two storage tanks installed before and after the batch unit, is discussed to show what implications exist between batch operation schemes and the capacities of the tanks.

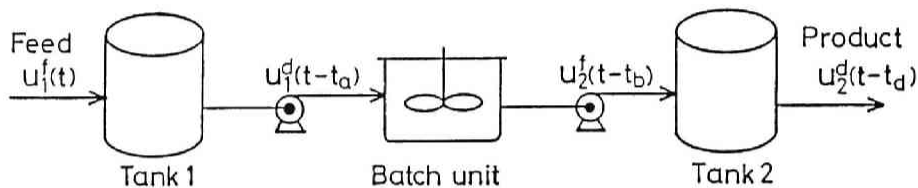


Figure 2-2. Example of a batch wise operated system

In the process schematically shown in Figure 2-2, the following assumptions are introduced.

- (i) The inlet flow rate into tank 1, U_1^f , and the outlet flow rate from tank 2, U_2^d , are constant.
- (ii) The maximum storage capacities of tank 1, V_1 , and tank 2, V_2 , are equal to the maximum hold-up in tank 1 and tank 2, respectively. And there is no initial inventory in each storage tank.
- (iii) The batch unit is operated according to the following steps.
 - The unit is filled to its full capacity, S , by a constant flow rate, U_1^d , from tank 1.

- The processing step is started and continues for holding time, T , without any inlet and outlet flow.
 - When the processing step is completed, the contents are discharged into tank 2 at a constant flow rate, U_2^f .
 - After the unit is emptied, a preparation period is necessary before the unit can be used for the next process cycle. The preparation period must be equal to or longer than T_p . During this preparation period, no flows to and from the unit are permitted.
- (iv) The process does not cause any volume changes in the flows.

With the above assumptions, we derive the following mathematical model of the process.

The cycle time of the batch unit, W , must be longer than or equal to its minimum cycle time, w :

$$W \geq w = (S/U_1^d) + T + (S/U_2^f) + T_p \quad (2-1)$$

where

S = the batch size of the batch item (= the equipment size of the batch unit),

U_i^j = capacities of the feed pump ($j=f$) and the discharge pump ($j=d$) of tank i per unit time,

T = the processing time of the batch item,

T_p = the minimum preparation period of the batch item.

In Eq.(2-1), S/U_1^d and S/U_2^f are the duration for the inlet and outlet flows to and from the batch unit in one cycle, respectively.

The input and the output functions to and from the batch unit, $u_1^d(t)$ and $u_2^f(t)$, are defined as follows:

$$u_i^j(t) = \begin{cases} U_i^j & ; \quad kW \leq t < kW + S/U_i^j \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (2-2)$$

((i = 1, j = d) or (i = 2, j = f))
(k = 0, 1, 2, ...)

Similarly, the input function to tank 1, $u_1^f(t)$, and the output function from tank 2, $u_2^d(t)$, are defined as follows:

$$u_i^j(t) = \begin{cases} U_i^j & ; \quad 0 \leq t \\ 0 & ; \quad t < 0 \end{cases} \quad (2-3)$$

(i = 1, j = f) or (i = 2, j = d)

Let the starting time of the inflow to tank 1 be the origin of the time axis. Then the hold-up in tank 1, $v_1(t)$, and in tank 2, $v_2(t)$, are expressed as follows:

$$\frac{dv_1(t)}{dt} = u_1^f(t) - u_1^d(t - t_a) \quad (2-4)$$

$$\frac{dv_2(t)}{dt} = u_2^f(t - t_b) - u_2^d(t - t_d) \quad (2-5)$$

where

t_a = the starting moment of the inflow to the batch unit in the first cycle,

t_b = the starting moment of the outflow from the batch unit in the first cycle,

t_d = the starting moment of the outflow from tank 2.

The right-hand side of Eq.(2-4) is a periodic function with the cycle time W for $t \geq t_a$. In order to continue the periodic operation of the process, $v_1(t)$ must also be a periodic function with the same cycle time W for $t \geq t_a$. Otherwise, overflowing or exhaustion of stored material in the tank happens in the course of time.

Since the duration of the inflow to the batch unit is shorter than the cycle time, the capacity of the feed pump to the batch unit, U_1^d , is greater than U_1^f . Then, the hold-up in tank 1, $v_1(t)$, is monotonically decreasing in the interval $(t_a, t_a + S/U_1^d)$ and monotonically increasing in the interval $(t_a + S/U_1^d, t_a + W)$. Therefore $v_1(t)$ takes the maximum value at $t = t_a$ and $t = t_a + W$, and takes the minimum value at $t = t_a + S/U_1^d$ in the interval $[t_a, t_a + W]$.

Letting the minimum value of $v_1(t)$ be zero, the maximum value of $v_1(t)$, that is, the volume of tank 1, V_1 , can be obtained as follows:

$$\begin{aligned} V_1 &= v_1(t_a + W) \\ &= v_1(t_a + S/U_1^d) + \int_{t_a + S/U_1^d}^{t_a + W} [u_1^f(t) - u_1^d(t - t_a)] dt \\ &= (W - S/U_1^d) U_1^f \end{aligned}$$

By taking into account the fact that $v_1(t_a) = v_1(t_a + W)$, the following relationships are derived.

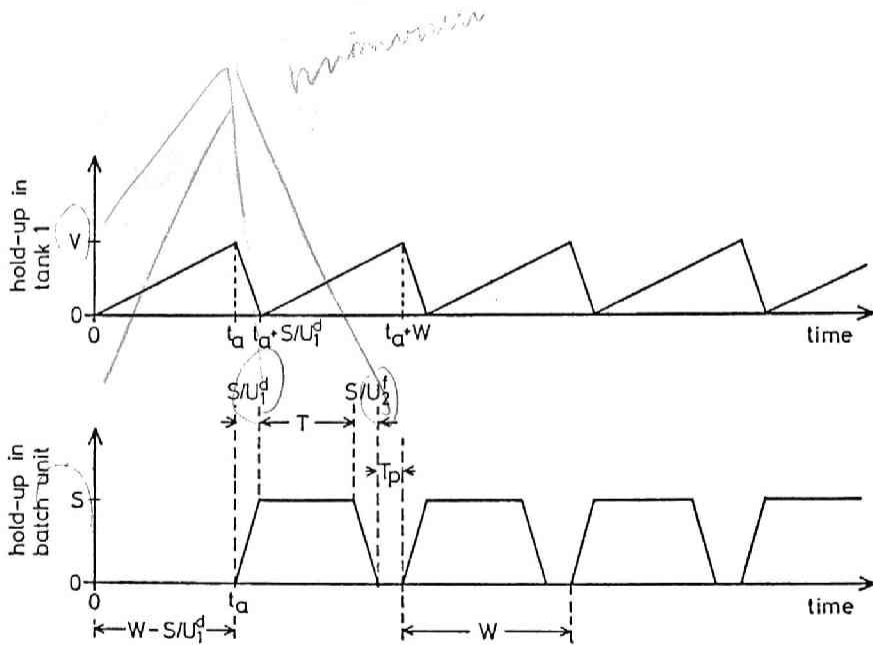
$$\begin{aligned}
 v_1(t_a) &= \int_0^{t_a} [u_1^f(t) - u_1^d(t - t_a)] dt \\
 &= U_1^f \cdot t_a - 0 \\
 &= v_1(t_a + W)
 \end{aligned}$$

$$t_a = W - (S/U_1^d) \quad (2-6)$$

$$\begin{aligned}
 v_1(t_a + W) - v_1(t_a) &= \int_{t_a}^{t_a + W} [u_1^f(t) - u_1^d(t - t_a)] dt \\
 &= U_1^f \cdot W - S = 0
 \end{aligned} \quad (2-7)$$

$$v_1 = (W - S/U_1^d) U_1^f = (1 - U_1^f/U_1^d) S \quad (2-8)$$

The changes of the hold-up in tank 1 and that in the batch unit are graphically shown in Figure 2-3.



Changes of the hold-up in
Figure 2-3. tank 1 and the batch unit

By the same procedure of determining the volume of tank 1, similar conditions are derived for tank 2:

$$V_2 = (1 - U_2^d/U_2^f)S \quad (2-9)$$

$$t_b = t_a + S/U_1^d + T = W + T \quad (2-10)$$

$$t_d = t_b \quad (2-11)$$

$$0 = S - U_2^d \cdot W \quad (2-12)$$

From Eqs.(2-7) and (2-12),

$$U_1^f = U_2^d \quad (2-13)$$

holds.

By eliminating W from Eqs.(2-1) and (2-7), the following inequality is obtained.

$$S \geq U_1^f \cdot U_1^d \cdot U_2^f (T + T_p) / (U_1^d \cdot U_2^f - U_1^f \cdot U_1^d - U_1^f \cdot U_2^f) \quad (2-14)$$

From Eqs.(2-8) and (2-9), it is clear that the volume of each storage tank becomes larger in proportion to the size of the batch equipment. Therefore, the total equipment cost of the batch item and storage tanks is minimized when the size of batch equipment, S , takes the minimum value which satisfies Eq.(2-14).

In actual design problems, the variable U_2^d is usually fixed as the production demand, and T and T_p are also fixed by technical reasons. Therefore, the optimal design problem is stated as follows:

Find the optimal equipment size, S , the optimal sizes of storage tanks, V_1 and V_2 , and the optimal capacities of feeding and discharging pumps, U_1^d and U_2^f , so as to minimize the following performance index:

$$P.I. = p(S) + \sum_{i=1}^2 q_i(V_i) + r_1(U_1^d) + r_2(U_2^f)$$

subject to

$$U_1^f = U_2^d \quad (2-13)$$

$$S = U_1^f \cdot U_1^d \cdot U_2^f (T + T_p) / (U_1^d \cdot U_2^f - U_1^f \cdot U_1^d - U_1^f \cdot U_2^f) \quad (2-15)$$

$$V_1 = (1 - U_1^f/U_1^d)S \quad (2-8)$$

$$V_2 = (1 - U_2^d/U_2^f)S \quad (2-9)$$

where p , q_i and r_i are all monotonically increasing functions.

From above equations, it is clear that there exist two degrees of freedom in this system. If U_1^d and U_2^f are chosen as the free variables, the other variables are successively determined.

3. A Process Including N-parallel Same Capacity Batch Units

In this section, the case where a batch section is composed of N-parallel same batch units, as shown in Figure 2-4 is treated.

In order to consider the design problem for this process, the following assumptions are added to the ones listed in the previous section.

- (v) All units in the N-parallel batch section are the same and are operated according to assumption (iii). Each unit is operated cyclically with a time delay in relation to the other units. Hereafter, the delay of the starting time of batch unit i in the first cycle compared to the time when the first batch is fed to the batch section is called the "phase difference of batch unit i " and is expressed by t_i . The starting time of each unit, that is, the phase difference of each unit

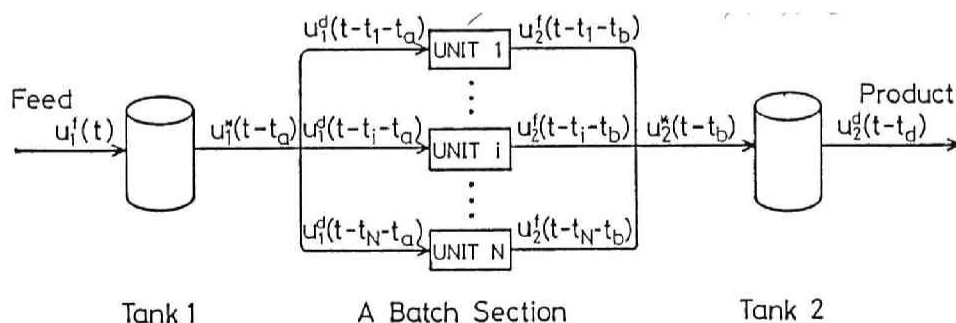


Figure 2-4. N-parallel batch system

can be arbitrarily chosen. However, since the cycle times of each unit is W , it is assumed that the phase difference of each unit is restricted such that

$$0 \leq t_i < W \quad (i = 1, 2, \dots, N)$$

- (vi) All feed pumps to batch units have the same capacity U_1^d , and it is greater than or equal to the inlet flow rate to tank 1, U_1^f . Similarly, all discharge pumps from batch units have the same capacity, U_2^f , and it is greater than or equal to the outlet flow rate from tank 2, U_2^d .

In this section, we set out the relationships between the variables, N , V_1 , V_2 , S , U_1^f , U_1^d , U_2^f , U_2^d , t_a , t_b , t_d and the N -dimensional vector $\#$ composed of the N -phase differences from the first to the N -th batch unit, and then the optimal phase differences of batch units, that is, the phase differences which minimize the sum of both tank capacities are analytically derived.

When the input and the output functions to and from each batch unit are given by Eq.(2-2), the input and the output functions to and from the N -parallel batch section, $u_1^*(t)$ and $u_2^*(t)$, are expressed as follows:

$$u_1^*(t) = \sum_{i=1}^N u_1^d(t - t_i) \quad (2-16)$$

$$u_2^*(t) = \sum_{i=1}^N u_2^f(t - t_i) \quad (2-17)$$

By using Eqs. (2-3), (2-16) and (2-17), the hold-up in tank 1 and tank 2 are given by the following differential equations.

$$\frac{dv_1(t)}{dt} = u_1^f(t) - u_1^*(t - t_a) \quad (2-18)$$

$$\frac{dv_2(t)}{dt} = u_2^*(t - t_b) - u_2^d(t - t_d) \quad (2-19)$$

When certain numbers are given to t_a and t , the change of the hold-up in tank 1 becomes as shown in Figure 2-5.

Since $u_1^d(t)$ is a periodic function with the cycle time W for $t \geq 0$, $u_1^*(t)$ also becomes a periodic function with the same cycle time for $t \geq W$. In order to continue the operation of the process for a long period, $v_1(t)$ must be a periodic function with the cycle time W for $t \geq t_a + W$. Then the following relationship is satisfied.

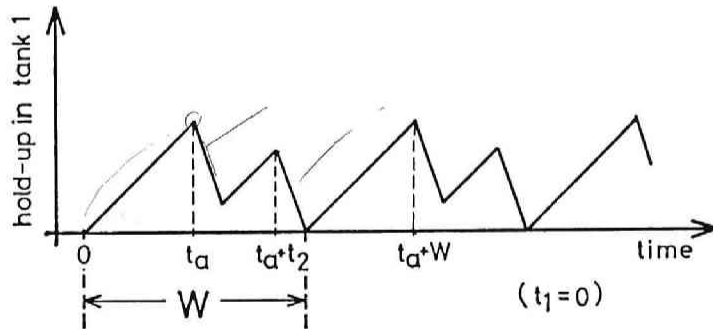


Figure 2-5. Change of the hold-up in tank 1 (N=2)

$$\begin{aligned}
\int_{t_a+W}^{t_a+2W} dv_1 &= \int_{t_a+W}^{t_a+2W} [u_1^f(t) - u_1^*(t-t_a)] dt \\
&= U_1^f \cdot W - \sum_{i=1}^N \int_W^{2W} u_1^d(t-t_i) dt \\
&= U_1^f \cdot W - \sum_{i=1}^N \int_{W-t_i}^{2W-t_i} u_1^d(t) dt \\
&= U_1^f \cdot W - N \cdot S \\
&= 0
\end{aligned} \tag{2-20}$$

Since

$$u_1^*(t) \leq u_1^*(t+W) \quad \text{for } \forall t < W, \text{ and}$$

$$u_1^*(t) = u_1^*(t+W) \quad \text{for } \forall t \geq W,$$

the following inequalities are satisfied.

$$\int_t^{t+W} [u_1^f(s) - u_1^*(s)] ds \geq 0 \quad \text{for } \forall t < W, \text{ and}$$

$$\int_t^{t+W} [u_1^f(s) - u_1^*(s)] ds = 0 \quad \text{for } \forall t \geq W.$$

From above inequalities, the following relationship is derived:

$$\begin{aligned}
\int_0^t [u_1^f(s) - u_1^*(s-t_a)] ds &\leq \int_0^{t+W} [u_1^f(s) - u_1^*(s-t_a)] ds \\
&= \int_0^{t+iW} [u_1^f(s) - u_1^*(s-t_a)] ds \\
&\text{for } \forall t \in [t_a, t_a+W] \quad ; (i = 2, 3, \dots)
\end{aligned}$$

Then, the maximum and minimum values of $v_1(t)$ are given by the following equations.

$$\begin{aligned}\max_{\{t\}}\{v_1(t)\} &= U_1^f \cdot t_a + \max_{W \leq t < 2W} \int_0^t [U_1^f - u_1^*(s)] ds \\ \min_{\{t\}}\{v_1(t)\} &= U_1^f \cdot t_a + \min_{0 \leq t < W} \int_0^t [U_1^f - u_1^*(s)] ds \\ &= 0\end{aligned}\quad (2-21)$$

From above two equations, the volume of tank 1 is derived, i.e.

$$\begin{aligned}V_1 &= \max_{\{t\}}\{v_1(t)\} \\ &= \max_{W \leq t < 2W} \int_0^t [U_1^f - u_1^*(s)] ds - \min_{0 \leq t < W} \int_0^t [U_1^f - u_1^*(s)] ds\end{aligned}\quad (2-22)$$

From Eq.(2-21), t_a is given by

$$t_a = - \left\{ \min_{0 \leq t < W} \int_0^t [U_1^f - u_1^*(s)] ds \right\} / U_1^f \quad (2-23)$$

For tank 2, the following relationships are also obtained in the same way.

$$U_2^d \cdot W = N \cdot S \quad (2-24)$$

$$V_2 = \max_{W \leq t < 2W} \int_0^t [u_2^*(s) - U_2^d] ds - \min_{0 \leq t < W} \int_0^t [u_2^*(s) - U_2^d] ds \quad (2-25)$$

$$t_d - t_b = - \left\{ \min_{0 \leq t < W} \int_0^t [u_2^*(s) - U_2^d] ds \right\} / U_2^d \quad (2-26)$$

From Eqs.(2-20) and (2-24),

$$U_1^f = U_2^d \quad (2-27)$$

holds.

By eliminating W from Eqs.(2-1) and (2-20) and rearranging with respect to S , the equipment size, S , must satisfy the following inequality:

$$S \geq U_1^f \cdot U_1^d \cdot U_2^f (T + T_p) / (N \cdot U_1^d \cdot U_2^f - U_1^f \cdot U_1^d - U_1^f \cdot U_2^f) \quad (2-28)$$

It is clear from Eqs.(2-18) and (2-19) that the tank volume becomes the function not only of the design variables such as S , U_1^d and U_2^f but also of the variables which describe the operation schedule such as t_a , t_b , t_d and $\mathbf{t}$. When U_2^d , T and T_p are given a priori, the optimal design problem is how to find the optimal values of N , S , U_1^f , U_1^d , U_2^f , t_a , t_b , t_d and $\mathbf{t}$ which satisfy Eqs.(2-22) to (2-28).

In the following, how the optimal values of t_a , t_b , t_d and $\mathbf{t}$ which minimize V_1 and V_2 are analytically obtained is shown. The main result of this section can be summarized in the following theorem.

[Theorem 2-1]

The optimal phase difference vector $\mathbf{t}$ which minimizes the volume of tank 1, V_1 , is equal to that which minimizes the volume of tank 2, V_2 , and is given by

$$\begin{aligned} \mathbf{t} &= (t_1, t_2, \dots, t_N) \\ &= (0, \frac{W}{N}, \frac{2 \cdot W}{N}, \dots, \frac{(N-1)W}{N}) \end{aligned} \quad (2-29)$$

[Proof of Theorem 2-1]

Without losing generality, it may be assumed that the following relationship exists between the phase differences,

$$t_1 = 0 \leq t_2 \leq t_3 \leq t_4 \leq \dots \leq t_N < W$$

since the numbering of any batch unit can be arbitrarily changed.

Let the set of all feasible phase difference vectors be P_0 , then P_0 can be divided into two subsets as follows:

$$P_0 = P_1 \dot{\cup} P_2$$

where

$$P_1 := \{t \in P_0 \mid t_i = (i-1)W/N, \quad i = 1, 2, \dots, N\}$$

$$P_2 := \{t \in P_0 \mid t_{k+1} - t_k < W/N \text{ for some } k, t_{N+1} = W\}$$

$\dot{\cup}$ = the direct sum

a) we first calculate the volume of tank 1 when the phase difference vector $t \in P_1$ is chosen.

From the definition of P_1 , $u_1^*(t)$ is expressed as follows:

$$u_1^*(t) = \sum_{i=1}^N u_1^d(t - \frac{i-1}{N} \cdot W)$$

From assumption vi) and Eq.(2-20), S/U_1^d satisfies the following inequality:

$$0 < S/U_1^d \leq W/N$$

Therefore, the following equation is satisfied for any $t \geq 0$.

$$u_1^d(t - \frac{N-1}{N} \cdot W) = u_1^d(t + \frac{W}{N})$$

By using the above equation, we can derive that $u_1^*(t)$ is a periodic function with the cycle time W/N for $t \geq 0$, i.e.

$$u_1^*(t + \frac{W}{N}) = u_1^d(t + \frac{W}{N}) + u_1^d(t) + \dots + u_1^d(t - \frac{N-2}{N}W)$$

$$\begin{aligned}
&= u_1^d(t - \frac{N-1}{N} \cdot W) + u_1^d(t) + \dots + u_1^d(t - \frac{N-2}{N} \cdot W) \\
&= u_1^*(t)
\end{aligned}$$

Therefore, $u_1^*(t)$ is expressed as follows:

$$u_1^*(t) = \begin{cases} U_1^d & ; iW/N \leq t < iW/N + S/U_1^d \\ 0 & ; \text{otherwise} \end{cases} \quad (2-30)$$

($i = 0, 1, 2, \dots$)

As $u_1^*(t)$ is a periodic function with the cycle time W/N for $t \geq 0$, the cycle time of $v_1(t)$ also becomes W/N for $t \geq t_a$.

From Eqs. (2-18), (2-20) and (2-30), $v_1(t)$ decreases monotonically in the interval $(t_a, t_a + S/U_1^d)$ and increases monotonically in the interval $(t_a + S/U_1^d, t_a + W/N)$. Therefore, $v_1(t)$ is the unimodal function in the interval $(t_a, t_a + W/N)$. Then $v_1(t)$ has maximum value at $t = t_a$ and $t = t_a + W/N$, and minimum value at $t = t_a + S/U_1^d$ in the interval $[t_a, t_a + W/N]$.

Integrating Eq. (2-18) over the interval $[t_a + S/U_1^d, t_a + W/N]$, we have

$$V_1^* = (1 - U_1^f/U_1^d)S$$

where

V_1^* = the volume of tank 1 for the phase difference vector $\mathbf{t} \in P_1$.

b) We next show that the volume of tank 1 is larger than V_1^* for any $\mathbf{t} \in P_2$.

From the definition of P_2 , there exists a certain k such that $t_{k+1} - t_k < W/N$. We consider $u_1^*(t)$ in the interval $[t_k, t_{k+1} + (S/U_1^d)] \dots \dots \dots$

The input functions of unit k and unit k+1 are given by

$$u_1^d(t-t_k) = \begin{cases} u_1^d & ; t_k \leq t < S/u_1^d + t_k \\ 0 & ; S/u_1^d + t_k \leq t \leq S/u_1^d + t_{k+1} \end{cases}$$

$$u_1^d(t-t_{k+1}) = \begin{cases} 0 & ; t_k \leq t < t_{k+1} \\ u_1^d & ; t_{k+1} \leq t \leq t_{k+1} + S/u_1^d \end{cases}$$

Integrating Eq. (2-18) over the interval $[t_a + t_k, t_a + t_{k+1} + (S/u_1^d)]$, we obtain

$$\begin{aligned} & v_1(t_a + t_{k+1} + S/u_1^d) - v_1(t_a + t_k) \\ & \leq (S/u_1^d + t_{k+1} - t_k)U_1^f - 2S \\ & < (S/u_1^d + W/N)U_1^f - 2S \\ & = - (1 - U_1^f/u_1^d)S . \end{aligned}$$

Then, the following relationship is obtained.

$$\begin{aligned} v_1(t_a + t_k) & > v_1(t_a + t_{k+1} + S/u_1^d) + (1 - U_1^f/u_1^d)S \\ & \geq v_1^* \end{aligned}$$

Now, we can conclude that the phase difference vector $\mathbf{t} \in P_1$ minimizes V_1 . We can also prove in the same manner as shown above that $\mathbf{t} \in P_1$ minimizes the volume of tank 2, V_2 .

(Q.E.D. of Theorem 2-1)

From Theorem 2-1, the volumes of tank 1 and tank 2 are minimized when each batch unit is operated with a delayed

starting time of W/N . Thus, the minimum volumes of tank 1, V_1^* , and tank 2, V_2^* , the starting moments of the outflow from tank 1 and the inflow to tank 2 in the first cycle, t_a and t_b , and the starting moment of the outflow from tank 2, t_d , can be obtained as follows:

$$V_1^* = (1 - U_1^f/U_1^d)S \quad (2-31)$$

$$V_2^* = (1 - U_2^d/U_2^f)S \quad (2-32)$$

$$t_a = W/N - S/U_1^d \quad (2-33)$$

$$t_b = t_a + S/U_1^d + T = W + T \quad (2-34)$$

$$t_d = t_b \quad (2-35)$$

As mentioned in the previous section, it is clear from Eqs.(2-31) and (2-32) that the total equipment cost of batch items and storage tanks is minimized when the size of batch equipment takes the minimum value which satisfies Eq.(2-28). Therefore, the optimal equipment size is given by the following equation.

$$S = U_1^f \cdot U_1^d \cdot U_2^f (T + T_p) / (N \cdot U_1^d \cdot U_2^f - U_1^f \cdot U_1^d - U_1^f \cdot U_2^f) \quad (2-36)$$

Consequently, the optimal design problem is how to find the optimal N , S , U_1^f , U_1^d , U_2^f , V_1^* and V_2^* which minimize the capital cost subject to the following four constraints:

$$U_1^f = U_2^d \quad (2-27)$$

$$S = U_1^f \cdot U_1^d \cdot U_2^f (T + T_p) / (N \cdot U_1^d \cdot U_2^f - U_1^f \cdot U_1^d - U_1^f \cdot U_2^f) \quad (2-36)$$

$$v_1^* = (1 - u_1^f/u_1^d)s \quad (2-31)$$

$$v_2^* = (1 - u_2^d/u_2^f)s \quad (2-32)$$

And the variables which denote the optimal operation schedule are given by the following equations:

$$t_a = W/N - S/u_1^d \quad (2-33)$$

$$t_b = t_a + S/u_1^d + T = W + T \quad (2-34)$$

$$t_d = t_b \quad (2-35)$$

$$t = (0 , \frac{W}{N} , \frac{2 \cdot W}{N} , \dots , \frac{(N-1)W}{N}) \quad (2-29)$$

From the above equations, it is clear that there exist three degrees of freedom in the system. If u_1^d , u_2^f and N are chosen as the free variables, the other variables are successively determined.

4. A Process Including N-Parallel Different Capacity Batch Units

In this section, the discussion is expanded to the case where the batch section is composed of N-parallel batch units but each unit i has a different volume S_i . The holding time and the preparation period of batch unit i are expressed by T_i and T_{pi} , respectively. And it is assumed that the cycle time of each batch item is equal to the minimum cycle time of its batch item. Other conditions for the formulation of the problem are the same those given in the previous section.

Under the above assumptions, relationships which exist among the variables $V_1, V_2, \{S_i\}, \{T_i\}, \{T_{pi}\}, \{W_i\}, U_1^f, U_1^d, U_2^f, U_2^d, t_a, t_b, t_d$ and t are discussed.

The cycle time of batch unit i, W_i , is given by

$$W_i = (S_i/U_1^d) + T_i + (S_i/U_2^f) + T_{pi} \quad (i=1,2,\dots,N) \quad (2-37)$$

The input and output functions of batch unit $i, u_{1i}^d(t)$ and $u_{2i}^f(t)$, are defined as follows:

$$u_{1i}^d(t) = \begin{cases} U_1^d & ; \quad mW_i \leq t < S_i/U_1^d + mW_i \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (2-38)$$

($i = 1, 2, \dots, N ; m = 0, 1, 2, \dots$)

$$u_{2i}^f(t) = \begin{cases} U_2^f & ; \quad mW_i \leq t < S_i/U_2^f + mW_i \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (2-39)$$

(i = 1, 2, ..., N ; m = 0, 1, 2, ...)

By using Eqs.(2-38) and (2-39), the input and output functions to and from the N-parallel batch section, $u_1^*(t)$ and $u_2^*(t)$, are expressed as follows:

$$u_1^*(t) = \sum_{i=1}^N u_{1i}^d(t - t_i) \quad (2-40)$$

$$u_2^*(t) = \sum_{i=1}^N u_{2i}^f(t - t_i) \quad (2-41)$$

Since the cycle time of batch unit i is W_i , t_i can be assumed to satisfy the following inequality:

$$0 \leq t_i < W_i \quad (i = 1, 2, \dots, N)$$

Then, $u_1^*(t)$ becomes a periodic function for $t \geq W^*$, where W^* is the maximum value of the cycle times, i.e.

$$W^* = \max_{\{i\}} \{W_i\} \quad (2-42)$$

Here, in order to continue the further discussion, we extend the usual concepts of the greatest common measure (G.C.M.) and the least common multiple (L.C.M.) in the field of rational numbers.

"multiple" and "measure" are defined at first:

If the quotient of the division of a rational number, x, by a rational number, y, is an integer, y is called an

aliquot part of x . Then, x is called a multiple of y and y is called a measure of x .

The extended G.C.M. and L.C.M. are defined as follows:

A rational number which is a common aliquot part of the rational numbers $x, y, \dots, z$, is called a common measure, and the greatest among the common measures is defined as the extended G.C.M. of the rational numbers, $x, y, \dots, z$. This G.C.M. is hereafter expressed by the symbol $\text{G.C.M.}(x, y, \dots, z)$.

If a certain rational number is a multiple of all of the rational numbers, $x, y, \dots, z$, this rational number is called a common multiple, and the least among the common multiples is defined as the extended L.C.M. of the rational numbers, $x, y, \dots, z$. This L.C.M. is hereafter expressed by the symbol $\text{L.C.M.}(x, y, \dots, z)$.

For example,

$$\text{G.C.M.}(48, 36) = 12 \quad ; \quad \text{G.C.M.}(4.8, 3.6) = 1.2 \quad ;$$

$$\text{L.C.M.}(48, 36) = 144 \quad ; \quad \text{L.C.M.}(4.8, 3.6) = 14.4 \quad .$$

By using the cycle time of each batch item, the cycle time of $u_1^*(t)$, W , is given by

$$W^o = \text{L.C.M.}(W_1, W_2, \dots, W_N) \quad (2-43)$$

The hold-up in tank 1, $v_1(t)$, is given by the following equation.

$$\frac{dv_1(t)}{dt} = u_1^f(t) - u_1^*(t-t_a) \quad (2-44)$$

Since $u_1^*(t)$ is a periodic function with the cycle time W^0 for $t \geq W^*$, the hold-up in tank 1, $v_1(t)$, must also be a periodic function with the cycle time W^0 for $t \geq t_a + W^*$ in order to continue the operation of the process without causing overflowing nor exhaustion of stored material. Therefore, by integrating Eq.(2-44) over the interval $[t_a + W^*, t_a + W^* + W^0]$, the following relationship is obtained.

$$U_1^f = \sum_{i=1}^N (S_i / W_i) \quad (2-45)$$

For tank 2, a similar relation is obtained from its hold-up equation, i.e.

$$U_2^d = \sum_{i=1}^N (S_i / W_i) \quad (2-46)$$

From Eqs.(2-45) and (2-46), it follows that

$$U_1^f = U_2^d \quad (2-47)$$

V_1 and V_2 are subsequently obtained in the same manner as explained in the previous section.

Summarizing the results obtained in the above discussion, the following relationships have been derived among the variables, V_1 , V_2 , $\{S_i\}$, $\{T_i\}$, $\{T_{pi}\}$, $\{W_i\}$, U_1^f , U_1^d , U_2^f , U_2^d and t .

$$W_i = S_i / U_1^d + T_i + S_i / U_2^f + T_{pi} \quad (i=1, 2, \dots, N) \quad (2-37)$$

$$W^0 = \text{L.C.M.}(W_1, W_2, \dots, W_N) \quad (2-43)$$

$$U_1^f = \sum_{i=1}^N (S_i / W_i) \quad (2-45)$$

$$U_1^f = U_2^d \quad (2-47)$$

$$V_1 = \max_{W^* \leq t < W^* + W^0} \int_0^t [U_1^f - u_1^*(s)] ds - \min_{0 \leq t < W^0} \int_0^t [U_1^f - u_1^*(s)] ds \quad (2-48)$$

$$V_2 = \max_{W^* \leq t < W^* + W^0} \int_0^t [u_2^*(s) - U_2^d] ds - \min_{0 \leq t < W^0} \int_0^t [u_2^*(s) - U_2^d] ds \quad (2-49)$$

$$W^* = \max_{\{i\}} \{W_i\} \quad (2-42)$$

$u_1^*(t)$ and $u_2^*(t)$ are defined by Eqs. (2-38) to (2-41).

In actual design problems, suitable values are usually assigned to the variables U_2^d , $\{T_i\}$ and $\{T_{pi}\}$. Therefore, by assuming the values of U_1^d , U_2^f , t and $\{S_i\}$ so as to satisfy Eqs. (2-45) and (2-47), all of the other variables and the capital cost of the process are determined.

5. Reduction of the Searching Domain

If $U_1^f, U_1^d, U_2^f, U_2^d, \{T_i\}, \{T_{pi}\}$ and $\{S_i\}$ are determined so as to satisfy Eqs.(2-45) and (2-47), the volumes of tank 1 and tank 2, V_1 and V_2 , are functions of the phase difference vector of the batch units. It is, therefore, necessary to search all possible phase differences to obtain the minimum values of V_1 and V_2 .

