

Visco-Resistive MHD Equilibrium and its Perturbation for the Vicinity of a Magnetic Neutral Point

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研究目的 (Research Objective):

The magnetic reconnection process is an energy conversion mechanism to convert magnetic energy to plasma thermal and kinetic energies in solar flares and geomagnetic substorms. In the past more than 40 years, the magneto-hydrodynamic (MHD) study of the magnetic reconnection was largely developed by numerical MHD simulations. However, the ubiquitous MHD simulations cannot exactly explore the basic characteristics of the magnetic reconnection beyond the CFL condition. For instance, the exact equilibrium of the magnetic reconnection is not yet studied because of the complicated nonlinearity. In this paper, introducing open boundary conditions, an analytically rigorous equilibrium for a non-uniform resistivity which is similar to that of Ugai,JPP1977 is introduced and the perturbation is numerically studied.

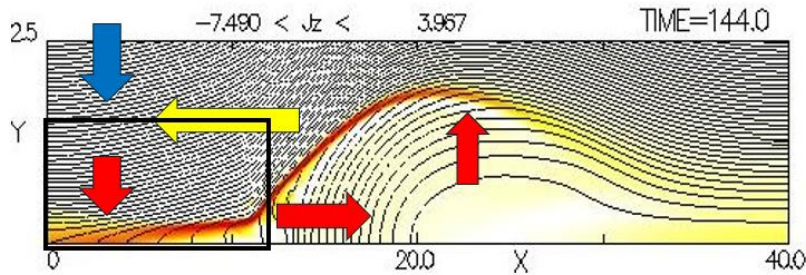


Fig.1: Spontaneous and external driven models (MHD simulation).

Fig.1 schematically explains how the magnetic reconnection is driven in MHD simulations. In Fig.1, the magnetic reconnection is caused at the origin, and then, a large scale plasmoid is ejected from the origin to the downstream, i.e., the right direction. Focusing on the rectangular black box around the origin, it is interpreted that the magnetic reconnection process is driven by the plasma inflow indicated by the blue or yellow arrow. Then, those arrows cause the plasma flows indicated by the red arrows. The yellow inflow (arrow) is originated in swelling of the plasmoid. This driven mechanism was firstly introduced by Ugai (e.g., Ugai,JPP1977, Ugai,GRL1987), and then, he named it “spontaneous model”. Recently, the spontaneous model is called “plasmoid (driven) instability” which is developed to a turbulent reconnection model. On the other hand, the blue inflow (arrow) was introduced by Hayashi,JGR1978. He named it “externally driven model”. In this model, it is unclear for what drives the magnetic reconnection and also for how it

is driven. However, in both models, it is important that the magnetic reconnection is driven by any MHD waves, such as Alfvén wave, generated at the upstream of the neutral point. In this paper, for simplicity, let us assume that the surroundings of the black box satisfy a kind of open boundary condition introduced in the next section. Hence, the outside of the black box is not considered in this paper.

計算手法 (Computational Aspects):

This paper starts from the 2D incompressible visco-resistive MHD equations (Shimizu, PoP2025, Loureiro, PoP2007) shown below. First, the analytically rigorous equilibrium is shown. Then, the perturbation is numerically studied.

$$\frac{\partial}{\partial t} \nabla_{\perp}^2 \varphi + \{\varphi, \nabla_{\perp}^2 \varphi\} = \{\psi, \nabla_{\perp}^2 \psi\} + \nu \left(\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} \right) \varphi + 2\nu \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} \quad (1)$$

$$\frac{\partial}{\partial t} \psi + \{\varphi, \psi\} = \eta \nabla_{\perp}^2 \psi + E_0 \quad (2)$$

Here, φ and ψ are respectively the potential of flow and magnetic fields. ν , η and E_0 are respectively viscosity, resistivity and the background electric field, which are constant. In this paper, the following equilibrium are studied.

$$\varphi_0 = \Gamma_0 x y \quad (3)$$

$$\psi_0 = \alpha x^2 + \beta y^2 \quad (4)$$

$$\eta = \frac{2\Gamma_0(\beta y^2 - \alpha x^2) - (E_0)}{(2\alpha + 2\beta)} \quad (5)$$

