

Dynamic load factor of concrete defects in hammer loading

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Abstract

The most reliable way to check the safety of elements against the spalling of concrete defects is to apply a load of sufficient magnitude. Nevertheless, the determination of a sufficiently large load is qualitative and based on the technician's intuition. Additionally, the response to impact loading is not the same as that to static loading. It would be useful to evaluate the response to a dynamic loading from an equivalent static loading. This paper investigates the dynamic load factor (DLF), defined as the ratio of the dynamic and static responses, adopting the response of a beam under impact loading and numerical analyses for more complex structures. Numerical results show that the most important variable in estimating the DLF is the ratio of the load time to the natural period of the first vibration mode. The DLF for displacements neglecting vibration damping can be estimated with sufficient accuracy in an undamped one-degree system. However, the DLF for stresses with damping, which is important for safety checks, is difficult to estimate. Nevertheless, its lower limit can be evaluated using the DLF of the undamped one-degree system. Although the risk of spalling is not considered to be high for defects with a first-order natural frequency exceeding 1000 Hz, a large load is likely to be applied by hammering. It is recommended not to hit hard to avoid excessive load during inspection.

Keywords: Plain concrete, Hammering test, Maintenance, Dynamic load factor, hammer loading

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1. Introduction

Periodic inspections are essential to maintain the functions of concrete structures for long service life. One of the most commonly used methods for inspecting concrete structures is the hammering test, where an inspector strikes a target using a steel hammer to evaluate the spalling risk through auditory and tactile sensations (Asakura and Kojima, 2003; Bergeson et al., 2015; Tsuno and Kishida, 2020). There are problems relating to this test, such as the lack of skilled technicians and the lack of unified evaluation criteria, and researchers have thus proposed alternative mechanical approaches that include an infrared method (Sakagami and Kubo, 2002; Clark et al., 2003; Maierhofer et al., 2006; Meola, 2007; Omar and Nehdi, 2017; Afshani et al., 2019; Ishikawa et al., 2021; Chun and Hayashi, 2021), mobile robotic system (Menendez et al., 2018; Loupos et al., 2018; Moreu et al., 2018; Nakamura et al., 2019) and non-contact acoustic exploration method (Mori et al., 2002; Zhu and Popovics, 2007; Oh et al., 2013; Sugimoto et al., 2019), and laser sensing method (Kurahashi et al., 2018; Yasuda et al., 2020, 2021; Wakata et al., 2022).

However, there is still no practical alternative to the hammering test. Although the alternative methods provide quantitative evaluations, there are challenges in terms of installation and operation cost, measurement speed, and the applicable range of machines. Another reason why the hammering test is preferred is that it concurrently serves as a load test. The most reliable way to check safety against spalling is to apply a load of sufficient magnitude. The risk of spalling can be considered low when the defect does not change after a load of sufficient magnitude is applied. Nevertheless, the sufficiently large load is qualitative and based on the technician’s intuition.

The hammering force can be quantified using an impact hammer. Judging from the measurement results, it is not difficult to make an impact of several kilonewtons (Yasuda, 2023). This indicates that a large impact load can be applied by hammering. Nevertheless, the response is not the same as that for a static load. It would be useful to evaluate the response of a dynamic loading with an equivalent static loading.

In the field of bridges, the determination of dynamic load factors (DLFs), defined as the ratio of the dynamic to static response, has been necessary for the evaluation of the dynamic loading due to vehicle travel (Bakht and Pinjarkar, 1989; Hwang and Nowak, 1991; Nassif and Nowak, 1995). These DLFs are known to be greater than 1. The dynamic load of a vehicle is a

moving load, whereas the dynamic load of a hammer should be treated as an impact load. The dynamic response of a system having one degree of freedom was well summarized in the book of Norris (1959). ADACHI (2021) studied the response of a fixed beam with both ends subjected to an impact load to consider the impact stress of objects. The study focuses on the point where the dynamic response is greater than the static response. Meanwhile, the dynamic response is expected to be smaller than the static response according to the results of hammering the concrete specimens (Yasuda, 2023). It is necessary to estimate what the expected value of the DLF is in the hammering test.

This paper investigates the DLF. Important variables in estimating the DLF are clarified from the response of a beam under impact loading. The DLF is then estimated for more complex structures through numerical analyses. Furthermore, the applicability of inspection using hammer loading is considered.

2. Theory

2.1. Theoretical response of a beam under impact loading

Consider a cantilevered beam of length L and thickness h , as shown in Fig. 1. The time-dependent impact load $F(t)$ is concentrated at a point $x = L$. The Euler–Bernoulli equation for the transverse displacement u_y of the beam with Young’s modulus E , moment of inertia I , mass density ρ , and constant cross-sectional area A is

$$EI \frac{\partial^4 u_y}{\partial x^4} + \rho A \frac{\partial^2 u_y}{\partial t^2} = 0. \quad (1)$$

When the beam has a rectangular cross section with a thickness of b , the moment of inertia I is

$$I = \frac{bh^3}{12}. \quad (2)$$

The initial conditions are

$$u_y(x, 0) = \frac{\partial u_y(x, 0)}{\partial t} = 0, \quad (3)$$

and the boundary conditions are

$$u_y(0, t) = \frac{\partial u_y(0, t)}{\partial x} = \frac{\partial^2 u_y(L, t)}{\partial x^2} = 0, -EI \frac{\partial^3 u_y(L, t)}{\partial x^3} = F(t). \quad (4)$$

It is assumed that the concentrated load that simulates the hammer impact can be expressed as

$$F_{(t)} = \frac{F_0}{2} \left\{ 1 - \cos \left(\frac{2\pi t}{T_i} \right) \right\} H(T_i - t), \quad (5)$$

where F_0 , T_i , and $H(t)$ are the load amplitude, load time, and Heaviside step function, respectively. The load time T_i becomes shorter as the exciting force strengthens. According to experimental results (Yasuda, 2022, 2023), T_i is estimated to be approximately 0.0004 s. Figure 2 shows the waveforms of the load $F_{(t)}$. The excitation frequency is up to approximately twice the reciprocal of T_i .

