

GOURSAT PROBLEM FOR A MICRODIFFERENTIAL OPERATOR OF FUCHSIAN TYPE AND ITS APPLICATION

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§0. INTRODUCTION.

The Goursat problem in the holomorphic (or the real analytic) category is treated by several authors and studied in depth. Moreover, C. Wagschal [W] extended the problem to the case of a system of integro-differential operators and obtained the Cauchy-Kovalevskaja type (*the unique solvability*) theorem. However, it seems that the study of the Goursat problem is not so satisfactory from the microlocal point of view. Therefore in this article, we treat a microdifferential operator of Fuchsian type with respect to several variables and consider the Goursat problem in the framework of holomorphic (or micro-) functions.

The notion of Fuchsian type (with respect to one variable) was introduced by M. S. Baouendi and C. Goulaouic [Ba-G] for a partial differential operator. This includes non characteristic type as a special case, and the Cauchy-Kovalevskaja type theorem was proved in [Ba-G]. Seeing this result, N. S. Madi [M] generalized Fuchsian type to several variables case

by the name of “a Goursat operator of several Fuchsian variables” and obtained the Cauchy-Kovalevskaja type theorem for the Goursat problem in the framework of holomorphic functions. Note that Y. Laurent-T. Monterio Fernandes [La-MF] and Z. Szmydt and B. Ziemian [Sz-Zi] gave different definitions of Fuchsian type with respect to several variables respectively. On the other hand, succeeding to Baouendi-Goulaouic [Ba-G], many mathematicians have obtained almost sufficient results in Fuchsian type with respect to one variable. For example, H. Tahara [Ta] treated a Fuchsian system in the sense of Volevič and proved the Cauchy-Kovalevskaja type theorem in the complex domain. Further, as an application he obtained the existence and uniqueness theorem on an initial value problem for a Fuchsian hyperbolic system in the framework of hyperfunctions. Moreover, he proved the existence theorem on a homogeneous initial value problem for a Fuchsian microhyperbolic system of microdifferential operators in the framework of microfunctions. On the other hand, T. Ôaku proved the existence theorem on an inhomogeneous initial value problem for a Fuchsian hyperbolic microdifferential operator in [O1] and the uniqueness theorem under the F-mildness condition (but without the hyperbolicity assumption) in [O3] in the framework of microfunctions (cf. [O2]).

In this article, we define a matrix of microdifferential operators of Fuchsian type with respect to several variables as a natural generalization of one variable case due to Tahara [Ta] or non-microlocal case due to Madi

[M]. Moreover, we prove the Cauchy-Kovalevskaja type theorem for the Goursat problem in the space of holomorphic functions under the action of microdifferential operators due to J. M. Bony and P. Schapira [Bo-Sc]. As an application we solve the Goursat problem in the framework of micro-(or hyper-)functions; we prove the existence theorem for sufficiently “regular” initial data under suitable assumptions.

§1. STATEMENT OF MAIN THEOREM.

In this article, we use the following notation: $\mathbb{N}$ denotes the set of natural numbers (not containing 0) and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. For a subset D of some topological space, $[D]$ denotes the closure. For natural numbers $M, N \in \mathbb{N}$, and a linear space L we denote by $\text{Mat}(M \times N; L)$ the space of matrices of size $N \times N$ whose components are in L . Further set

$$\left\{ \begin{array}{l} \text{Mat}(N; L) := \text{Mat}(N \times N; L), \\ L^N := \text{Mat}(1 \times N; L), \\ L^{\oplus N} := \text{Mat}(N \times 1; L). \end{array} \right.$$

In addition, if A has a norm $\| \cdot \|$, for $P = (P^{(\mu, \nu)})_{\mu, \nu=1}^N \in \text{Mat}(M \times N; A)$ we set $\|P\| := \max \{ \|P^{(\mu, \nu)}\|; 1 \leq \mu \leq M, 0 \leq \nu \leq N \}$. For natural numbers $d, n \in \mathbb{N}$, we use coordinates $\tau = (\tau_1, \dots, \tau_d) \in \mathbb{C}^d$ and $z = (z_1, \dots, z_n) \in \mathbb{C}^n$. Moreover for multi-indices $\gamma = (\gamma_1, \dots, \gamma_n)$ and