In other words, it is necessary to search in the set P,

$$P = \{ t \mid 0 \leq t_i < W_i, \quad i = 1, 2, \dots, N \} \quad (2-50)$$

Unfortunately this domain is too large. So here it is shown how small the domain can be reduced under the assumption that the hold-up in each tank which is needed for the start up and the shut down of the process can be disregarded.

Under the above assumption, the tank volumes which are needed for the cyclic operation can be expressed as follows:

$$V_1 = \max_{0 \leq t < W^0} \int_0^t [U_1^f - u_1^*(s+W^*)] ds - \min_{0 \leq t < W^0} \int_0^t [U_1^f - u_1^*(s+W^*)] ds \quad (2-51)$$

$$V_2 = \max_{0 \leq t < W^0} \int_0^t [u_2^*(s+W^*) - U_2^d] ds - \min_{0 \leq t < W^0} \int_0^t [u_2^*(s+W^*) - U_2^d] ds \quad (2-52)$$

In this section, it is presumed that $U_1^f, U_1^d, U_2^f, U_2^d, \{T_i\}, \{T_{pi}\}$ and $\{S_i\}$ are given so as to satisfy Eqs.(2-45) and (2-47) and tank volumes are given by Eqs.(2-51) and (2-52).

To simplify the discussion, it is first assumed that the batch section is composed of only two units. Since

$u_1^*(t)$ is a function of time t as well as of the phase differences t_1 and t_2 , the expression $u_1^*(t, t_1, t_2)$ is used instead of $u_1^*(t)$. $u_1^*(t, t_1, t_2)$ is hence defined as follows,

$$u_1^*(t, t_1, t_2) := u_{11}^d(t - t_1) + u_{12}^d(t - t_2) \quad (2-53)$$

Then, in order to derive the minimum value of the tank volumes which are given by Eqs.(2-51) and (2-52), it is sufficient to search in the domain given by the following theorem.

[Theorem 2-2]

When the input function to the batch section, $u_1^*(t, t_1, t_2)$, is given by Eq.(2-53), the minimum value of V_1 given by Eq.(2-51) can be obtained only by searching the following domain.

$$t_1 = 0$$

$$0 \leq t_2 < \text{G.C.M.}(W_1, W_2)$$

where

G.C.M. (X, Y) = the extended greatest common measure of X and Y.

[Proof of Theorem 2-2]

In order to explicitly clear the values of the phase differences, the following expression is used instead of V_1 :

$$V_1(t_1, t_2) := \max_{0 \leq t < W^0} \int_0^t [U_1^f - u_{11}^d(s + W^* - t_1) - u_{12}^d(s + W^* - t_2)] ds \\ - \min_{0 \leq t < W^0} \int_0^t [U_1^f - u_{11}^d(s + W^* - t_1) - u_{12}^d(s + W^* - t_2)] ds$$

$$= \max_{0 \leq t < W^0} \{v_1^*(t, t_1, t_2)\} - \min_{0 \leq t < W^0} \{v_1^*(t, t_1, t_2)\} \quad (2-54)$$

where $v_1^*(t, t_1, t_2)$ is defined as follows,

$$v_1(t, t_1, t_2) := \int_0^t [U_1^f - u_{11}^d(s+W^*-t_1) - u_{12}^d(s+W^*-t_2)] ds \quad (2-55)$$

When x_1 and x_2 are taken as the phase differences of batch unit 1 and batch unit 2, respectively, the tank volume $V_1(x_1, x_2)$ satisfies the following relationship:

$$\begin{aligned} V_1(x_1, x_2) &= \max_{0 \leq t < W^0} \{v_1^*(t, x_1, x_2)\} - \min_{0 \leq t < W^0} \{v_1^*(t, x_1, x_2)\} \\ &= \max_{-x_1 \leq t < W^0 - x_1} \{v_1^*(t, 0, x_2 - x_1)\} - \min_{-x_1 \leq t < W^0 - x_1} \{v_1^*(t, 0, x_2 - x_1)\} \end{aligned} \quad (2-56)$$

As $v_1^*(t, 0, x_2 - x_1)$ is a periodic function with respect to time t for $t \geq -x_1$, Eq. (2-56) can be rewritten as follows:

$$\begin{aligned} V_1(x_1, x_2) &= \max_{0 \leq t < W^0} \{v_1^*(t, 0, x_2 - x_1)\} - \min_{0 \leq t < W^0} \{v_1^*(t, 0, x_2 - x_1)\} \\ &= V_1(0, x_2 - x_1) \end{aligned}$$

By taking into account this result, only the phase difference of batch unit 2 is hereafter considered, i.e. the phase difference of batch unit 1 is set equal to zero. Since $x_1 \in [0, W_1]$ and $x_2 \in [0, W_2]$, $x_2 - x_1$ satisfies the following inequality:

$$-W_1 \leq x_2 - x_1 \leq W_2$$

Then, the theorem which has to be proved can be mathematically stated as follows:

"For $\forall x \in [-W_1, W_2]$, there exists some $x^* \in [0, G.C.M.(W_1, W_2)]$

such that $V_1(0, x) = V_1(0, x^*)$."

In the following, it is shown that $x^* = \text{mod}[x, \text{G.C.M.}(W_1, W_2)]$ satisfies the above equation.

We first prove the following lemma.

[Lemma 2-1]

Let X and Y be positive rational numbers. If A and A' be sets of rational numbers which are defined by the following relations, respectively,

$$A := \{iX + jY \mid i, j : \text{integers}\}$$

$$A' := \{k \cdot \text{G.C.M.}(X, Y) \mid k : \text{integers}\},$$

then, $A = A'$.

[Proof of Lemma 2-1]

We first prove that the minimum positive element of the set A is the extended greatest common measure of X and Y , i.e.,

$$Z := \min\{z \mid z > 0, z \in A\} = \text{G.C.M.}(X, Y)$$

For any $z \in A$, the remainder of z divided by Z also belongs to the set A . Since the remainder is smaller than Z , it should be zero. Therefore Z is a divisor of all the element of the set A , especially, $X = 1 \cdot X + 0 \cdot Y$ and $Y = 0 \cdot X + 1 \cdot Y$.

It is also evident that Z is divided by any common divisor of X and Y . Consequently, we obtain that $Z = \text{G.C.M.}(X, Y)$.

There exist some I and J which satisfy

$$I \cdot X + J \cdot Y = \text{G.C.M.}(X, Y).$$

So, we have

$$k \cdot \text{G.C.M.}(X, Y) = kI \cdot X + kJ \cdot Y \in A$$

for any $k \cdot \text{G.C.M.}(X, Y) \in A'$.

Conversely, for any $iX + jY \in A$, we have

$$iX + jY = \left[\frac{iX}{\text{G.C.M.}(X, Y)} + \frac{jY}{\text{G.C.M.}(X, Y)} \right] \cdot \text{G.C.M.}(X, Y) \in A'$$

(Q.E.D. of Lemma 2-1)

Now we return to the proof of Theorem 2-2.

From Lemma 2-1, there exist some positive integers I' and J' which satisfy

$$W_1 I' - W_2 J' = \text{G.C.M.}(W_1, W_2).$$

From the above equation, the following equations are hold for some positive integers, I and J .

$$x^* = \text{mod}[x, \text{G.C.M.}(W_1, W_2)]$$

$$x^* + W_1 I - W_2 J = x \quad (2-57)$$

By using Eq. (2-57), $V_1(0, x)$ can be rewritten as follows:

$$\begin{aligned} V_1(0, x) &= \max_{0 \leq t < W^0} \int_0^t [U_1^f - u_{11}^d(s+W^*) - u_{12}^d(s+W^*-x^*-W_1 I+W_2 J)] ds \\ &\quad - \min_{0 \leq t < W^0} \int_0^t [U_1^f - u_{11}^d(s+W^*) - u_{12}^d(s+W^*-x^*-W_1 I+W_2 J)] ds \\ &= \max_{-W_1 I \leq t < W^0 - W_1 I} \{v_1^*(t, -W_1 I, x^*-W_2 J)\} \\ &\quad - \min_{-W_1 I \leq t < W^0 - W_1 I} \{v_1^*(t, -W_1 I, x^*-W_2 J)\} \end{aligned}$$

Since $v_1^*(t, -W_1 I, x^*-W_2 J)$ is a periodic function for $t \geq -W_1 I$,

the following relationship is obtained.

$$V_1(0, x) = \max_{0 \leq t < W^0} \{v_1^*(t, -W_1 I, x^* - W_2 J)\} - \min_{0 \leq t < W^0} \{v_1^*(t, -W_1 I, x^* - W_2 J)\}$$

For $\forall t \geq 0$, $u_{11}^d(t + W^* + W_1 I) = u_{11}^d(t + W^*)$, and

$$u_{12}^d(t + W^* - x^* + W_2 J) = u_{12}^d(t + W^* - x^*).$$

Therefore, the following equation is satisfied for $\forall t \geq 0$.

$$v_1(t, -W_1 I, x^* - W_2 J) = v_1(t, 0, x^*)$$

Now we can conclude that

$$V_1(0, x) = V_1(0, x^*).$$

(Q.E.D. of Theorem 2-2)

The result obtained in this case can be extended to the general case of N-parallel batch units. The cycle times Z_i obtained by adding all the input functions from the first to the i-th unit, can be expressed by

$$Z_i = \text{L.C.M.}(Z_{i-1}, W_i) \quad i = 2, 3, \dots, N \quad (2-58)$$

$$Z_1 = W_1$$

By using these Z_i , the domain of the phase difference of batch unit $i+1$, t_{i+1} , can be reduced to

$$0 \leq t_{i+1} < \text{G.C.M.}(Z_i, W_{i+1}) \quad i = 1, 2, \dots, N-1 \quad (2-59)$$

because the situation is identical to the case where there exist only two batch units with cycle times Z_i and W_{i+1} , respectively.

Since the output function $u_{2i}^f(t)$ from each batch unit has the same cycle time as that of the input function $u_{1i}^d(t)$, it is easily verified that the searching domain of the phase difference t_i for minimizing V_2 is exactly the same domain for V_1 . Consequently, the following theorem can be obtained.

[Theorem 2-3]

It is assumed that the volumes of tank 1 and tank 2 are given by Eqs.(2-51) and (2-52), respectively. Then, the minimum values of both tank volumes can be obtained only by searching the following domain P^* .

$$P^* = \left\{ t \left| \begin{array}{l} t = (t_1, t_2, \dots, t_N) \\ 0 \leq t_i < \text{G.C.M.}(Z_{i-1}, W_i) \\ Z_i = \text{L.C.M.}(Z_{i-1}, W_i) \\ Z_1 = W_1, \quad t_1 = 0, \quad i = 2, 3, \dots, N \end{array} \right. \right\} \quad (2-60)$$

For example, it is assumed that $N = 4$ and the cycle time of each batch unit is given by the following equations,

$$W_1 = 2, W_2 = 3, W_3 = 4, W_4 = 5.$$

Then, the reduced searching domain, P^* , can be expressed as shown in Figure 2-6.

As regards the measure of P^* , the following theorem can be derived.

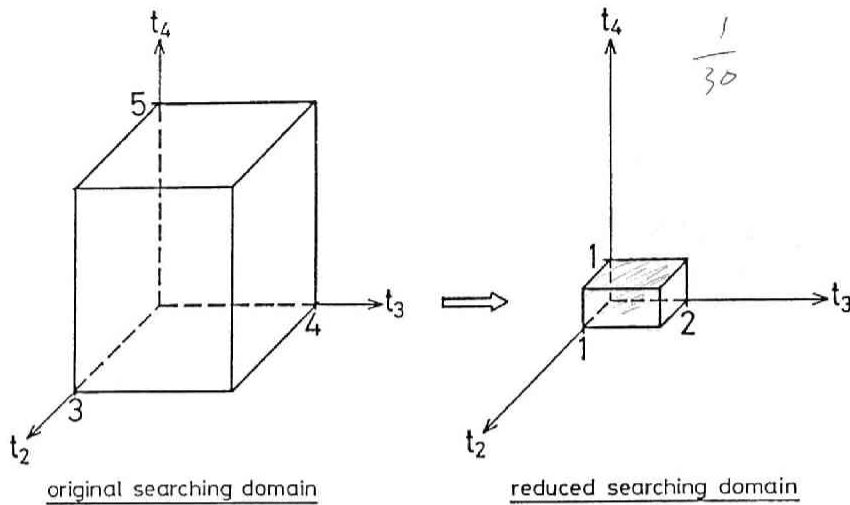


Figure 2-6. Reduction of the searching domain

[Theorem 2-4]

The $(N-1)$ -dimensional measure of P^* is constant and not affected by the order of $W_1, W_2, \dots, W_N$ for constructing the set P^* .

[Proof of Theorem 2-4]

In order to make the explanation simple, G.C.M.(x, y) and L.C.M.(x, y) are expressed as follows:

$$(x; y) := \text{G.C.M.}(x, y)$$

$$[x; y] := \text{L.C.M.}(x, y)$$

We also define

$$D(W_1, W_2, \dots, W_N) := (W_1; W_2) \times \prod_{i=2}^{N-1} ([W_1; W_2; \dots; W_i]; W_{i+1})$$

Then it will be shown that

$$D(W_1, W_2, \dots, W_N) = D(W_{\sigma(1)}, W_{\sigma(2)}, \dots, W_{\sigma(N)})$$

where σ is an arbitrary element of the permutation group of degree N . We first prove the following lemma.

[Lemma 2-2]

For any rational numbers, W_1 , W_2 and W_3 , the following equation is satisfied:

$$(W_1; W_2) ([W_1; W_2]; W_3) = (W_1; W_3) ([W_1; W_3]; W_2)$$

[Proof of Lemma 2-2]

We have,

$$\begin{aligned} ([W_1; W_2]; W_3) &= \left(\frac{W_1 W_2}{(W_1; W_2)} ; W_3 \right) \\ &= (W_1; W_3) \left(\frac{W_1}{(W_1; W_3)} \cdot \frac{W_2}{(W_1; W_2)} ; \frac{W_3}{(W_1; W_3)} \right) \\ &= (W_1; W_3) \left(\frac{W_2}{(W_1; W_2)} ; \frac{W_3}{(W_1; W_3)} \right) \end{aligned}$$

Similarly, we have

$$([W_1; W_3]; W_2) = (W_1; W_2) \left(\frac{W_3}{(W_1; W_3)} ; \frac{W_2}{(W_1; W_2)} \right)$$

Consequently,

$$(W_1; W_2) ([W_1; W_2]; W_3) = (W_1; W_3) ([W_1; W_3]; W_2)$$

(Q.E.D. of Lemma 2-2)

We now prove Theorem 2-4.

We have

$$\begin{aligned}
 D(W_1, W_2, W_3, \dots, W_N) &= (W_2; W_1) ([W_2; W_1]; W_3) \\
 &\quad \times \prod_{i=3}^{N-1} ([W_2; W_1; \dots; W_i]; W_{i+1}) \\
 &= D(W_2, W_1, W_3, \dots, W_N) .
 \end{aligned}$$

From Lemma 2-2, we have

$$\begin{aligned}
 D(W_1, W_2, W_3, \dots, W_N) &= (W_1; W_3) ([W_1; W_3]; W_2) \\
 &\quad \times \prod_{i=3}^{N-1} ([W_1; W_3; W_2; \dots; W_i]; W_{i+1}) \\
 &= D(W_1, W_3, W_2, \dots, W_N)
 \end{aligned}$$

For $i \geq 3$, from Lemma 2-2, we have

$$\begin{aligned}
 &D(W_1, W_2, \dots, W_N) \\
 &= (W_1; W_2) \cdot \left\{ \prod_{k=2}^{i-2} ([W_1; \dots; W_k]; W_{k+1}) \right\} ([W_1; \dots; W_{i-1}]; W_i) \\
 &\quad \times ([W_1; \dots; W_i]; W_{i+1}) \cdot \left\{ \prod_{k=i+1}^{N-1} ([W_1; \dots; W_k]; W_{k+1}) \right\} \\
 &= (W_1; W_2) \cdot \left\{ \prod_{k=2}^{N-1} ([W_1; \dots; W_k]; W_{k+1}) \right\} ([W_1; \dots; W_{i-1}]; W_{i+1}) \\
 &\quad \times ([W_1; \dots; W_{i-1}; W_{i+1}]; W_i) \\
 &\quad \times \left\{ \prod_{k=i+1}^{N-1} ([W_1; \dots; W_{i-1}; W_{i+1}; W_i; \dots; W_k]; W_{k+1}) \right\} \\
 &= D(W_1, \dots, W_{i-1}, W_{i+1}, W_i, W_{i+2}, \dots, W_N)
 \end{aligned}$$

Then, we obtain

$$D(W_1, W_2, \dots, W_N) = D(W_1, \dots, W_{i-1}, W_{i+1}, W_i, W_{i+2}, \dots, W_N)$$

for any i .

An arbitrary element of the permutation group can be replaced with the product of some transpositions, and any transposition can be also replaced by the product of some transpositions each of which is the permutation of the elements adjacent each other.

So we obtain

$$D(W_1, W_2, \dots, W_N) = D(W_{\sigma(1)}, W_{\sigma(2)}, \dots, W_{\sigma(N)})$$

(Q.E.D. of Theorem 2-4)

6. Results and Discussion

.. In this chapter, we have dealt with the problem of determining optimal process timings for batch units to minimize the storage capacity of tanks installed before and after the batch section so as to maintain continuous operation of the whole system.

For the case where a batch section is composed of N-parallel identical units, optimal scheduling is obtained analytically and it is shown that the operation of each batch unit has to be started with a constant delayed timing, W/N , so as to minimize the total capacity of the two storage tanks.

When the batch section consists of N-parallel batch units of different types, it is required to carry out a tremendous search calculations to obtain the optimal schedule of the batch section. It is, therefore, very important to be able to reduce the searching domain as small as possible. For this search, a very useful result (Theorem 2-3) is derived whereby the domain with respect to the phase difference of the starting time of each unit can be reduced to a domain such that $0 \leq t_i < \text{G.C.M.}(Z_{i-1}, W_i)$, where $Z_i = \text{L.C.M.}(Z_{i-1}, W_i)$, $Z_1 = W_1$, ($i=2,3,\dots,N$).

This result is valid for all batch operation schemes as long as they are operated periodically. Even when some batch units have side feed streams, this result can be applicable as far as the input and output functions to and

from the batch section are periodic functions. So this theorem has a very wide applicability in determining the optimal scheduling of many different type batch operations, at the same time, the theorem drastically reduces the necessary computation time for searching.

Nomenclature

N = number of parallel batch units

p = cost function of a batch unit

q_i = cost function of tank i

r_i = cost function of the feeding ($i=1$) and discharging ($i=2$)
pumps to and from the batch section

$S, (S_i)$ = batch size of a batch unit (unit i) (= equipment size
of a batch unit (unit i))

$T, (T_i)$ = holding time of the batch unit (unit i)

$T_p, (T_{pi})$ = preparation period of the batch unit (unit i)

t_a = starting moment of the inflow to the batch unit in
the first cycle

t_b = starting moment of the outflow from the batch unit
in the first cycle

t_d = starting moment of the outflow from tank 2

t_i = phase difference of unit i

$\mathbf{t}$ = phase difference vector

U_i^j = capacities of the feed pump ($j=f$) and the discharge pump
($j=d$) of tank i per unit time

u_1^f = input function to tank 1

$u_1^d, (u_{1i}^d)$ = input function from tank 1 to the batch unit (unit i)

u_1^* = input function from tank 1 to the batch section

$u_2^f, (u_{2i}^f)$ = output function from the batch unit (unit i) to
tank 2

u_2^d = output function from tank 2

u_2^* = output function from the batch section to tank 2

V_i = maximum storage capacity of tank i

v_i = hold-up function of tank i

$W, (W_i)$ = cycle time of the batch unit (unit i)

w = minimum cycle time of the batch unit

W^* = maximum value among the cycle times (see Eq.(2-42))

$W^o = \text{L.C.M.}(W_1, W_2, \dots, W_N)$

Other symbols

$\text{G.C.M.}(X, Y)$ = extended greatest common measure of X and Y

$\text{L.C.M.}(X, Y)$ = extended least common multiple of X and Y

Chapter 3

OPTIMAL DESIGN OF A BATCH PROCESS WITH INTERMEDIATE STORAGE TANKS

1. Introduction

In the previous chapter, we pointed out that the minimum capacity of an intermediate storage tank is a function not only of the batch sizes of parallel batch items but also of the process timing of parallel batch items. In other words, it is necessary to consider the operation schedule of the process in order to optimally design the batch process. In this chapter, by taking into account this fact, a more general design problem of a batch process is studied.

In a batch process consisting of many batch stages, the equipment size of each batch item has to be determined by taking into account the operation schedule of the whole process as well as the production capacity. Each piece of equipment in a continuous process can be designed depending only on the production rate.

For example, when two batch items are connected in a series, the outlet flow from the first item directly becomes the inlet flow to the second. Therefore, both items must be designed and operated in such a way that the amounts of production per batch in both items and their cycle times are identical.

Generally speaking, the minimal cycle time, that is, the time necessary for a batch item to treat material of a batch, varies according to the kinds of batch items. On the other hand, the whole process consisting of batch items connected in a series cannot be operated with a shorter

cycle time than the largest minimal cycle time of batch items. Therefore, the batch item with a shorter cycle time than that of the whole process has to be operated with a certain idle time. The process in which such a batch item exists is not preferable and can be improved from the point of view of the effective usage of batch items.

In order to reduce the idle time in a process, the batch item which is operated without any idle time has to be replaced by a set of parallel batch items, and/or intermediate storage tanks have to be installed at suitable places among the batch items. Therefore, a batch process is generally composed of many batch stages consisting of parallel batch items and some intermediate storage tanks.

The problem of the design of a single product batch process is stated as follows: "When the production rate is given, find the optimal number of batch items in each batch stage, the optimal size of each batch item and optimal volumes of intermediate storage tanks so as to minimize a given performance index."

In this chapter, the optimal design problem of a single product batch process with intermediate storage tanks is dealt with. First, the mathematical description of this problem is given by introducing several assumptions related to the batch operation. Needless to say, there are many uncertainties in actual batch operations. For example, the processing times in batch units are often not a constant but are variable with some frequency distribution. From this

viewpoint, a stochastic model might be necessary in order to completely express an actual batch operation. However, a deterministic model is used here to mathematically formulate the design problem of a batch process with intermediate storage tanks. Based on the deterministic model which is an idealized model in the sense that any uncertainties are not taken into account, the mathematical characteristics of this problem are fully analyzed, and some effective algorithms for determining the optimal solution are developed.

2. Formulation of the Problem

The optimal design problem of a single product batch process consisting of many batch stages and intermediate storage tanks as shown in Figure 3-1 can be stated as follows;

When the production requirement of the process is given, find the optimal number of parallel batch items in each batch stage, the optimal equipment size of each batch item, and the optimal volumes of intermediate storage tanks so as to minimize the performance index given by the following equation, i.e.

$$P.I. = \sum_{i=1}^B N_i p_i (S_i) + \sum_{j=1}^{K-1} q_j (V_j) \quad (3-1)$$

where

B = the number of batch stages

N_i = the number of parallel batch items in batch stage i

S_i = the batch size of batch stage i (= the equipment size of the batch item in batch stage i)

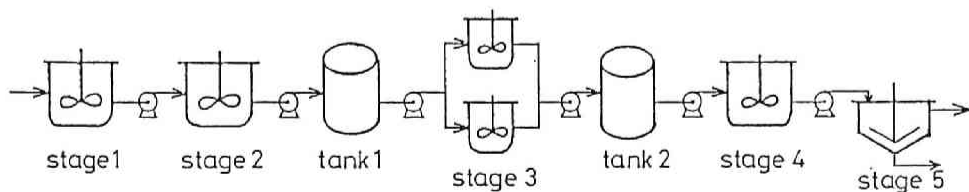


Figure 3-1. Schematic diagram of a general batch process

V_j = the volume of intermediate storage tank j
 $K-1$ = the number of possible places where an intermediate storage tank can be installed
 p_i, q_j = monotonically increasing functions.

In order to discuss how to solve the above problem, the following assumptions are first introduced:

- (i) The capacities of the feed and discharge pumps of the storage tanks are known.
- (ii) The process does not cause any volume changes in the flows.
- (iii) Each batch stage consists of one or more identical items of batch equipment in parallel.
- (iv) In each batch stage i , every batch item is periodically operated according to steps such as the filling, processing discharging, cleaning and perhaps waiting until the next filling step. Here, the time it takes to pass through all of the steps is called "the cycle time of the batch item, W_i ". The time required for a batch of material to pass completely through the steps excluding the waiting step is called "the minimal cycle time of the batch item, w_i ", and it is assumed to be a function of the equipment size of the batch item.
- (v) Every batch item is periodically operated with the same cycle time by delaying its starting moment at equal intervals. Each batch stage i with N_i batch items in parallel is charged in turn by a batch from the preceding

stage with the constant time interval W_i/N_i . This time interval is called "the stage cycle time". The amount of product produced in this time interval is also called "the batch size of the batch stage, S_i ". Figure 3-2 shows an illustrative example of the operation schedule of a batch stage consisting of two batch items in parallel.

As derived in the previous chapter, for the process where an intermediate storage tank is installed between the batch section consisting of parallel identical units and the continuous section, the capacity of the storage tank is minimized when parallel batch items are operated by delaying their starting moments at equal

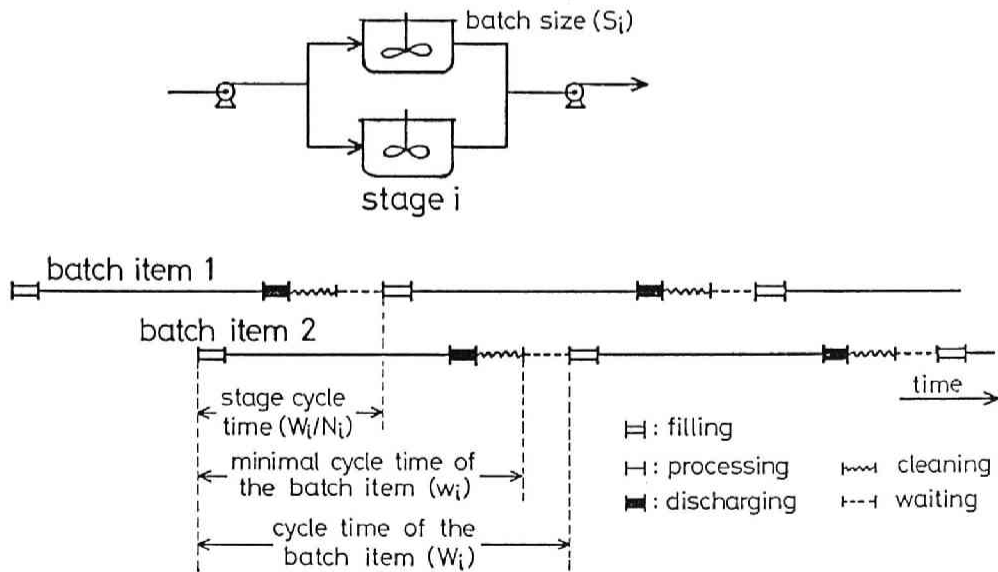


Figure 3-2. Operation schedule of batch stage i ($N_i = 2$)

- intervals. Strictly speaking, this result can not be used for the case where storage tank is installed between two batch stages. That is, by changing the operation schedule of the starting moment of each batch unit, the minimum volume of the intermediate storage tank may be reduced compared with the case where the starting moment is delayed at equal intervals. However, in order to reduce the degrees of freedom in the system, it is here assumed that the parallel units are operated by delaying their starting moments at equal intervals.
- (vi) The equipment size of the batch item and the batch size of the batch stage are measured by the same unit, and moreover both are identical in each batch stage.
- (vii) The "processing capacity of the batch item, c_i ," in batch stage i is given by Eq.(3-2), and it is assumed to be a monotonically increasing function of the batch size, S_i .

$$c_i(S_i) = S_i/w_i(S_i) \quad (3-2)$$

Under the assumptions introduced above, we develop an effective solution method for the problem stated above.

3. Relationships among the Variables in a Subprocess

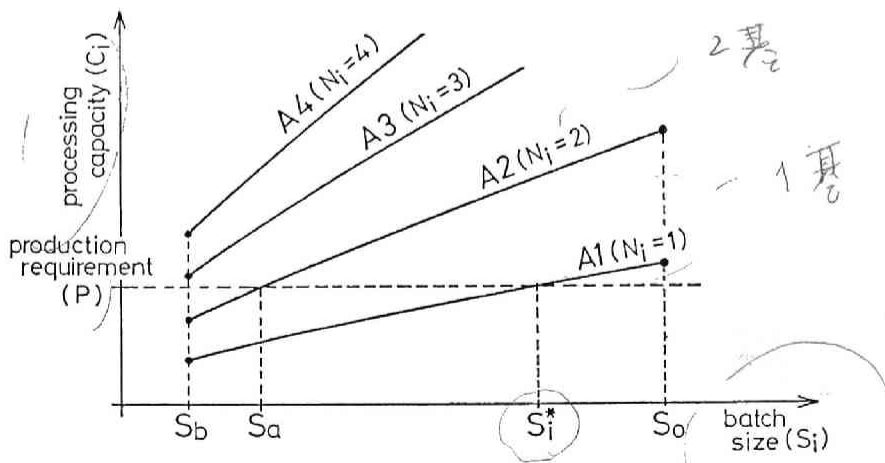
We first clarify the relationship between the number of parallel batch items, N_i , and the size of batch equipment, S_i , in batch stage i in order to satisfy the given production requirement.

By using Eq. (3-2), "the processing capacity of batch stage i , C_i ," is defined as follows;

$$C_i = c_i(S_i) \times N_i \quad (3-3)$$

Figure 3-3 shows the relationship between C_i and S_i by making N_i a parameter.

Curve A1 in Figure 3-3 shows the processing capacity of a batch item as a function of the batch size. This curve is usually obtained by actually measuring the minimal cycle time of the batch item with different batch sizes or



Relationship between the processing
Figure 3-3. capacity and the batch size

calculating the minimal cycle time using some suitable model equations. The black circles at both ends of a curve express the upper and lower bounds of the possible batch size imposed for technical or other reasons.

Curves A2, A3, and A4 are obtained from Curve A1 and Eq.(3-3) for $N_i=2, 3$ and 4, respectively.

By utilizing Figure 3-3, we show how the optimal solution can be derived. In order to satisfy the given production requirement, P , the necessary batch size has to be larger than S_i^* , S_a , S_b and S_p in cases where the batch stage consists of only one, two, three and four batch items, respectively.

By comparing the values of the performance index for these cases, the optimal solution can be easily obtained.

When N_i and S_i are fixed, the cycle time of each batch item, W_i , is given by

$$W_i = S_i \times N_i / P \quad (3-4)$$

and the operation schedule can also be determined.

Next, the process consisting of two batch stages (batch stage 1 and 2) connected in a series as shown in Figure 3-4 is considered.

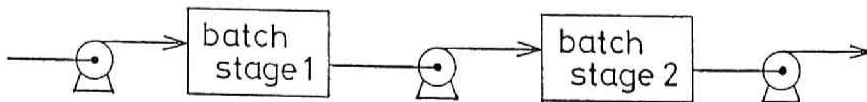


Figure 3-4. Process consisting of two batch stages

In the process taken up here, the outlet flow from batch stage 1 directly becomes the inlet flow to batch stage 2. Therefore, the cycle times and batch sizes of both batch stages have to be identical. For example, assume that the relationship between the processing capacities of both stages and the batch sizes is expressed as shown in Figure 3-5. If both stages consist of a single batch item and are not connected to each other, batch stage 1 and 2 can satisfy the given production requirement, P , by choosing S_1^* and S_2^* as its batch size, respectively. But both stages are actually connected in a series and, therefore, the batch sizes of both stages have to be equal to or larger than S_1^* .

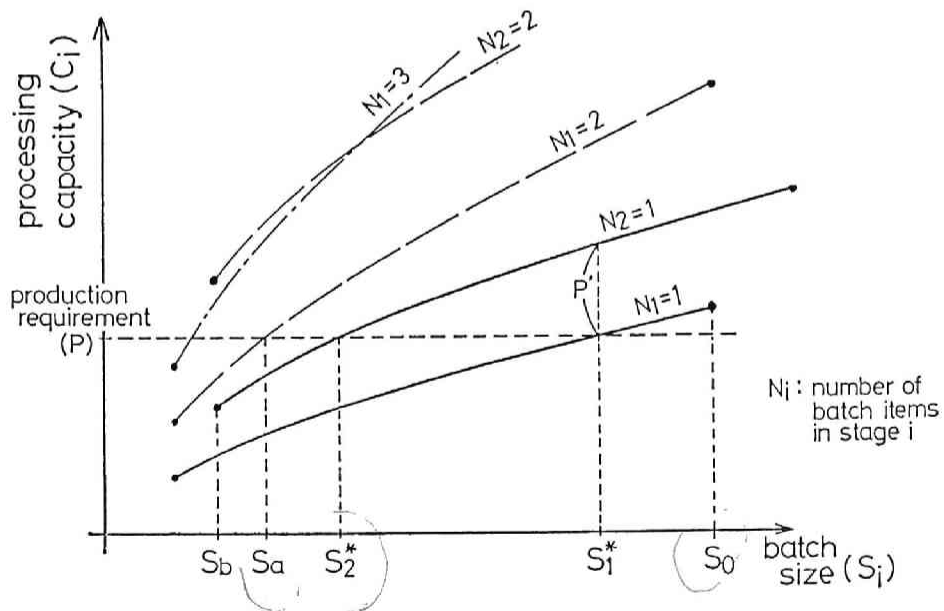


Figure 3-5. Processing capacities of both batch stages

Figure 3-6 shows the operation schedule when both batch stages consist of single batch item and are operated with the batch size of S_1^* . It is clear from this figure that the batch item in batch stage 1 is periodically operated with the minimal cycle time, S_1^*/P , but the batch item in batch stage 2 is operated with a fairly large cycle time compared with its minimal cycle time defined by $S_1^*/(P+P')$.