The Laplace transform is generally used to analytically solve the impact response of a beam. Applying the Laplace transform to Eq. (1) for the initial conditions given by Eq. (3), it follows that

$$EI \frac{\partial^4 \bar{u}_y}{\partial x^4} + \rho A s^2 \bar{u}_y = 0, \quad (6)$$

where the Laplace transform is defined as

$$\mathcal{L}[u_y(x, t)] = \bar{u}_y(x, s) = \int_0^\infty u_y(x, t) e^{-st} ds. \quad (7)$$

Meanwhile, the boundary conditions of Eq. (4) transform to

$$\bar{u}_y(0, s) = \frac{\partial \bar{u}_y(0, s)}{\partial x} = \frac{\partial^2 \bar{u}_y(L, s)}{\partial x^2} = 0, -EI \frac{\partial^3 \bar{u}_y(L, s)}{\partial x^3} = \bar{F}_{(s)}, \quad (8)$$

with

$$\bar{F}_{(s)} = \frac{F_0}{2} \left\{ \frac{1}{s} - \frac{T_i^2 s}{(T_i s)^2 + (2\pi)^2} \right\} (1 - e^{-T_i s}). \quad (9)$$

The general solutions of Eq. (6) are expressed as

$$\bar{u}_y = A_1 \cos \alpha x + A_2 \cosh \alpha x + A_3 \sin \alpha x + A_4 \sinh \alpha x, \quad (10)$$

where A_1 , A_2 , A_3 , and A_4 are complex coefficients determined by boundary conditions, and α is a complex constant that satisfies

$$\alpha^4 + \frac{\rho A}{EI} s^2 = 0. \quad (11)$$

From Eqs. (6), (8), and (10), the Laplace-transformed solutions are derived as

$$\overline{u}_y = \frac{-\overline{F}_{(s)}}{2(1 + \cos \alpha L \cosh \alpha L)EI\alpha^3} \{(\sin \alpha L + \sinh \alpha L)(\cos \alpha x - \cosh \alpha x) + (\cos \alpha L + \cosh \alpha L)(-\sin \alpha x + \sinh \alpha x)\}, \quad (12)$$

$$\overline{M} = -EI \frac{\partial^2 \overline{u}_y}{\partial x^2}, \quad (13)$$

where M is the bending moment of the beam.

In general, it is difficult to perform the Laplace inverse transform of the solutions analytically. Hence, the Laplace inverse transform is performed numerically adopting the fast Fourier transform (Krings and Waller, 1979). This allows for accurate solutions to be obtained rapidly.

Solutions for the both-ends-fixed beam and both-ends-supported beam can be derived in the same way. When the length of the beam is $2L$ and the concentrated load $F_{(t)}$ is applied at a point $x = L$, which is the center of the beam, the boundary conditions are as follows. For the both-ends-fixed beam,

$$u_y(0, t) = \frac{\partial u_y(0, t)}{\partial x} = \frac{\partial u_y(L, t)}{\partial x} = 0, -EI \frac{\partial^3 u_y(L, t)}{\partial x^3} = \frac{F_{(t)}}{2}. \quad (14)$$

For the both-ends-supported beam,

$$u_y(0, t) = \frac{\partial^2 u_y(0, t)}{\partial x^2} = \frac{\partial u_y(L, t)}{\partial x} = 0, -EI \frac{\partial^3 u_y(L, t)}{\partial x^3} = \frac{F_{(t)}}{2}. \quad (15)$$

2.2. Theoretical solution for free vibrations of beams

The general solution of Eq. (1) is expressed as

$$u_y = (B_1 \cos \beta x + B_2 \cosh \beta x + B_3 \sin \beta x + B_4 \sinh \beta x) e^{i2\pi f t}, \quad (16)$$

with

$$f = \frac{\beta^2}{2\pi} \sqrt{\frac{EI}{\rho A}}, \quad (17)$$

where B_1 , B_2 , B_3 , and B_4 are real coefficients determined by boundary conditions, β is the wavenumber, and f is the frequency.

For a cantilever beam under free vibration, the transverse displacement of the n -th mode $u_{y,n}$, which satisfies the boundary conditions

$$u_{y,n}(0, t) = \frac{\partial u_{y,n}(0, t)}{\partial x} = \frac{\partial^2 u_{y,n}(L, t)}{\partial x^2} = -EI \frac{\partial^3 u_{y,n}(L, t)}{\partial x^3} = 0, \quad (18)$$

is expressed as

$$u_{y,n} = C_n \{ \cos \beta_n x - \cosh \beta_n x + K(\sin \beta_n x - \sinh \beta_n x) \} e^{i2\pi f_n t}, \quad (19)$$

with

$$K = \frac{\cos \beta_n L + \cosh \beta_n L}{\sin \beta_n L + \sinh \beta_n L}, \quad (20)$$

where C_n is the amplitude. The wavenumbers β_n are roots of the transcendental equation

$$\cos \beta_n L \cosh \beta_n L + 1 = 0. \quad (21)$$

Solutions for the both-ends-fixed beam and both-ends-supported beam can be derived similarly.

2.3. Solutions for elements that are difficult to solve theoretically

Numerical analyses are performed for the responses of elements that are difficult to solve theoretically. Specifically, finite element method (FEM) analyses are conducted for the damped vibration responses of beams, rectangular plates, and concrete blocks with an inclined detachment. These analyses are conducted as linear elastic analyses in the commercial software ANSYS. The structure is meshed using a three-dimensional 20-node structural solid element (SOLID186).

In determining the dynamic response, transient analysis is carried out using mode superposition. This analysis is more computationally efficient than direct transient analysis. Moreover, it has the advantage that the damping ratio can be set for each mode in contrast with frequency-dependent damping, such as alpha (mass) damping and beta (stiffness) damping.

3. Numerical Results

The material properties of the concrete used in the numerical calculations are given in Table 1. In the following, the magnitude of the static loading is the same as that of dynamic loading F_0 . The subscript (s) is used to denote the solution for static loading.

3.1. Results for beams

Figure 3 shows the normalized responses of a cantilever beam for an impact loading of $T_i = 0.0004$ s. The displacements u_y and $u_{y(s)}$ are the values at the right end of the beam, where the load is applied. The moments M and $M_{(s)}$ are the values at the left end of the beam. The dimensions of the beam are $L = 0.1$ m and $h = 0.02$ m in (a) and $L = 0.3$ m and $h = 0.03$ m in (b). The natural frequencies of the first, second, and third modes in (a) are approximately 1200, 7300, and 20,000 Hz, respectively. Those in (b) are approximately 190, 1200, and 3400 Hz. Judging from the natural frequencies of the first modes, (a) and (b) correspond to defects with low and high risks of spalling, respectively. The first mode is mainly excited in (a), whereas the higher mode is also excited in (b). The response of the higher mode is greater in stress than in displacement. The maximum normalized value of the displacement is approximately 1.2 (larger than 1) in (a) whereas it is approximately 0.26 (smaller than 1) in (b). In both cases, the maximum normalized value of the moment is larger than that of the displacement. In particular in (b), the maximum normalized value of the moment is more than twice that of the displacement.

Figure 4 shows the DLF of the cantilever beam for the impact loading, where the DLF is defined as the maximum absolute value of the normalized response. T_1 is the natural period of the first vibration mode and is the reciprocal of the natural frequency of the first mode f_1 . Results for displacement and moment are calculated under two sets of conditions: (i) $T_i = 0.0004$ s and $h = 0.03$ m with L changing between 0.2 and 0.8 m and (ii) $T_i = 0.04$ s and $h = 0.02$ m with L changing between 0.2 and 2.0 m. Each DLF is plotted on the same line as a function of T_i/T_1 regardless of the conditions. The values of T_i/T_1 in Fig. 3 (a) and (b) are 0.48 and 0.076, respectively. The DLF of the displacement is a maximum 1.7 when T_i equals T_1 . The value approaches 1 as T_i becomes larger than T_1 and monotonically decreases as T_i becomes smaller than T_1 . The DLF of the moment is always larger than that

of the displacement. As T_i becomes smaller than T_1 , the difference between the two DLFs increases.

