

Homomorphisms of differentiable dynamical systems

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1) In this note we consider the following problems.

Let (M, φ_t) and (N, ψ_t) be differentiable dynamical systems (D.D.S.). Assume that there exists a homomorphism, i.e. differentiable mapping $\pi : M \rightarrow N$ such that $\pi \cdot \varphi_t = \psi_t \cdot \pi$ for all $t \in \mathbb{R}$. Under these assumptions, what relations can exist between the structures of (M, φ_t) and (N, ψ_t) ?

Then we obtain the following results. For the proofs, see [1].

2) Theorem 1. Let (M, φ_t) and (N, ψ_t) be D.D.S.'s and π be a homomorphism of (M, φ_t) to (N, ψ_t) .

If M is compact and the system (N, ψ_t) is minimal, then π is a surjective mapping of maximal rank, and as a consequent of it, M is the total space of a locally trivial fibre space over N , the system (φ_t) preserves the fibres, and the naturally induced system on the base space is isomorphic to (N, ψ_t) .

Theorem 2. Let $\pi : T^m \rightarrow N$ be a homomorphism of a quasi-periodic motion (T^m, τ_t) to D.D.S. (N, ψ_t) , and $r = \text{rank of } \pi$.

Then $\pi(T^m)$, image of π is an r -dimensional invariant submanifold of N , which is homeomorphic to an r -dimensional torus T^r , and the restricted system of (N, ψ_t) to $\pi(T^m) \subset N$, $(\pi(T^m), \psi_t|_{\pi(T^m)})$ is C^0 -isomorphic to some quasi-periodic motion $(T^r, \tilde{\tau}_t)$, i.e. there exists a homeomorphism h of T^r to $\pi(T^m)$ such that

$$h \cdot \tilde{\tau}_t = \psi_t|_{\pi(T^m)} \cdot h \quad \text{for all } t.$$

Here (T^m, τ_t) is called a quasi-periodic motion, when

$T^m = \{ (x^1, x^2, \dots, x^n) : x^i \in \mathbb{R} \pmod{1}, i=1, 2, \dots, n \}$, and

$\tau_t : (x^1, \dots, x^n) \mapsto (x^1 + \omega^1 t, \dots, x^n + \omega^n t) \pmod{1}$, where $\omega^1, \dots, \omega^n$ are rationally independent.

References

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