We develop an algorithm for determining the optimal solution by taking into account a constraint such that the batch sizes of both batch stages must be identical.

We first assume that each batch stage consists of only one batch item. It is clearly not optimal to make the batch size larger than S_1^* and, therefore, the optimal batch sizes of the batch stages must be S_1^* .

When we try to choose a batch size smaller than S_1^* , batch stage 1 is the bottleneck, as is clear from Figure 3-5. By installing batch items in parallel in batch stage 1, it becomes possible to choose a batch size smaller than S_1^* .

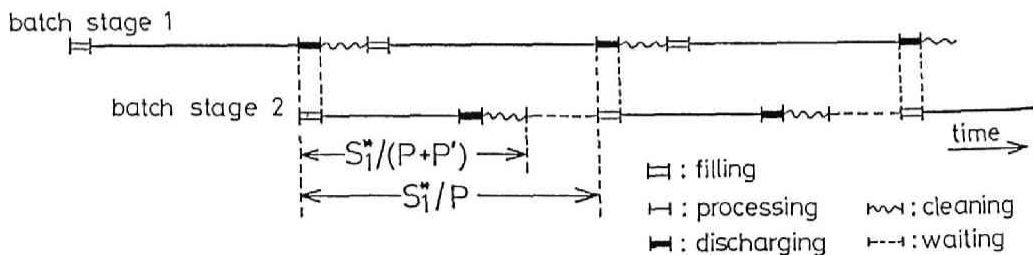


Figure 3-6. Operation schedule of a batch process

When we set batch stage 1 and 2 with two batch items in parallel and as a single batch item, respectively, the minimum batch size necessary to satisfy the given production requirement is given by S_2^* as shown in Figure 3-5.

By increasing the number of parallel batch items in the bottlenecked batch stage, it is possible to operate the process at a smaller batch size. By repeating this procedure so as to make the batch size as small as possible in the technically feasible range, we can find the necessary number of parallel batch items in both batch stages and the minimal batch size, one after another. The results for the example in Figure 3-5 can be summarized as shown in Table 3-1.

Any combination of the numbers of parallel batch items in both batch stages other than those given in Table 3-1 (for example, $N_1=1$ and $N_2=2$) cannot be optimal. Therefore, the optimal solution can be found by calculating the value of the performance index for each case shown in Table 3-1 and comparing these four cases.

Table 3-1. Candidates for optimal solution

number of batch items		minimal batch size
N_1	N_2	
1	1	S_1^*
2	1	S_2^*
2	2	S_a
3	2	S_b

In this section, we took up the process consisting of two batch stages in order to make the explanation simpler, but the algorithm developed here can be applicable to more general cases where the process consists of an arbitrary number of batch stages connected in a series. When the process consists of many batch stages, a large number of combinations of the numbers of parallel batch items in the batch stages must be considered. But depending on the algorithm developed here, the combinations which are impossible candidates for the optimal solution are eliminated and, the optimal solution can, thus, be more easily obtained.

4. A Batch Process with an Intermediate Storage Tank

In the previous section, the process where the batch stage is directly connected in a series was considered. In such a process, there are often cases where the batch size is too large for some stages, while the amount of material enough to satisfy the given production requirement can be treated even in a smaller batch size.

Such inefficiency can be eliminated by installing an intermediate storage tank between batch stages. By installing a storage tank, the outlet flow from the batch stage before the tank is stored in the tank and does not immediately become the inlet flow of the batch stage after the tank. Therefore, the cycle times and batch sizes can be chosen arbitrarily in both batch stages so as to satisfy the given production requirement. The intermediate storage tanks provide large flexibility in the design and operation of a batch process.

In this section, we take up a process consisting of two batch stages each of which includes only one batch item and a tank, as shown in Figure 3-7. By utilizing the example, we first derive several theorems for determining the volume of the tank. Then we develop a procedure for solving the optimal design problem.

In order to satisfy the given production requirement, P , the batch size of each batch stage, S_i , must satisfy the following relationship:

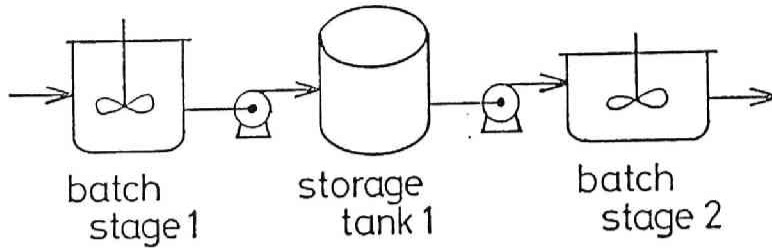


Figure 3-7. Process consisting of two stages and a tank

$$S_i / w_i(S_i) \geq P \quad (i = 1, 2) \quad (3-5)$$

where

w_i = the function representing the minimal cycle time of the batch item in batch stage i .

Then, the cycle time of each batch item, w_i , is given by

$$w_i = S_i / P \quad (i = 1, 2) \quad (3-6)$$

When the batch sizes, S_1 and S_2 and the cycle times, w_1 and w_2 , at both batch stages are determined to satisfy Eqs.(3-5) and (3-6), both the accumulation of the inlet flow to and the depletion of outlet flow from the tank can be expressed by functions which have piecewise constant derivatives with respect to time, as shown in Figure 3-8. The gradients of the slanting parts of the two broken lines are determined by the capacities of the feed and discharge pumps of the tank, respectively. t_1 and t_2 in the figure express the starting moments of the inflow from batch stage 1 to the

tank and the starting moment of the discharge from the tank to batch stage 2 in the first cycle, respectively. Hereafter, the time lag between t_1 and t_2 is called "the discharge lag" of the tank. The vertical distance between the two broken lines represents the hold-up in the tank at every moment, and the maximum distance gives the minimum value of the necessary tank volume. By shifting the discharge lag, $t_2 - t_1$, in Figure 3-8, the hold-up in the tank and its maximum value change. In other words, the volume of the tank is a function not only of the batch sizes and the cycle times of both batch stages, but also of the process timing of both stages, that is, the discharge lag, $t_2 - t_1$.

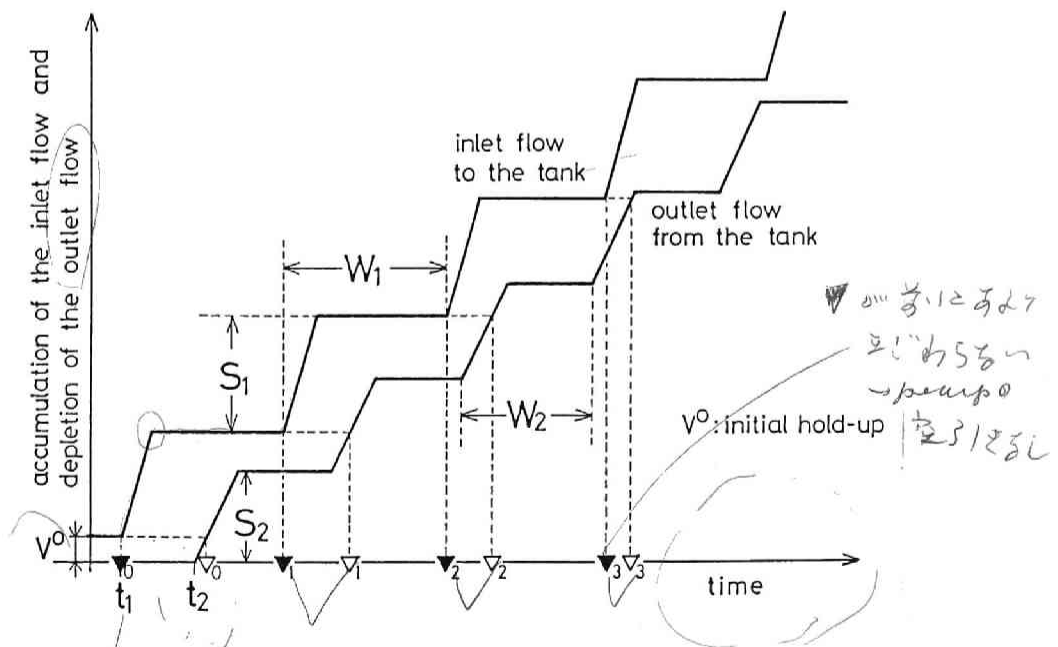


Figure 3-8. Accumulation of the inlet flow to and the depletion of the outlet flow from the tank

In order to simplify the further discussion of how to obtain the minimum value of the tank volume, V , it is, hereafter, assumed that the capacity of the feed pump of the tank is greater than that of the discharge pump of the tank. This assumption is, however not essential in the following discussion.

In Figure 3-8, we represent the time when the accumulation of the inlet flow to the tank exceeds $V^0 + iS_1$ and the time when the depletion of the outlet flow from the tank exceeds $V^0 + iS_1$ by the symbols ∇_i and ∇_i , respectively. Then, it can be verified that the depletion of the outlet flow from the tank never exceeds the accumulation of the inlet flow to the tank, that is, the exhaustion of stored material never occurs if $t_2 - t_1$ is chosen in such a way that the time ∇_i is always earlier than the time ∇_i for every i .

The relationship discussed above can be mathematically expressed as follows:

The input function to and the output function from the tank, $u^f(t)$ and $u^d(t)$, are defined by the following equations.

$$u^f(t) = \begin{cases} U^f & ; \quad iW_1 \leq t \leq iW_1 + S_1/U^f \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (3-7)$$

($i = 0, 1, 2, \dots$)

$$u^d(t) = \begin{cases} U^d & ; \quad iW_2 \leq t \leq iW_2 + S_2/U^d \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (3-8)$$

($i = 0, 1, 2, \dots$)

where

U^j = the capacities of the feed ($j=f$) and the
discharge ($j=d$) pumps of the tank per unit time.

By using above functions, the differential equation
that the hold-up in the tank $v(t)$ must satisfy can be
derived as follows:

$$\frac{dv(t)}{dt} = u^f(t - t_1) - u^d(t - t_2) \quad (3-9)$$

Then, the hold-up in the tank, $v(t)$, is given by

$$v(t) = v^0 + \int_0^t [u^f(s-t_1) - u^d(s-t_2)] ds$$

where

v^0 = the initial inventory in the storage tank.

From Eq.(3-9) and the assumption that $U^f > U^d$, the
hold-up in the tank, $v(t)$, takes on local minimum values at
 $t = t_1 + iW_1$, where i = any non-negative integer. Therefore,
 $v(t) \geq 0$ holds for any t if and only if the following inequality
is satisfied for any non-negative integer i .

$$v(t_1 + iW_1) = v^0 + iS_1 - \int_0^{t_1 + iW_1} u^d(s-t_2) ds \geq 0 \quad (3-10)$$

($i = 0, 1, 2, \dots$)

From Eq.(3-10), we can derive the condition in which the
exhaustion of stored material never occurs.

[Theorem 3-1a]

It is presumed that the capacity of the feed pump, U^f , is greater than that of the discharge pump, U^d , and that the batch sizes, S_1 and S_2 , and the cycle times, W_1 and W_2 , of both batch units are determined to satisfy the given production requirement P . Then both batch items can be operated in a steady cyclic condition without causing the exhaustion of stored material in the tank if and only if the discharge lag $t_2 - t_1$ and the initial hold-up V^0 are chosen so as to satisfy the following relationship:

$$(1 - P/U^d) \{ S_2 - (1-h) \cdot \text{G.C.M.}(S_1, S_2) \} - V^0 \leq (t_2 - t_1)P \quad (3-11)$$

where

$$h = \text{mod}[V^0 / \text{G.C.M.}(S_1, S_2), 1]$$

$\text{G.C.M.}(X, Y)$ = the extended greatest common measure of X and Y ; (see Chapter 2),

$$\text{mod}(X, Y) := X - \text{trunc}(X/Y) \cdot Y$$

$$\text{trunc}(X) := \text{the largest integer} \leq X.$$

[Proof of Theorem 3-1a]

From Eq. (3-10), the time when the depletion of the outlet flow from the tank, i.e. $\int_0^t u^d(s - t_2) ds$ exceeds $V^0 + iS_1$, is given by

$$t_2 + \text{trunc}[(V^0 + iS_1)/S_2] \cdot W_2 + \text{mod}(V^0 + iS_1, S_2)/U^d.$$

Therefore, Eq. (3-10) is satisfied if and only if the following inequality holds:

$$t_1 + iW_1 \leq t_2 + \text{trunc}[(V^0 + iS_1)/S_2] \cdot W_2 + \text{mod}(V^0 + iS_1, S_2)/U^d$$

$$(i = 1, 2, \dots) \quad (3-12)$$

Here, following values are defined;

$$L := S_1 / \text{G.C.M.}(S_1, S_2) = W_1 / \text{G.C.M.}(W_1, W_2) \quad (3-13)$$

$$M := S_2 / \text{G.C.M.}(S_1, S_2) = W_2 / \text{G.C.M.}(W_1, W_2) \quad (3-14)$$

$$H := \text{trunc}[V^0 / \text{G.C.M.}(S_1, S_2)] \quad (3-15)$$

$$h := \text{mod}[V^0 / \text{G.C.M.}(S_1, S_2), 1] \quad (3-16)$$

By using Eqs.(3-13) to (3-16), Eq.(3-12) is rewritten as follows:

$$\begin{aligned} t_2 - t_1 \geq \{iL - \text{trunc}[(H+h+iL)/M] \cdot M\} \cdot W_1 / L \\ - \text{mod}(H+h+iL, M) \cdot \text{G.C.M.}(S_1, S_2) / U^d \end{aligned} \quad (3-17)$$

Since both $H+iL$ and M are integers, we obtain

$$\text{trunc}[(H+h+iL)/M] = \text{trunc}[(H+iL)/M] \quad (3-18)$$

$$\text{mod}(H+h+iL, M) = \text{mod}(H+iL, M) + h \quad (3-19)$$

From Eqs.(3-13) to (3-19), the following equation is derived:

$$(t_2 - t_1)^P \geq \{\text{mod}(H+iL, M) + h\} (1 - P/U^d) \cdot \text{G.C.M.}(S_1, S_2) - V^0 \quad (3-20)$$

Equation (3-20) is satisfied for any integer i if and only if Eq.(3-20) is satisfied for some i which maximizes the value of $\text{mod}(H+iL, M)$. So the problem is to derive the integer which maximizes the value of $\text{mod}(H+iL, M)$.

From Lemma 2-1 which is shown in Chapter 2, there exist some positive integers I and J which satisfy the following equation;

$$I \cdot L - J \cdot M = (M - H - 1) \cdot \text{G.C.M.}(L, M)$$

From the definition of L and M, G.C.M.(L,M) = 1. Therefore, we have the following result.

$$I \cdot L - J \cdot M = M - H - 1, \text{ and}$$

$$\text{mod}(H+I \cdot L, M) = M - 1.$$

For any integer i, $\text{mod}(H+iL, M)$ is less than or equal to $M-1$.

Then, we have

$$\max_{\{i\}} \{\text{mod}(H+iL, M)\} = M - 1. \quad (3-21)$$

Consequently we can derive Eq.(3-11).

(Q.E.D. of Theorem 3-1a)

Next, the condition in which the storage tank does not overflow is explained by using the figure.

In Figure 3-9, the dotted line is obtained by making parallel displacement of the broken line representing the depletion of the outlet flow from the tank by the tank size V along the ordinate. The time when the value of the dotted line becomes $V^0 + iS_1$ and that when the accumulation of the inlet flow to the tank becomes $V^0 + iS_1$ are represented by the black and white circles, $\bullet_i$ and O_i , respectively. In that case, overflowing never occurs if the tank volume V and the discharge lag $t_2 - t_1$ are chosen in such a way that the time $\bullet_i$ is always earlier than the time O_i for every integer i.

The relationship stated above is mathematically described as follows;

From Eq.(3-9), the hold-up in the tank, $v(t)$, takes on

local maximum values at $t = t_1 + (S_1/U^f) + iW_1$, where $i =$ any non-negative integer. Therefore, $V \geq v(t)$ holds for any t , if and only if the following inequality is satisfied for any non-negative integer i :

$$\begin{aligned}
 V &\geq v(t_1 + S_1/U^f + iW_1) \\
 &= v^0 + (i+1)S_1 - \int_0^{t_1 + (S_1/U^f) + iW_1} u^d(s-t_2) ds \quad (3-22) \\
 &\quad (i = 0, 1, 2, \dots)
 \end{aligned}$$

From this result, we can derive the condition in which the overflowing from the tank does not occurs.

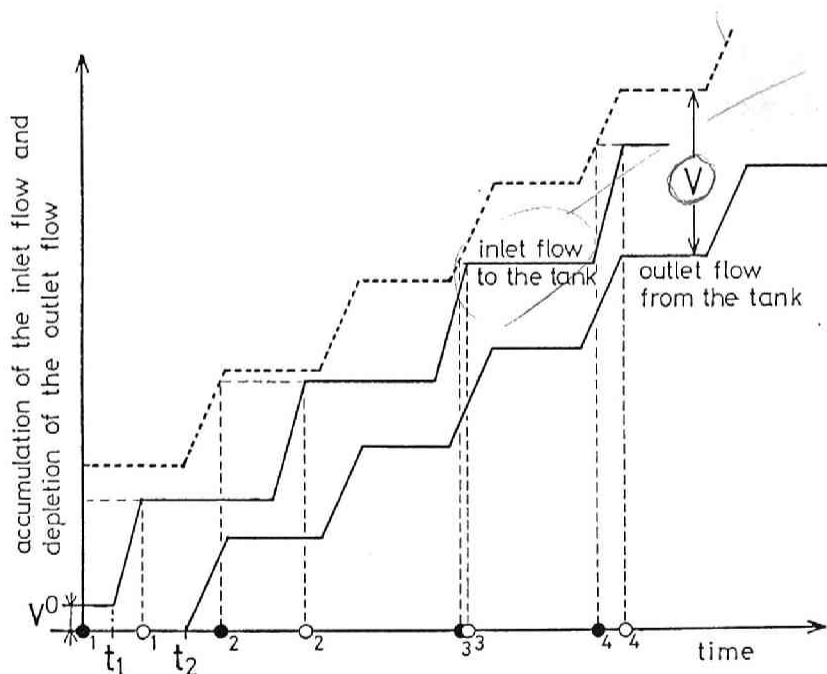


Figure 3-9. Geometric explanation of Theorem 3-1b

[Theorem 3-1b]

It is presumed that the capacity of the feed pump, U^f , is greater than that of the discharge pump, U^d , and that the batch sizes, S_1 and S_2 , and the cycle times, W_1 and W_2 , at both batch units are determined to satisfy the given production requirement P . Then both batch items can be operated in a steady cyclic condition without causing the overflowing from the tank if and only if the discharge lag, $t_2 - t_1$, the initial hold-up, V^0 , and the tank volume, V , are chosen so as to satisfy the following relationship:

$$(t_2 - t_1)P \leq V - V^0 - (1 - P/U^f)S_1 + (1 - r)(1 - P/U^d)\text{G.C.M.}(S_1, S_2) \quad (3-23)$$

where

$$r = \text{mod}[(V - V^0)/\text{G.C.M.}(S_1, S_2), 1]$$

[Proof of Theorem 3-1b]

Equation (3-22) is satisfied if and only if the time when the depletion of the outlet flow from the tank becomes $V^0 + (i+1)S_1 - V$ is earlier than or equal to $t_1 + (S_1/U^f) + iW_1$. That is, the tank does not overflow if and only if the following inequality is satisfied for any non-negative integer i :

$$\begin{aligned} t_1 + S_1/U^f + iW_1 &\geq t_2 + \text{trunc}\{[V^0 + (i+1)S_1 - V]/S_2\} \cdot W_2 \\ &+ \text{mod}[V^0 + (i+1)S_1 - V, S_2]/U^d - \delta_i \end{aligned} \quad (3-24)$$

($i = 0, 1, 2, \dots$)

where

$$\delta_i = \begin{cases} w_2 - S_2/U^d ; \text{ mod}[V^0 + (i+1)S_1 - V, S_2] = 0 \\ 0 ; \text{ otherwise} \end{cases}$$

Here, R and r are defined by the following equations.

$$R := \text{trunc}[(V - V^0)/\text{G.C.M.}(S_1, S_2)] \quad (3-25)$$

$$r := \text{mod}[(V - V^0)/\text{G.C.M.}(S_1, S_2), 1] \quad (3-26)$$

By using L, M, R and r which are defined by Eqs.(3-13), (3-14), (3-25) and (3-26), Eq.(3-24) is rewritten as follows:

$$\begin{aligned} t_2 - t_1 \leq & i w_1 + S_1/U^f - \text{trunc}\{[(i+1)L - R - r]/M\} \cdot w_2 \\ & - \text{mod}[(i+1)L - R - r, M] \cdot \text{G.C.M.}(S_1, S_2)/U^d + \delta_i \end{aligned} \quad (3-27)$$

If r which is determined by Eq.(3-26) is not equal to zero, δ_i becomes zero for every i. In this case, Eq.(3-27) is rewritten as follows:

$$\begin{aligned} (t_2 - t_1)P \leq & \{\text{mod}[(i+1)L - R - 1, M] + 1 - r\} (1 - P/U^d) \cdot \text{G.C.M.}(S_1, S_2) \\ & - (1 - P/U^f) S_1 + V - V^0 \end{aligned} \quad (3-28)$$

From the fact that $\text{G.C.M.}(L, M) = 1$,

$$\min_{\{i\}} \{\text{mod}[(i+1)L - R - 1, M]\} = 0$$

Then, it can be concluded that Eq.(3-28) is satisfied for any integer i if and only if the following equation is satisfied:

$$\begin{aligned} (t_2 - t_1)P \leq & (1 - r) (1 - P/U^d) \cdot \text{G.C.M.}(S_1, S_2) - (1 - P/U^f) S_1 \\ & + V - V^0 \end{aligned} \quad (3-29)$$

If $r = 0$, Eq. (3-27) can be rewritten as follows:

$$(t_2 - t_1)P \leq \delta_i P + \text{mod}[(i+1)L-R, M] (1-P/U^d) \cdot \text{G.C.M.}(S_1, S_2) - (1-P/U^f)S_1 + V - V^0 \quad (3-30)$$

For any integer i , following inequality is satisfied.

$$\begin{aligned} & \delta_i P + \text{mod}[(i+1)L-R, M] (1-P/U^d) \cdot \text{G.C.M.}(S_1, S_2) \\ & \geq (1-P/U^d) \cdot \text{G.C.M.}(S_1, S_2) \end{aligned}$$

This inequality shows that Eq. (3-30) holds for any integer i if Eq. (3-29) holds. From Eq. (3-29), Eq. (3-23) is derived.

(Q.E.D. of Theorem 3-1b)

For the case where the capacity of the feed pump is smaller than that of the discharge pump, the condition in which the exhaustion of stored material never occurs can be stated as follows:

"The exhaustion of stored material never happens if and only if the time when the accumulation of the inlet flow becomes iS_2 is earlier than the time when the depletion of the outlet flow becomes iS_2 for every integer i ."

Moreover, the condition in which the overflowing from the tank never occurs can be stated as follows:

"The overflowing from the tank never occurs if and only if the time when the depletion of the outlet flow exceeds iS_2 is earlier than the time when the accumulation of the inlet flow exceeds $V+iS_2$ for every non-negative integer i ."

Even for these cases, we can also derive the results

similar to that shown in Theorem 1a and Theorem 1b. The results for both cases that $U^f > U^d$ and $U^f \leq U^d$ can be summarized as the following theorem.

[Theorem 3-2]

It is presumed that the batch sizes and cycle times of both batch stages are determined so as to satisfy the given production requirement, and that the initial inventory in a storage tank is given a priori. Then both stages can be operated in a steady cyclic condition without causing either the overflow or the exhaustion of stored material in the tank if and only if the capacity of the storage tank, V , and the discharge lag, $t_2 - t_1$, satisfy the following inequalities:

$$(1-P/U^d)S_2 - V^0 - (1-b)(1-h) \cdot \text{G.C.M.}(S_1, S_2) \leq (t_2 - t_1)P \quad (3-31)$$

tank volume for t_2, t_1 & V & $t_2 - t_1$

$$\leq V - V^0 - (1-P/U^f)S_1 + (1-b)(1-r) \cdot \text{G.C.M.}(S_1, S_2) \quad (3-32)$$

where

$$b = P/\min(U^f, U^d) \quad (3-33)$$

$$h = \text{mod}[V^0/\text{G.C.M.}(S_1, S_2), 1] \quad (3-16)$$

$$r = \text{mod}[(V - V^0)/\text{G.C.M.}(S_1, S_2), 1] \quad (3-26)$$

Next the minimum value of the tank volume is derived by using the result of Theorem 3-2.

By eliminating $(t_2 - t_1)P$ from Eqs.(3-31) and (3-32), Eqs.(3-31) and (3-32) can be rewritten as follows:

$$\begin{aligned} & [(1-P/U^f)S_1 + (1-P/U^d)S_2 - V^0]/G.C.M.(S_1, S_2) - (1-b)(2-h) \\ & \leq R + br \end{aligned} \quad (3-34)$$

where

$$R = \text{trunc}[(V - V^0)/G.C.M.(S_1, S_2)] \quad (3-25)$$

If batch sizes of both batch units, capacities of the feed and discharge pumps and the initial hold-up in the tank are given, the value of the left-hand side of Eq.(3-34) can be calculated. From Eqs.(3-25) and (3-26), the capacity of the tank is given by

$$V = V^0 + (R+r)G.C.M.(S_1, S_2).$$

Then the minimum value of the necessary tank volume can be obtained by minimizing $R+r$ under the constraint of Eq.(3-34). Let the left-hand side of Eq.(3-34) be Q . Then R must be equal to or greater than $\text{trunc}(Q)$, since $b \cdot r < 1$. If $\text{mod}(Q, 1) < b$, $\text{trunc}(Q) + \text{mod}(Q, 1)/b$ is the minimum value of $R+r$ which satisfies Eq.(3-34). If $\text{mod}(Q, 1) \geq b$, R must be greater than $\text{trunc}(Q)$. Therefore, $\text{trunc}(Q)+1$ is the minimum value of $R+r$ which satisfies Eq.(3-34) for this case. On the other hand, $R+r$ must be greater than or equal to zero from the definitions of R and r . Consequently, we obtain the following theorem.

[Theorem 3-3]

When the batch sizes and the cycle times of both batch stages are determined so as to satisfy the given production

requirement, the minimum tank volume is given by the following equation:

$$V = \{ \text{trunc}(Q') + \min[\text{mod}(Q', 1)/b, 1] \} \text{G.C.M.}(S_1, S_2) + V^0 \quad (3-35)$$

where

$$Q = [(1 - P/U^f)S_1 + (1 - P/U^d)S_2 - V^0] / \text{G.C.M.}(S_1, S_2) - (1-b)(2-h)$$

$$Q' = \max(Q, 0)$$

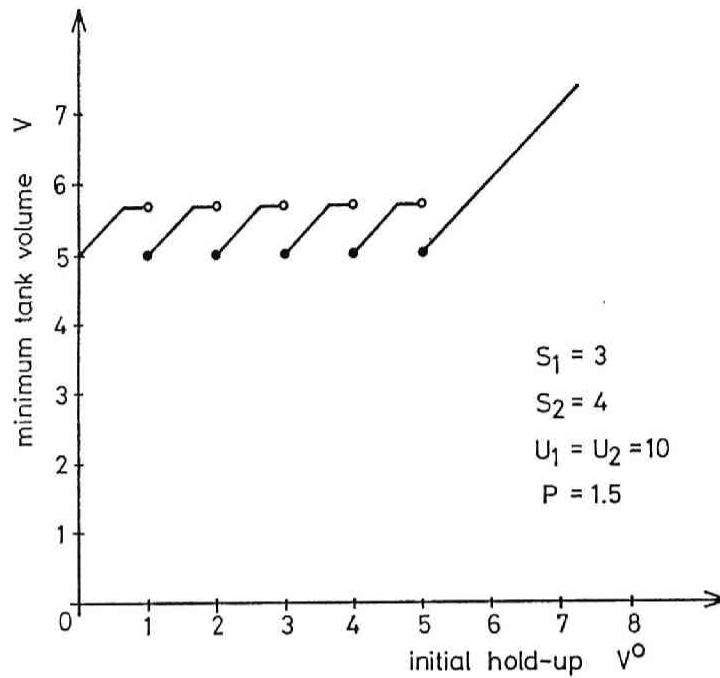
$$h = \text{mod}[V^0 / \text{G.C.M.}(S_1, S_2), 1]$$

$$b = (P / \min(U^f, U^d))$$

From Eq.(3-35), it is clear that the tank volume V is a function not only of the batch sizes of both batch stages but also of the initial hold-up in the tank. When the batch sizes of both stages are given, the relationship between the tank volume given by Eq.(3-35) and the initial hold-up is shown in Figure 3-10. There are many cases where the initial hold-up is not given a priori and the appropriate value can be chosen as the initial hold-up. In this case, the following result can be obtained.

[Corollary 3-1]

It is assumed that the initial hold-up can be arbitrarily chosen. Then, the initial hold-up V^0 which minimizes the tank volume V given by Eq.(3-35) is zero.



Relationship between the tank
 Figure 3-10. volume and the initial hold-up

[proof of Corollary 3-1]

Let D and d be real numbers which satisfy the following equations.

$$D = \text{trunc}\{[(1-P/U^f)S_1 + (1-P/U^d)S_2] / \text{G.C.M.}(S_1, S_2) - 2(1-b)\}$$

$$d = [(1-P/U^f)S_1 + (1-P/U^d)S_2] / \text{G.C.M.}(S_1, S_2) - 2(1-b) - D$$

By using above equations, Q can be rewritten as follows:

$$Q = D + d - H - bh$$

i) The case where $Q > 0$ is first considered.

In this case, the tank volume V is given by

$$V = \{\text{trunc}(D+d-bh) + \min[\text{mod}(d-bh, 1)/b, 1] + h\} \cdot \text{G.C.M.}(S_1, S_2)$$

According to values of b and d , the tank volume V can be expressed more simply as follows:

Case i-1) $d \geq 2b$

$$V = (D+1+h) \cdot \text{G.C.M.}(S_1, S_2)$$

Case i-2) $2b > d \geq b$

$$V = \begin{cases} (D+1+h) \text{G.C.M.}(S_1, S_2) & ; 0 \leq h < d/b - 1 \\ (D+d/b) \text{G.C.M.}(S_1, S_2) & ; d/b - 1 \leq h < 1 \end{cases}$$

Case i-3) $b > d \geq 0$

$$V = \begin{cases} (D+d/b) \text{G.C.M.}(S_1, S_2) & ; 0 \leq h < d/b \\ (D+h) \text{G.C.M.}(S_1, S_2) & ; d/b \leq h < 1 \text{ and } (1+d)/b \geq 1 \\ & \text{or } d/b \leq h < (1+d)/b - 1 \text{ and } (1+d)/b < 1 \\ [D-1+(1+d)/b] \text{G.C.M.}(S_1, S_2) & ; (1+d)/b - 1 \leq h < 1 \\ & \text{and } (1+d)/b < 1. \end{cases}$$

For these three cases, it is clear that $H = h = 0$ gives the minimum value of V .

ii) Next the case where $Q \leq 0$ is considered.

In this case, the minimum value of the tank volume is equal to the initial hold-up in the tank. From the constraint that $Q \leq 0$, V^0 must satisfy the following inequality:

$$V^0 \geq [D + \min(d/b, 1)] \text{G.C.M.}(S_1, S_2)$$

The value of the right-hand side of above inequality is equal to the tank volume which is given for the case where $H = h = 0$.

[Q.E.D. of Corollary 3-1]

From Theorem 3-3, the following corollary is also derived.

[Corollary 3-2]

It is assumed that the initial hold-up in the tank V^0 is zero, and that the feed and discharge pumps have the same capacity U which satisfies the following inequality:

$$U/P \geq [S_1 + S_2 - \text{G.C.M.}(S_1, S_2)] / \text{G.C.M.}(S_1, S_2).$$

Then the minimum tank volume is given by

$$V = S_1 + S_2 - 2 \cdot \text{G.C.M.}(S_1, S_2)$$

$S_1 = S_2$
 $\rightarrow V = 0$
 (3-36)

Corollary 3-2 shows that the minimum tank volume is given by Eq.(3-36) if the cycle time of each batch stage is sufficiently long compared with the time necessary for feeding to and discharging from the tank.

For the case in which $U^f = U^d$ and $V^0 = 0$, the value of $S_1 + S_2 - 2 \cdot \text{G.C.M.}(S_1, S_2)$ is greater than or equal to the minimum tank volume V which is given by Eq.(3-35) regardless of the capacities of pumps. Therefore, the tank volume V given by Eq.(3-36) is large enough to continue the steady operation of the process for the case where $U^f = U^d$ and $V^0 = 0$.

Consequently, the optimal design problem of a process shown in Figure 3-7 can be mathematically formulated as follows:

Minimize

$$P.I. = p_1(S_1) + p_2(S_2) + q(V) \quad (3-37)$$

subject to

$$S_i / w_i(S_i) \geq P \quad (i = 1, 2) \quad (3-5)$$

$$V = f(S_1, S_2) \quad (3-38)$$

where

f = a function representing the tank volume obtained by Theorem 3-3 or its corollary.

Since the tank volume is determined as a discontinuous function of the batch sizes, it is almost impossible to analytically solve the above problem. So, dependence on some direct search method is unavoidable.