Figure 5 shows the normalized responses of the cantilever beam for $T_i/T_1 = 4.0$. The horizontal axis is the normalized time, where the time t is divided by the natural period of the first mode T_1 . When the load time T_i is 0.0004 s, the natural frequency of the first mode becomes 10000 Hz, which is a high frequency assuming concrete defects. The beam deforms according to the impact load, which can be roughly regarded as a quasi-static response.

Figure 6 shows the DLF of the impact loading for the two other types of beam. The length of the beam is $2L$ and the concentrated load $F(t)$ is applied at the center of the beam. The DLF of the moment for the both-ends-fixed beam is shown in two places, where the value of $|M_{(s)}|$ is a maximum. The displacement results are almost the same as the results for the cantilever beam shown in Fig. 4. In contrast, the moment results differ depending on the boundary conditions and position. In the case of the both-ends-fixed beam, the DLF of the moment at $x = 0$ is always larger than that of the displacement.

Figure 7 compares the displacement and moment distributions of the first vibration mode and the static loading. These are the results obtained when the maximum displacement is 1 mm. The displacement distributions of the first vibration mode and the static loading are similar in all cases. In contrast, there is a difference in the moment distributions. The distribution of the static loading changes linearly whereas the distribution of the first vibration mode changes curvilinearly. In the cases of the cantilever beam and both-ends-fixed beam, the absolute value of the moment at $x = 0$ for the first vibration mode is approximately 20% larger than that for the static loading.

Figure 8 compares the normalized responses of the cantilever beam between the beam theory solutions and FEM solutions. The beam has a length L of 0.3 m and thickness h of 0.03 m. The damping ratio for each mode is zero. In beam theory, σ_{xx} is derived from the moment ($\sigma_{xx} = 0.5Mh/I$), and the values of $\sigma_{xx}/\sigma_{xx(s)}$ and $M/M_{(s)}$ are exactly the same. In FEM analysis, a cantilever beam with a square cross section is assumed. The left end of the beam is completely fixed in all directions, and the load is applied to the upper center of the right corner of the beam. Displacements u_y and $u_{y(s)}$ are the values at the lower center of the right corner of the beam, and the stresses σ_{xx} and $\sigma_{xx(s)}$ are the values at the upper center of the left end of the beam. Solutions obtained from the beam theory and FEM show good

agreement, although their difference increases with time. This difference is attributed to the slightly different natural frequencies.

Figure 9 shows normalized responses of the cantilever beam for various damping ratios. The damping ratio of defects in a reinforced concrete tunnel lining is at most 10% ($\xi = 0.10$) (Yasuda et al., 2021). A larger damping ratio corresponds to a smaller response. A comparison of the results for $\xi = 0.00$ and $\xi = 0.10$ shows that the maximum displacement response is reduced by 18% and the maximum stress response is reduced by 41%. The stress is more susceptible than the displacement to the damping ratio.

3.2. Results for a rectangular plate

Concrete defects with a risk of spalling often have a plate-like structure. Hence, the response of a rectangular plate is considered. The numerical model is shown in Fig. 10. The length, thickness, and depth are 0.30, 0.03, 0.15 m, respectively. The length and thickness are the same as the dimensions in the previous beam analysis. The load $F_{(t)}$ expressed by Eq. (5) is applied to the corners of the top edge of the plate. The load time T_i is 0.0004 s.

Figure 11 shows normalized responses of the plate when the plane of $x = 0$ is completely fixed. The displacement $u_{y(s)}$ is the value at point a, where $u_{y(s)}$ is the maximum, and the stress $\sigma_{xx(s)}$ is the value at point A, where $\sigma_{xx(s)}$ is the maximum. The frequency of the lowest mode is approximately 190 Hz, and T_i/T_1 is 0.076. There is a difference in the normalized responses depending on the location. The responses do not necessarily decrease with the distance from the load point. At point C, the negative peak amplitude is larger than the positive peak amplitude. Although it is an estimation result obtained for three points, the DLF is estimated to be 0.24 for displacement and 0.48 for stress. Compared with the results of $\xi = 0.10$ in Fig. 9, the DLFs are larger in both displacement and stress.

Figure 12 shows normalized responses when the planes of $x = 0$ and $z = 0.075$ m are completely fixed. The displacement $u_{y(s)}$ is the value at point a, and the stress $\sigma_{xx(s)}$ is the value at point c. The frequency of the lowest mode is approximately 940 Hz, and T_i/T_1 is 0.38. The DLF of displacement is 1.0 for $\xi = 0.00$ and 0.76 for $\xi = 0.10$, and the DLF of stress is 1.5 for $\xi = 0.00$ and 1.1 for $\xi = 0.10$. The decrease in the DLF due to damping is 24% for displacement and 27% for stress. Compared with the results in Fig. 9, the decrease of stress is small. Compared with the result of $T_i/T_1 = 0.38$ in Fig. 4, the DLF for $\xi = 0.00$ is almost the same in displacement and larger in stress.

3.3. Results for a concrete block with an inclined detachment

The response of a defect with a more complex structure than a defect with a plate-like structure is considered. With reference to the past experiments (Yasuda, 2022, 2023), a concrete block with an inclined detachment is considered. The numerical model is as shown in Fig. 13. The bottom of the concrete block (400 mm $\times$ 175 mm rectangle) is completely fixed. The adhesion area is 100 mm in length and 20 mm in width. The load $F_{(t)}$ expressed by Eq. (5) is applied in the direction normal to the concrete surface. The load amplitude and load time T_i are 1 N (compression) and 0.0004 s, respectively. Note that defects with such a high risk of spalling are judged to require repair or reinforcement promptly in the visual inspection stage and should not remain in the field.

Figure 14 shows the first principal stress distribution in the dynamic loading when the stress is a maximum. Tension is taken to be positive. The frequency of the lowest mode is approximately 69 Hz, and T_i/T_1 is 0.028. The times t for the results in (a) and (b) are 0.0058 and 0.0053 s, respectively. Figure 15 shows the first principal stress distribution in the static loading. The results of Figs. 14 and 15 are obtained with the same FEM mesh. The stress distributions in Figs. 14 and 15 are completely different. The stress concentration is more localized in dynamic loading. The DLF is roughly 0.2 for $\xi = 0.00$ and 0.1 for $\xi = 0.10$. The DLF of the stress for $\xi = 0.10$ is almost the same as the DLF of the displacement for $T_i/T_1 = 0.028$ in Fig. 4.

4. Discussion

The most important variable in estimating the DLF is the ratio of the load time to the natural period of the first (lowest) vibration mode, T_i/T_1 , as shown in Figs. 4 and 6. The DLF of the displacement is almost the same regardless of the beam boundary conditions. Field measurements of defects requiring follow-up assessment in unreinforced plain concrete sections (Yasuda, 2023) show that the lowest mode frequencies are mostly below a few kilohertz, with a lower limit of a few hundred Hertz. Assuming the load time T_i is 0.0004 s, the approximate range of T_i/T_1 for the hammering test is from 0.1 to 1.0.