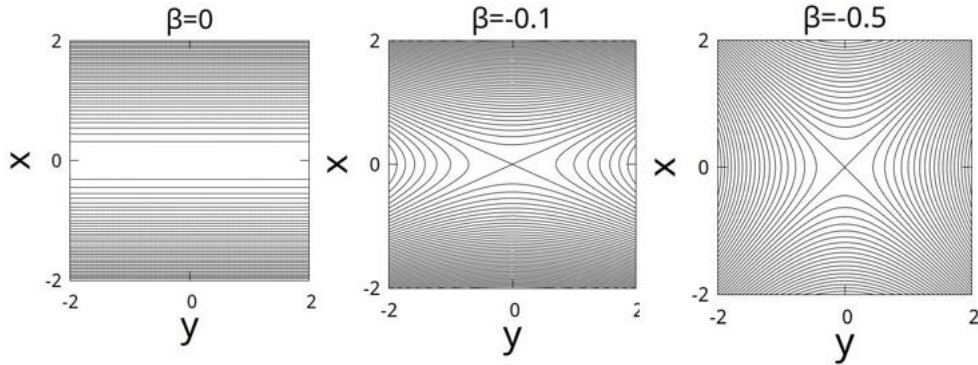


Fig.2: The β variation of equilibrium magnetic fields.

Here, Γ_0 is the acceleration rate of the equilibrium plasma outflow jet from the neutral point. α and β are the parameters to determine the magnetic field configuration. Let us keep $\alpha=0.5$ through this paper. The β variation changes the separatrix angle of the magnetic field, as shown in Fig.2. In this equilibrium, the current density $J_z=2\alpha+2\beta$ is uniform and the reconnection rate, i.e., the electric field ηJ_z , at the neutral point, is kept to be constant, even when β changes. Note that, in $\beta=-0.5$, the magnetic reconnection is not stopped because the rate is kept to be constant with the $J_z=0$ and $\eta=+\infty$. Fig.3 shows the flow field and non-uniform resistivity. As shown in Fig.3, this resistivity takes negative values

beyond a value of x and y , which is unphysical. Hence, the introduction of open boundary condition is important to define the finite size black box (Fig.1).

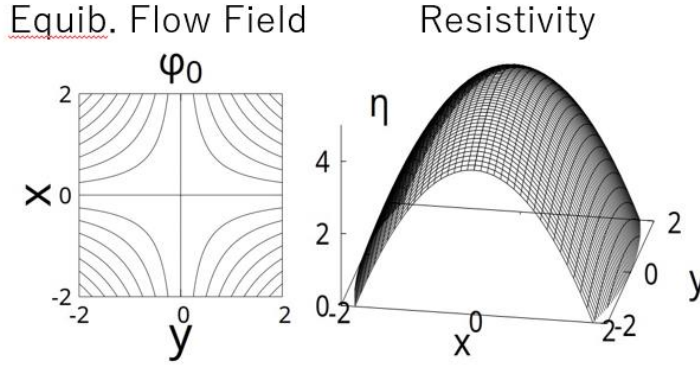


Fig.3: Equilibrium flow field and non-uniform resistivity.

Next, let us consider the following perturbations on the equilibrium eqs.(3)-(5).

$$\psi_1 = (\Psi_0(x) + \Psi_2(x)(ky)^2 + \Psi_4(x)(ky)^4 + \Psi_6(x)(ky)^6 + \dots)e^{\gamma t} \quad (6)$$

$$\varphi_1 = (\Phi_1(x)(ky) + \Phi_3(x)(ky)^3 + \Phi_5(x)(ky)^5 + \Phi_7(x)(ky)^7 + \dots)e^{\gamma t} \quad (7)$$

Here, we take the WKB approximation (Loureiro, PoP2007), where the acceleration rate $k=k(T)=k_0 e^{-\Gamma_0 t}$ of the perturbation plasma outflow jet can vary as time proceeds with the linear growth rate γ . Inserting eqs.(3), (4), (6) and (7) into eqs.(1) and (2), we obtain the following linear perturbation equations.

$$\Psi_0 \gamma - \Gamma_0 x \Psi_0' - 2\alpha x \Phi_1 k = (C - Ax^2)(\Psi_0'' + 2\Psi_2 k^2) \quad (8)$$

$$\begin{aligned} & (\Phi_1'' k + 6\Phi_3 k^3) \gamma - \Gamma_0 (\Phi_1'' k + 18\Phi_3 k^3) + \Gamma_0 (\Phi_1' k + 6\Phi_3 k^3) - \Gamma_0 x (\Phi_1''' k + 6\Phi_3' k^3) \\ & = 2\alpha x (2\Psi_2'' k^2 + 24\Psi_4 k^4) - 2\beta (\Psi_0''' + 2\Psi_2' k^2) + v \Phi_1'''' + 120v \Phi_5 k^5 + 12v \Phi_3'' k^3 \end{aligned} \quad (9)$$

$$\begin{aligned} & (\Psi_2 k^2 \gamma - 2\Psi_2 k^2 \Gamma_0) + 2\Psi_2'' k^2 \Gamma_0 - \Gamma_0 x \Psi_2' k^2 + 2\beta \Phi_1' k - 6\alpha x \Phi_3 k^3 \\ & = B(\Psi_0'' + 2\Psi_2 k^2) + (C - Ax^2)(\Psi_2'' k^2 + 12\Psi_4 k^4) \end{aligned} \quad (10)$$