Let's explain how the optimal batch sizes and the optimal tank volume are determined, by using a simple example. In this example, it is assumed that the processing capacities of both batch stages are given as shown in Figure 3-11, and at the same, the necessary tank volume is given by Eq.(3-36). As is clear from Figure 3-11, the batch sizes, of both batch stages 1 and 2 have to be equal to or greater than 100 and 30, respectively, so as to satisfy the given production requirement P . By increasing the batch sizes, S_1 and S_2 ,

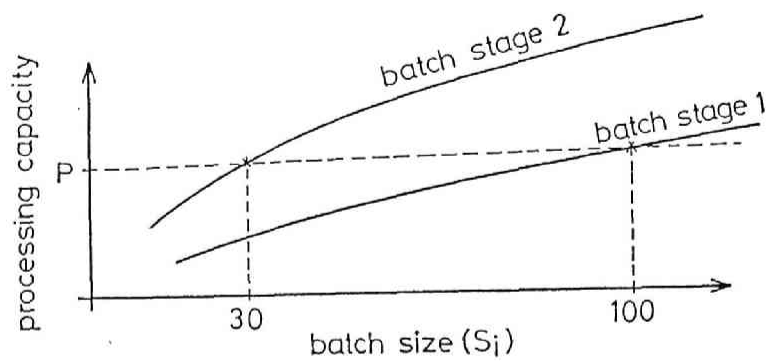


Figure 3-11. Processing capacities of both batch stages

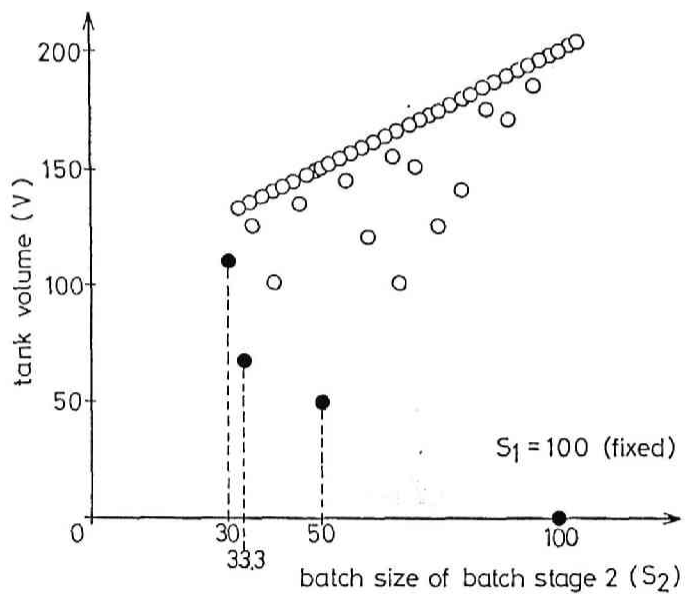


Figure 3-12. Relationship between the tank volume and the batch size

more than the minimum values, the necessary volume of the tank may decrease drastically when G.C.M. of S_1 and S_2 becomes very large, as is easily understood from Eq.(3-36).

For example, when S_1 is fixed at 100, the necessary tank volume V changes as shown in Figure 3-12. The necessary tank volume increases monotonically at almost every point in accordance with the increase of S_2 . But at several points, in this case when S_2 is 33.3, 50 and 100, the necessary tank volume V decreases drastically as shown by the black points in the figure. Therefore, the high possibility exists of being able to decrease the total investment cost resulting from the drastic decrease of the necessary tank volume at these three points. In other words, we can find the optimal batch sizes of both batch stages and the optimal volume of the tank by checking very few candidates for the optimal solution, as shown here.

In regard to this point, the following theorem can be derived.

[Theorem 3-4]

When the tank volume, V , is determined by Eq.(3-36), the optimal batch sizes of both batch stages which minimize Eq.(3-37), satisfy either of the following relationships:

$$\begin{cases} S_1 = S_1^* \\ S_2 = S_2^* \end{cases} \quad (3-39)$$

or

$$\begin{cases} S_1 = S_1^* \\ S_2 = S_1^* \cdot \text{trunc}[(M^*N'/N^*)+1]/N' \end{cases} \quad (3-40)$$

($N' = 1, 2, \dots, N^*-1$)

or

$$\begin{cases} S_1 = S_2^* \cdot \text{trunc}[(N^*M'/M^*)+1]/M' \\ S_2 = S_2^* \end{cases} \quad (3-41)$$

($M' = 1, 2, \dots, M^*-1$)

where

S_1^* = the minimal batch size of batch stage 1

$$N^* = S_1^* / \text{G.C.M.}(S_1^*, S_2^*) \quad (3-42)$$

$$M^* = S_2^* / \text{G.C.M.}(S_1^*, S_2^*) \quad (3-43)$$

[Proof of Theorem 3-4]

We give the proof according to the following four cases.

Case 1) $S_1 = S_1^*, S_2 = S_2^*$

We first consider the case where the batch size of each batch stage is equal to its minimal batch size, respectively. From Eqs.(3-42) and (3-43), we have

$$S_2^*/S_1^* = M^*/N^* \quad (3-44)$$

And the tank volume V^* is given by

$$V^* = S_1^* + S_2^* - 2 \cdot \text{G.C.M.}(S_1^*, S_2^*) = [(M^*+N^*-2)/N^*]S_1^* \quad (3-45)$$

Case 2) $S_1 = S_1^*, S_2 > S_2^*$

We next enumerate the candidates for the optimal batch sizes for the case where S_2 is greater than S_2^* .

Let

$$S_2 = (M''/N')S_1^*$$

where M'' and N' are both natural numbers and they are relatively prime. Then the tank volume V is given by the following equation.

$$V = S_1^* + S_2 - 2 \cdot \text{G.C.M.}(S_1^*, S_2) = [(N' + M'' - 2)/N']S_1^* \quad (3-46)$$

As S_2 is greater than S_2^* , performance index is smaller than Case 1 only if V is smaller than V^* . And, by Eqs.(3-44) to (3-46), $V < V^*$ satisfies only if $N^* > N'$. When N' is fixed to a certain number, both V and S_2 can be minimized by making M'' as small as possible so far as M'' satisfies the following inequality.

$$(M''/N')S_1^* > (M^*/N^*)S_1^* \quad (3-47)$$

The minimal value of M'' which satisfies Eq.(3-47) is given by

$$M'' = \text{trunc}(M^*N'/N^* + 1)$$

As the result, for this case the candidates for the optimal batch sizes are given by Eq.(3-40).

Case 3) $S_1 > S_1^*$, $S_2 = S_2^*$

For this case, we can prove in the same manner as shown in Case 2, that the candidates for the optimal batch sizes of both batch stages are given by Eq.(3-41).

Case 4) $S_1 > S_1^*$, $S_2 > S_2^*$

We finally show that for this case there are no candidates for the optimal batch sizes.

Let S_1 ($> S_1^*$) and S_2 ($> S_2^*$) be arbitrary batch sizes of batch

stage 1 and 2, respectively. Then, there exist some natural numbers m and n which satisfy

$$S_2/S_1 = m/n, \quad (3-48)$$

where m and n are assumed to be relatively prime.

In this case, the tank volume V is given by

$$V = S_1 + S_2 - 2 \cdot \text{G.C.M.}(S_1, S_2) = [(m+n-2)/n]S_1 \quad (3-49)$$

The batch sizes of both batch stages, S_1 and S_2 , satisfy one of the following two equations.

$$S_2/S_1 \geq S_2^*/S_1^* \quad (3-50)$$

or

$$S_2/S_1 < S_2^*/S_1^* \quad (3-51)$$

So, we first show that the batch sizes, S_1 and S_2 , which satisfy Eq.(3-50) are not optimal.

Let

$$S_2' = (S_2/S_1)S_1^* \quad (3-52)$$

then S_2' satisfies the following inequality.

$$S_2^* \leq S_2' < S_2 \quad (3-53)$$

From Eqs.(3-48) and (3-52), we have

$$S_2'/S_1^* = m/n. \quad (3-54)$$

Eq.(3-54) shows that when the batch sizes of both batch stages are S_1^* and S_2' , respectively, the tank volume V' is given by the following equation.

$$\begin{aligned}
V' &= S_1^* + S_2^* - 2 \cdot \text{G.C.M.}(S_1^*, S_2^*) \\
&= [(m+n-2)/n] S_1^* \\
&< V
\end{aligned} \tag{3-55}$$

Eqs.(3-53) and (3-55) show that the batch sizes, S_1 and S_2 , which satisfy Eq.(3-50) are not optimal. And we can prove in the same manner as shown above that the batch sizes which satisfy Eq.(3-51) are not optimal.

Consequently we can conclude that every candidate for the optimal batch sizes of both batch stages is given by Eqs.(3-39) to (3-41).

(Q.E.D. of Theorem 3-4)

Theorem 3-4 ensures that the optimal solution can be obtained by comparing no more than (M^*+N^*-1) number of cases.

Irrespective of the pump capacities, the tank volume determined by Eq.(3-36) is large enough to guarantee the smooth operation of the whole process. Therefore, the algorithm developed here could be widely used for the design of more general processes with an intermediate storage tank.

5. A General Batch Process Consisting of Multi-Batch Stages

In this section, the way that the relationships and the algorithms developed in the previous sections can be applied to solve the problem of the optimal design of a general batch process, as shown in Figure 3-1, is demonstrated.

First, consider the relationship between the batch size and the number of batch items in each stage. A series of batch stages between two consecutive tanks is hereafter called "a subprocess". The batch size of each stage, S_i , in a subprocess should be identical. This batch size, that is, the amount of production of the subprocess per batch, is called "the batch size of its subprocess". i.e.

$$S_i = \tilde{S}_j \quad (i \in G_j ; j = 1, 2, \dots, K) \quad (3-56)$$

where

$\tilde{S}_j$ = the batch size of subprocess j ,

G_j = a set of suffixes representing the order of batch stages in subprocess j .

In order to satisfy the given production requirement, P , the batch size, S_i , and the number of parallel batch items, N_i , in batch stage i have to satisfy the following relationship:

$$S_i N_i / w_i(S_i) \geq P \quad (3-57)$$

When the batch size of each stage is once fixed, the minimum number of parallel batch items in each stage can be

uniquely determined from Eq.(3-57). It is also clear that in each batch stage the installation of redundant parallel items more than the minimum number obtained from the above relationship, is not optimal. Therefore, the optimal number of parallel batch items in each stage is expressed by the following function of the batch size of each stage:

$$N_i = \lceil P \cdot w_i(S_i) / S_i \rceil, \quad (i = 1, 2, \dots, B) \quad (3-58)$$

where

$\lceil x \rceil :=$ the minimum integer $\geq x$.

For example, it is assumed that a subprocess consists of two batch stages and that the processing capacities of both stages are given as shown in Figure 3-5. Then, the optimal number of parallel batch items in each stage can be derived as a function of the batch size as shown in Table 3-2.

Table 3-2. Number of parallel batch items and batch size

number of batch items		range of batch size
N_1	N_2	$(S_i ; i=1,2)$
1	1	$S_1^* \leq S_i \leq S_0$
2	1	$S_2^* \leq S_i < S_1^*$
2	2	$S_a \leq S_i < S_2^*$
3	2	$S_b \leq S_i < S_a$

Next, how to determine the volume of an intermediate storage tank between two subprocesses is considered. The volume of each storage tank j , V_j , can be obtained as a function of the batch sizes of the subprocesses before and after the tank as shown in Theorem 3-3, i.e.

$$V_j = f_j(\tilde{S}_j, \tilde{S}_{j+1}) \quad (j = 1, 2, \dots, K-1) \quad (3-59)$$

From the above discussion, the optimal design problem can be mathematically formulated as a problem aimed at finding the number of parallel batch items, N_i , the equipment size of the batch item (= batch size), S_i , in each batch stage and the volumes of storage tanks, V_j , so as to minimize Eq.(3-1), subject to Eqs.(3-56) to (3-59).

By assuming the batch sizes of all of the subprocesses, all of the variables in the process and the performance index can be calculated from Eqs.(3-1), (3-56) to (3-59). Therefore, the optimal solution to the problem formulated above can be obtained by letting the batch sizes of all of the subprocesses be free variables and performing the search in the feasible domain of these free variables.

Since the number of parallel batch items and the storage tank volume determined from Eqs.(3-58) and (3-59) are not continuous functions of batch sizes, the searching procedure has to be performed by applying a direct search method.

Here, two practical cases will be considered, and algorithms which can be used to find the exactly optimal solution in a very small number of searching steps will be developed.

First, a case is considered where the process consists of two subprocesses and an intermediate storage tank, and the tank volume can be determined by Eq.(3-36). By utilizing an example, the way that the optimal solution can be obtained is shown. It is assumed that the process consists of three batch stages and an intermediate storage tank, as shown in Figure 3-13, and that the performance index is given by the following equation:

$$P.I. = \sum_{i=1}^3 N_i \cdot a_i \cdot S_i^{\alpha} + b \cdot V^{\beta} \quad (3-60)$$

where

$$a_1 = 3, \quad a_2 = 2, \quad a_3 = 3, \quad b = 1, \quad \alpha = \beta = 0.7 .$$

Then the optimal solution can be obtained by the following procedure:

- i) By applying the algorithm mentioned above to each subprocess before and after the tank, find the relationship

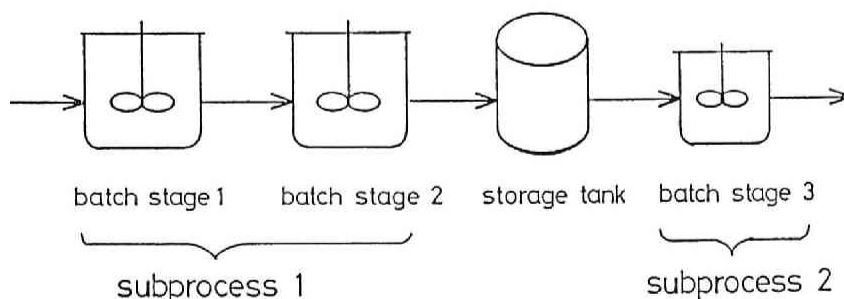


Figure 3-13. Process consisting of two subprocesses and a tank

between the batch size and the minimum number of parallel batch items in each batch stage. Next, make the result obtained for each subprocess into a table similar to that shown in Table 3-2. For the example shown in Figure 3-13, it is assumed here that two tables are obtained as shown in Table 3-3. The first part of each row in the table shows the minimum number of parallel batch items necessary for each batch stage, and the second part shows the range of batch size for which each batch stage can be composed of parallel batch items the number of which is indicated in the first part of the same row.

Table 3-3. Number of parallel batch items and range of batch size

3-3a. subprocess 1			3-3b. subprocess 2	
number of batch items		range of batch size	number of batch items	range of batch size
N_1	N_2	$\tilde{S}_1 = S_1 = S_2 \text{ [m}^3\text{]}$	N_3	$\tilde{S}_2 = S_3 \text{ [m}^3\text{]}$
1	1	$10 \leq \tilde{S}_1 \leq 15$	1	$5 \leq \tilde{S}_2 \leq 10$
2	1	$6 \leq \tilde{S}_1 < 10$	2	$2.5 \leq \tilde{S}_2 < 5$
2	2	$4 \leq \tilde{S}_1 < 6$		
3	2	$3 \leq \tilde{S}_1 < 4$		

ii) Choose a row from each table. Then, each subprocess can be composed of batch stages each of which consists of parallel batch items the number of which is indicated in the first part of that row.

For example, we choose the first rows in Table 3-3a and 3-3b which correspond to the case where $N_1=N_2=N_3=1$. Then, the feasible range of the batch size for each subprocess is given as follows:

$$10 \leq \tilde{S}_1 \leq 15, \quad 5 \leq \tilde{S}_2 \leq 10 \quad [m^3]$$

iii) By applying the algorithm developed based on Theorem 3-4 to the subprocesses constructed above, and performing a search with respect to the batch sizes of both subprocesses, find the optimal batch size of each subprocess, the optimal volume of the tank and the value of the performance index.

For the case in which $N_1=N_2=N_3=1$, the minimum batch size of subprocess 1, S_1^* , is $10 m^3$, and that of subprocess 2, S_2^* , is $5 m^3$. From Theorem 3-4, the candidates for the optimal solution can be obtained as follows:

The candidate which is derived from Eq.(3-39) is

$$\tilde{S}_1 = 10 m^3, \quad \tilde{S}_2 = 5 m^3.$$

Then, the volume of the tank which is determined from Eq.(3-36) is $5 m^3$.

The value of N^* and M^* in Eqs.(3-42) and (3-43) are 2 and 1, respectively. Therefore, the value of N' which satisfies Eq.(3-40) is 1. When $N' = 1$, $\tilde{S}_1$, $\tilde{S}_2$ and V are

obtained from Eqs.(3-40) and (3-36) as follows:

$$\tilde{S}_1 = 10 \text{ m}^3, \quad \tilde{S}_2 = 10 \text{ m}^3, \quad V = 0 \text{ m}^3.$$

Since $M^* = 1$, there is no M' which satisfies Eq.(3-41). Therefore, in this case there are only two pairs of candidates of the optimal batch sizes and the tank volume:

$$(\tilde{S}_1 = 10 \text{ m}^3, \tilde{S}_2 = 5 \text{ m}^3, V = 5 \text{ m}^3)$$

$$(\tilde{S}_1 = 10 \text{ m}^3, \tilde{S}_2 = 10 \text{ m}^3, V = 0 \text{ m}^3)$$

By substituting the value obtained above into Eq.(3-60), we can find the pair which minimizes the performance index.

iv) Choose a different row from each table and return to step iii).

In the case where $N_1 = 2$, $N_2 = 1$, and $N_3 = 1$, the pairs of $\tilde{S}_1$ and $\tilde{S}_2$ which satisfy Eqs.(3-39), (3-40) and (3-41) can be obtained as shown in Table 3-4. The value of the performance index for the given $\tilde{S}_1$ and $\tilde{S}_2$ is also shown in Table 3-4. In this case, the performance index is minimized for $\tilde{S}_1 = 6 \text{ m}^3$, $\tilde{S}_2 = 6 \text{ m}^3$ and $V = 0 \text{ m}^3$.

Repeat this procedure until the selection of a row from each table covers all of the combinations of the rows in the two tables. Then, for all the pairs of N_1 , N_2 and N_3 , pairs of $\tilde{S}_1$, $\tilde{S}_2$ and V which minimize the given performance index can be finally obtained as shown in Table 3-5. Among the solutions obtained and shown in Table 3-5, find the one which minimizes the performance index. This is the optimal solution. For this example, the optimal solution is given

Candidates for the optimal solution
Table 3-4. (for the case where $N_1=2$, $N_2=N_3=1$)

batch size of subprocess		tank volume	performance index
$\tilde{S}_1$	$\tilde{S}_2$	V	P.I.
6 [m ³]	6 [m ³]	0 [m ³]	38.56
6	5	9	41.95
6.25	5	8.75	42.67
6.67	5	8.33	43.85
7.5	5	7.5	46.14

Relationship between the number of
Table 3-5. batch items and the performance index

number of batch items			batch size of subprocess		tank volume	performance index
N_1	N_2	N_3	$\tilde{S}_1$	$\tilde{S}_2$	V	P.I.
1	1	1	10 [m ³]	5 [m ³]	5 [m ³]	37.40
2	1	1	6	6	0	38.56
2	2	1	5	5	0	40.11
3	2	1	3	6	3	40.72
1	1	2	10	2.5	7.5	40.55
2	1	2	6	3	3	43.14
2	2	2	4.5	2.5	6	43.56
3	2	2	3	3	0	41.00

as follows.

$$N_1 = N_2 = N_3 = 1,$$

$$S_1 = S_2 = 10 \text{ m}^3, S_3 = 5 \text{ m}^3, \quad V = 5 \text{ m}^3.$$

So far, it has been assumed that the equipment size at each batch stage can be chosen arbitrarily. But it often happens that only batch items in standard sizes are available. Therefore, for a general batch process system as shown in Figure 3-1, it is necessary to consider the case where the batch size of each subprocess can take only discrete values.

From the facts that the performance index is given by a sum of the term related to each batch stage and the one related to the intermediate storage tank, and that the tank volume is determined only by the batch sizes of the subprocesses before and after the tank, the optimal solution can be obtained by applying "Dynamic Programming".

Rewrite the performance index given by Eq.(3-1). Let h_j denote a part of the performance index related to storage tank j and subprocess $j+1$ adjacent to it. Then h_j is expressed by the following equation:

$$\begin{aligned} h_j(\tilde{S}_j, \tilde{S}_{j+1}) := & \sum_{i \in G_{j+1}} \|P \cdot w_i(\tilde{S}_{j+1}) / \tilde{S}_{j+1}\| \cdot p_i(\tilde{S}_{j+1}) \\ & + (1 - \delta_{0j}) q_j(f_j(\tilde{S}_j, \tilde{S}_{j+1})) \end{aligned} \quad (3-61)$$

$$(j = 0, 1, 2, \dots, K-1)$$

where

δ_{ij} = Kronecker's delta

$\tilde{S}_0 = 0.$

Let

$$H_j(\tilde{S}_{j+1}) := \min_{\{\tilde{S}_0, \tilde{S}_1, \dots, \tilde{S}_j\}} \left\{ \sum_{i=0}^j h_i(\tilde{S}_i, \tilde{S}_{i+1}) \right\} \quad (3-62)$$

then

$$H_j(\tilde{S}_{j+1}) = \min_{\{\tilde{S}_j\}} [h_j(\tilde{S}_j, \tilde{S}_{j+1}) + H_{j-1}(\tilde{S}_j)] \quad (3-63)$$

By introducing the functions defined above, the problem formulated by Eqs.(3-1), (3-56), (3-57), (3-58) and (3-59) can be restated as follows:

Minimize

$$H_{K-1}(\tilde{S}_K)$$

subject to Eqs.(3-56) to (3-63).

$H_j(\tilde{S}_{j+1})$ can be determined by calculating Eq.(3-63) for discrete values of the batch size of subprocess j . Therefore, the optimal value of the performance index H_{K-1} can be obtained by calculating H_j one after another for all subprocesses from 1 to K .

6. Discussion and Conclusion

The problem of the optimal design of general processes consisting of multi-batch stages and intermediate storage tanks has been dealt with.

First, a simple process consisting of two batch stages was taken up and an algorithm was developed for determining the optimal number of parallel batch items and the optimal equipment size of the batch item by utilizing the fact that the batch size and the cycle time of each stage must be identical.

Next, a process consisting of two batch stages and one storage tank was considered. By installing a storage tank between two batch stages, the cycle times and batch sizes of the batch stages before and after the tank can be arbitrarily chosen. It was shown that the minimum tank volume can be given by a function of the batch sizes of both stages. A theorem was derived such that if the time necessary for feeding to and discharging from the tank is fairly short compared with the cycle time of each batch stage, the tank volume can be given by a very simple equation as follows:

$$\text{tank volume} = S_1 + S_2 - 2 \cdot \text{G.C.M.}(S_1, S_2) \quad (3-36)$$

where S_1 and S_2 are the batch sizes of the batch stages before and after the tank.

For a general process, it was shown that the number of parallel batch items, the size of its equipment and the

volume of a storage tank can be determined by a function of the batch sizes of the subprocesses, respectively.

For two practical cases where the process consists of two subprocesses and a storage tank the volume of which can be determined by Eq.(3-36), and where only standard batch items are available, algorithms by which the optimal solution can be easily obtained were derived.

In this study, it is assumed that the capacities of the feed and discharge pumps are known. The pump capacity affects the cycle time of a batch item. Therefore, when the cost of pumps cannot be neglected compared to the cost of batch items and storage tanks, the optimal design problem taken up here could be solved by utilizing a two level approach. In it the determination of the capacities of the pumps are performed at the upper level, and the determination of the number of batch items, the size of its equipment and the volumes of the storage tanks is done at the lower level.

Moreover, an attempt was made to develop an effective method of solution without assuming any concrete functions about the equipment cost, the minimal cycle time nor processing capacity of each batch item. Thus, the algorithms developed in this chapter have wide applicability in the solution of more general batch processes.

In order to mathematically express a batch process, a deterministic model was used, and some interesting results were derived. However, in an actual batch process there are many uncertainties. The processing times vary,

environmental conditions change, and production requirements and/or production schedules change with the market environment. Therefore, much effort has to be devoted to develop some effective design methods which enable us to make rational design and operation decisions for any kind of batch process, even though there are various uncertainties. The design problem of a flexible batch process is taken up in Chapter 4.

Nomenclature

B = number of batch stages

C_i = processing capacity of batch stage i

c_i = processing capacity of the batch item in batch stage i

$f, (f_j)$ = function representing the volume of the tank (tank j)

G_j = a set of suffixes representing the order of batch stages in subprocess j

K = number of subprocesses

N_i = number of parallel batch items in batch stage i

P = production requirement per unit time

p_i = cost function of a batch equipment in batch stage i

$q, (q_j)$ = cost function of intermediate storage tank (tank j)

S_i = batch size of batch stage i

(= equipment size of a batch item in batch stage i)

$\tilde{S}_i$ = batch size of subprocess i

t_1 = starting moment of the inflow from batch stage 1 to the tank in the first cycle

t_2 = starting moment of the discharge from the tank to batch stage 2

U^j = capacities of the feed pump ($j=f$) and the discharge pump ($j=d$) of the tank per unit time

u^j = input ($j=f$) and output ($j=d$) functions to and from the tank

$V, (V_j)$ = volume of the intermediate storage tank (tank j)

v^0 = initial hold-up in the tank

v = hold-up in the tank

w_i = cycle time of the batch item in batch stage i

w_i = minimum cycle time of the batch item in batch stage i

Other symbols

G.C.M.(X,Y) = extended greatest common measure of X and Y

$\text{mod}(X,Y) = X - \text{trunc}(X/Y) \cdot Y$

$\text{trunc}(X)$ = the largest integer which is equal to or less
than X

$\lceil X \rceil$ = the minimum integer which is equal to or greater
than X

Chapter 4

DESIGN OF A FLEXIBLE BATCH PROCESS WITH INTERMEDIATE STORAGE TANKS

1. Introduction

Many batch processes are utilized for producing products with high added values. Such processes usually necessitate so-called sophisticated operations, such as reactions with complex reaction-paths and/or long reaction times, and the processing of solid materials which are very difficult to handle.

In such a sophisticated batch operation, there are many uncertainties. Often there are cases in which the processing time and/or cleaning time of each batch unit becomes shorter or longer than the scheduled ones. It also happens that the amount of material processed in a batch, that is, the batch size, often changes from the preassigned nominal value.

When each batch unit is connected in a series, the outlet flow from the precedent batch unit directly becomes the inlet flow to the next unit. Therefore, the variations in processing time and/or batch size in a certain unit affect the operation schedule and the batch size of the whole process. That is, these variations cause problems such that the operation schedules and/or the batch sizes of other batch units have to be readjusted. From the practical viewpoint, however, such readjustments are unfavorable because usually operators are forced to do some excessive work which sometimes leads to dangerous mis-operations.

In some batch units, it might be impossible to amend the preassigned schedule, i.e. it is absolutely necessary to

strictly operate those batch units in accordance with the fixed schedule.

In designing a batch process in which the variations mentioned above might easily occur, it is indispensable to develop some countermeasures so as to decrease the unfavorable effect caused by the variations. As a promising countermeasure to avoid the propagation of such unfavorable influences on the other batch stages, it is most effective to install an intermediate storage tank between batch stages. By installing a storage tank between two batch stages, the outlet flow from the precedent stage is stored in the tank and does not immediately become the inlet flow to the batch stage following the tank. Therefore, the storage tank installed between two batch stages can absorb any influence due to the variation in the operation which occurred in the previous batch stage and prevent the propagation of the influence on the batch stage following the tank.

In Chapter 3, the optimal design procedure was developed for a batch process consisting of many batch stages and intermediate storage tanks without taking into account the uncertainties in each operation. In this chapter, we deal with the problem of how to design a flexible batch process in which the storage tanks never overflow nor run out of stored material, and moreover the preassigned regular operations are kept in other batch stages, even though the batch size and the processing and cleaning times of a batch stage vary due to various uncertain causes and/or mis-operations.

2. Description of the Problem

A general single product batch process consists of many batch stages and intermediate storage tanks as shown in Figure 4-1. In this chapter, the problem of how to design a flexible batch process which can absorb unfavorable effects due to some uncertainties is considered.

In order to clarify the problem, the following assumptions are first introduced:

- i) The capacities of the feed and discharge pumps of the batch stages are known.
- ii) Each batch stage consists of one or more identical items of batch equipment in parallel. Every batch item in a batch stage is periodically operated with the same cycle time by delaying its starting moment at equal intervals.
- iii) The minimal cycle time of the batch item, w_i , is a function of the batch size of the batch stage, S_i . The processing capacity of the batch item, c_i , is a

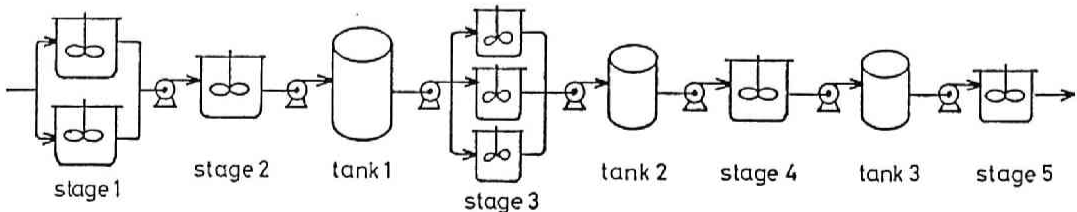


Figure 4-1. Schematic diagram of a general batch process

monotonically increasing function of the batch size of the batch stage, S_i .

- iv) The process does not cause any volume changes in the flow, and the equipment size of the batch item and the batch size of the batch stage are measured by the same unit.
- v) Possible places where an intermediate storage tank can be installed are known. A series of batch stages between two consecutive tanks is called "a subprocess".

In the process shown in Figure 4-1, many different kinds of variations may occur in its real operation due to various uncertainties. However, here we consider only the two most essential, different kinds of variations, that is, the variation in the operation schedule and that in the batch size.

In order to make the discussion clearer, the following assumptions are further introduced:

- vi) Even though variations in the duration times of the operation steps such as the filling, processing, discharging and cleaning of a certain batch unit in a subprocess occur due to some uncertain causes, the operation of the subprocess can be continued by readjusting the operation schedules of the other batch units contained in the same subprocess. Therefore, the variations in the starting moment of the outflow from a tank to the first batch unit in a subprocess (i.e. the starting moment of the outflow from a tank) and in the

starting moment of the inflow from the last batch unit in a subprocess to a tank (i.e. the starting moment of the inflow to a tank) only are considered in this study. The ranges of delay and advance in the starting moments of the inlet flow to and the outlet flow from each storage tank from the original scheduling time are moreover assumed to be given a priori.

When a variation in the starting moment of the inflow to the tank occurs in a certain batch, the process is presumed to be operated according to the operation schedule as shown in the following: In every periodical operation after the variation has occurred, the starting moment of the inflow to the tank is shifted from the originally scheduled starting moment by the size of the variation. On the other hand, the starting moment of the outlet flow from the tank is not changed but is kept exactly the same as that of the original schedule. The variation in the starting moment of the inflow to the tank is defined as "allowable" if and only if the tank does not overflow nor run out of stored material even though the whole process is periodically operated according to the readjusted schedule mentioned above.

As for the variation in the starting moment of the outlet flow from the tank, the "allowable variation" is similarly defined as the variation in the starting moment of the inlet flow to the tank.

vii) In regard to the variations in batch sizes, the upper limit of these variations is known a priori. Moreover, the upper and lower bounds of the total sum of the variations in the batch size of a batch that is flowed into and discharged from a subprocess over the whole production period are all known.

When the variation in the batch size of a batch of material that is flowed into the tank occurs, the process is presumed to be operated according to the preassigned schedule as shown in the following: In every periodical operation after the variation in the batch size has occurred, the batch sizes of a batch that is flowed into and from the tank are identical with the original batch sizes before the variation occurred. The starting moments in the inflow to and outflow from the tank are not changed and follow the preassigned schedule, irrespective of whether a variation in the batch size has occurred or not. The variation in the batch size of a batch that is flowed into the tank is defined as "allowable" if and only if the tank does not overflow nor run out of stored material even though the whole process is operated according to the schedule explained above.

As for the variation of the batch size in a batch of material discharged from the tank, the "allowable variation" is defined in a way similar to the case of the variation in the batch size of a batch flowing into

the tank.

viii) The flexibility of a batch process may be evaluated by many different kinds of measures. In this study, it is assumed that the flexibility of a batch process could be measured by the size of the regions of "allowable variations" in the operation schedule and the batch size.

The problem of the design of a flexible batch process is essentially multi-objective. It is necessary to simultaneously solve two optimization problems which are mutually contradictory: one is how to minimize the investment cost of the process, and the other is how to maximize the flexibility of the process. These two objectives related to completely different attributes, flexibility and cost, cannot be evaluated by the same parameters. Consequently, this design problem has to be handled either as an optimization problem of how to increase the flexibility of the process within a given available budget for construction, or as that of how to decrease the construction cost of the process which has a given flexibility.

In this study, the problem of how to design a batch process which has a certain desired flexibility so as to minimize the required construction cost is dealt with. In other words, the problem is how to minimize the construction cost of a batch process when the ranges of the allowable variations with respect to the operation schedule and/or the batch sizes are given a priori.

3. Minimum Capacity of the Storage Tank

In this section, a process consisting of two subprocesses and a tank as shown in Figure 4-2 is considered, and the minimum capacity of the storage tank is derived such that the tank does not overflow nor run out of stored material even though the variations of the operation schedule and/or the batch size of a subprocess may occur.

First, the relationships between variables, such as the number of parallel batch items and the batch size in each batch stage, the volume of the storage tank, etc., are shown.

The batch size of every batch stage in a subprocess must be equal to the batch size of the subprocess, i.e.

$$S_i = \tilde{S}_j \quad (i \in G_j ; j = 1, 2) \quad (4-1)$$

where

$\tilde{S}_j$ = the batch size of subprocess j ,

G_j = a set of suffixes representing the order of batch stages in subprocess j .