Figure 16 compares the DLF of the displacement of beams and the DLF of the undamped one-degree system. The DLF of the undamped one-degree

system is derived as (Norris, 1959)

$$\text{DLF} = \max \left[\frac{2\pi}{T_1} \int_0^t F(t') \sin \frac{2\pi}{T_1} (t - t') dt' \right], \quad (22)$$

where $F(t)$ is an arbitrary function; here, the function expressed by Eq. (5) is used. The results for the beams and one-degree system are roughly the same when T_i/T_1 is larger than 0.1. This is because the response of the first mode is dominant and the displacement distribution of the first vibration mode and that of the static loading are similar, as shown in Fig. 7. Such a result is considered to hold regardless of the structural shape because the effect of the inertial term is not large in the first mode. In fact, the DLF of the displacement of the plate for $T_i/T_1 = 0.38$ and $\xi = 0.00$, which is shown in Fig. 12, is almost the same as that of the beams. Therefore, the DLF for displacements without damping can be estimated with sufficient accuracy using an undamped one-degree system.

When considering the failure of concrete, the stress is more important than the displacement. However, the DLF of the stress is more difficult to estimate than the DLF of the displacement. The DLF of the stress (or moment) depends on the beam boundary conditions, as shown in Figs. 4 and 6. The DLF of stress at locations where the beams are fixed is greater than the DLF of displacement. Meanwhile, the DLF of stress at locations where the beams are not fixed is not necessarily greater than the DLF of displacement. Dynamic loading is presumed to cause larger stress concentration at fixed boundaries, as predicted in Fig. 7. The adhered area of the actual concrete defect can be regarded as a fixed boundary rather than a free boundary. Hence, the DLF of the stress is presumed to be larger than the DLF of the displacement in actual defects.

Stress is more sensitive than displacement to higher-order modes because it is related to the displacement gradient. There is mode superposition when higher-order modes are excited. As a result, areas of localized stress concentration appear as shown in Figs. 14 and 15, and the difference between the dynamic and static responses becomes large. This is the reason why the difference between the DLF of stress and the DLF of displacement increases as T_i/T_1 decreases as shown in Fig. 4.

The DLF of stress is always larger than that of displacement, as shown in Figs. 4 and 6 (b). The former is approximately 10% larger for $T_i/T_1 = 1.0$ and approximately 90% larger for $T_i/T_1 = 0.1$. Given that the DLF of stress of

the vibration attenuation for a damping ratio ξ of 10% is 1.1 for $T_i/T_1 = 0.38$ and 0.34 for $T_i/T_1 = 0.076$, as shown in Figs. 12 (b) and 9 (b), the DLF of stress with damping is expected to be as large as or larger than the DLF of displacement without damping. Moreover, considering that the approximate range of T_i/T_1 for hammering test is from 0.1 to 1.0 and the results of Fig. 16, the lower limit of the DLF of stress with damping can be estimated from the DLF of the undamped one-degree system expressed in Eq. (22), and the DLF of the stress for the hammering test is expected to be at least 0.3.

The dependence of the fracture strength on the strain rate is also important when considering concrete failure. The strength of tensile fracture due to hammering is considered to be at most twice that of static loading (Pajak, 2011). Even taking this into account, the equivalent static load for hammering is expected to be greater than 15% (0.15) of the maximum excitation force. When the actual excitation force is measured, it is not difficult to apply an impact of several kilonewtons. Hence, a load equivalent to its own weight can be applied by hammering for defects of at least several tens of kilograms. For defects with the natural frequency of the primary mode exceeding 1000 Hz, the DLF is expected to be greater than 0.5, and thus a load equivalent to the defects' own weight can be applied even to defects weighing 100 kg. Although the risk of spalling is not considered to be high for defects with a first-order natural frequency exceeding 1000 Hz, a large load is likely to be applied by hammering.

It is inclined to hammer hard during inspection to prevent spalling. However, excessive loading may promote failure. Therefore, it is recommended not to hit hard to avoid excessive load during the inspection. For this purpose, the inspector should actually use an impact hammer at least once to grasp how much maximum excitation force is generated by hammering. In addition, it should be remembered that, as shown in Figs. 4 and 16, the load on the defect (DLF) varies depending on the state of the defect (natural frequency). Although defects with natural frequencies below a few hundred Hertz are likely to require repair or reinforcement at the visual inspection stage, such defects have low DLF and it is difficult to estimate whether they can be sufficiently loaded by hammering.

The above discussion was based on elastic vibration theory and laser measurements at defects in unreinforced plain concrete. Thus, defects where delamination is highly developed and the presence of a firm adhesion area is not obvious should be considered out of the scope of this study. The effect of changes in the stiffness of the concrete due to mixing and other factors does

not significantly affect the discussion results since it depends on the root of the stiffness, as expected from the Eq. (17).

Although a large load can be applied by hammering, there are points to note. The dynamic stress concentration is more localized than the static stress concentration, as shown in Figs. 14 and 15. Additionally, large stress concentrations occur only for short periods of time. Hence, hammer loading may cause gradual progression of the failure, whereas static loading causes abrupt failure. In fact, in experiments using concrete specimens with the shape shown in Fig. 13, where the hammer excitation force gradually increases until spalling occurs, concrete fracture progresses gradually and eventually spalling occurs (Yasuda, 2023). In the case that a fracture propagates under the loading of a hammer, there may be spalling when the equivalent magnitude of the load is applied statically. Incidentally, this concrete specimen did not fail up to a maximum hammering force of approximately 5000 N. Considering that the specimen with half the width of the adhesion area failed at 501 N when loaded at the center of the specimen (Yasuda, 2022), the specimen shown in Fig. 13 is expected to fail at a static load of approximately 500 N. The T_i/T_1 of this specimen is approximately 0.03, and the DLF can be estimated to be approximately 0.1. This is consistent with the results in Fig. 16.

When measured with a device such as a laser Doppler vibrometer, the gradual progression of the failure can be grasped (Yasuda, 2023). However, the gradual progression is challenging to notice using the human senses. Therefore, it is not always possible to guarantee that a defect has a low risk of spalling through hammer loading. Measures other than the use of the human senses are needed to perform an inspection with high accuracy.

5. Conclusions

This study investigated the DLF of hammer loading in the hammering test. The DLF was estimated for beams, a rectangular plate, and a concrete block with an inclined detachment. Furthermore, the applicability of inspection using hammer loading was evaluated. The following conclusions were drawn from the results of the study.

- (1) The most important variable in estimating the DLF is T_i/T_1 , and the approximate range of T_i/T_1 in the hammering test is 0.1 to 1.0.

- (2) The DLF of displacement without damping can be estimated using a simple equation assuming an undamped one-degree system because the response of the first mode is dominant and the effect of the inertial term is not large in the first mode.
- (3) The lower limit of the DLF of the stress with damping can be estimated using the DLF of the undamped one-degree system.
- (4) The equivalent static load for hammering is expected to be greater than 15% of the maximum excitation force.
- (5) Although the risk of spalling is not considered to be high for a first-order natural frequency exceeding 1000 Hz, a large load is likely to be applied by hammering. It is recommended not to hit hard to avoid excessive load during inspection.
- (6) It is not always possible to guarantee that the defect has a low risk of spalling through hammer loading because hammer loading may cause the gradual progression of the failure and it is challenging to notice the gradual progression only using the human senses.

