

Velocity correlations in dense granular shear flows

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概要 分子動力学法を用いて、高密度領域での粉体剪断流の数値実験を行い、速度揺らぎの同時空間相関を調べた。その結果、速度相関は、粒子直径程度の空間スケールで振動的な振舞を示していることを見いだした。また、衝突時の相対速度は、相関の無い場合に予想される値よりも低かった。この速度相関が、エネルギー散逸率や圧力におよぼす影響を調べた。

abstract We investigate the instantaneous spatial correlations of velocity in sheared inelastic hard sphere systems using the molecular dynamics simulations. We find that the velocity correlation oscillates in the particle diameter scale, and the relative velocity of colliding particles is smaller than the value expected for random collisions. We study the effect of the velocity correlations on the constitutive relations.

Granular materials can flow like fluids under appropriate external force, but its rheology has not been understood yet. One of the models of the granular material is the inelastic hard sphere system, and the constitutive relations have been derived from the kinetic theory, which takes into account the static density correlations but not the velocity correlations. The obtained constitutive relations are known to give a quantitative agreement for the flow with rather low-density and small inelasticity, but disagree for the denser flow [1]. One of the possible origins of the disagreement is the effect of the instantaneous spatial velocity correlations enhanced by the inelastic collisions, which results in the locally aligned velocity fluctuations. We study the velocity correlations and their effect on the constitutive relations numerically.

We simulated the inelastic hard sphere systems with the restitution coefficient e_p , the particle mass m , and the particle diameter σ , under shear in three dimensions. A constant shear rate γ is applied to the x direction in the plane perpendicular to the z -axis by using Lees-Edwards boundary conditions. The system reaches steady state after long enough waiting time; We analyze the steady uniform shear flow with the average velocity $\langle \vec{c}(\vec{r}) \rangle = \vec{v} = (\gamma z, 0, 0)$. Here, $\vec{c}$ represents particle's instantaneous velocity, and $\langle \dots \rangle$ represents average. Note that the kinetic temperature $T = m \langle (\vec{c} - \vec{v})^2 \rangle / 3$ is not controlled by the heat bath but determined by the balance between the energy injection from the shear and the energy loss due to inelasticity, and γ^{-1} is the only parameter to give the time scale.

We found that the normal relative velocity of colliding pair of particles, v_{cn} , is much smaller than the value expected from random collisions. If the velocity fluctuation of a particle $\vec{c} - \vec{v}$

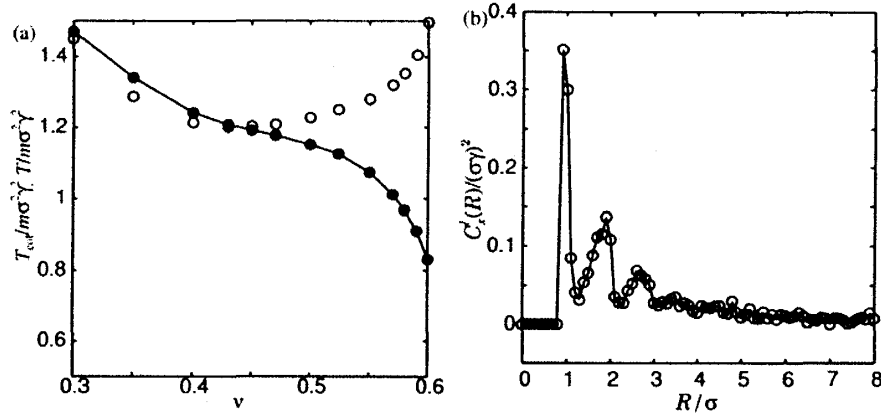


Figure 1: (a) Temperature T (open circles) and T_{col} (filled circles with line) versus the packing fraction ν . (b) The longitudinal velocity correlation in the x direction $C_x^l(R)$ for $\nu = 0.58$. The data with $e_p = 0.92$ are shown.

obeys Boltzmann distribution with the temperature T and collisions occur randomly, we expect $\langle v_{\text{cn}}^2 \rangle = 4T/m$. In Fig. 1(a), $T_{\text{col}} \equiv m \langle v_{\text{cn}}^2 \rangle / 4$ and T are plotted against the packing fraction ν . We find that T_{col} decrease with the packing fraction ν , while T increase with ν for $\nu > 0.5$. This suggests particles tend to move collectively in the denser region.

We also measured the instantaneous velocity correlation function defined as $C_{\alpha,\beta}(\vec{R}) = \sum_{i,j} [(c_{\alpha,i} - v_{\alpha})(c_{\beta,j} - v_{\beta})\delta(\vec{R} - (\vec{r}_i - \vec{r}_j))] / \sum_{i,j} [\delta(\vec{R} - (\vec{r}_i - \vec{r}_j))]$, where α and β take x, y or z , and i and j identify the particles. We find that the longitudinal correlations $C_{\alpha}^l(R) = C_{\alpha,\alpha}(R\vec{e}_{\alpha})$ show larger amplitudes than other components; here, $\vec{e}_{\alpha}$ represents the unit vector in the α direction. In Fig. 1(b), the longitudinal correlation in the x direction $C_x^l(R)$ is shown. We see that the correlation at the particle diameter σ is positive, which is consistent with the fact that $T_{\text{col}} < T$. It is also found that the correlation shows an oscillation, whose wavelength is of order of the particle diameter. Similar oscillations are found in the y and z directions also. Analysis of the sheared Langevin system suggests that this velocity correlation is related to the radial distribution function [2], which also oscillates in the scale of the particle diameter.

The colliding velocity affects strongly on the energy dissipation rate Γ , because the energy is lost at the collision. T_{col} also determines the momentum transfer at the collision, which have large contribution to the pressure P in the dense region. We compared Γ and P with the constitutive relation from the kinetic theory $\Gamma = \Gamma(\nu, T)$ and $P = P(\nu, T)$, and found that, though $\Gamma(\nu, T)$ and $P(\nu, T)$ is larger than the simulation data in the dense region, $\Gamma(\nu, T_{\text{col}})$ and $P(\nu, T_{\text{col}})$ agree with the data well in the whole density region. The analysis on effect of the velocity correlation on the shear stress is left for the future study.

[1] N. Mitarai and H. Nakanishi, Phys. Rev. Lett. **94**, 128001 (2005).

[2] A. Yoshimori, private communication.