$\alpha = (\alpha_1, \dots, \alpha_d)$, we set

$$\left\{ \begin{array}{ll} \partial_z^\gamma := \partial_{z_1}^{\gamma_1} \dots \partial_{z_n}^{\gamma_n}, & \partial_\tau^\alpha := \partial_{\tau_1}^{\alpha_1} \dots \partial_{\tau_d}^{\alpha_d}, \\ z^\gamma := z_1^{\gamma_1} \dots z_n^{\gamma_n}, & \tau^\alpha := \tau_1^{\alpha_1} \dots \tau_d^{\alpha_d}, \\ \gamma! := \gamma_1! \dots \gamma_n!, & \alpha! := \alpha_1! \dots \alpha_d!, \\ |\gamma| := \sum_{j=1}^n \gamma_j, & |\alpha| := \sum_{j=1}^d \alpha_j, \end{array} \right.$$

as usual. For vectors $R = (R_1, \dots, R_d)$ and $R' = (R'_1, \dots, R'_d) \in \mathbb{R}^d$, we define an order relation as follows:

$$\begin{aligned} R' &\leq R \stackrel{\text{def.}}{\iff} R'_j \leq R_j \quad \text{for all } j, \\ R' &< R \stackrel{\text{def.}}{\iff} R' \leq R \text{ and } R' \neq R, \\ R' &\prec R \stackrel{\text{def.}}{\iff} R'_j < R_j \quad \text{for all } j. \end{aligned}$$

For a vector $r = (r_1, \dots, r_d) \in \mathbb{R}^d$, we set $[r]_+ := ([r]_+, \dots, [r]_+)$, where $[r_j]_+ = \max\{r_j, 0\}$. We fix $m^{(\nu)} = (m_1^{(\nu)}, \dots, m_d^{(\nu)})$ and $k^{(\nu)} = (k^{(1)}, \dots, k^{(N)}) \in \mathbb{N}_0^d$ with $m^{(\nu)} \geq k^{(\nu)}$ ($1 \leq \nu \leq N$) and set $m = (m^{(1)}, \dots, m^{(N)})$ and $k = (k^{(1)}, \dots, k^{(N)}) \in (\mathbb{N}_0^d)^N$. For any N -tuple of (generalized) functions $f(z, \tau) = {}^t(f_1(z, \tau), \dots, f_N(z, \tau))$, we mean $f = O(\tau^{m-k})$ by

$$\partial_{\tau_j}^i f_\nu|_{\tau_j=0} = 0 \quad (1 \leq j \leq d, 1 \leq \nu \leq N, 0 \leq i \leq m_j^{(\nu)} - k_j^{(\nu)} - 1).$$

Set $\mathbf{1}_d := (1, \dots, 1) \in \mathbb{N}^d$. For a vector $R = (R_1, \dots, R_d) \in \mathbb{R}^d$ with $0 \prec R$, we set $B(R) := \{\tau \in \mathbb{C}^d; |\tau_j| < R_j \ (1 \leq j \leq d)\}$. Let $V \subset \mathbb{C}^n$ be

a relatively compact open neighborhood of the origin and h_0 a positive number. We set

$$U = \{(z; \zeta) \in T^*\mathbb{C}^n; z \in V, \zeta_1 = 1, |\zeta_j| < h_0 (2 \leq j \leq n)\}.$$

We denote the sheaf of rings of microdifferential operators of finite order (resp. of order at most ν) by $\mathcal{E}$ (resp. $\mathcal{E}(\nu)$) as usual.