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References

- ADACHI, T., 2021. Fundamental issues and the applications of impact engineering. *Journal of the Society of Materials Science, Japan* 70, 712–718 (in Japanese).
- Afshani, A., Kawakami, K., Konishi, S., Akagi, H., 2019. Study of infrared thermal application for detecting defects within tunnel lining. *Tunnelling and Underground Space Technology* 86, 186 – 197.
- Asakura, T., Kojima, Y., 2003. Tunnel maintenance in japan. *Tunnelling and Underground Space Technology* 18, 161 – 169.
- Bakht, B., Pinjarkar, S., 1989. Dynamic testing of highway bridges—a review. *Transportation Research Record* .

- Bergeson, W., Ernst, S.L., et al., 2015. Tunnel operations, maintenance, inspection, and evaluation (TOMIE) manual. Technical Report. United States. Federal Highway Administration.
- Chun, P.j., Hayashi, S., 2021. Development of a concrete floating and delamination detection system using infrared thermography. *IEEE/ASME Transactions on Mechatronics* 26, 2835–2844.
- Clark, M., McCann, D., Forde, M., 2003. Application of infrared thermography to the non-destructive testing of concrete and masonry bridges. *NDT & E International* 36, 265 – 275.
- Hwang, E.S., Nowak, A.S., 1991. Simulation of dynamic load for bridges. *Journal of structural engineering* 117, 1413–1434.
- Ishikawa, M., Tsukagoshi, M., Kasano, H., Nishino, H., 2021. Influence of composition and surface discoloration of concrete on active thermographic nondestructive inspection. *Measurement* 168, 108395.
- Krings, W., Waller, H., 1979. Contribution to the numerical treatment of partial differential equations with the laplace transformation—an application of the algorithm of the fast fourier transformation. *International Journal for Numerical Methods in Engineering* 14, 1183–1196.
- Kurahashi, S., Mikami, K., Kitamura, T., Hasegawa, N., Okada, H., Kondo, S., Nishikino, M., Kawachi, T., Shimada, Y., 2018. Demonstration of 25-hz-inspection-speed laser remote sensing for internal concrete defects. *Journal of Applied Remote Sensing* 12, 015009.
- Loupos, K., Doulamis, A.D., Stentoumis, C., Protopapadakis, E., Makantasis, K., Doulamis, N.D., Amditis, A., Chrobocinski, P., Victores, J., Montero, R., et al., 2018. Autonomous robotic system for tunnel structural inspection and assessment. *International Journal of Intelligent Robotics and Applications* 2, 43–66.
- Maierhofer, C., Arndt, R., Röllig, M., Rieck, C., Walther, A., Scheel, H., Hillemeier, B., 2006. Application of impulse-thermography for non-destructive assessment of concrete structures. *Cement and Concrete Composites* 28, 393 – 401.

- Menendez, E., Victores, J.G., Montero, R., Martínez, S., Balaguer, C., 2018. Tunnel structural inspection and assessment using an autonomous robotic system. *Automation in Construction* 87, 117 – 126.
- Meola, C., 2007. A new approach for estimation of defects detection with infrared thermography. *Materials Letters* 61, 747 – 750.
- Moreu, F., Ayorinde, E., Mason, J., Farrar, C., Mascarenas, D., 2018. Remote railroad bridge structural tap testing using aerial robots. *International Journal of Intelligent Robotics and Applications* 2, 67–80.
- Mori, K., Spagnoli, A., Murakami, Y., Kondo, G., Torigoe, I., 2002. A new non-contacting non-destructive testing method for defect detection in concrete. *NDT & E International* 35, 399–406.
- Nakamura, S., Yamashita, A., Inoue, F., Inoue, D., Takahashi, Y., Kamimura, N., Ueno, T., 2019. Inspection test of a tunnel with an inspection vehicle for tunnel lining concrete. *Journal of Robotics and Mechatronics* 31, 762–771.
- Nassif, H., Nowak, A.S., 1995. Dynamic load spectra for girder bridges. *Transportation Research Record* .
- Norris, C.H., 1959. Structural design for dynamic loads. McGraw-Hill.
- Oh, T., Popovics, J.S., Ham, S., Shin, S.W., 2013. Improved interpretation of vibration responses from concrete delamination defects using air-coupled impact resonance tests. *Journal of Engineering Mechanics* 139, 315–324.
- Omar, T., Nehdi, M.L., 2017. Remote sensing of concrete bridge decks using unmanned aerial vehicle infrared thermography. *Automation in Construction* 83, 360–371.
- Pajak, M., 2011. The influence of the strain rate on the strength of concrete taking into account the experimental techniques .
- Sakagami, T., Kubo, S., 2002. Development of a new non-destructive testing technique for quantitative evaluations of delamination defects in concrete structures based on phase delay measurement using lock-in thermography. *Infrared Physics & Technology* 43, 311 – 316.

- Sugimoto, K., Sugimoto, T., Utagawa, N., Kuroda, C., 2019. Detection of resonance frequency of both the internal defects of concrete and the laser head of a laser doppler vibrometer by spatial spectral entropy for noncontact acoustic inspection. *Japanese Journal of Applied Physics* 58, SGGB15.
- Tsuno, K., Kishida, K., 2020. Evaluation of spalling of concrete pieces from tunnel lining employing joint shear model. *Tunnelling and Underground Space Technology* 103, 103456.
- Wakata, S., Hosoya, N., Hasegawa, N., Nisikino, M., 2022. Defect detection of concrete in infrastructure based on rayleigh wave propagation generated by laser-induced plasma shock waves. *International Journal of Mechanical Sciences* , 107039.
- Yasuda, N., 2022. Vibration characteristics of spalling defects in concrete lining. *Tunnelling and Underground Space Technology* 124, 104441.
- Yasuda, N., 2023. Hammering sound of concrete with defects and spalling risk. *Tunnelling and Underground Space Technology* 131, 104789.
- Yasuda, N., Misaki, N., Shimada, Y., Kotyaev, O., 2021. Applicability of non-contact inspection using laser ablation-induced vibration in a reinforced concrete tunnel lining. *Tunnelling and Underground Space Technology* 113, 103977.
- Yasuda, N., Misaki, N., Shimada, Y., Yamaoka, D., 2020. Detection and characteristics estimation of defects in concrete structures using laser ablation-induced vibration. *Tunnelling and Underground Space Technology* 103, 103460.
- Zhu, J., Popovics, J.S., 2007. Imaging concrete structures using air-coupled impact-echo. *Journal of engineering mechanics* 133, 628–640.

Table 1: Material properties of concrete.