Here, the following parameters are defined to simplify the expressions.

$$A = \frac{\Gamma_0 \alpha}{\alpha + \beta}, B = \frac{\Gamma_0 \beta}{\alpha + \beta}, C = \frac{-E_0}{2\alpha + 2\beta} \quad (11)$$

Eq.(8) is obtained for the y^0 order term. Eqs.(9) and (10) are respectively for the y^1 and y^2 order terms. In this paper, Ψ_0 , Φ_1 , and Ψ_2 are only solved with $\Phi_3=\Psi_4=\Phi_5=0$. Eqs.(8)-(10) are numerically solved by the shooting technique (Shimizu, PoP2025) with the numerical high precision.

研究成果 (Accomplishments) :

In this section, let us set $\Gamma_o = k_o = 1$, $\alpha = 0.5$, $\beta = -0.25$, $\nu = 0.001$, $\Psi_0(0) = 1$ and $E_o = -16$. In addition, assuming the symmetric field condition around the neutral point, $\Psi_0'(0) = \Psi_0'''(0) = \Psi_2'(0) = \Psi_2'''(0) = 0$ and $\Phi_1(0) = \Phi_1''(0) = 0$ are set. Then, when three control parameters; γ , $\Phi_1'(0)$, $\Phi_1'''(0)$, and $\Psi_0''(0)$, are given, a set of $\Psi_0(x)$, $\Phi_1(x)$, and $\Psi_2(x)$ is numerically obtained from eqs.(8)-(10). Most of the obtained sets will be physically meaningless. To uniquely select one from the sets, one of the following upstream open boundary conditions is set.

$$\Psi_0(X_c) = \Phi_1(X_c) = \Psi_2(X_c) = 0 \quad (12)$$

$$\Psi_0(X_c) = \Phi_1'(X_c) = \Psi_2(X_c) = 0 \quad (13)$$

Eq.(12) is the boundary condition to allow the plasma inflow of the yellow arrow of Fig.1, i.e., which is for the spontaneous model. On the other hand, Eq.(13) is the boundary condition to allow the blue arrow of Fig.1, i.e., which is for the externally driven model. The following numerical results are for eq.(13) but qualitatively the same as the case of eq.(12). X_c is defined at the point where either of eq.(12) or (13) is satisfied. Hence, X_c itself is interpreted to be additional control parameter to uniquely determine a set of $\Psi_0(x)$, $\Phi_1(x)$, and $\Psi_2(x)$.

Fig.4 shows the numerical result in the case of $\gamma = 2$, $\Phi_1'(0) = 2.89$, $\Phi_1'''(0) = -0.18$, and $\Psi_0''(0) = -0.2$. The left figure shows the profiles of $\Psi_0(x)$, $\Phi_1(x)$, and $\Psi_2(x)$. Eq.(13) is exactly satisfied at $X_c = 2.7$. The right figure shows the J_z profile on x-y plane, which has a single-dome shape. Since the current sheet is not bifurcated, this result may correspond to the Sweet-Parker model. The linear growth rate $\gamma = 2(\sim 0.8X_c)$ is determined to assume that the perturbed plasma inflow is grown at the roughly 80% of Alfvén wave speed measured at X_c , which is equal to the equilibrium magnetic field's intensity, but this result is not sensitive for the γ value.