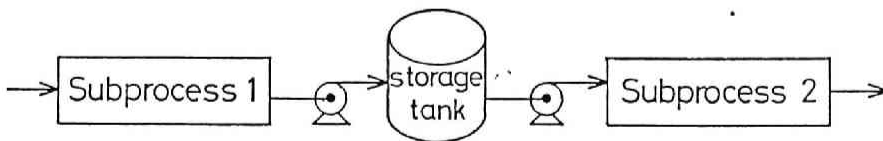


Figure 4-2. A process consisting of two subprocesses

In order to satisfy the given production requirement per unit time, P , the minimum number of parallel batch items in each batch stage is given as a function of the batch size of the batch stage, i.e.

$$N_i = \| P \cdot w_i(S_i) / S_i \| \quad (i = 1, 2, \dots, B) \quad (4-2)$$

where

B = the number of batch stages

$\|x\|$ = the minimum integer which is greater than or equal to x .

Batch sizes and cycle times of both subprocesses must be determined so as to satisfy the material balance of the whole process:

$$P = \tilde{S}_1 / \tilde{W}_1 = \tilde{S}_2 / \tilde{W}_2 \quad (4-3)$$

where

$\tilde{W}_j$ = the cycle time of subprocess j .

The input function to and the output function from the tank, u^f and u^d , are defined as follows:

$$u^k(t) = \begin{cases} U^k ; & i\tilde{W}_j \leq t < i\tilde{W}_j + \tilde{S}_j / U^k \\ 0 ; & \text{otherwise} \end{cases} \quad (4-4)$$

(($k = f$ and $j = 1$) or ($k = d$ and $j = 2$)),
($i = 0, 1, 2, \dots$)

where

U^k = the capacities of the feed ($j=f$) and the discharge ($j=d$) pumps of the tank per unit time.

Then, the hold-up in the tank, $v(t)$, is given by:

$$v(t) = v^0 + \int_0^t [u^f(\tau - t_1) - u^d(\tau - t_2)] d\tau \quad (4-5)$$

where

v^0 = the initial inventory in the storage tank

t_1 = the starting moment of the inflow from subprocess
1 to the tank in the first cycle

t_2 = the starting moment of the discharge from the tank
to subprocess 2 in the first cycle.

The tank does not overflow nor run out of stored material if and only if the hold-up in the tank satisfies the following inequalities for any time t :

$$0 \leq v(t) \leq V \quad (4-6)$$

where

V = the volume of the storage tank.

We number the variations which occurred in the process according to their occurrence times. Here, occurrence times are defined as follows: For the delay in the starting moments of the inflow to and the outflow from the tank, it is defined as the time when the inflow to or the outflow from the tank is scheduled. For the advance in the starting moments of the inflow to and the outflow from the tank, it is defined as the time when the inflow or the outflow actually starts, respectively. The occurrence time of the variation in the batch size is defined as the time when the inflow or

the outflow of a batch of material with varied batch size to or from the tank begins.

By using four kinds of variables, the i -th variation which occurred in the process can be expressed in terms of an array such as $(\Delta t^i, \Delta T^i, \Delta s^i, \Delta S^i)$:

where

$\Delta t^i, \Delta T^i$ = the amounts of variations in the starting moments of the inflow to and the outflow from the tank for the i -th variation, respectively,
 $\Delta s^i, \Delta S^i$ = the amounts of variations in the batch sizes of a batch which was flowed into and was discharged from the tank for the i -th variation, respectively.

The size of the i -th variation in the starting moment is measured by the magnitude of the delay or advance from the time schedule operated after the $(i-1)$ -th variation occurred; i.e., the starting moments of the inflow to and the outflow from the tank after the variation has occurred are shifted by

$$\sum_{j=1}^i \Delta t^j \text{ and } \sum_{j=1}^i \Delta T^j$$

from the original schedule in which no variations are assumed to occur. Here, the positive value is taken for the delay, and the negative one is taken for the advance in starting moment. The variations in the batch sizes are measured as deviations from the preassigned batch sizes of both subprocesses. The positive value is taken for the increase and the

negative one is taken for the decrease in the batch size.

Here it is assumed that each variation consists of only one kind of variation. For every i , an array for i -th variation, $(\Delta t^i, \Delta T^i, \Delta s^i, \Delta S^i)$, has only one non-zero element. For example, if the i -th variation which occurred in the process is the delay of the starting moment of the inflow to the tank, Δt^i takes a positive value, and all of the other variables, such as ΔT^i , Δs^i and ΔS^i are equal to zero, i.e., i -th variation can be expressed by the array $(\Delta t^i, 0, 0, 0)$. If two kinds of variations occur at one time, for example, variations in batch size and in the starting moment of the inlet flow into the tank occur simultaneously, these variations are handled as if two variations expressed by the arrays $(\Delta t^i, 0, 0, 0)$ and $(0, 0, \Delta s^{i+1}, 0)$, have occurred.

From the assumption which was introduced in the previous section, the sum of amounts of each kind of variation satisfies the following constraints:

$$\left. \begin{aligned} \Delta t^L &\leq \sum_{i=1}^k \Delta t^i \leq \Delta t^U & (k = 1, 2, \dots) \\ \Delta T^L &\leq \sum_{i=1}^k \Delta T^i \leq \Delta T^U & (k = 1, 2, \dots) \\ \Delta s^L &\leq \sum_{i=1}^k \Delta s^i \leq \Delta s^U & (k = 1, 2, \dots) \\ \Delta S^L &\leq \sum_{i=1}^k \Delta S^i \leq \Delta S^U & (k = 1, 2, \dots) \end{aligned} \right\} \quad (4-7)$$

These lower and upper bounds have to be determined by taking into account many kinds of factors, such as the frequency and the magnitude of the variations and/or the characteristics of the operation of the process, such as a property in which its operation schedule is easy or not to rearrange.

We now consider the condition that the variables t_1 , t_2 and V must satisfy so that the tank does not overflow nor run out of stored material even though a delay of the starting moment of the inflow to the tank occurs.

When the starting moment of the inflow to the tank is delayed Δt^1 from the original schedule, the hold-up in the tank changes with time as follows:

$$v(t) = v^0 + \int_0^t [u^f(\tau - t_1) - u^d(\tau - t_2)] d\tau \quad (4-8a)$$

$$\text{for } 0 \leq t < t_a$$

$$v(t) = v^0 + \int_0^{t_a} u^f(\tau - t_1) d\tau - \int_0^t u^d(\tau - t_2) d\tau \quad (4-8b)$$

$$\text{for } t_a \leq t < t_a + \Delta t^1$$

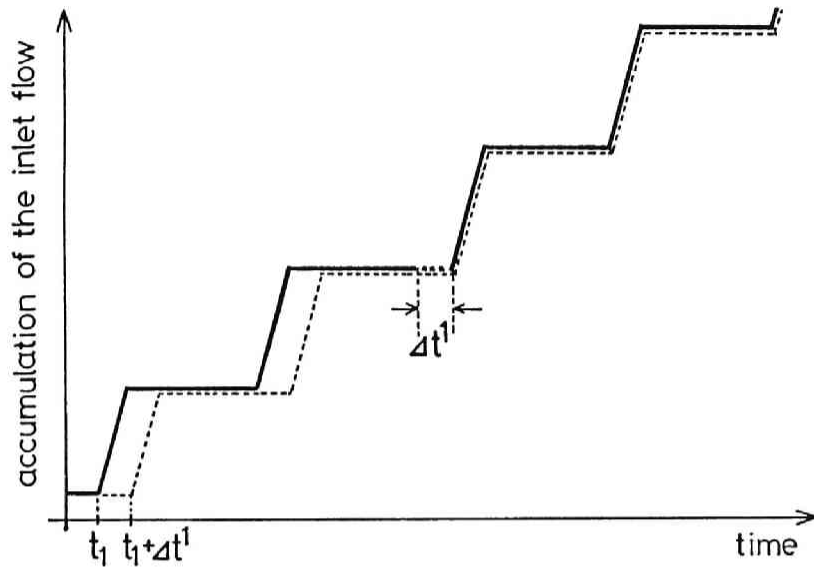
$$v(t) = v^0 + \int_0^t [u^f(\tau - t_1 - \Delta t^1) - u^d(\tau - t_2)] d\tau \quad (4-8c)$$

$$\text{for } t_a + \Delta t^1 \leq t$$

where

t_a = the time when the first variation takes place.

The changes of the accumulation of the inflow to the tank can be expressed by the bold broken line in Figure 4-3.



.. Figure 4-3. The accumulation of the inlet flow to the tank

The fine dotted line in the figure shows the accumulation of the inflow to the tank for the case in which the starting moment of the tank in the first cycle is delayed Δt^1 . It is clear from this figure that after the variation has occurred, these two lines coincide with each other.

Therefore, $v(t)$ defined by Eq.(4-8) satisfies the following relationship for any t .

$$\begin{aligned}
 & v^0 + \int_0^t [u^f(\tau - t_1) - u^d(\tau - t_2)] d\tau \\
 & \geq v(t) \geq v^0 + \int_0^t [u^f(\tau - t_1 - \Delta t^1) - u^d(\tau - t_2)] d\tau
 \end{aligned}$$

By taking into account this fact, we can derive the following result.

"The variation Δt^1 is allowable if and only if the tank does not overflow nor run out of stored material even though the starting moment of the inflow to the tank in the first cycle is delayed for the same length of time as the variation Δt^1 from the original starting moment".

In other words, when $v(t)$ defined by Eq.(4-8a) satisfies Eq.(4-6) for any t , the variation Δt^1 is allowable if and only if $v(t)$ defined by Eq.(4-8c) satisfies Eq.(4-6) for any t . This result was derived by Oi et al.^[1] for the process consisting of parallel batch units, a storage tank and a continuous section.

By substituting t_1 for $t_1 + \Delta t^1$ in Eq.(4-8c), Eq.(4-8c) is equal to Eq.(4-5). Therefore, in order to continue the operation of the process without overflow or exhaustion of stored material irrespective of the variation Δt^1 , variables t_1 , t_2 and V must be chosen in such a way that Theorem 3-2 which has been derived in Chapter 3 is satisfied even though $t_1 + \Delta t^1$ is substituted for t_1 .

From Theorem 3-2 in Chapter 3, this condition is mathematically expressed as follows:

$$\begin{aligned} (1-P/U^d)\tilde{S}_2 - (1-b)(1-h) \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) - V^0 \\ \leq [t_2 - (t_1 + \Delta t^1)]P \\ \leq V - V^0 - (1-P/U^f)\tilde{S}_1 + (1-b)(1-r) \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) \end{aligned}$$

where

$$h = \text{mod}[V^0/\text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

$$r = \text{mod}[(V-V^0)/\text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

$$b = P/\min(U^f, U^d)$$

G.C.M.(X,Y) = extended greatest common measure of X and Y
(see Chapter 2)

$$\text{mod}(X,Y) = X - \text{trunc}(X/Y) \cdot Y$$

$$\text{trunc}(X) = \text{largest integer} \leq X.$$

Even for the other variations related to an operation schedule, such as the advance of the starting moment of the flow into the tank, and the delay and/or advance in the starting moment of the outlet flow from the tank, we can derive the same kind of result as shown here.

Next, we consider the variation of the batch size of a subprocess. If the batch size of some batch that is flowed into the tank increases by Δs^1 at $t=t_a$, the hold-up in the tank changes with time as follows:

$$v(t) = V^0 + \int_0^t [u^f(\tau-t_1) - u^d(\tau-t_2)] d\tau \quad (4-9a)$$

for $0 \leq t < t_a$

$$v(t) = V^0 + \int_0^{t_a} u^f(\tau-t_1) d\tau + (t-t_a)U^f - \int_0^t u^d(\tau-t_2) d\tau$$

for $t_a \leq t < t_b$ (4-9b)

$$v(t) = V^0 + \Delta s^1 + \int_0^t [u^f(\tau-t_1) - u^d(\tau-t_2)] d\tau \quad (4-9c)$$

for $t_b \leq t$

where

$$t_b = t_a + (\tilde{S}_1 + \Delta s^1) / U^f$$

For any t , $v(t)$ determined by Eq.(4-9) satisfies the following relationship:

$$V^0 + \int_0^t [u^f(\tau - t_1) - u^d(\tau - t_2)] d\tau$$

$$\leq v(t)$$

$$\leq V^0 + \Delta s^1 + \int_0^t [u^f(\tau - t_1) - u^d(\tau - t_2)] d\tau$$

Therefore, if $v(t)$ given by Eq.(4-9a) satisfies Eq.(4-6) for any t , the variation in the batch size of a batch that is flowed into the tank, Δs^1 , is allowable if and only if $v(t)$ determined by Eq.(4-9c) satisfies Eq.(4-6) for any t . By substituting V^0 for $V^0 + \Delta s^1$ in Eq.(4-9c), Eq.(4-9c) is equal to Eq.(4-5). Therefore, Δs^1 is allowable if and only if the following inequalities are satisfied.

$$(1 - P/U^d) \tilde{S}_2 - (1 - b)(1 - h') \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) - (V^0 + \Delta s^1)$$

$$\leq (t_2 - t_1)P$$

$$\leq V - (V^0 + \Delta s^1) - (1 - P/U^f) \tilde{S}_1 + (1 - b)(1 - r') \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2)$$

where

$$h' = \text{mod}[(V^0 + \Delta s^1) / \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

$$r' = \text{mod}[(V - V^0 - \Delta s^1) / \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

Even for the other variations, such as the decrease in the batch size of a batch of material that is flowed into the tank, and the increase and/or decrease of the batch size of a batch discharged from the tank, we can derive the same kind of result by repeating a similar discussion.

The hold-up in the tank after the n -th variation is over, $v^n(t)$, is given by

$$v^n(t) = v^0 + \sum_{i=1}^n (\Delta s^i - \Delta S^i) + \int_0^t [u^f(\tau - t_1 - \sum_{i=1}^n \Delta t^i) - u^d(\tau - t_2 - \sum_{i=1}^n \Delta T^i)] d\tau \quad (4-10)$$

By substituting v^0 , t_1 and t_2 for $v^0 + \sum_{i=1}^n (\Delta s^i - \Delta S^i)$, $t_1 + \sum_{i=1}^n \Delta t^i$ and $t_2 + \sum_{i=1}^n \Delta T^i$ in Eq.(4-10), respectively, Eq.(4-10) is equal to Eq.(4-5). Consequently, we can derive the following theorem from the above discussion and Theorem 3-2 in Chapter 3.

[Theorem 4-1]

It is assumed that the $(n-1)$ -th variation is allowable. Then the n -th variation, $(\Delta t^n, \Delta T^n, \Delta s^n, \Delta S^n)$, is allowable if and only if the following relationship is satisfied.

$$(1-P/U^d)\tilde{S}_2 - (1-b)(1-h') \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) - v^0 - \sum S^n \leq (t_2 - t_1 + \sum T^n)P \quad (4-11a)$$

$$\leq v - v^0 - \sum S^n - (1-P/U^f)\tilde{S}_1 + (1-b)(1-r') \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) \quad (4-11b)$$

where

$$h' = \text{mod}[(V^0 + \Sigma S^n)/\text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

$$r' = \text{mod}[(V - V^0 - \Sigma S^n)/\text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

$$\Sigma S^n = \sum_{i=1}^n (\Delta s^i - \Delta S^i)$$

$$\Sigma T^n = \sum_{i=1}^n (\Delta T^i - \Delta t^i)$$

$$b = P/\min(U^f, U^d)$$

The left-hand side of Eq.(4-11a) is a monotonically decreasing function with respect to ΣS^n , and the right-hand side is a monotonically increasing function with respect to ΣT^n . Therefore, Eq.(4-11a) is satisfied for any variations restricted by Eq.(4-7) if and only if Eq.(4-11a) is satisfied for the variations such that $\Sigma S^n = \Delta s^L - \Delta s^U$ and $\Sigma T^n = \Delta T^L - \Delta t^U$ hold. Similarly, Eq.(4-11b) is satisfied for any variations restricted by Eq.(4-7) if and only if Eq.(4-11b) is satisfied for the variations such that $\Sigma S^n = \Delta s^U - \Delta s^L$ and $\Sigma T^n = \Delta T^U - \Delta t^L$ hold. By substituting the above values for ΣS^n and ΣT^n in Eq.(4-11) and by eliminating $t_2 - t_1$ in Eq.(4-11), the constraint that the capacity of the tank must satisfy is derived as follows:

$$\begin{aligned} & (1 - P/U^f)\tilde{S}_1 + (1 - P/U^d)\tilde{S}_2 - (1-b)(2-h'') \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) \\ & - V^0 + (\Delta s^U - \Delta s^L) + (\Delta t^U + \Delta T^U - \Delta t^L - \Delta T^L)P \\ & \leq (R'' + br'') \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) \end{aligned} \quad (4-12)$$

where

$$h'' = \text{mod}[(V^0 + \Delta S^L - \Delta S^U) / \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

$$R'' = \text{trunc}[(V - V^0 - \Delta S^U + \Delta S^L) / \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2)]$$

$$r'' = \text{mod}[(V - V^0 - \Delta S^U + \Delta S^L) / \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

If the ranges of variations of operation schedules and batch sizes, and the batch sizes themselves are given for both subprocesses, the value of the left-hand side of Eq.(4-12) can be calculated. Therefore, the minimum capacity of the tank is given as a minimum value of

$$V^0 + (R'' + r'') \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) + (\Delta S^U - \Delta S^L)$$

which satisfies Eq.(4-12). That is, the following theorem is derived.

[Theorem 4-2]

When the batch sizes and the cycle times of both subprocesses are determined so as to satisfy the given production requirement, and the ranges of variations are also given by Eq.(4-7), the minimum tank volume is given by the following equation.

$$V = \{\text{trunc}(Q') + \min[\text{mod}(Q', 1)/b, 1]\} \cdot \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) + V^0 + (\Delta S^U - \Delta S^L) \quad (4-13)$$

where

$$Q = [(1 - P/U^f)\tilde{S}_1 + (1 - P/U^d)\tilde{S}_2 - V^0 + (\Delta S^U - \Delta S^L) + (\Delta t^U + \Delta T^U - \Delta t^L - \Delta T^L)P] / \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2) - (1 - b)(2 - h'')$$

$$Q' = \max(Q, 0)$$

$$h'' = \text{mod}[(V^0 + \Delta S^L - \Delta S^U) / \text{G.C.M.}(\tilde{S}_1, \tilde{S}_2), 1]$$

$$b = P / \min(U^f, U^d)$$

The proof of Theorem 4-2 is exactly similar to that of Theorem 3-3 in Chapter 3.

4. Mathematical Formulation

In previous sections, relationships among the variables in the process consisting of two subprocesses and a tank are clarified. By using the results obtained in previous sections, the optimal design problem for a general batch process as shown in Figure 4-1 can be mathematically formulated as follows:

Find the optimal number of parallel batch items in each batch stage, N_i , the optimal equipment size of each batch item, E_i , and the optimal volumes of intermediate storage tanks, V_j , so as to minimize

$$P.I. = \sum_{k=1}^B N_k \cdot p_k(E_k) + \sum_{k=1}^{K-1} q_k(V_k) \quad (4-14)$$

subject to

$$S_k = \tilde{S}_m = P \cdot \tilde{W}_m \quad (k \in G_m) \quad (4-15)$$

$$E_i = S_i + \Delta S_i^M \quad (4-16)$$

$$N_i = \lceil P \cdot w_i(S_i) / S_i \rceil \quad (4-2)$$

$$V_j = \{ \text{trunc}(Q_j^f) + \min[\text{mod}(Q_j^f, 1)/b_j, 1] \} \text{G.C.M.}(\tilde{S}_j, \tilde{S}_{j+1}) + V_j^0 + (\Delta S_j^U - \Delta S_j^L) \quad (4-17)$$

$$Q_j = [(1 - P/U_j^f) \tilde{S}_j + (1 - P/U_j^d) \tilde{S}_{j+1} - V_j^0 + (\Delta S_j^U - \Delta S_j^L) + (\Delta t_j^U + \Delta T_j^U - \Delta t_j^L - \Delta T_j^L)P] / \text{G.C.M.}(\tilde{S}_j, \tilde{S}_{j+1}) - (1 - b_j)(2 - h_j'') \quad (4-18)$$

$$Q_j^! = \max(Q_j, 0) \quad (4-19)$$

$$b_j = P / \min(U_j^f, U_j^d) \quad (4-20)$$

$$h_j'' = \text{mod}[(V_j^0 + \Delta s_j^L - \Delta s_j^U) / \text{G.C.M.}(\tilde{S}_j, \tilde{S}_{j+1}), 1] \quad (4-21)$$

$$(m = 1, 2, \dots, K ; i = 1, 2, \dots, B ; j = 1, 2, \dots, K-1)$$

where

B = number of batch stages

K = number of subprocesses

p_i = cost function of a batch unit in batch stage i

q_j = cost function of intermediate storage tank j

Δs_i^M = the maximum value of the variation in the batch
size of batch stage i

Subscript j of the symbols $V_j, V_j^0, U_j^f, U_j^d, \Delta t_j^U, \Delta t_j^L, \Delta T_j^U, \Delta T_j^L, \Delta s_j^U, \Delta s_j^L, \Delta S_j^U$ and ΔS_j^L , means that these symbols are related to storage tank j (see Nomenclature).

5. Solution Method

From Eqs.(4-2), (4-15) and (4-16), the number of parallel batch items and the equipment size of the batch item in each batch stage can be obtained as functions of the batch size of the subprocess. From Eq.(4-17), the volume of the tank can be given as a function of the batch sizes of subprocesses before and after the tank.

The batch size of a subprocess is given as a function of the cycle time of the subprocess. Therefore, by assuming the cycle time of all of the subprocesses, all of the variables in the process and the performance index can be calculated. For the process shown in Figure 4-1, Figure 4-4 shows how all the variables are uniquely determined as functions of the cycle times of the subprocesses. In Figure 4-4, each vertex corresponds to a variable or a set of variables. An arc leading from vertex A to vertex B means that the variable corresponding to vertex B is a function of the variables corresponding to vertex A. Therefore, the optimal solution of the problem formulated above can be obtained by letting the cycle times of all of the subprocesses be free variables and by performing the search in the feasible domain of these free variables.

Since the number of parallel batch items and the volume of the storage tank determined from Eqs.(4-2) and (4-17) are not continuous functions of batch sizes, the searching procedure has to be performed by applying a certain direct search method.

So far, it has been assumed that the cycle time of each batch stage can be chosen arbitrarily. But in a real batch process there are many cases where the cycle time of each batch stage can take only discrete values. For example, assume that the process is on a 24-hour job and the minimum cycle time of a subprocess is 23 hours. In this case, we may take 24 hours as the cycle time of the subprocess even though the process can be cyclically operated in the interval of 23 hours.

We next consider the case where the cycle time of each subprocess can take only discrete values. From Figure 4-4, it can easily be understood that the part of the performance index related to subprocess j is a function of the cycle time of subprocess j and that the other part related to storage tank j is a function of the cycle times of both subprocess j and $j+1$. Therefore, "Dynamic Programming" can be easily applied to obtain the optimal solution.

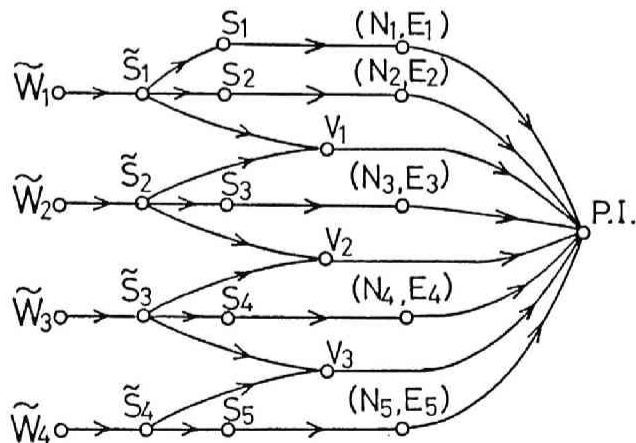


Figure 4-4. Relationships among the variables

6. Numerical Example

In this example, the whole process is assumed to have the same structure as shown in Figure 4-1. It is here assumed that the item of equipment in stage 5 is operated continuously with a fixed processing rate so as to satisfy the production requirement. It is also assumed that there is a sufficient supply of raw material for batch stage 1 and that the cycle time of each subprocess can only take discrete values which are multiples of 2 hours. The performance index is given by

$$P.I. = \sum_{i=1}^4 N_i \cdot a_i \cdot E_i^{0.7} + \sum_{j=1}^3 0.6 \cdot V_j^{0.7} \quad (4-22)$$

where

$$a_1 = a_4 = 1.5, \quad a_2 = 1.0, \quad a_3 = 2.0 .$$

Other data are given as follows:

The minimal cycle time of a batch item in each stage is given by

$$w_i(S_i) = y_i + 2.0 \cdot S_i \quad (i = 1, 2, 3, 4)$$

where

$$y_1 = 50, \quad y_2 = 20, \quad y_3 = 30, \quad y_4 = 8 \quad [\text{hr}] .$$

The capacity of each pump and production requirement take the following values:

$$U_1^f = U_1^d = 30, \quad U_2^f = U_2^d = U_3^f = 20, \quad U_3^d = P = 0.1 \quad [\text{m}^3/\text{hr}]$$

The initial hold-up in each storage tank is assumed to be

zero, i.e.,

$$v_j^0 = 0 \quad [m^3] \quad (j = 1, 2, 3)$$

The upper limit of the variations in batch sizes and the upper and lower bounds of the total sum of the variations are given by the following equations:

$$\Delta S_i^M = 0.05 S_i \quad ; \quad \Delta s_j^U = 0.2 \tilde{S}_j \quad ; \quad \Delta s_j^L = -0.2 \tilde{S}_j \quad ;$$

$$\Delta S_k^U = 0.2 \tilde{S}_{k+1} \quad ; \quad \Delta S_k^L = -0.2 \tilde{S}_{k+1} \quad ;$$

$$\Delta t_j^U = \Delta T_k^U = 4 \text{ [hr]} \quad ; \quad \Delta t_j^L = \Delta T_k^L = -2 \text{ [hr]} \quad ;$$

$$(i = 1, 2, 3, 4 \ ; \ j = 1, 2, 3 \ ; \ k = 1, 2)$$

A continuous unit can be regarded as a batch unit the cycle time of which is equal to the time required for filling the batch unit. Therefore, the capacity of the storage tank between batch stage 4 and the continuous section can be calculated by using Eq.(4-18).

By calculating the performance index given by Eq.(4-22) for all of the combinations of the cycle time values of subprocess 1, 2 and 3, and by selecting the minimum value among them, the optimal solution can be obtained. However, this approach is inefficient and time-consuming, especially in high dimensional problems. In such a case, dynamic programming can play an essential role in reducing searching numbers of the cycle time values for finding the optimal solution.

In order to formulate the problem, the following functions

are first introduced:

$$f_1(\tilde{W}_1, \tilde{W}_2) := \sum_{i=1}^3 N_i \cdot a_i \cdot E_i^{0.7} + 0.6 \cdot V_1^{0.7}$$

$$f_2(\tilde{W}_2, \tilde{W}_3) := \sum_{i=4}^5 N_i \cdot a_i \cdot E_i^{0.7} + \sum_{j=2}^3 V_j^{0.7}$$

For each of the values of the cycle time of subprocess 2, the cycle time of subprocess 1 is determined so as to minimize $f_1(\tilde{W}_1, \tilde{W}_2)$. Then,

$$\min_{\{\tilde{W}_1\}} \{f_1(\tilde{W}_1, \tilde{W}_2)\}$$

becomes a function of the cycle time of subprocess 2. Next, for each value of the cycle time of subprocess 3, the optimal cycle time of subprocess 2 is searched so as to minimize

$$f_2(\tilde{W}_2, \tilde{W}_3) + \min_{\{\tilde{W}_1\}} \{f_1(\tilde{W}_1, \tilde{W}_2)\}.$$

By calculating the value of

$$\min_{\{\tilde{W}_2\}} \{f_2(\tilde{W}_2, \tilde{W}_3) + \min_{\{\tilde{W}_1\}} \{f_1(\tilde{W}_1, \tilde{W}_2)\}\}$$

for all of the candidates of the cycle time values in subprocess 3, and by selecting the minimum value among them, the optimal solution can be obtained. That is, the minimum value of the performance index given by Eq.(4-22) can be obtained by calculating the following formula:

$$\min_{\{\tilde{W}_3\}} \{ \min_{\{\tilde{W}_2\}} \{ f_2(\tilde{W}_2, \tilde{W}_3) + \min_{\{\tilde{W}_1\}} \{ f_1(\tilde{W}_1, \tilde{W}_2) \} \} \} \quad (4-23)$$

In this problem, the performance index takes the minimum value for $\tilde{W}_1 = 28$ [hr], $\tilde{W}_2 = 18$ [hr] and $\tilde{W}_3 = 10$ [hr].

Then, the batch sizes of subprocesses and the value of the performance index are calculated as follows:

$$\tilde{S}_1 = 2.8 \text{ [m}^3\text{]}, \tilde{S}_2 = 1.8 \text{ [m}^3\text{]}, \tilde{S}_3 = 1.0 \text{ [m}^3\text{]},$$

$$\text{P.I.} = 21.49 .$$

The optimal values of other design variables are shown in Figure 4-5.

In order to show how the solution changes depending upon whether variations are taken into account or not, the optimal solution for the case where no variations are assumed to occur in the process is also shown in Figure 4-6.

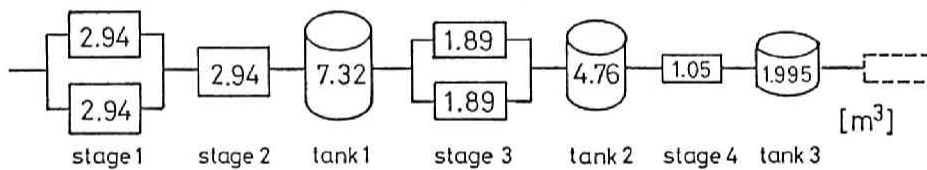


Figure 4-5. Equipment sizes of batch units and volumes of tanks

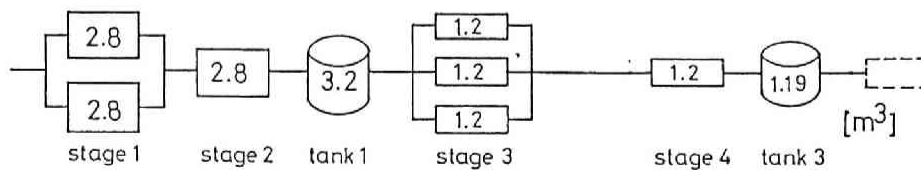


Figure 4-6. Result without any consideration on variations

The optimal solution for this case can be easily obtained by applying the exact same optimization procedure as that mentioned above, assuming that all of the upper and lower bounds on the variations are zero.

Needless to say, the value of the construction cost for the process shown in Figure 4-6, (P.I.=18.78), is smaller than that of the optimal process shown in Figure 4-5, (P.I.=21.49). However, the process shown in Figure 4-6 has not flexibility. On the other hand, the optimal process obtained by taking into account variations has a favorable flexibility, in other words, it can absorb the variations shown in the beginning of this section, though it requires slightly higher construction cost.

Now consider the case where the design variables of the process are already given. It is easily understood from Eq.(4-11) that whether or not the overflow or exhaustion of stored material in tank 1 will occur after the n -th variation is over depends upon the values of $t_2 - t_1 + \Sigma T^n$ and ΣS^n . Therefore, by graphically displaying the range of a pair of values of $t_2 - t_1 + \Sigma T^n$ and ΣS^n which satisfy Eq.(4-11), whether the variations that occurred in the process are allowable or not can easily be judged.

When the process is designed as shown in Figure 4-5, the allowable range of variations is given in Figure 4-7. In that figure, the abscissa shows the value of $t_2 - t_1 + \Sigma T^n$, and the ordinate shows the value of ΣS^n . The rectangle described in broken lines shows the preassigned range of

variations. The black circle $\bullet^i$ in Figure 4-7 corresponds to the i -th variation given by the following:

$$(\Delta t_1^1, \Delta T_1^1, \Delta s_1^1, \Delta S_1^1) = (0, 0, 0.2\text{m}^3, 0)$$

$$(\Delta t_1^2, \Delta T_1^2, \Delta s_1^2, \Delta S_1^2) = (0, 3\text{hr}, 0, 0)$$

$$(\Delta t_1^3, \Delta T_1^3, \Delta s_1^3, \Delta S_1^3) = (0, 0, 0, -0.2\text{m}^3)$$

For the above variations, Figure 4-7 shows that tank 1 does not overflow nor run out of stored material even if these variations occur.

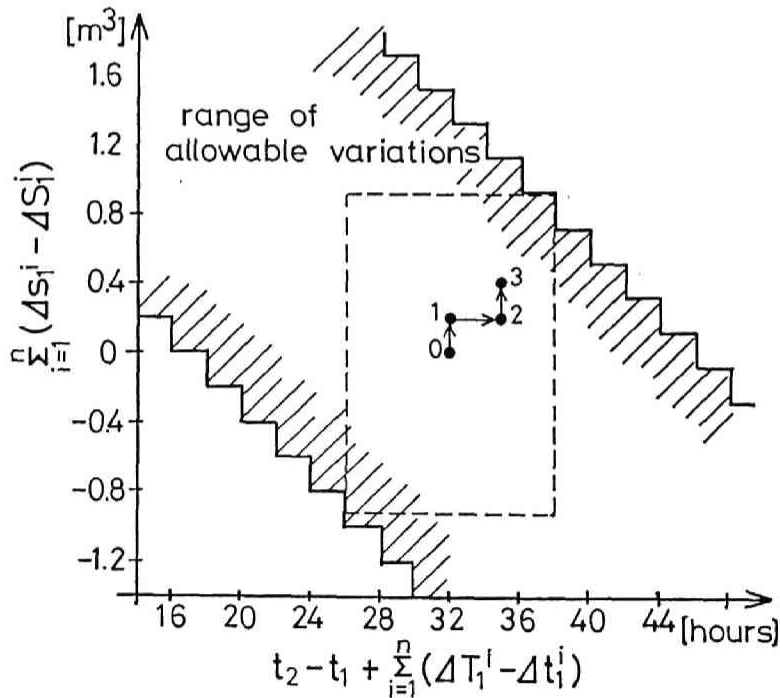


Figure 4-7. Range of allowable variations

7. Conclusion

The problem of the design of a flexible batch process which consists of several subprocesses composed of a certain number of batch units and intermediate storage tanks has been dealt with. In order to consider the design problem of a flexible batch process, it is first necessary to clarify the following two points. One is the way of defining "flexibility" itself and the kind of measure to be used to evaluate the magnitude of the flexibility. The other is determining what performance index has to be used in order to express the two kinds of objectives which are mutually contradictory: Minimization of the construction cost and maximization of the flexibility of the process.