1.1 Definition. Let $P(z, \tau; \partial_z, \partial_\tau) = (P^{(\mu, \nu)}_{\mu, \nu=1}(z, \tau; \partial_z, \partial_\tau))^N$ be a matrix in $\text{Mat}(N; \Gamma([U \times B(R)]; \mathcal{E}_{\mathbb{C}^{n+d}}))$; that is, each $P^{(\mu, \nu)}$ is a microdifferential operator of finite order defined in some neighborhood of $[U \times B(R)]$. Then, P is said to be of *Fuchsian type with weight (k, m)* (with respect τ -variables) if it has the following form:

$$P^{(\mu, \nu)}(z, \tau; \partial_z, \partial_\tau) = \sum_{0 \leq \alpha \leq m^{(\nu)}} P^{(\mu, \nu)}_\alpha(z, \tau; \partial_z) \partial_\tau^\alpha,$$

where each $P^{(\mu, \nu)}_\alpha$ is a microdifferential operator with holomorphic parameters τ and satisfies the following:

- (1) The order $\text{ord } P^{(\mu, \nu)}_\alpha$ of $P^{(\mu, \nu)}_\alpha$ is at most $|m^{(\nu)}| - |\alpha|$;
- (2) There exist $P^{1, (\mu, \nu)}_\alpha(z, \tau; \partial_z)$ and $P^{2, (\mu, \nu)}_\alpha(z, \tau; \partial_z)$ ($0 \leq \alpha \leq m^{(\nu)}$) such that $\text{ord } P^{1, (\mu, \nu)}_\alpha \leq 0$ and

$$\begin{aligned} P^{(\mu, \nu)}_\alpha(z, \tau; \partial_z) = & \tau^{[\alpha - m^{(\nu)} + k^{(\nu)}]_+} P^{1, (\mu, \nu)}_\alpha(z, \tau; \partial_z) \\ & + \tau^{[\alpha - m^{(\nu)} + k^{(\nu)} + 1_d]_+} P^{2, (\mu, \nu)}_\alpha(z, \tau; \partial_z). \end{aligned}$$

1.2 Remark. (1) The Fuchsian property above is invariant under any coordinate change of z -variables, or more generally an arbitrary quantized contact transformation for $(z; \zeta)$ -variables.

(2) The Fuchsian type defined in Definition 1.1 is a natural generalization of differential operators of Fuchsian type introduced by Madi [M]; that is, if P is a differential operator of Fuchsian type in the sense of Definition 1.1, then P is of Fuchsian type in the sense of Madi. Further if $d = N = 1$, a microdifferential operator of Fuchsian type is nothing but of Fuchsian type defined by Tahara (see [Ta]).

Let $T^{(\nu)} = (T_1^{(\nu)}, \dots, T_d^{(\nu)})$ ($1 \leq \nu \leq N$) be indeterminates and set

$$\tilde{T} := (T^{(1)}, \dots, T^{(N)}).$$

If P is of Fuchsian type with weight (k, m) , we define the *indicial polynomial* of P by

$$\mathcal{I}_P(z; \zeta; \tilde{T}) := \det \left(\sum_{m^{(\nu)} - k^{(\nu)} \leq \alpha \leq m^{(\nu)}} \sigma_0(P_\alpha^{1, (\mu, \nu)})(z, 0; \zeta) \mathcal{I}_\alpha(T^{(\nu)}) \right),$$

where $\mathcal{I}_\alpha(T^{(\nu)}) = \prod_{j=1}^d \mathcal{I}_{\alpha_j}(T_j^{(\nu)})$ with

$$\mathcal{I}_{\alpha_j}(T_j^{(\nu)}) := \begin{cases} T_j^{(\nu)}(T_j^{(\nu)} - 1) \cdots (T_j^{(\nu)} - \alpha_j + 1) & (\alpha_j \geq 1), \\ 1 & (\alpha_j = 0). \end{cases}$$

Let $A(z, \tau; \partial_z)$ be a microdifferential operator of finite order with holomorphic parameters τ defined in a neighborhood of $[U \times B(R)]$. Let