Young's modulus (GPa)	Poisson's ratio	Density (kg/m ³)
30	0.20	2300

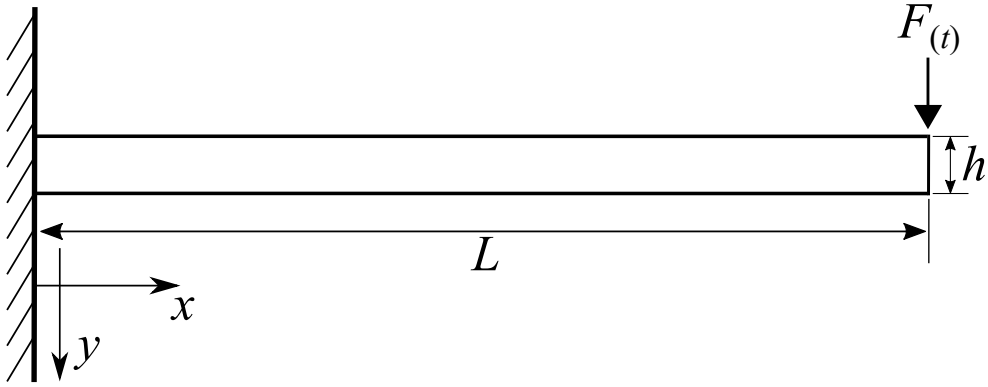


Figure 1: Cantilever beam subject to impact loading.

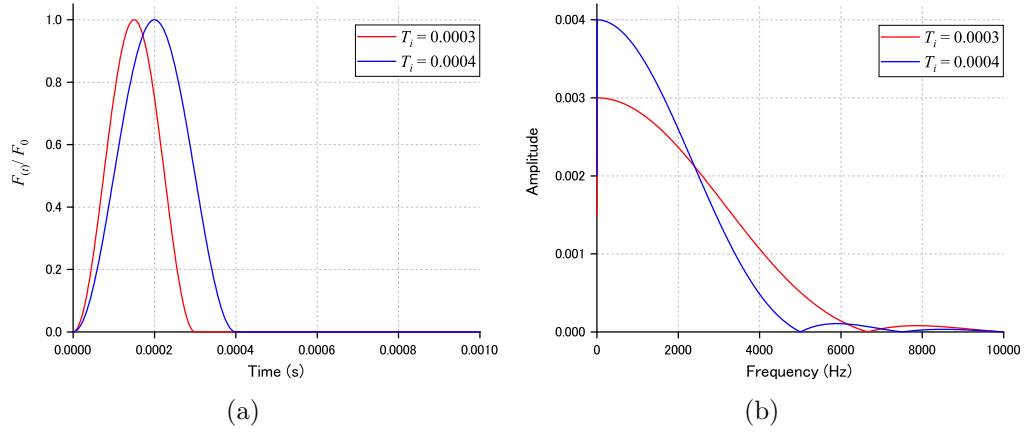


Figure 2: Waveforms of the impact load: (a) time history and (b) Fourier spectra.

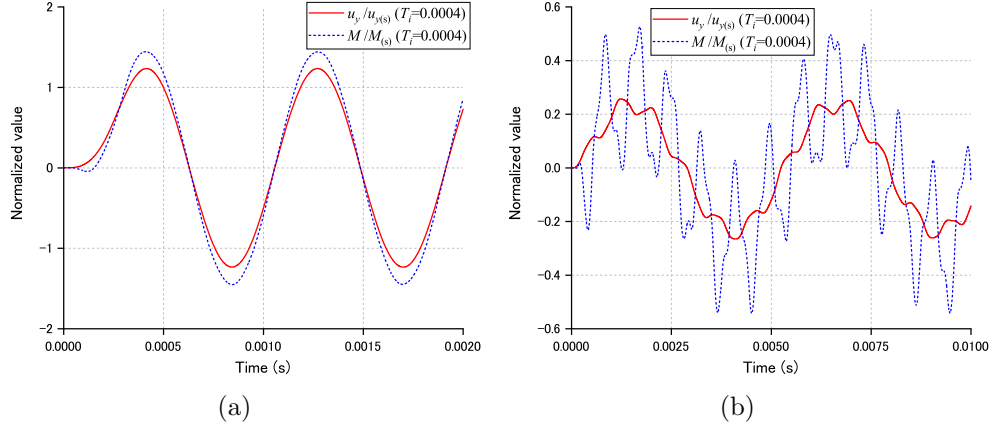


Figure 3: Normalized responses of the cantilever beam for impact loading with (a) $L = 0.1$ m and $h = 0.02$ m and (b) $L = 0.3$ m and $h = 0.03$ m.

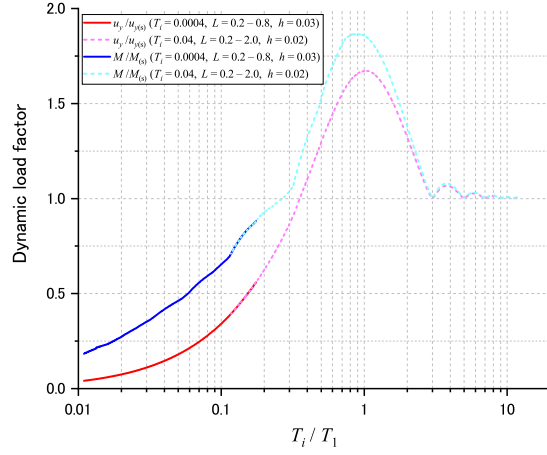


Figure 4: Dynamic load factor of the cantilever beam for impact loading.