As uncertain variations in the batch process, we considered the variations related to the operation schedule of a subprocess and the variations of its batch size. The "allowable variation" in a subprocess is defined as the one which does not cause the overflow nor the running out of the stored material in the tank, even though the operation schedules and the batch sizes of the other subprocesses are not readjusted.

The larger the volume of the storage tank installed between subprocesses becomes, the larger the range of the allowable variation becomes. In this study, we derived the quantitative relationship between the size of variations in the operation schedule and in the batch sizes, and the

necessary volume of the intermediate storage tank so as to make those variations allowable ones (Theorem 4-2).

By using this theorem, the volume of the tank which used to be estimated only through empirical knowledge could be determined more rationally. Presuming that the range of the allowable variations is given a priori, we mathematically formulated the problem of the design of a batch process which makes not only every variation within the given range "allowable" but also minimizes the construction cost. We then proposed an effective solution method.

In order to search for the optimal solution of this design problem, we cannot help but depend upon direct searching with respect to the free variables the dimension of which is equal to the number of subprocesses. When the cycle time or the batch size of each subprocess takes only discrete values, the optimal solution of the problem can be easily obtained by applying a Dynamic Programming Approach.

Here, we took up a batch process consisting of a certain number of batch stages and storage tanks, and derived several interesting results. The results obtained can be applicable to a more general process which consists of batch stages, continuous sections and storage tanks, as demonstrated by using an example.

An approach to the problem of the design of a flexible batch process was proposed. The definition of flexibility introduced in this chapter only represents an aspect of the manifold attributes with which the so-called "flexible

process" has to be essentially endowed. In this study, it is assumed that only "hard" countermeasures are available in order to increase the flexibility of the process. In other words, the problem of how to rationally estimate the design margin of the design variables is considered so as to absorb every unfavorable effect due to various uncertain variations.

Needless to say, "soft" countermeasures such as readjusting the operation schedule or installing sophisticated control systems are also very effective in enhancing the flexibility of the process. From now on, much effort has to be devoted to developing effective design methods which enable the design of flexible processes which are fully supported by both countermeasures.

Nomenclature

B = number of batch stages

$b = P/\min(U^f, U^d)$

c_i = processing capacity of the batch item in batch stage i

E_i = equipment size of a batch item in batch stage i

G_j = a set of suffixes representing the order of batch stage
in subprocess j

K = number of subprocesses

N_i = number of parallel batch items in batch stage i

P = production requirement per unit time

p_i = cost function of a batch equipment in batch stage i

q_j = cost function of intermediate storage tank j

S_i = batch size of batch stage i

$\tilde{S}_i$ = batch size of subprocess i

$\Delta S_j^L, \Delta S_j^U, (\Delta S_j^L, \Delta S_j^U)$ = lower and upper bounds of the sums of
amounts of variations in the batch size of a batch
discharged from the tank (tank j)

$\Delta s_j^L, \Delta s_j^U, (\Delta s_j^L, \Delta s_j^U)$ = lower and upper bounds of the sums of
amounts of variations in the batch size of a batch
which is flowed into the tank (tank j)

ΔS_i^M = maximum value of the variation of the batch size of
batch stage i

$\Delta s^i, \Delta S^i$ = amounts of variations in the batch sizes of a
batch which is flowed into and discharged from the tank
for the i -th variation

t_1, t_2 = starting times of the inflow to and the discharge
from the tank in the first cycle

t_a = time when the first variation takes place

$\Delta T_j^L, \Delta T_j^U, (\Delta t_j^L, \Delta t_j^U)$ = lower and upper bounds of the sums of amounts of variations in the starting moment of the discharge from the tank (tank j)

$\Delta t_j^L, \Delta t_j^U, (\Delta t_j^L, \Delta t_j^U)$ = lower and upper bounds of the sums of amounts of variations in the starting moment of the inflow to the tank (tank j)

$\Delta t^i, \Delta T^i$ = amounts of variations in the starting moment of the inflow to and the discharge from the tank for the i-th variation

$U_j^f, (U_j^f)$ = capacity of the feed pump of the tank (tank j) per unit time

$U_j^d, (U_j^d)$ = capacity of the discharge pump of the tank (tank j) per unit time

u^f = input function to the tank

u^d = output function from the tank

$V, (V_j)$ = volume of the intermediate storage tank (tank j)

$V^0, (V_j^0)$ = initial hold-up in the tank (tank j)

v = hold-up in the tank

W_j = cycle time of subprocess j

w_i = minimum cycle time of the batch item in batch stage i

Other symbols

G.C.M. (X,Y) = extended greatest common measure of X and Y

trunc(X) = largest integer $\leq X$

mod(X,Y) = $X - \text{trunc}(X/Y) \cdot Y$

$\| X \|$ = minimum integer $\geq X$

Literature Cited

1. Oi, K., Itoh, H. & Muchi, I; "Improvement of Operational Flexibility of Batch Units by a Design Margin", Computers Chem. Eng., vol. 3, pp. 177-184, (1979).

Chapter 5

OPTIMAL SCHEDULING OF A CYCLICALLY OPERATED BATCH PROCESS WITH UTILITY CONSTRAINTS

1. Introduction

In the previous chapters, the design problems of batch processes are discussed. However, in a batch process, how to schedule the operation of each unit becomes a large problem even though the size of each batch unit is already determined. In this chapter, a cyclically operated batch process is taken up. And the problem of determining the starting moments of batch units so as to smooth the peak consumption of utilities is discussed.

In a batch process, many batch processing units are simultaneously operated. Each piece of batch equipment needs different amounts of utilities, such as manpower, electricity, steam and water etc. for its operation. The amount of these utilities necessary differ, according not only to the kind of batch item but also to the different step in the operation such as charging, processing or discharging. Thus, the total amount of each utility changes with time in a fairly large scale as shown in Figure 5-1.

In order to assure the stable operation of the process, the capacity of each utility supply system must be large enough to be able to meet its peak demand. If it is possible to smooth the necessary amount of a utility over time by properly scheduling the operations of the process, the capacity of a utility supply system can be reduced without impairing the over-all productivity of the process. Therefore, the way of scheduling the starting moment of each

A utility consumption

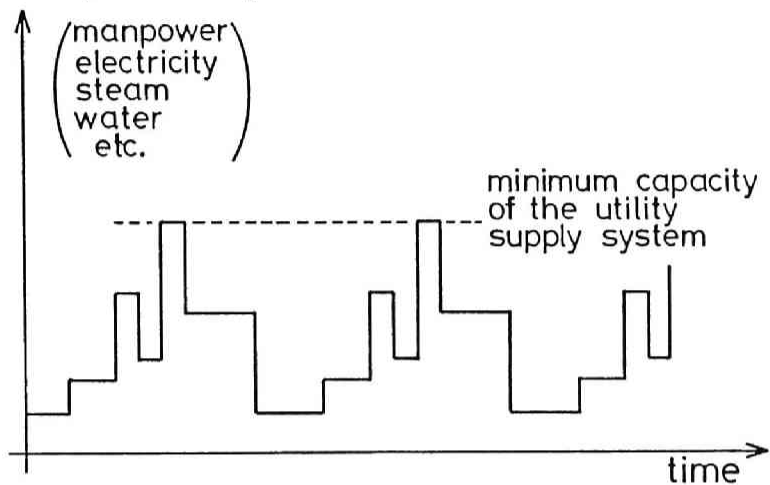


Figure 5-1. Profile of the utility consumption

batch item so as to smooth the peak consumption of utility is of great importance.

In most of batch processes, products are produced by cyclically operating the process as mentioned in Chapter 1. So, the methods developed for solving the project scheduling are not applicable to solve the scheduling problem of a batch process. Because most of the solution procedures in the project scheduling problem use the arrow diagram to indicate the precedence order of processing steps. If the precedence order of processing steps for a cyclically operated

batch process is expressed by using an arrow diagram, we must distinguish between the first batch of a processing step and the second batch of the same processing step. In this case the arrow diagram becomes a very complicated one.

Furthermore, as fluid materials are mainly handled in a batch process, whether a storage tank is installed between two batch stages or not strongly affects the operation scheduling of the process.

In this chapter, by taking into account the characteristics of batch processes mentioned above, an effective algorithm is derived to solve the utility smoothing problem of a cyclically operated batch process, that is, the problem of scheduling the process so as to smooth the peak consumption of utilities.

2. Formulation of the Utility Smoothing Problem

2-1. Operation module and operation train

We here introduce the following assumptions prior to discussing how to solve the utility smoothing problem.

The process is cyclically operated with a fixed cycle time for a long period. It is also assumed that the time necessary for each operation, such as charging, processing and discharging etc., is known, and the consumption rate of a utility required for each operation is piecewise constant.

By introducing above assumptions, a series of operations necessary for producing a specified final product can be redivided into a new series of segments each of which corresponds to an operation step or a part of consecutive operation steps during which the consumption rate of each utility is constant. Each segment during which the consumption rate of each utility is constant is hereafter called an "operation module".

Some operation module must be started immediately after the precedent operation module is completed. A series of operation modules which must be successively operated without any waiting time is hereafter called an "operation train".

Figure 5-2 shows an example of a cyclically operated batch process. For the process shown in Figure 5-2, the raw material is heated and fed into reactor 1. The intermediate product processed in reactor 1 is filtrated and once held in a tank, and then fed into reactor 2. After the processing

is finished in reactor 2, the product is discharged to a storage tank. In this process, the final product is produced by repeating above operation steps cyclically. Figure 5-3 shows an example of the operation schedule for the process.

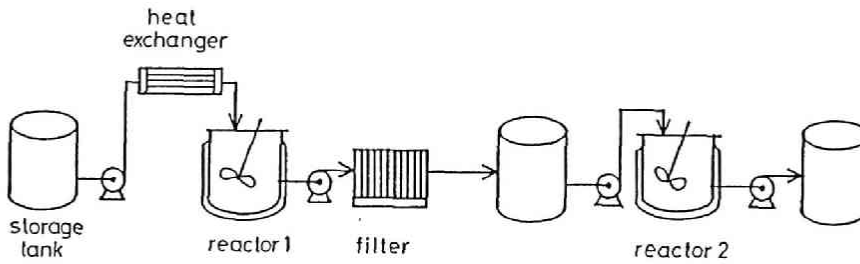


Figure 5-2. Cyclically operated batch process

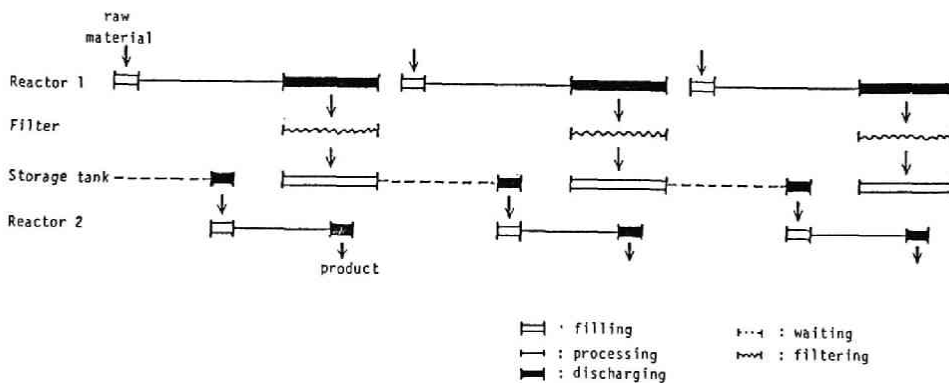
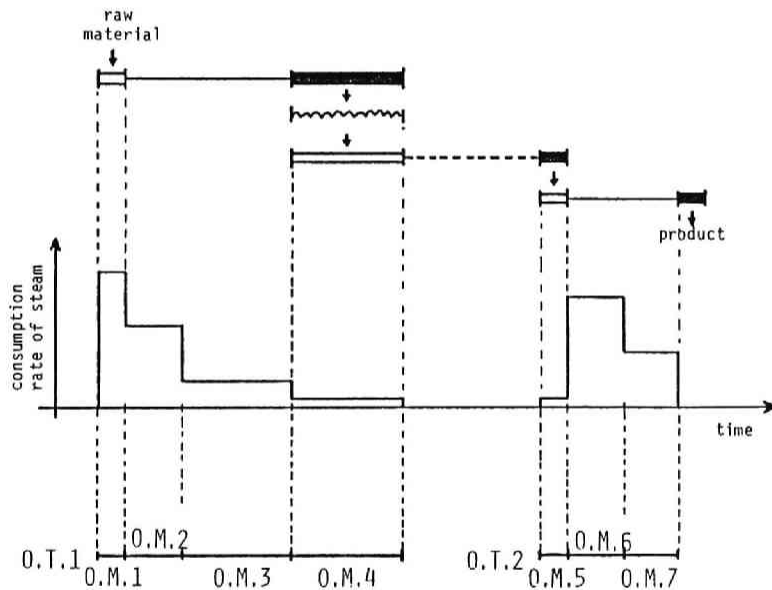


Figure 5-3. Operation schedule for a cyclically operated batch process

For this process, an example of the change of steam consumption to produce a batch of product is shown in Figure 5-4. If only the consumption of the steam must be smoothed, a series of operations of the process can be divided into seven operation modules as shown in Figure 5-4. Where, O.M.i and O.T.j mean the i-th operation module and the j-th operation train, respectively. In this case, a sequence of four operation modules from O.M.1 to O.M.4 makes up an operation train, O.T.1.



Operation schedule and the profile of the
Figure 5-4. steam consumption for a batch of product

2-2. Formulation of the utility smoothing problem

By introducing the words "operation module" and "operation train", the data which are required to formulate the utility smoothing problem can be summerized as follows:

- 1) names of operation modules which belong to each operation train and their precedence order,
- 2) the duration of each operation module and the consumption rate of each utility for its operation module,
- 3) the cycle time of the process and the performance index given by a function of the maximum value of each utility consumption.

The starting moment of the first operation module in O.T.j is hereafter called the starting moment of O.T.j. If the starting moment of O.T.i is once decided, the starting moments of all of the operation modules in O.T.i are automatically determined, because all of the operation modules in an operation train have to be successively operated without any waiting time between every two consecutive operation modules. Once the starting moments of all of the operation trains are decided, the profile of a utility consumption over time and its maximum value are uniquely determined.

Therefore, the utility smoothing problem is stated as follows:

"Find the starting moments of all of the operation trains which minimize the performance index given by a

function of the maximum value of each utility consumption".

2-3. Mathematical formulation

The operation of the process is assumed to be divided into L operation trains each of which consists of N_i operation modules. Let $M_{i,j}$ be the j -th operation module in O.T.i. For $M_{i,j}$, the function which expresses the consumption rate of utility k over time is defined as follows:

$$f_{i,j,k}(t) = \begin{cases} U_{i,j,k} & ; \quad nW \leq t < nW + P_{i,j} \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (5-1)$$

$$n : \text{integer} \quad ; \quad i = 1, 2, \dots, L ;$$

$$j = 1, 2, \dots, N_i ; \quad k = 1, 2, \dots, K ;$$

where

$U_{i,j,k}$ = the consumption rate of utility k per unit time
which is required to execute $M_{i,j}$,

$P_{i,j}$ = the processing time of $M_{i,j}$,

W = the cycle time of the process,

K = the number of the different kinds of utilities.

The starting moments of all of the operation modules in O.T.i is calculated by determining the starting moment of O.T.i, i.e.

$$s_{i,1} = t_i$$

$$s_{i,j+1} = s_{i,j} + P_{i,j} \quad (j = 1, 2, \dots, N_i-1) \quad (5-2)$$

where

$s_{i,j}$ = the starting moment of $M_{i,j}$,

t_i = the starting moment of O.T.i.

Then, the consumption rate of utility k which is required to operate O.T.i becomes a function only of the starting moment of O.T.i, and expressed as follows:

$$F_{i,k}(t - t_i) = \sum_{j=1}^{N_i} f_{i,j,k}(t - s_{i,j}) \quad (5-3)$$

By using above equation, the maximum value of the consumption of utility k , G_k , is given by the following equation:

$$G_k(\mathbf{t}) = \max_{\{\mathbf{t}\}} \sum_{i=1}^L F_{i,k}(t - t_i) \quad (5-4)$$

where

$\mathbf{t} = (t_1, t_2, \dots, t_L)$: the vector consisting of the starting moments of all of the operation trains.

Hereafter, $\mathbf{t}$ is called "an operation schedule of the process". Then, the utility smoothing problem is mathematically formulated as follows:

"Find the operation schedule $\mathbf{t}$ which minimizes the following performance index, subject to Eqs.(5-1) to (5-4);

$$P.I. = g(G_1(\mathbf{t}), G_2(\mathbf{t}), \dots, G_K(\mathbf{t})) \quad (5-5)$$

where

g = the monotonically non-decreasing function.

3. Solution Procedure

3-1. Procedure to derive the exact optimal solution

The L-dimensional function $G_k(t)$ in Eq.(5-5) takes a piecewise constant value on each t_i -axis ($i = 1, 2, \dots, L$). Thus, any nonlinear programming technique which utilizes the derivative of a function cannot be applied to solve the problem formulated in the previous section.

A direct search method, therefore, has to be utilized for obtaining the optimal solution. In such a case, the problem is how to clarify the searching points for which direct searching has to be performed, and at the same time, how to reduce the number of such searching points as much as possible.

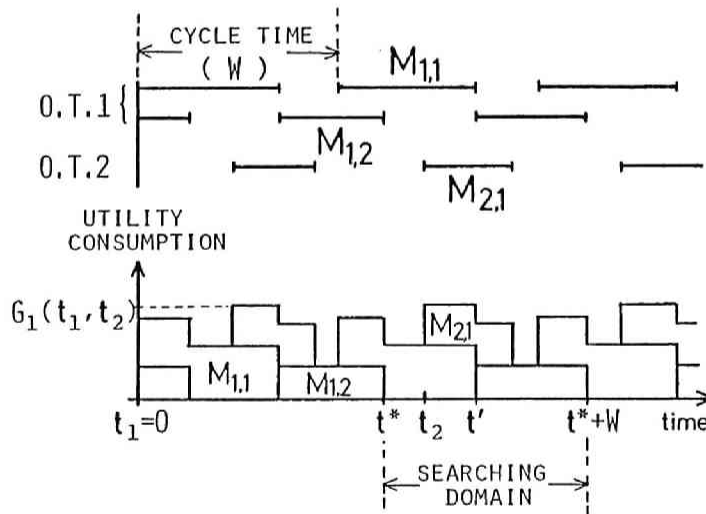
In order to clear the basic idea for the further discussion, we use a simple example where the operation of the process consists of only two operation trains and, moreover, only one kind of utility is necessitated. Figure 5-5 shows an operation schedule of this process and the profile of a utility consumption corresponding to the schedule.

Without losing generalities, the origin of the time axis can be taken at the starting time of O.T.1, that is, $t_1 = 0$. The problem is one of finding the optimal starting moment of O.T.2, t_2 , so as to minimize the peak consumption of the utility, $G_1(t) = G_1(0, t_2)$.

As O.T.2 is cyclically operated, it is sufficient to perform the searching with respect to t_2 in the interval

$[t^*, t^*+W)$, where t^* is the ending moment of $M_{1,2}$ and W is the cycle time of the process.

For the operation schedule shown in Figure 5-5, the peak consumption of the utility, $G_1(t)$, does not increase even if the starting moment of O.T.2, t_2 , is advanced up to the ending moment of $M_{1,2}$, t^* . That is, the relationship such that $G_1(0, t_2) \geq G_1(0, t^*)$ holds for any $t_2 \in [t^*, t')$. By repeating the discussion similarly, it is easily understood that the relationship such that $G_1(0, t_2) \geq G_1(0, t')$ also holds for any $t_2 \in [t', t^*+W)$.



Schedule of the operation consisting
Figure 5-5. of two operation trains

Consequently, the searching domain for t_2 , $[t^*, t^*+W)$, can be divided into two parts such as $[t^*, t')$ and $[t', t^*+W)$. The minimum values of the peak consumption of the utility for these two subdomains can be given by $G_1(0, t^*)$ and $G_1(0, t')$, respectively.

Therefore, t^* and t' can be considered as the representatives of these two subdomains. In other words, t^* and t' represent all of the schedules of O.T.2 which exist in the interval $[t^*, t')$ and $[t', t^*+W)$, respectively. Then, the optimal solution can be obtained by comparing the value of the performance index for these two points, t^* and t' .

In a general utility smoothing problem, the searching domain is given by a $(L-1)$ -dimensional cube, each edge of which has a length of W , since all of the operation trains are cyclically operated with the cycle time W . If this original searching domain can be divided into many subdomains each of which has a representative t^* such that the inequalities $G_k(t) \geq G_k(t^*)$, $k=1, 2, \dots, K$, hold for any t in that subdomain, it is sufficient to perform the searching with respect to these representatives in order to find the optimal solution. Therefore, the next problem is how to generate a set of such representatives.

In the example shown in Figure 5-5, each representative shows an operation schedule such that the ending moment of an operation module in O.T.1 coincides with the starting moment of the operation module in O.T.2. This relationship is extended to a general case. That is, if the ending

moment of a certain operation module in O.T.i coincides with the starting moment of some operation module in O.T.j, we can say that there is a "relationship from O.T.i to O.T.j" and express this relationship by an arrow such that $O.T.i \rightarrow O.T.j$.

We can define T_0 as a set of the operation schedules such that there is an arrow progression from O.T.1 to each O.T.j, ($j = 2, 3, \dots, L$). Then, this T_0 gives a set of the representatives discussed above. Consequently, the following theorem can be derived.

[Theorem 5-1]

Let T_0 be a set of operation schedules such that there is an arrow progression from O.T.1 to every other operation trains. Then, the optimal schedule which minimizes the performance index given by a function of the maximum value of each utility consumption always exists in T_0 .

[Proof of Theorem 5-1]

Let $\mathfrak{t}^* = (0, t_2^*, \dots, t_L^*)$ be an arbitrary operation schedule.

We first define the following sets:

$Q(\mathfrak{t}^*)$ is a set of suffixes representing the order of operation trains each of which does not have an arrow progression from O.T.i. This set is a function of the operation schedule $\mathfrak{t}^*$.

$C(\mathfrak{t}^*)$ is a set of operation modules each of which belongs to one of the operation trains whose suffixes are elements of $Q(\mathfrak{t}^*)$, and $\bar{C}(\mathfrak{t}^*)$ is the complement of $C(\mathfrak{t}^*)$; i.e.,

$$C(\mathfrak{t}^*) = \{M_{i,j} \mid i \in Q(\mathfrak{t}^*), \quad j \in \underline{N_i}\}$$

$$\bar{C}(\mathfrak{t}^*) = \{M_{i,j} \mid i \notin Q(\mathfrak{t}^*), \quad j \in \underline{N_i}\}$$

where

$$\underline{X} := \{1, 2, \dots, X\} : \text{a set of natural numbers } \leq X.$$

$E(\mathfrak{t}^*)$ is a set of ending moments of operation modules which belong to $\bar{C}(\mathfrak{t}^*)$; i.e.,

$$E(\mathfrak{t}^*) = \{e \mid e = \text{mod}(t_i^* + \sum_{j=1}^m P_{i,j}, W), \quad M_{i,m} \in \bar{C}(\mathfrak{t}^*)\}$$

where

$$\text{mod}(X, Y) := X - Y \cdot \text{trunc}(X/Y),$$

$$\text{trunc}(X) := \text{the maximum integer } \leq X.$$

$S(\mathfrak{t}^*)$ is a set of starting moments of operation modules which belong to $C(\mathfrak{t}^*)$; i.e.,

$$S(\mathfrak{t}^*) = \{s \mid s = \text{mod}(t_i^* + \sum_{j=0}^{m-1} P_{i,j}, W), \quad M_{i,m} \in C(\mathfrak{t}^*), \quad P_{i,0} = 0\}$$

$D(\mathfrak{t}^*)$ is a set of the difference between an ending moment of an operation module in $\bar{C}(\mathfrak{t}^*)$ and a starting moment of an operation module in $C(\mathfrak{t}^*)$; i.e.,

$$D(\mathfrak{t}^*) = \{\Delta t \mid \Delta t = \text{mod}(e - s, W), \quad e \in E(\mathfrak{t}^*), \quad s \in S(\mathfrak{t}^*)\}$$

Δt^* and $\mathfrak{t}'$ are defined as follows:

$$\Delta t^* = \min_{\Delta t \in D(\mathfrak{t}^*)} \{\Delta t\}$$

$$\mathfrak{t}' = (0, t_2', t_3', \dots, t_L')$$

where

$$t'_i = \begin{cases} t_i^* & ; i \notin Q(t^*) \\ t_i^* - \Delta t^* & ; i \in Q(t^*) \end{cases}$$

We first prove that the following inequality is satisfied for any $k \in \underline{K}$.

$$G_k(t^*) \geq G_k(t')$$

Let e_m and e_n be elements which belong to $E(t^*)$ such that in the interval (e_m, e_n) , there exist no elements which belong to $E(t^*)$. $\sum_{i \in Q(t^*)} F_{i,k}(t - t_i^*)$ is monotonically non-increasing in the interval $[e_n, e_n + \Delta t^*)$. $\sum_{i \in Q(t^*)} F_{i,k}(t - t_i^*)$ is monotonically non-decreasing in the interval (e_m, e_n) . Therefore, there exists some $\delta > 0$ which satisfies the following two equations.

$$\max_{e_m \leq t < e_n} \left\{ \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^*) \right\} = \sum_{i \in Q(t^*)} F_{i,k}(e_n - \delta - t_i^*)$$

$$\max_{e_n \leq t < e_n + \Delta t^*} \left\{ \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^*) \right\} = \sum_{i \in Q(t^*)} F_{i,k}(e_n - \delta - t_i^*)$$

If $e_n - e_m \leq \Delta t^*$, the following inequality is derived from above two equations.

$$\begin{aligned} & \max_{e_m \leq t < e_n} \left\{ \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^*) + \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^* + \Delta t^*) \right\} \\ & \leq \sum_{i \in \underline{L}} F_{i,k}(e_n - \delta - t_i^*) \end{aligned} \quad (5-6)$$

If $e_n - e_m > \Delta t^*$, we can also derive the following inequalities:

$$\begin{aligned}
& \max_{e_n - \Delta t^* \leq t < e_n} \left\{ \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^*) + \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^* + \Delta t^*) \right\} \\
& \leq \sum_{i \in \underline{L}} F_{i,k}(e_n - \delta - t_i^*) \quad (5-7)
\end{aligned}$$

$$\begin{aligned}
& \max_{e_m \leq t < e_n - \Delta t^*} \left\{ \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^*) + \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^* + \Delta t^*) \right\} \\
& \leq \max_{e_m \leq t < e_n - \Delta t^*} \left\{ \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^* + \Delta t^*) + \sum_{i \in Q(t^*)} F_{i,k}(t - t_i^* + \Delta t^*) \right\} \\
& \leq \max_{e_m \leq t < e_n} \left\{ \sum_{i \in \underline{L}} F_{i,k}(t - t_i^*) \right\} \quad (5-8)
\end{aligned}$$

Eqs. (5-6) to (5-8) show that the following inequality is satisfied for any $k \in \underline{K}$:

$$\max_{e_m \leq t < e_n} \left\{ \sum_{i \in \underline{L}} F_{i,k}(t - t_i^*) \right\} \geq \max_{e_m \leq t < e_n} \left\{ \sum_{i \in \underline{L}} F_{i,k}(t - t_i') \right\}$$

Here, e_m and e_n have been able to be chosen arbitrarily in so far as there exist no elements which belong to $E(t^*)$ in the interval (e_m, e_n) . Therefore, for other pair of elements in $E(t^*)$, similar results can be obtained. Consequently, the following relationship can be derived for any $k \in \underline{K}$.

$$\begin{aligned}
G_k(t^*) &= \max_{\{t\}} \left\{ \sum_{i \in \underline{L}} F_{i,k}(t - t_i^*) \right\} \\
&\geq \max_{\{t\}} \left\{ \sum_{i \in \underline{L}} F_{i,k}(t - t_i') \right\} = G_k(t')
\end{aligned}$$

When t' is chosen as an operation schedule, the ending moment of a certain operation module in $\bar{C}(t^*)$ coincides with the starting moment of some operation module in $C(t^*)$. That is, there exists

an arrow progression between O.T.1 and some operation train in $C(\#^*)$. This means that the number of elements which belong to $Q(\#')$ is less than the number of elements which belong to $Q(\#^*)$.

If $Q(\#') \neq \emptyset$, by repeating the procedure stated above, the number of operation trains each of which has an arrow progression from O.T.1 can be increased without increasing the maximum consumption rate of each utility. Consequently, for any $\#^*$ there exists some $\#_0 \in T_0$ which satisfies the following inequality:

$$G_k(\#^*) \geq G_k(\#_0) ; \quad k \in \underline{K} .$$

[Q.E.D. of Theorem 5-1]

We next show how to generate all of the elements in T_0 . In order to make the explanation easier, a simple example is used. In this example, the operation of the process can be divided into three operation trains. O.T.1 and O.T.3 consist of only one operation module, respectively, and O.T.2 consists of two operation modules. The duration of each operation module and the cycle time of the process are given as follows:

$$P_{1,1} = 6.0 , \quad P_{2,1} = 2.6 , \quad P_{2,2} = 4.0 , \quad P_{3,1} = 3.0 , \\ W = 10.0 .$$

All of the elements in T_0 can be generated by executing the following steps:

- 1) Rooted direct trees where a vertex and an arrow represent an operation train and the "relationship" defined above, respectively, are considered. First, all of the directed

trees rooted at vertex O.T.1 are generated. As shown in Figure 5-6, there are three kinds of directed trees rooted at vertex O.T.1 for this example. For each of the directed trees, the following steps are executed.

2) For each arrow such that $O.T.i \rightarrow O.T.j$, a Cartesian product of a set of operation modules which belong to O.T.i and that of operation modules which belong to O.T.j is generated:

$$O.T.i \times O.T.j := \{(M_{i,m}, M_{j,n}) \mid M_{i,m} \in O.T.i, M_{j,n} \in O.T.j\}$$

From directed graph A in Figure 5-6, the following two Cartesian products are generated.

$$O.T.1 \times O.T.2 = \{(M_{1,1}, M_{2,1}), (M_{1,1}, M_{2,2})\}$$

$$O.T.2 \times O.T.3 = \{(M_{2,1}, M_{3,1}), (M_{2,2}, M_{3,1})\}$$

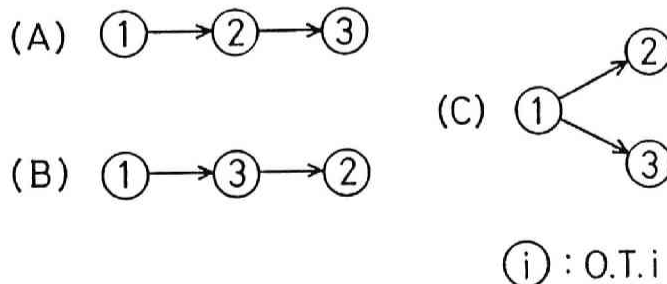


Figure 5-6. All of the rooted directed trees consisting of three vertices

3) Choose an ordered pair of operation modules from each Cartesian product. For a selected pair of operation modules, $(M_{i,m}, M_{j,n}) \in O.T.i \times O.T.j$, the starting moments of O.T.i and O.T.j, t_i and t_j , are determined such that the ending moment of $M_{i,m}$ coincides with the starting moment of $M_{j,n}$. As there is an arrow progression between O.T.1 and every other operation train, the starting moments of all of the operation trains are uniquely determined by choosing a pair of operation modules from each Cartesian product.

For example, $(M_{1,1}, M_{2,1})$ and $(M_{2,1}, M_{3,1})$ are selected from Cartesian products $O.T.1 \times O.T.2$ and $O.T.2 \times O.T.3$, respectively. Then, the following operation schedule which belongs to T_0 is derived.

$$t_1 = (0, 6.0, 8.6)$$

4) Repeat step 3 for every combinations of the pairs of operation modules. In this example, the following four operation schedules are derived from directed graph A.

$$t_1 = (0, 6.0, 8.6) \quad , \quad t_2 = (0, 6.0, 2.6) \quad ,$$

$$t_3 = (0, 3.4, 6.0) \quad , \quad t_4 = (0, 3.4, 0).$$

5) By executing step 2 to step 4 for every directed tree, all of the operation schedules which belong to T_0 can be generated. In this example, the following operation schedules are derived from directed graphs B and C in Figure 5-6.

$$t_5 = (0, 3.0, 6.0) \quad , \quad t_6 = (0, 0.4, 6.0) \quad ,$$

$$t_7 = (0, 6.0, 6.0).$$

So, the optimal solution can be obtained by searching for only 7 cases.

When the number of operation trains is relatively small, all of the elements in T_0 can easily be obtained by executing above five steps. And the optimal solution can be derived by calculating the maximum value of each utility consumption and the performance index for every element in T_0 , and by choosing the element which minimizes the performance index.

3-2. Algorithm to derive a suboptimal solution

The number of rooted directed trees with L vertexes is L^{L-2} [1]. Therefore, if the operation of the process consists of many operation trains, it would require an astronomical computing time to generate all of the elements in T_0 . So, it is very important to develop an effective algorithm by which a suboptimal solution can be obtained within a reasonable computing time.