$c \in \mathbb{C}$ and set $\Sigma := \{z \in \mathbb{C}^n; z_1 = c\}$. Let $\Omega \subset V$ be an open convex set and assume that Ω is h_0 - Σ -flat in the sense of Bony-Schapira; that is, if $z \in \Omega$, $w \in \Sigma$ and $h_0|z_j - w_j| \leq |z_1 - w_1|$ ($2 \leq j \leq n$), then it follows that $w \in \Omega \cap \Sigma$. Let $f(z, \tau)$ be a holomorphic function defined on $\Omega \times B(R)$. If $p \in \mathbb{N}$, there exists a unique holomorphic function $g(z, \tau)$ on

$$\begin{cases} \partial_{z_1}^p g(z, \tau) = f(z, \tau), \\ \partial_{z_1}^j g|_{z_1=c} = 0 \quad (0 \leq j \leq p-1). \end{cases}$$

Then, we define $(\partial_{z_1}^{-p})_\Sigma f(z, \tau) := g(z, \tau)$; that is,

$$(\partial_{z_1}^{-p})_\Sigma f(z, \tau) := \int_c^{z_1} \frac{(z_1 - w_1)^{p-1}}{(p-1)!} f(w_1, z') dw_1,$$

where $z' := (z_2, \dots, z_n)$. We write formally

$$A(z, \tau; \partial_z) = \sum_{\gamma_1 \in \mathbb{Z}, \gamma_2, \dots, \gamma_n \in \mathbb{N}_0} A_\gamma(z, \tau) \partial_z^\gamma.$$

Then, applying the argument as in Bony-Schapira [Bo-Sc] regarding τ as holomorphic parameters, we find that

$$\begin{aligned} A_\Sigma f(z, \tau) &:= \sum_{\gamma_1, \dots, \gamma_n \in \mathbb{N}_0} A_\gamma(z, \tau) \partial_z^\gamma f(z, \tau) \\ &+ \sum_{\gamma_1 < 0, \gamma_2, \dots, \gamma_n \in \mathbb{N}_0} A_\gamma(z, \tau) (\partial_{z_1}^{\gamma_1})_\Sigma \partial_{z'}^{\gamma'} f(z, \tau) \end{aligned}$$

is holomorphic on $\Omega \times B(R)$. Let s be a parameter with $0 < s < 1$. We fix a point $z_0 \in \Omega \cap \Sigma$ and set

$$\Omega_s := \{s(z - z_0) + z_0 \in \mathbb{C}^n; z \in \Omega\}$$

Consider the following condition:

[A-1]. There exist a positive constant $C > 0$ and a neighborhood W of $[U]$ such that for any $(z; \zeta) \in [W]$ and $\beta = (\beta^{(1)}, \dots, \beta^{(N)}) \in (\mathbb{N}_0^d)^N$ with $\beta^{(\nu)} \geq m^{(\nu)} - k^{(\nu)}$ ($1 \leq \nu \leq N$)

$$|\mathcal{I}_P(z; \zeta; \beta)| \geq C \prod_{\nu=1}^N (\beta^{(\nu)} + \mathbf{1}_d)^{m^{(\nu)}}.$$

Note that if $N = 1$, then [A-1] is a natural generalization of Madi's condition which is similar to the "Fuchsian ellipticity condition" due to Szmydt-Ziemian [Sz-Zi].