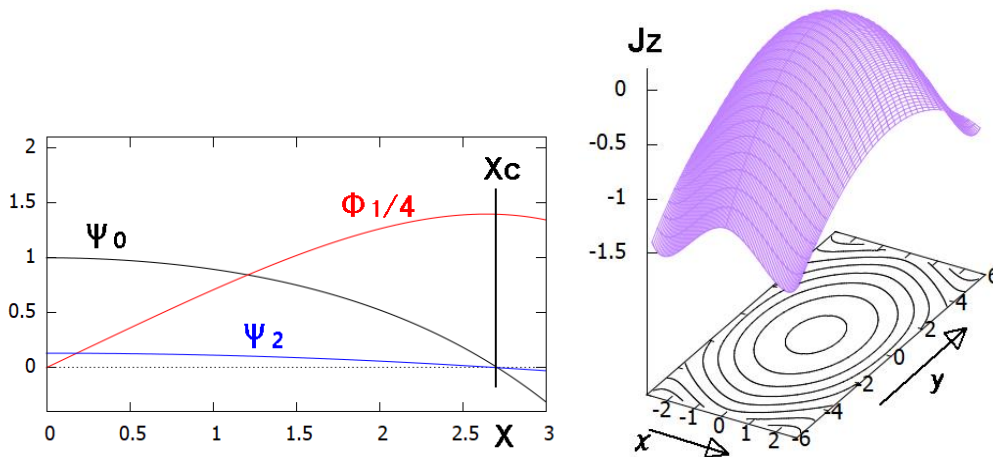


Fig.4: Sweet-Parker type. The J_z (right figure) is not bifurcated.

Fig.5 shows the case of $\gamma = 0.7$, $\Phi_1'(0) = 187$, $\Phi_1'''(0) = -980$, and $\Psi_0''(0) = -2$, resulting in

$X_c=0.92$ as shown in the left figure. The right figure shows that J_z profile which has a double-cusp shape (see the contour on the bottom). Since the current sheet J_z is bifurcated in the y direction, this result may correspond to the Petschek model. The linear growth rate $\gamma=0.7(\sim 0.8X_c)$ is determined to assume that the plasma inflow perturbation is grown at the roughly 80% of Alfvén wave speed at X_c . Also, this result is not sensitive for the γ value.

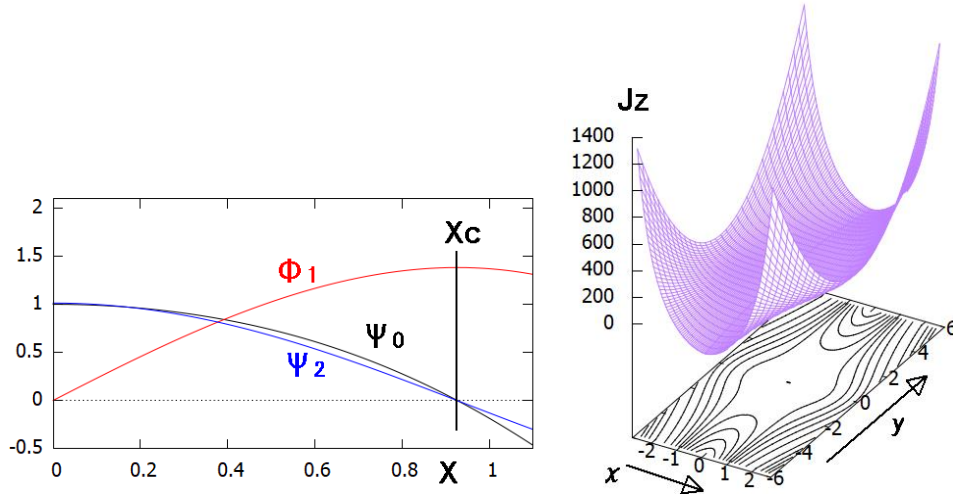


Fig.5: Petschek type. The J_z (right figure) is bifurcated in the y direction.

In comparison with Fig.4 and 5, Ψ_2 in Fig.5 is much larger than that of Fig.4 and comparable with Ψ_0 . It suggests that the convergence of eq.(6) is wrong. If so, the higher order terms, such as Ψ_4 , Ψ_6 ,, cannot be ignored. At this point, this paper is a preliminary result.

Summary:

When resistivity is non-uniformly localized around a magnetic neutral point, 2D incompressible visco-resistive MHD equilibrium was analytically derived and then the perturbation was studied by assuming the open boundary condition (eqs.(12) and (13)). Then, it was shown that two types of perturbation are obtained, which may correspond respectively to the Sweet-Parker and Petschek models. This resistivity is similar to what was introduced by Ugai,JPP1977. Ugai reported that the non-uniform resistivity results in the Petschek model. In contrast, this paper suggests that the non-uniform resistivity may result in the Sweet-Parker model.

公表状況 (Publications) :

(口頭)

1. 清水徹、The dissipation effect variations in resistive MHD shock waves, SGEPSS2025 秋大会、神戸、2025/11/26
2. 清水徹、磁気中性点近傍の磁気流体的摂動解析, STE シミュレーション研究会、東京、2026/3/10