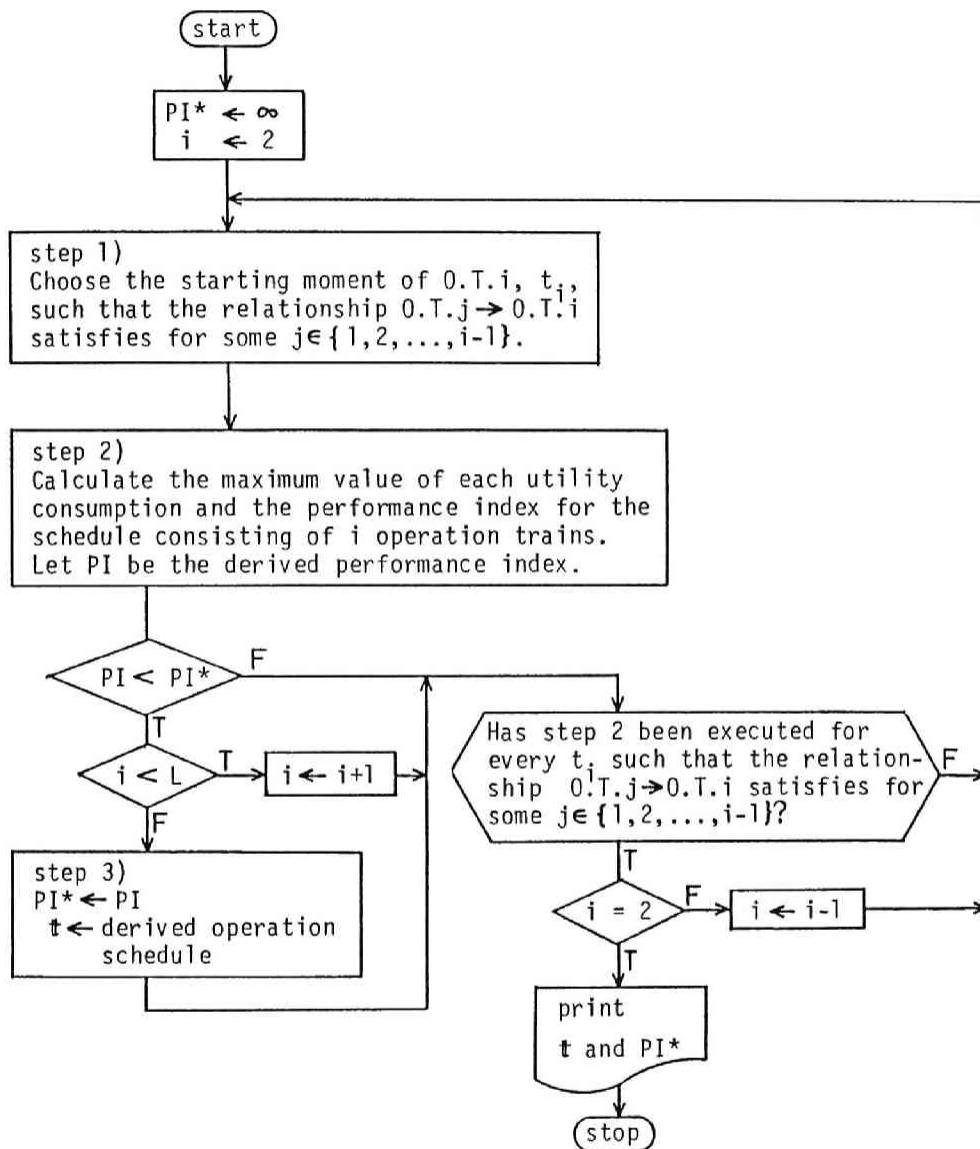
Burgess and Killebrew^[2] proposed a solution procedure for a utility smoothing problem of a cyclically operated process. In their procedure, one dimensional search on the starting moments of operation trains is repeated until the performance index does not decreased even if the starting moment of any operation train is perturbed. By applying this procedure, the solution can be obtained in a short computing time, though it is not ensured that the solution converges to the true optimum.

In solving many different cases of the problem, it has often been experienced that for many different elements in T_0 , in other words, for many different operation schedules, the performance index takes the same value. This fact suggests that the optimal solution can be probably found by only checking a part of T_0 .

By taking into account this fact, we here propose an algorithm in which the searching calculation is performed for the elements in T_0 as much as possible within a limited computing time. In the proposed algorithm, the searching calculation is performed only for the operation schedules such that the starting moment of an operation module in O.T.i+1, ($i=1, 2, \dots, L-1$), coincides with the ending moment of some operation module in one of the operation trains from O.T.1 to O.T.i. The flow chart of the proposed algorithm is shown in Figure 5-7.

In this algorithm, the operation schedule is arranged successively from the starting moment of O.T.2 to that of O.T.L. So, the Branch and Bound method is effectively utilized in order to speed up the searching procedure. Figure 5-8 shows all of the rooted directed trees consisting of four operation trains. In the algorithm proposed here, the searching calculation is performed only for the trees which are indicated by bold lines. The ratio of the number of points searched by using this algorithm to the total number of the elements in T_0 is given by $(L-1)!/L^{L-2}$.

In order to show the effectiveness of the proposed



Flow chart of the algorithm to
 Figure 5-7. derive a suboptimal solution

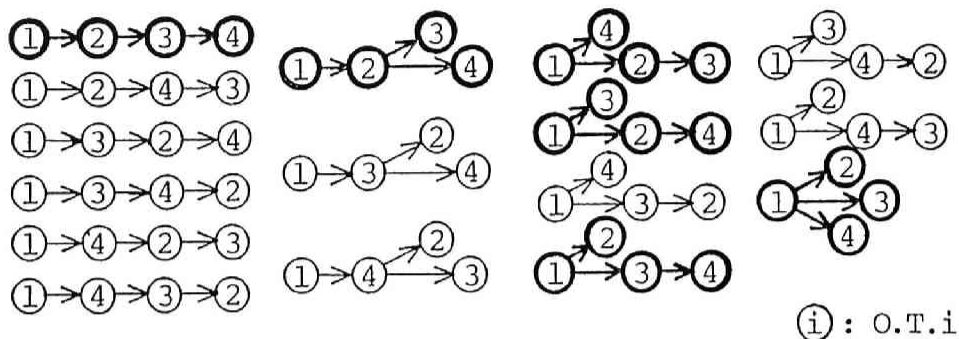


Figure 5-8. All of the rooted directed trees consisting of four vartices

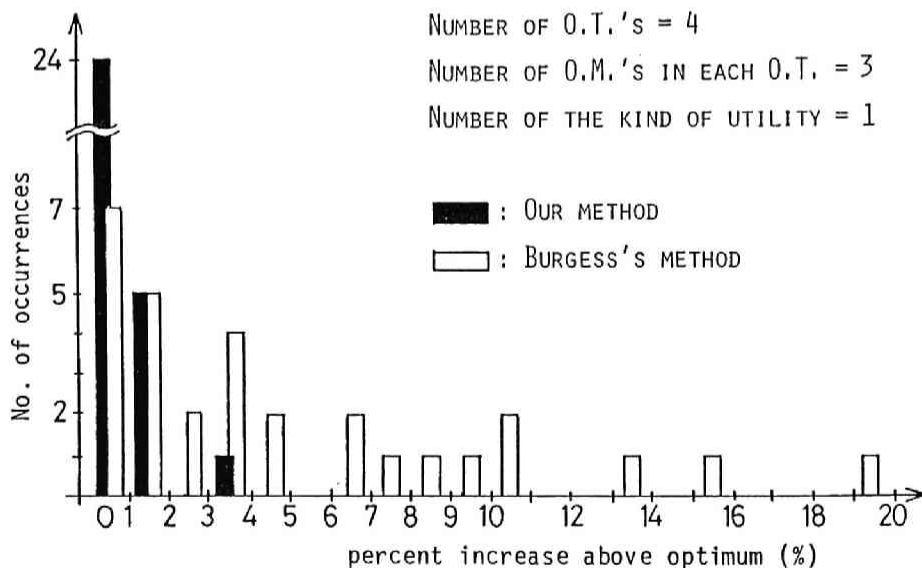


Figure 5-9. Frequency distribution of percent increase above optimum

procedure, we solved 30 numerical examples by applying both algorithms; one is the algorithm proposed here and the other is the algorithm by Burgess and Killebrew. For the comparison, the true optimal solutions are also calculated for all examples by applying the algorithm explained in the previous section. In these examples, it is commonly assumed that the operation of the process consists of four operation trains each of which contains three operation modules. The other conditions such as the necessary amount of the utility for each operation module and the duration of each operation module, etc. are given as shown in Table 5-1.

Table 5-1. Scheduling data for 30 examples

number of utilities	1
cycle time of the process	100
duration of each operation module	random variable which is rectangularly distributed on the range (10, 60)
consumption rate of the utility to execute each operation module	random variable which is rectangularly distributed on the range (1,31)
performance index	maximum value of the utility consumption

The result is shown in Figure 5-9. In this figure, the abscissa shows relative errors of the value of the performance index of the suboptimal solution from that of the true optimal solution. In 24 out of these 30 examples, the suboptimal solutions obtained by applying the algorithm proposed here coincided with the true optimal solutions. And in 7 out of these 30 examples, the suboptimal solutions obtained by applying the Burgess's algorithm coincided with the optimal solutions. This result shows the effectiveness of the algorithm proposed here.

The suboptimal solutions for these examples were obtained by the algorithm proposed here in $1/6$ of the computing time spent obtaining the optimal solution, though the computing time to derive the suboptimal solutions by the algorithm proposed here was 10 times as long as that by the Burgess's method.

4. Extention of the Problem to More General Cases

4-1. Restriction on the starting moment of an operation train

In previous sections, it has been assumed that the starting moment of each operation train can be chosen arbitrarily. However, in an actual scheduling problem, there are many cases where the starting moment of an operation train is restricted within a specified range.

For example, for the process shown in Figure 5-2, it is assumed that only a batch of material can be stored in the intermediate storage tank. Then, the intermediate storage tank must be emptied before the inflow of the filtrated material of the next batch to the tank is started. That is, the starting moment of the inflow to reactor 2, in other words, the starting moment of O.T.2, t_2 , must satisfy the following inequality:

$$t_1 + \sum_{i=1}^4 P_{1,i} \leq t_2 \leq t_1 + \sum_{i=1}^3 P_{1,i} + W - P_{2,1} \quad (5-9)$$

where

$P_{i,j}$ = the processing time of $M_{i,j}$,

W = the cycle time of the process.

In the algorithms proposed in the previous sections, the representatives of the operation schedules for which direct searching must be performed are first enumerated, and the performance index is calculated for each of these representatives. So, it is easy to judge whether or not each representative satisfies the restriction on the starting

moment of an operation train such as Eq.(5-9).

Therefore, the proposed procedures can easily be applied to solve the scheduling problem which has the restrictions on the starting moments of some operation trains. Furthermore, by adding the constraints to the scheduling problem, the number of representatives for which searching must be performed may be reduced.

There exists a representative $t^* \in T_0$ for any operation schedule t which satisfies the restriction such that

$t_i' \leq t_i \leq t_i''$, if the inequality

$$\{ t \mid t \in T_0, t_i = t_i' \} \neq \emptyset$$

is satisfied.

Thus, the following theorem can be derived.

[Theorem 5-2]

It is assumed that the starting moment of each O.T.i, t_i , has a constraint such that $t_i' \leq t_i \leq t_i''$. And it is also assumed that the following condition is satisfied:

$$\{ t \mid t \in T_0, t_i = t_i', i = 1, 2, \dots, L \} \neq \emptyset.$$

Then, the optimal schedule which minimizes the performance index given by a function of the maximum value of each utility consumption always exists in T_0' defined by the following equation:

$$T_0' = \{ t \mid t \in T_0, t_i' \leq t_i \leq t_i'', i = 1, 2, \dots, L \} \quad (5-10)$$

4-2. Restriction on the capacity of a utility supply system

There are many cases where the capacity of a utility supply system has been determined before the scheduling problem is solved. If the capacity of a utility supply system is relatively small, the utility smoothing problem must be solved under the restriction that the maximum consumption rate of the utility does not exceed the capacity. As the enumeration methods are used in the proposed algorithms, it is easy to solve the utility smoothing problem with the restriction on the maximum value of each utility consumption. And we can easily prove that even though such restrictions exist, the optimal solution of the utility smoothing problem always exists in T_0 which is defined at Theorem 5-1.

When a batch unit can be multiply used at many processing steps, we must schedule the operation such that the batch unit is not used at different two processing steps simultaneously. In this case, it is convenient to regard the batch unit as a kind of utility. By solving the problem on condition that the maximum value of the consumption rate of this utility is restricted, we can avoid such overlap in the usage of a unit.

The manpower can also be considered as a kind of utility. There are not a few cases where the upper bound on the consumption rate of this utility changes with time. For example, for the case in which the cycle time of the process is 24 hours, the upper bound on the consumption rate of the manpower is assumed to be given as shown in Figure 5-10. In

this case, by introducing a dummy operation train consisting of three operation modules as shown in Figure 5-11, the problem can be reduced to that with a constant upper bound on the consumption rate of the utility.

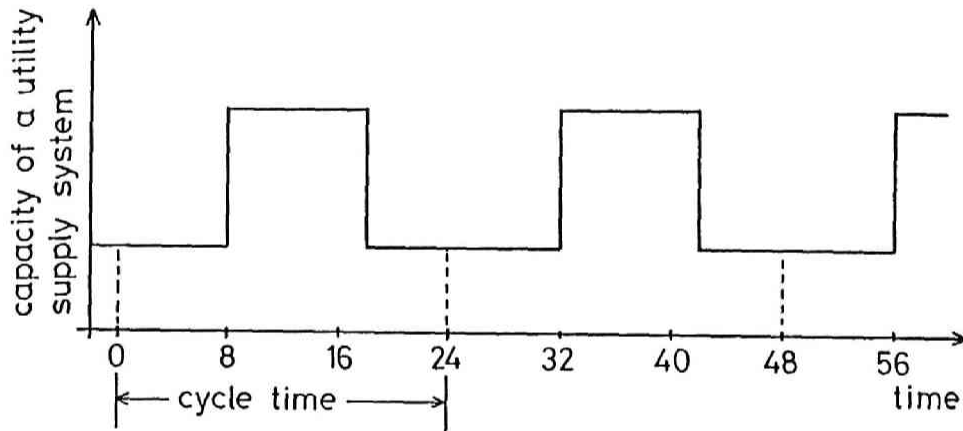


Figure 5-10. Change of the capacity of a utility supply system

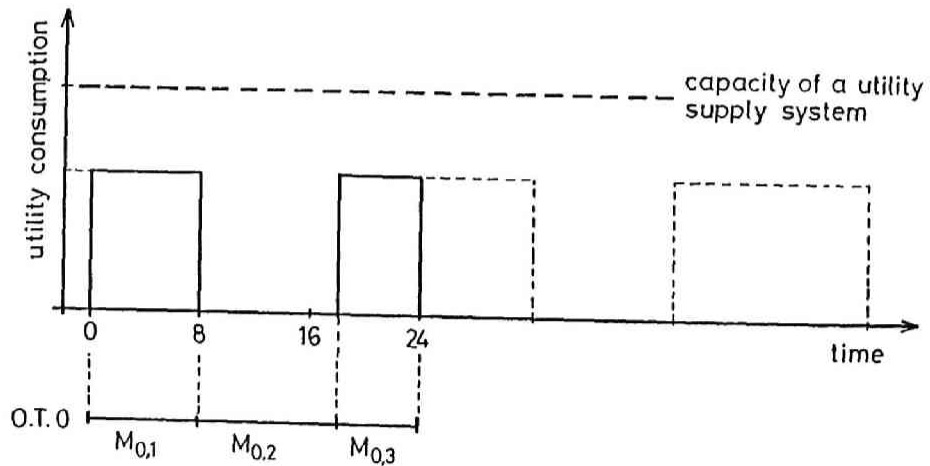


Figure 5-11. Dummy operation train consisting of three operation modules

5. Conclusion

We dealt with the utility smoothing problem of a cyclically operated batch process. In order to solve this problem a direct search method has to be utilized, since the necessary amount of a utility is represented by a function which takes a piecewise constant value.

In applying a direct search method, the most critical problem is that of reducing the searching domain. We derived a theorem relevant to the set of searching points each of which gives the minimum value to the performance index in a certain sub-area of the searching domain. Then, it was clarified that this set can be generated by using rooted directed graphs. It, however, takes an extremely long computing time to generate these graphs when the size of the problem becomes large. By taking into account this fact, an effective algorithm was developed in order to obtain a suboptimal solution in a very short computing time.

The algorithm developed here is applicable in handling more general cases such as;

- i) A process consisting of many production lines,
- ii) A process which has an upper limit on the supply of each kind of utility, and
- iii) A process where some batch units are multiply used.

Nomenclature

$F_{i,k}$ = function which shows the consumption rate of utility

k which is required to operate O.T.i

$f_{i,j,k}$ = function which shows the consumption rate of utility

k for $M_{i,j}$

G_k = maximum value of the consumption of utility k

g = performance index (monotonically non-decreasing function)

K = number of the different kind of utilities

L = number of operation trains

$M_{i,j}$ = operation module which is processed the j th in O.T.i

N_i = number of operation modules in O.T.i

O.T.i = operation train i

$P_{i,j}$ = processing time of $M_{i,j}$

$s_{i,j}$ = starting moment of $M_{i,j}$

T_0 = set of operation schedules defined at Theorem 5-1

$t = (t_1, t_2, \dots, t_L)$: operation schedule of the process

t_i = starting moment of O.T.i

$U_{i,j,k}$ = consumption rate of utility k per unit time which
is required to execute $M_{i,j}$

W = cycle time of the process

Other symbols

$A \times B$ = Cartesian product of A and B

O.T.i $\rightarrow$ O.T.j = relationship such that the ending moment of

a certain operation module in O.T.i coincides with

the starting moment of some operation module in O.T.j

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Chapter 6

OPERATION SCHEDULING OF A MULTI-PRODUCT BATCH PROCESS WITH INTERMEDIATE STORAGE TANKS

1. Introduction

A multi-product process consists of items of batch and/or continuous equipment which are operated intermittently, and sequentially produces various products. In such a multi-product process, there are many alternatives to the order of the production of each product and to the time needed for its production to satisfy the given overall production requirement. That is, there are many different production schedules which satisfy the production requirement.

In the field of Industrial Engineering, many studies have been done to solve such scheduling problems. However, it was recently clarified that the great majority of scheduling problems are "NP-complete", that is, the solution time for obtaining the exact optimal solution cannot be bounded by the polynomial in the characteristic size of the problem^[1].... Therefore, much effort has been devoted to the development of effective scheduling algorithms so as to obtain feasible schedules in the shortest possible time and/or interactive man-computer scheduling systems by introducing many heuristics^{[2]-[5]}.

One of the prominent characteristics of chemical processes is that fluid materials are mainly handled in a process. Therefore, the storage capacity of fluid materials strongly affects the operation scheduling of the process^{[6]-[8]}.

In this chapter, the scheduling problem of a multi-product process consisting of many production stages is

considered by taking into account the capacities of the storage tanks for intermediate products and final products. A scheduling algorithm is proposed by which the feasible production schedules of each stage can be derived so as to satisfy the medium term production requirement of each product.

In many cases, products produced in a multi-product batch process are final products which come into the market directly. It is common that the market demands for these products are uncertain and change with time, and that the derived schedule is seldom implemented in its entirety without revision. So, it is very important to develop the scheduling procedure in due consideration of how to modify the previous schedule.

By taking into account the characteristics mentioned above, the interactive man-computer system is developed to solve the scheduling problem of the multi-product process.

2. Problem Formulation

The process consisting of many production stages and intermediate storage tanks as shown in Figure 6-1 is taken up.

In order to clarify the problem, the following assumptions are first introduced:

- (i) Each production stage consists of batch or continuous items of equipment. Many different kinds of products can be produced in each production stage by changing its "operation scheme", that is, by changing its operation and/or feed materials.
- (ii) Each operation scheme of a production stage can be mathematically specified by the following variables:

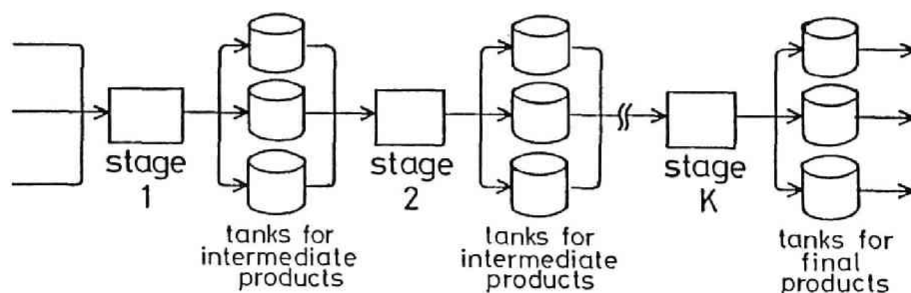


Figure 6-1. A process consisting of many production stages

- 1) the kinds of materials needed in its stage and their consumption rates,
- 2) the kinds of products produced in its stage and their production rates,
- 3) the operating cost per unit time in its stage.

Here, it is assumed that both the consumption rate of raw materials and the production rate of products take continuous values for each operation scheme, although in a batch stage products are produced in batch-wise. However, for the problem such that the total planning period is fairly long compared with the cycle time of a batch stage, this assumption will be justified.

- (iii) The change-over cost depends only on the consecutive operation schemes before and after the switching, and the time necessary for switching from one operation scheme to another is negligible.
- (iv) The market demand for each product produced in the final production stage (i.e. final product) is piecewise constant. Each period in which the demand for each product is constant is hereafter called a "production period".
- (v) The sufficient amount of each raw material for the process is available for the whole production period.
- (vi) Only one kind of material is stored in each storage tank. The upper and lower bounds on the capacity of

each tank are known a priori.

Under the assumptions introduced above, the scheduling problem of the process shown in Figure 6-1 can be stated as follows:

"Find the sequence of operation schemes and their run lengths in each production stage so as to satisfy the given production requirements and minimize the sum of the operation cost and the change-over cost in the whole process".

In this problem, the number of feasible sequences of operation schemes in each production stage grows exponentially with the increase in the number of necessary operation schemes. Therefore, it becomes almost impossible to obtain the exact optimal solution in a reasonable computing time. So it is of great importance to develop an effective algorithm by which the feasible solution can be derived in the shortest possible time.

3. Mathematical Formulation

In the process shown in Figure 6-1, the hold-up of a final product is determined depending only on the execution order of operation schemes and their run lengths in the final production stage, if the market demand of each final product is known a priori. On the other hand, the hold-up of an intermediate product in an arbitrary production stage i can be determined by the execution order of operation schemes and their run lengths in both production stages i and $i+1$.

In order to expedite the solution procedure for obtaining feasible solutions, the following rule is first introduced:

"The determination of the execution order of operation schemes and their run lengths is done stage by stage backward from the final production stage (stage K) to the first one (stage 1)."

By deciding the execution order of operation schemes and their run lengths in the final production stage K , the demand for the precedent stage $K-1$ can be determined. Based on the above rule, the solution of the whole process can be obtained by solving an equal number of subproblems as the stage numbers.

The subproblem being solved at each production stage can be decomposed into two problems; one is the problem of finding the execution order of operation schemes, and the other is the problem of deciding the run length of each

operation scheme the execution order of which has already been fixed.

In this section, assuming that the execution order of operation schemes is already determined, the problem of how to find the optimal run length of each operation scheme is considered. The problem of how to determine the execution order of operation schemes will be examined in the next section.

To simplify the explanation, it is assumed that the process consists of only one production stage, and only two different kinds of products are produced in the production stage.

Let's assume that an operation scheme (operation scheme 1) is executed from the origin of the time axis. Then, the run length of operation scheme 1, t_1 , must satisfy the following relationship:

$$V^L + D_1 t_1 \leq V^0 + R_1 t_1 \leq V^U + D_1 t_1 \quad (6-1)$$

where

$V^0 = (v_1^0, v_2^0)^T$: vector representing the initial hold-up in the tanks for products 1 and 2,

$V^L = (v_1^L, v_2^L)^T$: vector representing the lower bounds of the possible hold-up in the tanks for products 1 and 2,

$V^U = (v_1^U, v_2^U)^T$: vector representing the upper bounds of the possible hold-up in the tanks for products 1 and 2,

$D_i = (d_{i,1}, d_{i,2})^T$: vector representing the market demand on products 1 and 2 during production period i ,

$R_i = (r_{i,1}, r_{i,2})^T$: vector representing the production rates of products 1 and 2 resulting from the execution of operation scheme i ,

Superscript "T" means transpose of a vector.

In Eq.(6-1), the inequality of the left hand side shows the constraint on the lower bounds of the hold-up in the tanks, and the inequality of the right hand side shows the constraint on the upper bounds of the hold-up.

The relationship of Eq.(6-1) can be graphically expressed as shown in Figure 6-2. This figure shows the variations in the total production and the market demand for the final products with respect to time. The thin line in the figure represents the total depletion of each product with respect to time. And the bold line in the figure represents the total accumulation of the inflow to the tank resulting from the execution of operation scheme 1. The gradient of the bold line shows the production rate of each product. The vertical distance between two solid lines represents the hold-up of the tank at every moment.

Two dotted lines in Figure 6-2 represent the lower and upper bounds of the hold-up in the tank, respectively. That is, the hold-up in the tank does not exceed its lower and upper bounds if the bold line in the figure does not intersect

either of these two dotted lines. In this example, the hold-up for product 2 exceeds its upper bound at time t_1^M , if operation scheme 1 is executed.

Consider the case where operation scheme 2 is performed in the wake of operation scheme 1. Then, the run length of operation scheme 2, t_2 , must satisfy the following relationship:

$$V^L - V^0 \leq (R_1 - D_1)t_1 + (R_2 - D_1)t_2 \leq V^U - V^0 \quad (6-2)$$

Here, note that the run lengths of both operation schemes, t_1 and t_2 , are treated as independent variables.

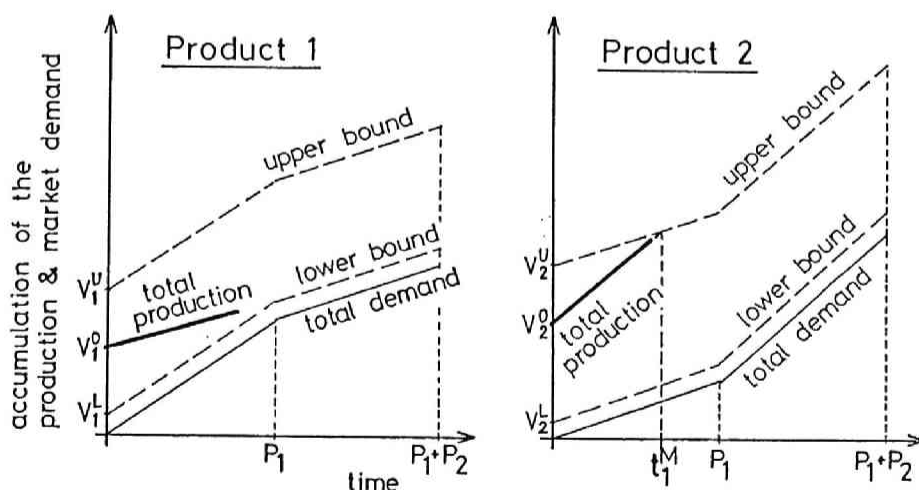


Figure 6-2. Total production & market demand for both products (No. of operation scheme = 1)

Let's assume that the market demand of the final product during production period 1 is satisfied by executing operation schemes 1 and 2. Then, operation scheme 2 can be successively executed even in production period 2, in so far as the following relationship holds:

$$\begin{aligned} v^L - v^0 &\leq (R_1 - D_1)t_1 + (R_2 - D_1)t_2 + (R_2 - D_2)t'_2 \\ &\leq v^U - v^0 \end{aligned} \quad (6-3)$$

$$t_1 + t_2 = p_1 \quad (6-4)$$

where

t'_2 = the run length of operation scheme 2 during production period 2,

p_i = the length of production period i .

If the market demand of the final products during the overall production period (production period 1 + production period 2) can be fulfilled by successively executing operation schemes 1, 2 and 3, the run length of each operation scheme must satisfy the following relationships:

$$v^L - v^0 \leq (R_1 - D_1)t_1 \leq v^U - v^0 \quad (6-1)'$$

$$v^L - v^0 \leq (R_1 - D_1)t_1 + (R_2 - D_1)t_2 \leq v^U - v^0 \quad (6-2)$$

$$v^L - v^0 \leq (R_1 - D_1)t_1 + (R_2 - D_1)t_2 + (R_2 - D_2)t'_2 \leq v^U - v^0 \quad (6-3)$$

$$\begin{aligned} v^L - v^0 &\leq (R_1 - D_1)t_1 + (R_2 - D_1)t_2 + (R_2 - D_2)t'_2 + (R_3 - D_2)t_3 \\ &\leq v^U - v^0 \end{aligned} \quad (6-5)$$

$$t_1 + t_2 = p_1 \quad (6-4)$$

$$t_2' + t_3 = p_2 \quad (6-6)$$

When t_1 , t_2 , t_2' and t_3 satisfy the above restrictions, the total production and market demand for final products 1 and 2 change with respect to time as shown in Figure 6-3.

The operation cost is given by the following equation:

$$C_1 t_1 + C_2 (t_2 + t_2') + C_3 t_3 \quad (6-7)$$

where

C_i = operation cost per unit time of operation scheme i .

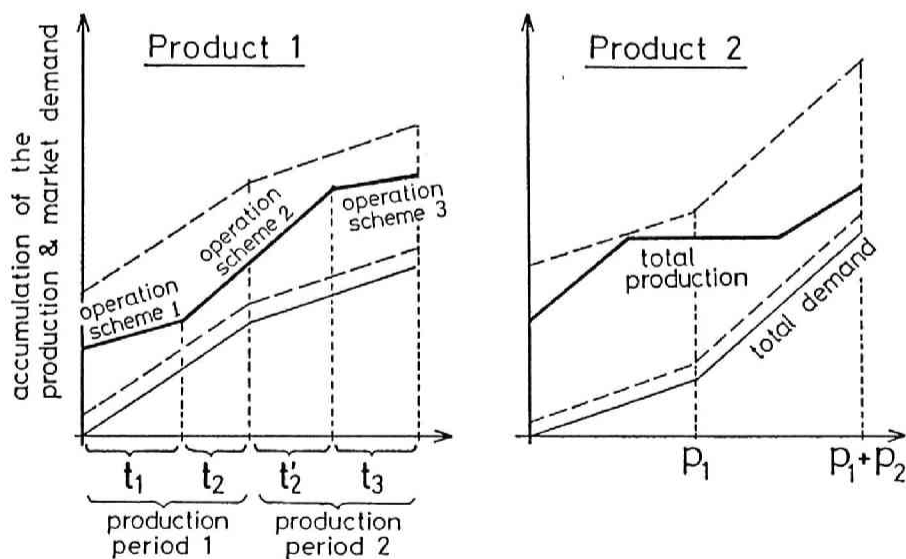


Figure 6-3. Total production & market demand for both products (No. of operation schemes = 3)

Eq.(6-1)' to (6-7) are all linear with respect to t_1 , t_2 , t_2' and t_3 . Therefore, the run lengths of the operation schemes, that is, t_1 , (t_2+t_2') and t_3 which minimize the total operation cost given by Eq.(6-7), can easily be obtained by utilizing the Linear Programming (L.P.) technique.

4. Scheduling Algorithm

The "operation sequence" will be defined as a set of operation schemes the execution order of which is already decided. By utilizing the terminology defined above, the problem of determining the execution order of the operation schemes at each production stage can be restated as one of selecting an operation sequence from feasible operation sequences at that production stage.

Here, it is necessary to notice that the number of feasible operation sequences is astronomical. For example, even if the number of operation schemes is 10, and each operation sequence has fewer than five operation schemes, the number of feasible operation sequences reaches the following number:

$$\sum_{i=0}^4 10 \cdot (10 - 1)^i \cong 7 \times 10^4.$$

Therefore, in a practical case where the number of operation schemes is fairly large, it is almost impossible to perform the solution procedure of an L.P. problem for all of the feasible operation sequences in a reasonable computing time. By taking into account the difficulty mentioned above, the following algorithm for determining the execution order of operation schemes at a production stage is proposed. Here, it is assumed that m is the number of operation schemes which are executed in an operation sequence, and n is the number of production period for which the execution order of

operation schemes is being determined.

Step 1) Let both m and n be one. We first derive the candidates for the operation scheme which is first executed. For all of the operation sequences each of which consists of only one operation scheme, find the maximum run length in such a way that neither overflowing nor exhaustion of stored material in the storage tank occurs. This procedure corresponds to finding the maximum value of the run length of each operation scheme which satisfies Eq.(6-1).

Step 2) Arrange the operation sequences each of which consists of m operation schemes in decreasing order of run length. Without losing generality, it is assumed that the run length of operation sequence i , s_i , satisfies the following relationship:

$$s_1 \geq s_2 \geq \dots \geq s_R \geq \dots \geq s_Q \geq \dots \quad ; \quad (R \leq Q)$$

where

s_i = the maximum run length of operation sequence i .

Let T_n be the sum of the lengths of production periods from production period 1 to production period n . If s_1 is longer than T_n , jump to step 7.

Step 3) As a part of candidates for the operation schedule, choose R operation sequences in decreasing order with respect to their maximum run lengths, i.e. operation sequences from operation sequence 1 to operation sequence R are selected.

Step 4) For each of $Q-R$ operation sequences from operation

sequence $R+1$ to operation sequence Q , find the minimum operation cost subject to the constraint that the run length of the operation sequence is equal to or longer than the run length of operation sequence Q , s_Q . This minimum value can easily be obtained by solving an L.P. problem. From these Q - R operation sequences, choose W - R operation sequences in increasing order with respect to the sum of the operation cost and the change-over cost of the operation schemes.