1.3 Theorem. Let P be a matrix of microdifferential operators defined in a neighborhood of $[U \times B(R)]$. Assume that P is of Fuchsian type with weight (k, m) . and satisfies [A-1]. Then, there exist constants $r_0 > 0$ and $\overset{\circ}{R}$ with $0 < \overset{\circ}{R} \leq R$ such that the following hold:

Take arbitrary h and r with $0 < h < h_0$ and $0 < r < r_0$ respectively. Let Ω be any h - Σ -flat open convex subset of V with $\text{dia } \Omega \leq r$, where dia denotes the diameter. Then, there exists a constant δ such that for any $\tilde{R}$ with $0 < \tilde{R} \leq \overset{\circ}{R}$ it follows that for any holomorphic functions $f(z, \tau) = {}^t(f_1(z, \tau), \dots, f_N(z, \tau))$ and $g(z, \tau) = {}^t(g_1(z, \tau), \dots, g_N(z, \tau))$ on $\Omega \times B(\tilde{R})$, there exists a unique holomorphic solution

$$u(z, \tau) = {}^t(u_1(z, \tau), \dots, u_N(z, \tau))$$

of the Goursat problem

$$(G.P.) \quad \begin{cases} P_{\Sigma} u = f, \\ u - g = O(\tau^{m-k}), \end{cases}$$

and each $u_\nu(z, \tau)$ ($1 \leq \nu \leq N$) is holomorphic on

$$\bigcup_{0 < s < 1} \left(\Omega_s \times \left\{ \tau \in B(\tilde{R}); \prod_{j=1}^d |\tau_j| < \delta (1-s)^{|m|} \right\} \right),$$

where $|m| := \sum_{\nu=1}^N \sum_{j=1}^d m_j^{(\nu)}$.

We can prove Theorem 1.3 by applying techniques of [O1] and [W].

1.4 Remark. Assume that P is a differential operator. Then Theorem 1.3 is (essentially) obtained by Madi [M] (cf. [La-MF]).

§2. APPLICATIONS.

Let M be $\mathbb{R}_x^n \times \mathbb{R}_t^d$ with its complexification $X := \mathbb{C}_z^n \times \mathbb{C}_\tau^d = Y \times \mathbb{C}^d$ and π_M the canonical projection $T_M^*X \rightarrow M$. Set $N := \mathbb{R}^n \cong M \cap \{t = 0\} \hookrightarrow M$, $L := X \cap \{\operatorname{Im} z = 0\} = \mathbb{R}^n \times \mathbb{C}^d$, $\tilde{A} := T_L^*X \cong T_N^*Y \times \mathbb{C}^d$ and $\Lambda := T_M^*X \cap \tilde{A}$. We denote the sheaf of microfunctions on T_M^*X (resp. T_N^*Y) by $\mathcal{C}_M$ (resp. $\mathcal{C}_N$) as usual. Further, let $\mathcal{CO}_L$ be the sheaf of microfunctions with holomorphic parameters on $\tilde{A}$; that is,

$$\mathcal{CO}_L := \mu_L(\mathcal{O}_X) \otimes or_{N/Y}[n],$$

where μ_L denotes Sato's microlocalization functor along L and $or_{N/Y}$ denotes the relative orientation sheaf (see [K-Sc] and [S-K-K]). The sheaf $\mathcal{B}_M$ of hyperfunctions on M and the sheaf $\mathcal{BO}_L$ of hyperfunctions with

holomorphic parameters on L are defined by $\mathcal{B}_M := \mathcal{C}_M|_M$ and $\mathcal{BO}_L := \mathcal{CO}_L|_L$ respectively. Let ρ be a natural mapping

$$N \times_M T_M^* X \ni (x, 0; \sqrt{-1}(\langle \xi, dx \rangle + \langle \eta, dt \rangle)) \mapsto (x; \sqrt{-1} \langle \xi, dx \rangle) \in T_N^* Y.$$

Then, we have the following canonical morphisms:

$$\mathcal{CO}_L|_A \hookrightarrow \mathcal{C}_M|_A, \quad \rho!(\mathcal{C}_M|_{N \times_M T_M^* X}) \longrightarrow \mathcal{C}_N.$$

Set $p_0 := (0; \sqrt{-1} dx_1) \in T_N^* Y$ and assume that $P(x, t; \partial_x, \partial_t)$ is a matrix of microdifferential operators of Fuchsian type with weight (k, m) defined in some neighborhood of $\rho^{-1}(p_0)$, then the following morphism is induced:

$$P: \rho!(\mathcal{C}_M|_{N \times_M T_M^* X})_{p_0} \longrightarrow \rho!(\mathcal{C}_M|_{N \times_M T_M^* X})_{p_0},$$

where $\rho!(\mathcal{C}_M|_{N \times_M T_M^* X})_{p_0}$ denotes the stalk at p_0 .