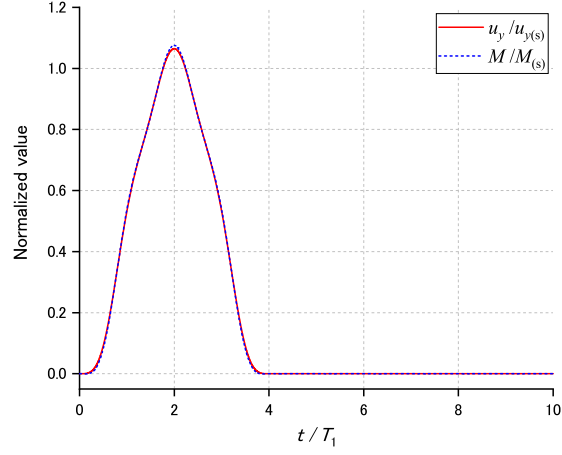


Figure 5: Normalized responses of the cantilever beam for $T_i/T_1 = 4.0$

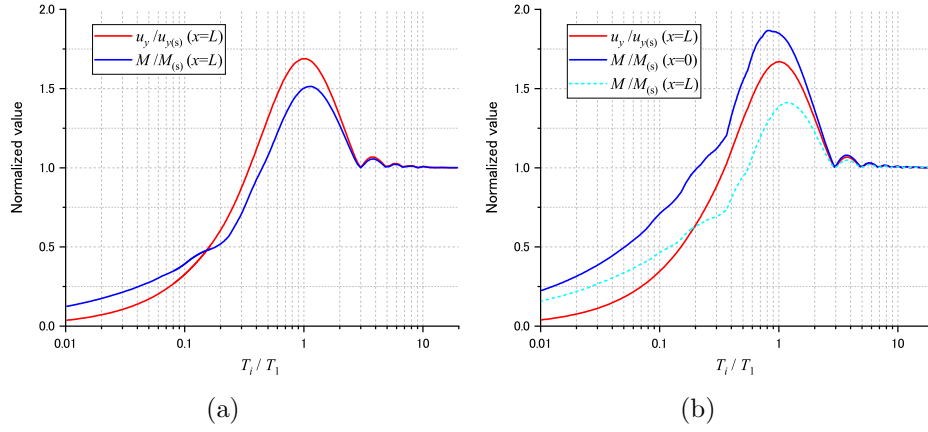
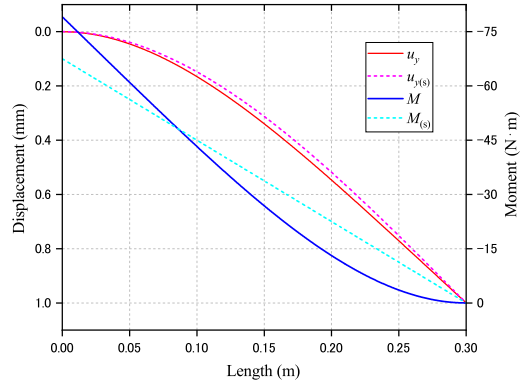
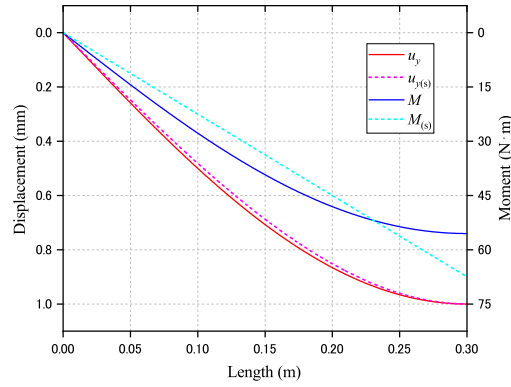


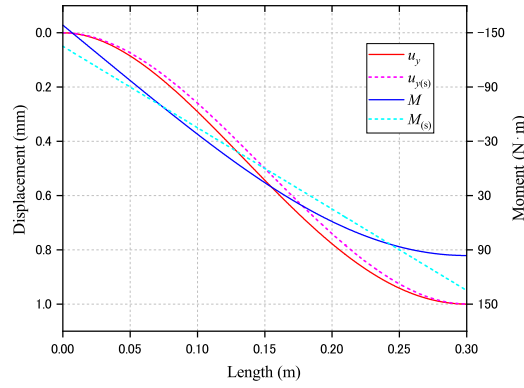
Figure 6: Dynamic load factor of (a) the both-ends-supported beam and (b) the both-ends-fixed beam in impact loading. The length of the beam is $2L$ and the concentrated load is applied at the center of the beam.



(a)



(b)



(c)

Figure 7: Comparison of the displacement and moment distributions of the first vibration mode and the static loading; (a) cantilever beam; (b) both-ends-supported beam; and (c) both-ends-fixed beam. The length of the beam L is 0.3 m and the thickness h is 0.03 m. The maximum displacement is 1 mm.

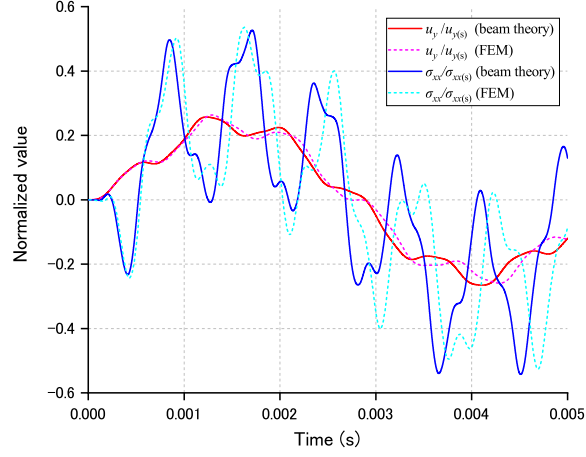


Figure 8: Comparison of the normalized responses of a cantilever beam between the beam theory and FEM. The damping ratio for each mode is zero.

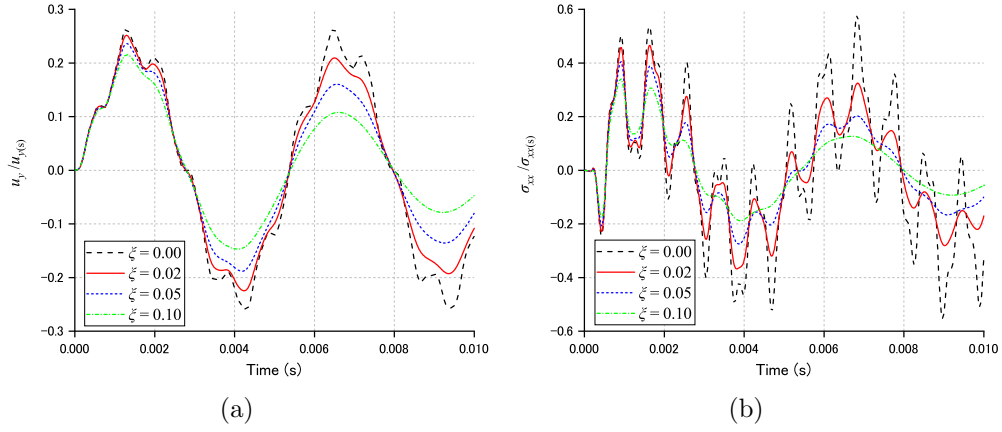


Figure 9: Normalized responses of a cantilever beam for various damping ratios: (a) displacement and (b) axial stress.

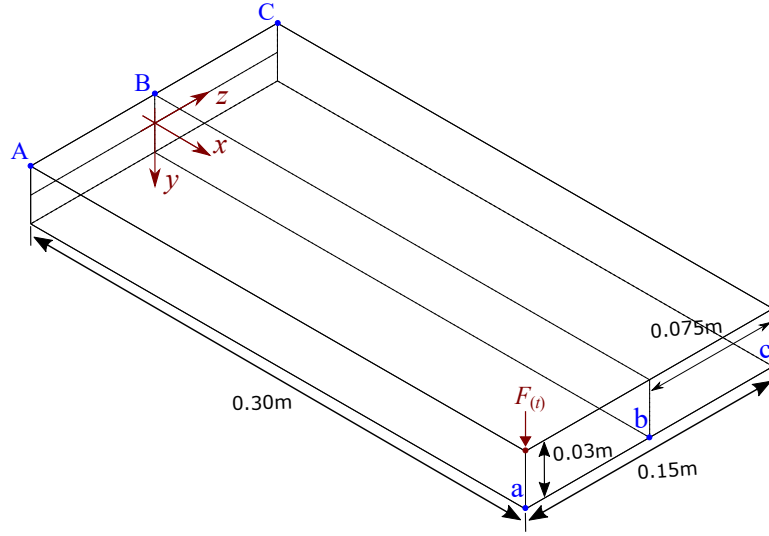


Figure 10: Plate model.