Step 5) W operation sequences selected at step 3 and step 4 are considered as the candidates for the operation schedule consisting of m operation schemes. These W operation sequences are hereafter called "dominant sequences" consisting of m operation schemes. Consider new operation sequences which can be generated by adding one more operation scheme to the dominant sequence obtained in step 3 and step 4. And find the maximum run length of each newly generated operation sequence as mentioned above.

Step 6) Increase the value of m by one (i.e., $m \leftarrow m+1$). And return to step 2.

Step 7) For each of operation sequences the run length of which is longer than T_n , find the minimum operation cost subject to the constraint that the run length of the operation sequence is equal to T_n . Then, choose W operation sequences in increasing order with respect to the sum of the operation cost and the change-over cost of the operation schemes. These W operation sequences are also called "dominant sequences" consisting of m operation schemes. If n is equal

to the number of production periods, jump to step 10.

Step 8) The market demand of products between production period 1 and production period n is satisfied by executing m operation schemes. In this case, the last operation scheme, in other words, the m -th operation scheme executed in each operation sequence can be successively executed even in production period $n+1$. So, recalculate the maximum run length of each operation sequence by adding the constraints such as Eqs. (6-3) and (6-4).

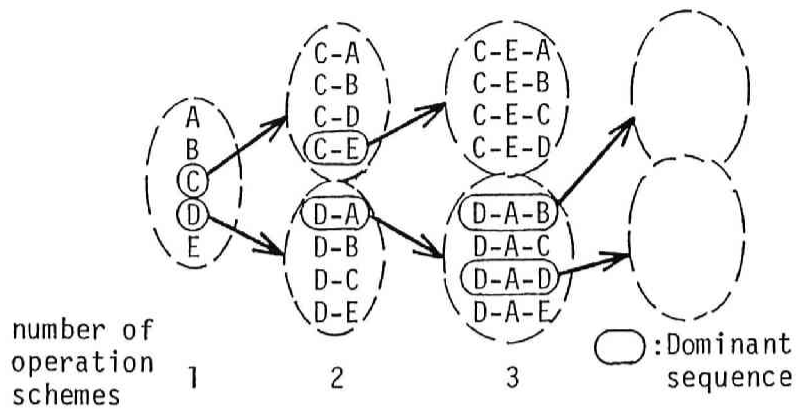
Step 9) Increase the value of n by one (i.e., $n \leftarrow n+1$). And return to step 2.

Step 10) When n is equal to the number of production periods, operation sequences derived at step 7 are feasible solutions. So, find the optimal operation sequence which minimizes the sum of the operation cost and the change-over cost among them.

By executing 10 steps mentioned above, the execution order of operation schemes and their run lengths at a production stage are determined.

Figure 6-4 shows the set of operation sequences to which the optimization calculation is performed, when the number of operation schemes is 5, and the number of dominant sequences, W , is 2.

In this algorithm, the constants, Q , R and W are given in advance by taking into account the size of the problem and the performance of the available computer etc..



A,B,C,D,E : operation schemes

Figure 6-4. Operation sequences being searched

Once the execution order of the operation schemes and their run lengths in a production stage are determined, the demand for the products in the previous production stage is automatically determined. Therefore, by performing step 1 through step 10 at each production stage backward from the final stage to the first one, the execution order of the operation schemes and their run lengths can be determined at every production stage.

5. Computer Package for the Scheduling of a Multi-product Process

5-1. Overall structure

In order to execute the algorithm proposed at the previous section, the interactive man-computer scheduling system is developed. The "production scheduling system of a multi-product process" developed here is simply called PSMP. PSMP is written in FORTRAN 77 language, and it has been developed for use in the interactive mode.

The computer package PSMP consists of the three main blocks as shown in Figure 6-5. The first block consists of subprograms for the data input and the data output. The

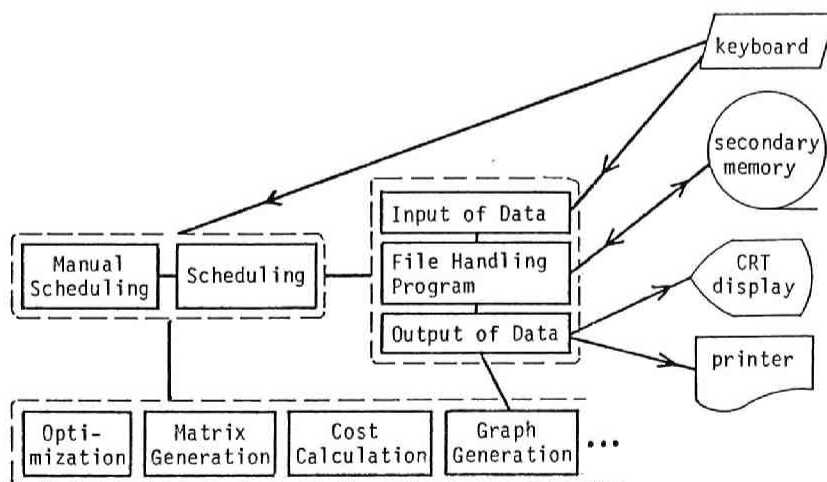


Figure 6-5. Structure of computer package PSMP

second block is the main scheduling block. And the third consists of peripheral subroutines which are used at the first and second blocks.

Each operation which can be selected by the user is initiated by inputting a command from the keyboard. The commands which are available in this package are listed in Table 6-1. The meaning of these commands are explained more precisely during the course of this section.

5-2. Input of data

The objective of this subsection is to describe the way in which input data are stored in internal memory.

Data input operation is initiated by the command DATAIN. On reading this command, the program reads the number of stages, and data required for the scheduling are inputted stage by stage from stage 1 to the final stage. Data required for the scheduling of a stage are listed as follows:

- Number of product types,
- Number of raw material types,
- Initial stock level of each storage tank,
- Lower bound of the storage in each tank,
- Upper bound of the storage in each tank,

For each operation scheme,

- Production rate of each product,
- Consumption rate of each raw material,
- Operation cost per unit time,
- Change-over cost from the operation scheme to each of other operation schemes,

Only for the final stage,
Number of production periods,
Length of each production period.

Table 6-1. Commands available in the computer package PSMP

Command	Description
DATAIN	Reads the input data from the terminal keyboard and stores them in internal memory
LTFDAT	Prints the input data stored in secondary memory as a file
LTMDAT	Prints the input data in main internal memory
CGFDAT	Modifies a part of data stored in secondary memory as a file
CGMDAT	Modifies a part of data in main internal memory
LTFILE	Prints the list of names of files formed in secondary memory
DLFILE	Deletes a file in secondary memory
PRFSOL	Prints the scheduling result stored in secondary memory as a file
RUN	Executes the scheduling algorithm and prints the derived schedules and stores them in secondary memory as a file
STOP	Stops the execution of the program

These data are stored in main internal memory and also stored in secondary memory (auxiliary memory) as a file. The list of file names is also stored in secondary memory as a file.

Data stored in secondary memory can easily be modified by using the command CGFDAT. Similarly, data in main internal memory can be modified by using the command CGMDAT.

5-3. Scheduling routine

The scheduling routine is initiated by the command RUN. If data needed for scheduling have not been stored in main internal memory, by inputting the name of data file, scheduling data stored in secondary memory are read and stored in main memory. Then, the scheduling is executed according to the algorithm mentioned in the previous section. Feasible solutions derived are printed and if necessary, they are stored in secondary memory as a file.

A part of the operation schedule derived here can be modified by using the manual scheduling routine which is included in the scheduling program. In a manual scheduling routine, by inputting the execution order of operation schemes, the maximum run length for the operation sequence can be derived. Therefore, the user of the system can interactively derive the operation schedule by changing the execution order of operation schemes. It is also possible to fix the run lengths of a part of operation schemes in the operation sequence.

5-4. Output of data stored in secondary memory

The command `LTFILE` provides the printed output of names of files which are stored in secondary memory.

The output of the input data stored in secondary memory, that is, the data which have been read in under the command `DATAIN` can be obtained by inputting the command `LTFDAT` and the name of the file. A sample output of the input data is shown in Tables 6-2 and 6-3 for the numerical example which is formulated and solved at the next section.

In this scheduling algorithm, several feasible solutions can be derived for one scheduling problem, and they are stored in secondary memory as a file. So, the output of the scheduling results stored in secondary memory can be provided by inputting the command `PRFSOL`, the name of the file and the number of a feasible solution. An example of the output of the scheduling result is also shown in Tables 6-4 and 6-5. Moreover, the variations of the hold-up in each storage tank with respect to time can be graphically expressed as shown in Figures 6-8 and 6-9.

A file formed in secondary memory can be deleted by the command `DLFILE`.

6. Numerical Example

In this section, the process consisting of two production stages as shown in Figure 6-6 is taken up, and the way the algorithm proposed here can be applied is shown.

In the distillation column at production stage 1, three different products can be produced by changing the plates for the side cuts. In the batch equipment at production stage 2, three different final products are produced by changing the feed materials. The market demand for this process changes for every one month.

Then, the problem is to find the operation schedule for the period of 2 months so as to minimize the sum of the operation cost and the change-over cost.

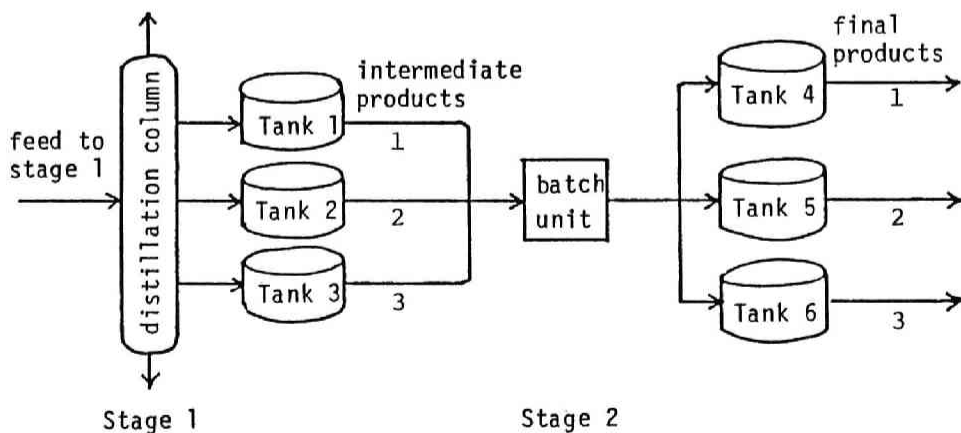


Figure 6-6. A process consisting of a distillation column and a batch unit

In order to store the scheduling data in main memory, the command DATAIN is first inputted. Then, the system interactively indicates the variable for which data must be inputted. Figure 6-7 shows how the scheduling data are inputted and stored in secondary memory as a file.

The printed output of the input data are depicted in Tables 6-2 and 6-3. These data can be obtained by the command LTFDAT. In this example, it is assumed that three operation schemes are available for both stages.

The scheduling algorithm is initiated by the command RUN. For this example, the operation schedule of stage 2 is first derived so as to satisfy the production demand. Then the scheduling for stage 1 is executed.

As candidates of the operation schedule for stage 2, three feasible solutions are obtained by using the scheduling algorithm. The result of a feasible schedule which minimizes the sum of the operation cost and the change-over cost is shown in Table 6-4.

By deciding the operation schedule of stage 2, the production demand for stage 1 can be determined. So, the operation schedule of stage 1 is next derived so as to satisfy the production demand for intermediate products. When the schedule which is shown in Table 6-4 is adopted as the operation schedule of stage 2, only one feasible schedule is obtained as candidate of the operation schedule of stage 1.

The operation schedule of stage 1 is shown in Table 6-5. As is clear from Tables 6-5 and 6-4, the operation schedule

at the production stages 1 and 2 is switched 5 and 6 times respectively, in order to satisfy the given market demand for final products.

Figures 6-8 and 6-9 show the graphic output of the result. Figure 6-8 shows the variations in the hold-up of final products with respect to time. Figure 6-9 shows the variations in the total production and the market demand for the intermediate products with respect to time. Dotted lines in these figures represent the lower and the upper bounds of the hold-up in the tank.

```
READY
INPUT COMMAND NAME
00610 ?
DATAIN
```

```
** DATA INPUT ROUTINE (DATAIN) **
```

```
INPUT THE NUMBERS OF FIRST AND LAST SUBPROCESSES
OF WHICH YOU WANT TO STORE THE DATA
(FROM SUBPROCESS I TO SUBPROCESS J)
INPUT I AND J (212)
00760 ?
1 2
```

```
*****
* SCHEDULING DATA FOR SUBPROCESS 1 *
*****
```

```
NUMBER OF PRODUCT TYPES
03220 ?
3
```

```
NUMBER OF RAW MATERIALS =
03250 ?
1
```

```
IF YOU WANT TO STORE THE DATA IN SECONDARY MEMORY,
INPUT THE DATA OF DATA FILE (A6)
OTHERWISE INPUT 99
04070 ?
DATADD
```

```
*****
* SCHEDULING DATA FOR SUBPROCESS 2 *
*****
```

```
NUMBER OF PRODUCT TYPES =
03220 ?
3
```

** : console input

Figure 6-7. List of a part of data input operation

Table 6-2. List of scheduling data for stage 1

```
*****
*   SCHEDULING DATA FOR SUBPROCESS 1   *
*   NAME OF DATA FILE = DATADD.D1   *
*****
```

```
NUMBER OF PRODUCT TYPES = 3
NUMBER OF RAW MATERIALS  1
NUMBER OF OPE. SCHEMES   3
NUMBER OF PROD. PERIODS  = 0
```

< LOWER AND UPPER BOUNDS OF THE HOLD-UP AND
INITIAL STOCK LEVEL IN STORAGE TANKS > (ton)

PRODUCT NO.	1	2	3
LOWER BOUND	100.00	100.00	100.00
UPPER BOUND	1200.00	1200.00	1200.00
INITIAL STOCK LEVEL	600.00	700.00	800.00

< PRODUCTION AND CONSUMPTION RATES > (ton/day)

OPERATION SCHEME NO.	1	2	3
PRODUCT NO.			
1	70.00	0.0	60.00
2	70.00	100.00	0.0
3	0.0	40.00	80.00
RAW MATERIAL NO.			
1	140.00	140.00	140.00

< CHANGE-OVER COST AND OPERATION COST >

CHANGE-OVER COST (MY)	TO	1	2	3
FROM	1	0.0	100.00	50.00
OP. SCHEME	2	50.00	0.0	100.00
	3	100.00	100.00	0.0
OPERATION COST (MY/day)		1.90	2.00	2.10

Table 6-3. List of scheduling data for stage 2

```
*****
*   SCHEDULING DATA FOR SUBPROCESS 2   *
*   NAME OF DATA FILE = DATADD.D2    *
*****
```

```
NUMBER OF PRODUCT TYPES      3
NUMBER OF RAW MATERIALS      3
NUMBER OF OPE. SCHEMES       3
NUMBER OF PROD. PERIODS      2
```

< DEMAND DATA FOR SUBPROCESS 2 >

PRODUCTION PERIOD NO.	1	2
PRODUCTION PERIOD (day)	30.00	30.00
PRODUCT NO.	PRODUCTION REQUIREMENT (ton/day)	
1	50.00	50.00
2	60.00	30.00
3	30.00	60.00

< LOWER AND UPPER BOUNDS OF THE HOLD-UP AND
INITIAL STOCK LEVEL IN STORAGE TANKS > (ton)

PRODUCT NO.	1	2	3
LOWER BOUND	50.00	50.00	50.00
UPPER BOUND	1200.00	1200.00	1200.00
INITIAL STOCK LEVEL	700.00	700.00	700.00

< PRODUCTION AND CONSUMPTION RATES > (ton/day)

OPERATION SCHEME NO.	1	2	3
PRODUCT NO.			
1	120.00	0.0	0.0
2	0.0	180.00	0.0
3	0.0	0.0	140.00
RAW MATERIAL NO.			
1	120.00	0.0	0.0
2	0.0	180.00	0.0
3	0.0	0.0	140.00

< CHANGE-OVER COST AND OPERATION COST >

CHANGE-OVER COST (M¥) TO	1	2	3
FROM			
1	0.0	50.00	100.00
OP. SCHEME 2	100.00	0.0	50.00
3	70.00	120.00	0.0
OPERATION COST (M¥/day)	0.70	0.60	0.70

Table 6-4. List of scheduling result for stage 2

```
*****
* OPERATION SCHEDULE FOR SUBPROCESS 2
* SOLUTION NUMBER = 1
* NAME OF SCHEDULING DATA FILE = DATADD.D2
* NAME OF DEMAND DATA FILE = DATADD.D2
*****
```

TOTAL COST = 0.3805181E+03 (M¥)
 OPERATION COST = 0.4051807E+02
 CHANGE-OVER COST = 0.3400000E+03

*** OPERATION SCHEDULE FOR SUBPROCESS 2 ***

PRODUCTION PERIOD	OPE-SCHEME NO.	OPERATION TIME (day)	STARTING MOMENT(day)
1	1	7.14	0.0
1	2	7.74	7.14
1	3	8.60	14.88
1	1	6.52	23.48
2	1	5.16	30.00
2	2	7.08	35.16
2	3	11.62	42.24
2	1	6.14	53.86
TOTAL SCHEDULING PERIOD -			60.00

*** STOCK LEVEL *** (ton)

TIME :	PRODUCT NO. 1	2	3
0.0 (day)	700.00	700.00	700.00
7.14	1200.00	271.43	485.71
14.88	813.10	1200.00	253.57
23.48	382.90	683.77	1200.00
30.00	838.96	292.86	1004.55
35.16	1200.00	138.13	695.08
42.24	846.04	1200.00	270.33
53.86	265.00	851.37	1200.00
60.00	694.97	667.10	831.46

*** ACCUMURATIONS OF PRODUCTS *** (ton)

TIME :	PRODUCT NO. 1	2	3
0.0 (day)	700.00	700.00	700.00
7.14	1557.14	700.00	700.00
14.88	1557.14	2092.86	700.00
23.48	1557.14	2092.86	1904.54
30.00	2338.96	2092.86	1904.54
35.16	2957.88	2092.86	1904.54
42.24	2957.88	3367.10	1904.54
53.86	2957.88	3367.10	3531.46
60.00	3694.96	3367.10	3531.46

Table 6-5. List of scheduling result for stage 1

```
*****
*   OPERATION SCHEDULE FOR SUBPROCESS 1   *
*   SOLUTION NUMBER           = 1         *
*   NAME OF SCHEDULING DATA FILE   DATADD.D1 *
*-----*
*   DEMAND DATA ARE CALCULATED FROM THE SCHEDULING *
*   RESULT OF THE PREVIOUS SUBPROCESS             *
*   NAME OF SOLUTION FILE           SOLTDD.S2 *
*   NUMBER OF SOLUTION USED         = 1         *
*****
```

TOTAL COST = 0.6205569E+03 (MY)
 OPERATION COST = 0.1205569E+03
 CHANGE-OVER COST 0.5000000E+03

*** OPERATION SCHEDULE FOR SUBPROCESS 1 ***

PRODUCTION PERIOD	OPE-SCHEME NO.	OPERATION TIME (day)	STARTING MOMENT (day)
1	1	7.14	0.0
2	1	7.74	7.14
3	1	0.42	14.88
3	2	3.76	15.30
3	3	4.43	19.06
4	3	11.67	23.48
5	3	0.0	35.16
5	1	2.92	35.16
5	2	4.15	38.08
6	2	10.07	42.24
6	3	1.55	52.31
7	3	6.14	53.86
TOTAL SCHEDULING PERIOD =			60.00

*** STOCK LEVEL *** (ton)

TIME :	PRODUCT NO. 1	2	3
0.0 (day)	600.00	700.00	800.00
7.14	242.86	1200.00	800.00
14.88	784.52	348.81	800.00
15.30	813.68	377.97	741.68
19.06	813.68	754.08	365.58
23.48	1079.26	754.08	100.00
35.16	378.88	754.08	1033.83
35.16	370.88	754.08	1033.83
38.08	583.62	432.34	1033.83
42.24	583.62	100.00	1200.00
52.31	583.62	1106.87	193.13
53.86	676.75	1106.87	100.00
60.00	308.21	1106.87	591.39

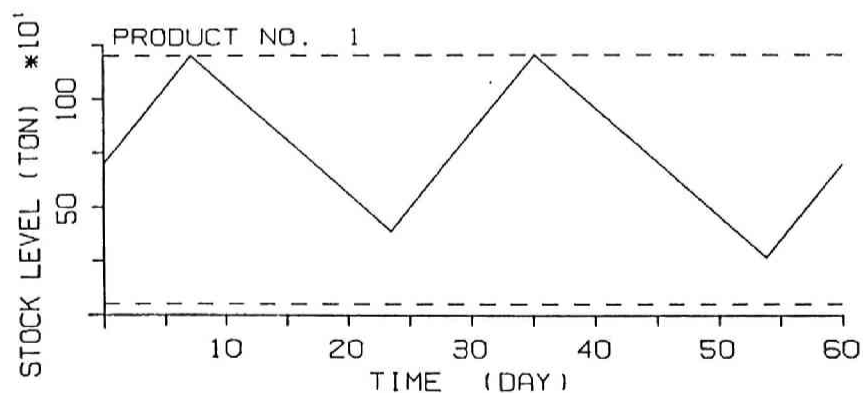
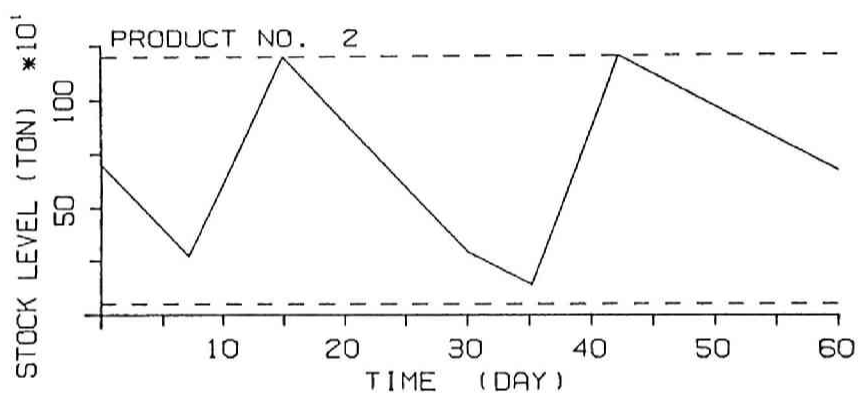
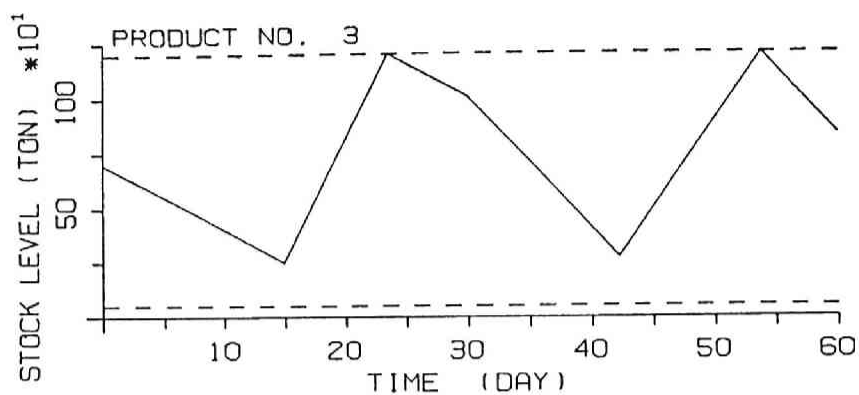


Figure 6-8. Variations in the hold-up of final products

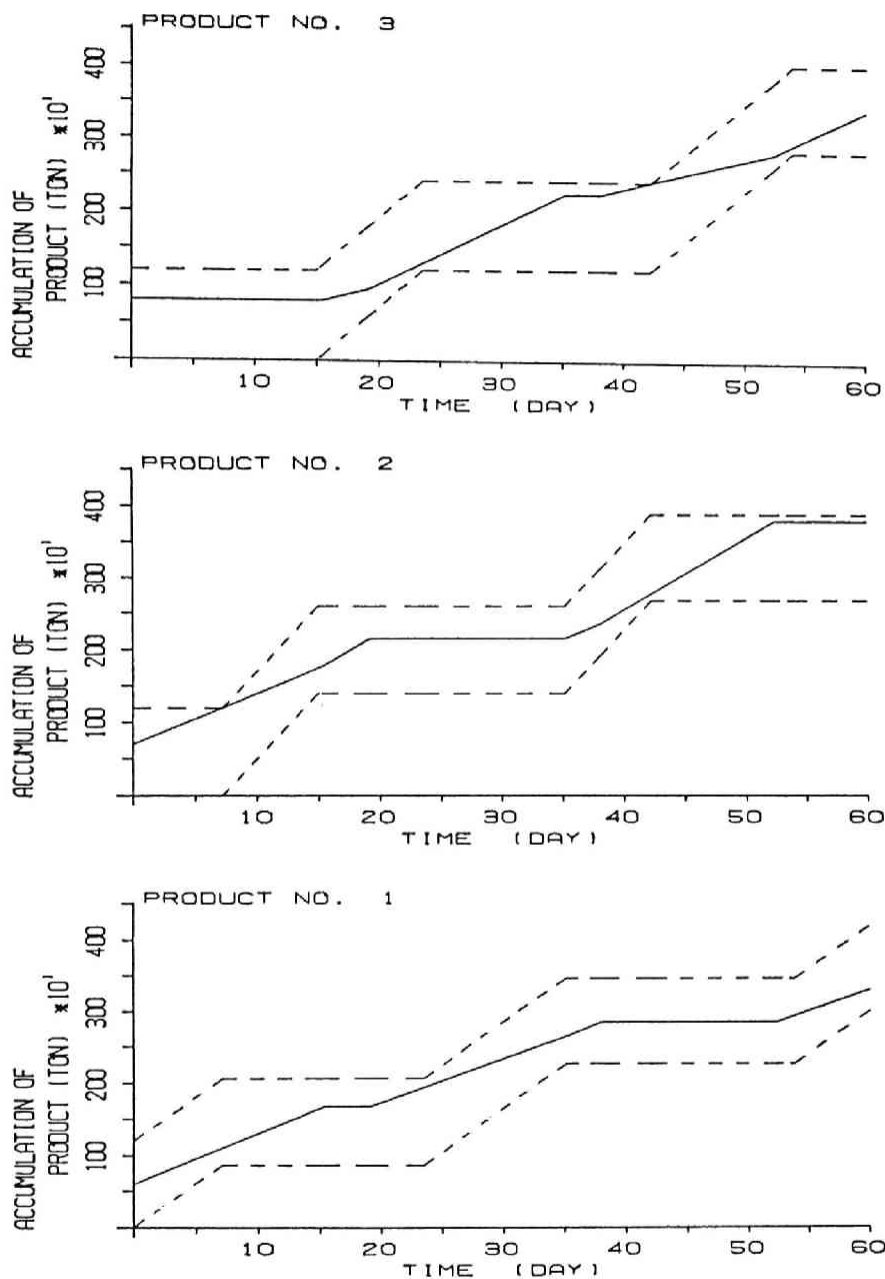


Figure 6-9. Variations in the total production and the market demand for intermediate products

7. Conclusion and the Extension of the Problem

The scheduling problem of a multi-product process which consists of many production stages is considered. An effective algorithm was developed to derive feasible production schedules which minimize the sum of the operation cost and the change-over cost. In this algorithm, the production schedule of the process is successively determined stage by stage backward from the final production stage to the first one.

The word "operation scheme" was introduced to express the operation of each production stage. By utilizing the terminology "operation scheme", the problem which must be solved at each stage resolves itself into the problem to decide the execution order of operation schemes and their run lengths.

The number of the execution order of the operation schemes, that is, the number of operation sequences which must be searched becomes astronomical in a practical case. Therefore, in the procedure proposed here, the number of searching points is restricted to a reasonable number from the point of view of the actual computation procedure by introducing some heuristics. However, in order to derive good feasible solutions, the run length of each operation scheme is treated as an independent variable in the calculation procedure.

The way of thinking utilized in developing this algorithm

is applicable even to cases where the following conditions are imposed.

i) The change-over time between two operation schemes cannot be neglected.

Even to this case, inequalities related to the lower and the upper bounds of the hold-up in the tank are linear with respect to the run lengths of operation schemes executed. For example, Eqs. (6-1)' and (6-2) can be rewritten as follows:

$$\begin{aligned} (R_1 - D_1)t_1 &\leq V^U - V^0 \\ V^L - V^0 &\leq (R_1 - D_1)t_1 - D_1 E_{12} \\ (R_1 - D_1)t_1 - D_1 E_{12} + (R_2 - D_1)t_2 &\leq V^U - V^0 \\ V^L - V^0 &\leq (R_1 - D_1)t_1 - D_1 E_{12} + (R_2 - D_1)t_2 - D_1 E_{23} \end{aligned}$$

where

E_{ij} = the change-over time from operation scheme i to operation scheme j .

Therefore, the algorithm proposed here is applicable without any modification for this case.

ii) The production level of each operation module can be changed.

It is assumed that R_i is a vector representing the production rates of products resulting from the execution of operation scheme i at full capacity. And it is also assumed that the production level of operation scheme i , k_i , can be changed in a following range:

$$k_i^L \leq k_i \leq k_i^U$$

Then, the production rates of products for operation scheme i is expressed by $k_i R_i$.

Even to this case, inequalities related to the lower and the upper bounds of the hold-up in the tank are linear with respect to the run lengths of operation schemes executed. For example, Eqs.(6-1)' and (6-2) can be rewritten as follows:

$$V^L - V^0 \leq (k_1^L R_1 - D_1) t_{11} + (k_1^U R_1 - D_1) t_{12} \leq V^U - V^0$$

$$\begin{aligned} V^L - V^0 \leq & (k_1^L R_1 - D_1) t_{11} + (k_1^U R_1 - D_1) t_{12} \\ & + (k_2^L R_2 - D_1) t_{21} + (k_2^U R_2 - D_1) t_{22} \leq V^U - V^0 \end{aligned}$$

$$t_{11} + t_{12} = t_1$$

$$t_{21} + t_{22} = t_2$$

As is clear from above inequalities, the algorithm proposed here is applicable even to this case.

Nomenclature

C_i = operation cost per unit time of operation scheme i
 D_i = vector representing the market demands on products
during production period i
 E_{ij} = change-over time from operation scheme i to operation
scheme j

K = number of production stages

p_i = length of production period i

R_i = vector representing the production rates of products
resulting from the execution of operation scheme i

s_i = maximum run length of operation sequence i

T_i = sum of the lengths of production periods from production
period 1 to production period i

t_i = run length of operation scheme i

V^0 = vector representing the initial hold-up in the tanks

V^L = vector representing the lower bounds of the possible
hold-up in the tanks

V^U = vector representing the upper bounds of the possible
hold-up in the tanks

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Chapter 7

CONCLUSION

The design and scheduling problem of batch processes have been studied in this thesis. In a batch process, the main equipment items are operated batch-wise. So, the formulation of the design problem differs from that of a continuous process. Moreover, as the batch item is operated batch-wise, the scheduling of the process becomes a large problem.

One of the dominant characteristics of chemical processes is that the fluid materials are mainly handled in the process. Therefore, storage tanks are indispensable to holding the products or intermediate products in the process. And the places where storage tanks are installed and the capacities of these tanks strongly affect the design and scheduling of the process. By taking account of this fact, the design and scheduling problems of batch processes with intermediate storage tanks have been studied in this thesis.

From Chapters 2 to 4, the design problem of a single product batch process was studied. In these chapters, it was stressed that the minimum size of an intermediate storage tank depends not only on the batch sizes of batch stages before and after the tank but also on the operation schedule of the process.

In Chapter 2, a simple process consisting of periodically operated parallel batch units and intermediate storage tanks installed before and after the batch section was taken up. And the problem of determining the schedule of the parallel batch operations and the tank capacities was discussed. For

the case where a batch section is composed of parallel identical units, optimal scheduling was obtained analytically. Then the case in which each batch unit has different cycle time was dealt with. For this case, it was shown that the search domain necessary can be reduced to an extremely small size.

In Chapter 3, the problem of the optimal design and operation of a single product batch process consisting of many batch stages and intermediate storage tanks was dealt with. The minimum volume of an intermediate storage tank was analytically derived as a function of batch sizes of batch stages before and after the tank. Then, effective algorithms which can be used to find the optimal solution in a very small number of searching steps were developed for practical cases.

One of the functions of intermediate storage tanks is to mitigate the effects of many kinds of variations. In Chapter 4, the design problem of a batch process with a fixed degree of flexibility was solved. As uncertain variations, the variations related to the operation schedule and to the batch size of each batch stage were considered. And the quantitative relationship between the size of allowable variations and the necessary volume of the storage tank was derived.

In these three chapters mentioned above, it was assumed that a batch item was cyclically operated according to four steps such as the charging, processing, discharging and cleaning. And by using this simplified model, many valuable

results have been analytically derived.

In Chapters 5 and 6, the scheduling problems of batch processes were studied. As explained in Chapter 1, there are many differences between the scheduling for chemical processes and that for non-chemical processes. Therefore, the scheduling techniques developed for non-chemical processes can not directly apply the scheduling problems of chemical processes. Most of these differences are caused by the characteristics of the batch chemical processes such that the process is cyclically operated and the fluid materials are mainly handled in the process.

In Chapter 5, the utility smoothing problem of a cyclically operated batch process was studied. By installing a storage tank between two batch units, the starting moment of the latter batch unit can be arbitrarily chosen. By using this flexibility, the problem of how to smooth the peak consumption of utilities was discussed. In order to derive the optimal solution, a direct search method was used, and an effective algorithm was also developed to obtain a suboptimal solution in a very short computing time.

In Chapter 6, the scheduling problem of a multi-product process consisting of many production stages and storage tanks was studied. In such a process, the storage capacity of fluid materials strongly affects the operation scheduling of the process. Here an algorithm was developed to derive feasible production schedules which minimize the sum of the operation and the change-over costs. By using this algorithm, an interactive scheduling system was developed.

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