Consider the following condition:

[A-2]. $\det(\sigma_{|m(\nu)|}(P^{(\mu, \nu)})(z, \tau; \zeta, \eta)) = \tau^{\tilde{k}} \tilde{P}(z, \tau; \zeta, \eta)$ for a function $\tilde{P}$ ($\tilde{k} := \sum_{\nu=1}^N k^{(\nu)} \in \mathbb{N}_0^d$) which satisfies the following condition:

There exist positive constants h_0 , M and ν_i with $\nu_i \geq 1$ ($1 \leq i \leq d$) such that $\tilde{P}(z, t; \zeta, \eta)$ never vanishes on the set

$$\{(z, t; \zeta, \eta) \in \mathbb{C}^n \times \mathbb{R}^d \times \mathbb{C}^n \times \mathbb{C}^d; |z|, |t| < h_0,$$

$$|\zeta_j| < h_0 |\zeta_1| \ (2 \leq j \leq n), \ |\operatorname{Im}(\eta_i/\zeta_1)| = \nu_i \lambda \ (1 \leq i \leq d)$$

$$\text{for } \exists \lambda > M \left(\sum_{j=1}^n |\operatorname{Im} z_j| + \sum_{j=2}^n |\operatorname{Im}(\zeta_j/\zeta_1)| \right) \}.$$

2.1 Remark. Condition [A-2] is satisfied if

[A-3]. $\tilde{P}(x, t; \xi, \eta) = \prod_{j=1}^d P_j(x, t; \xi, \eta_j)$ and each P_j is of degree $\sum_{i=1}^N m_j^{(i)}$ and hyperbolic with respect to the direction t_j (cf. Kashiwara-Kawai [K-K]).

2.2 Theorem. Assume that P satisfies [A-1] and [A-2]. Then, for any microfunctions with holomorphic parameters

$$f(x, t), g(x, t) \in \rho_* (\mathcal{CO}_L|_{N \times_M T_M^* X})_{p_0}^{\oplus N},$$

there exists a microfunction

$$u(x, t) \in \rho_! (\mathcal{C}_M|_{N \times_M T_M^* X})_{p_0}^{\oplus N}$$

such that u is a solution of the Goursat problem

$$(G.P.) \quad \begin{cases} P(x, t; \partial_x, \partial_t) u(x, t) = f(x, t), \\ u(x, t) - g(x, t) = O(t^{m-k}). \end{cases}$$

Outline of Proof of Theorem 2.2 is as follows: First, choosing suitable defining functions, we can solve (G.P.) in a complex open set by using Theorem 1.3. Next, we can apply the holomorphic continuation method due to Kashiwara-Kawai [K-K] by assumption [A-1].

2.3 Remark. The author does not know how to prove the uniqueness of $u(x, t)$ in Theorem 2.2 (cf. [O2] and [O3]).

2.4 Corollary. *Let P be a matrix of an analytic differential operators of Fuchsian type defined on a neighborhood of $(x, t) = (0, 0)$. Assume that [A-1] and [A-3]. Then, for any holomorphic hyperfunctions with holomorphic parameters*

$$f(x, t), g(x, t) \in (\mathcal{BO}_L|_M)_0^{\oplus N},$$

there exists a hyperfunction

$$u(x, t) \in (\mathcal{B}_M)_0^{\oplus N}$$

such that u has t as real analytic parameters and is a solution of the Goursat problem

$$(G.P.) \quad \begin{cases} P(x, t; \partial_x, \partial_t) u(x, t) = f(x, t), \\ u(x, t) - g(x, t) = O(t^{m-k}). \end{cases}$$

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