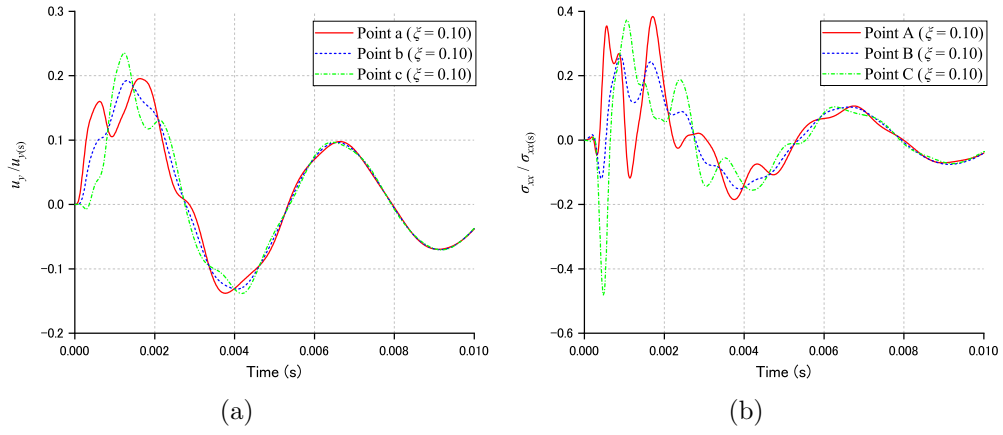


Figure 11: Normalized responses of a plate when the plane of $x = 0$ is completely fixed: (a) displacement and (b) stress.

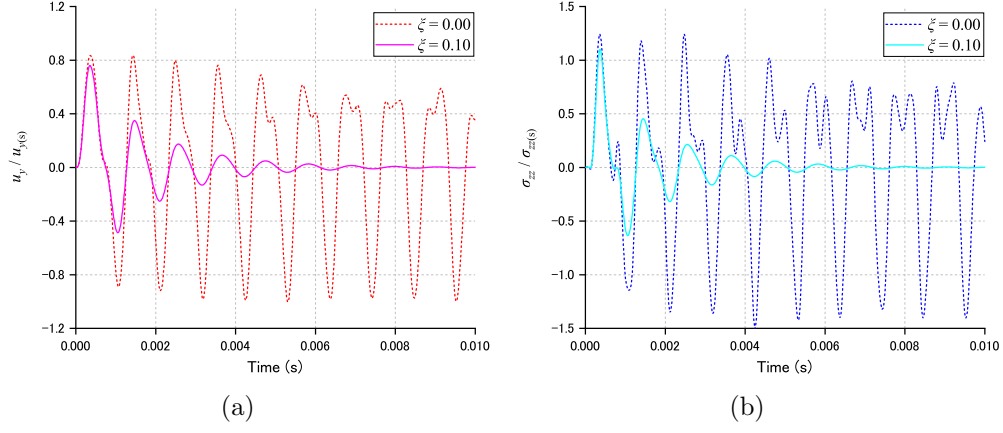


Figure 12: Normalized responses of a plate when the planes of $x = 0$ and $z = 0.075$ m are completely fixed: (a) displacement at point a and (b) stress at point c.

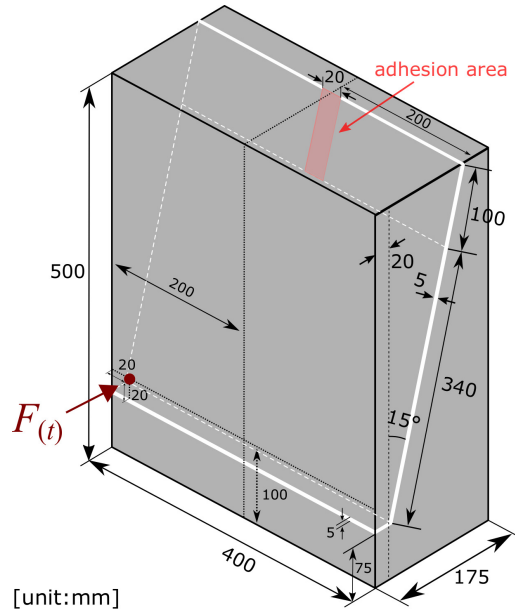


Figure 13: Model of a concrete block with an inclined detachment.

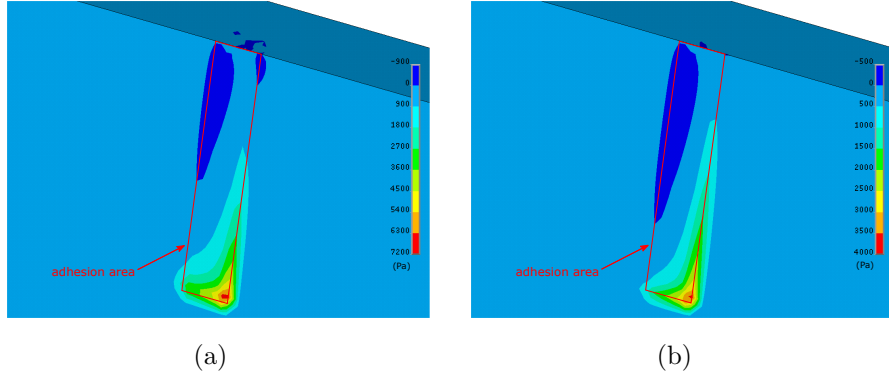


Figure 14: First principal stress distribution in dynamic loading when the stress is a maximum: (a) $\xi = 0.00$ and (b) $\xi = 0.10$.

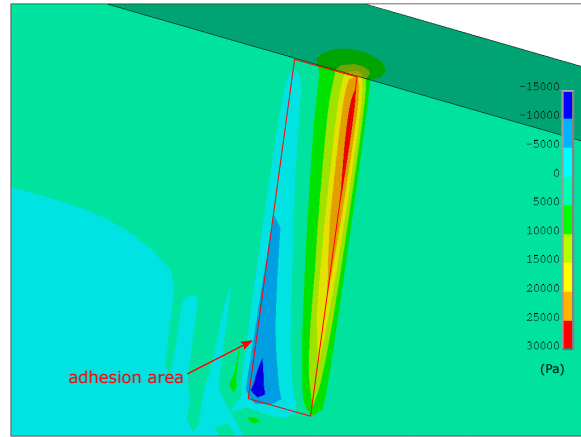


Figure 15: First principal stress distribution in static loading.

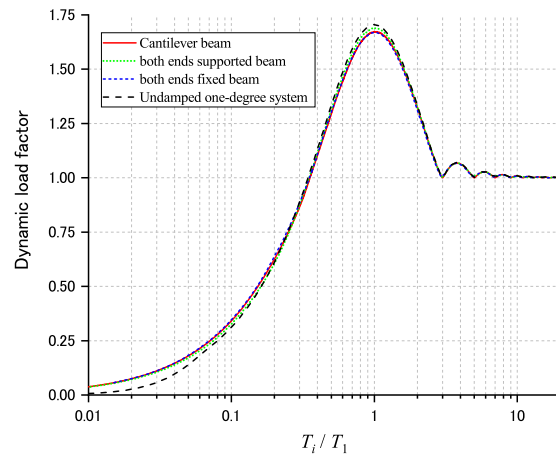


Figure 16: Comparison of the DLF of the displacement of beams and DLF of the undamped one-degree system.