

Aerodynamic Characteristics of Airfoils with Fractal Structures in Low-Reynolds-Number Regime

Jinyu Li^{1*}, Koji Nagata^{2*} and Tomoaki Watanabe²

^{1*}Department of Aerospace Engineering, Nagoya University, Chikusa, Nagoya, 464-8603, Aichi, Japan.

^{2*}Department of Mechanical Engineering and Science, Kyoto University, Nishikyo, Kyoto, 615-8540, Kyoto, Japan.

*Corresponding author(s). E-mail(s): li.jinyu.g7@s.mail.nagoya-u.ac.jp;
nagata.koji.2y@kyoto-u.ac.jp;

Contributing authors: watanabe.tomoaki.w6@f.mail.nagoya-u.ac.jp;

Abstract

This study investigates the aerodynamic characteristics of airfoils partially equipped with fractal structures (“fractal airfoils”) in the low-Reynolds-number regime. Wind tunnel experiments are conducted using symmetric and frat airfoils under both steady and unsteady conditions. The aerodynamic performance is examined with measurements of the lift and drag forces and flow visualization using the smoke-wire method. The results are compared with airfoils partially equipped with slits to produce the same porosity (“slit airfoils”). The results show that fractal structures significantly influence the flow characteristics around the airfoil and lift and drag forces. The lift-to-drag ratio, C_l/C_d , of the fractal airfoil is higher than that of the slit airfoil for low angles of attack at lower Reynolds number. The fractal parts effectively reduce the occurrence of a dynamic stall in unsteady conditions, resulting in a more stable C_l curve. The results suggest that fractal structures have significant potential in future aviation applications.

Keywords: Airfoil, Aerodynamics, Wind Tunnel Experiments, Fractal

Nomenclature

A_j	=	surface area of a bar at iteration j [mm ²]
AoA	=	angle of attack [deg]
AR	=	aspect ratio of an airfoil (= 3)
B	=	a constant (= 4 in this study) describing the fractal pattern
b	=	airfoil's span length (= 300 mm)
c	=	chord length (= 100 mm)
C_d	=	drag coefficient
C_l	=	lift coefficient
C_{lmax}	=	maximum value of lift coefficient
D_f	=	fractal dimension
F_x	=	drag force [N]
F_y	=	lift force [N]
L_j	=	the length of a bar at iteration j [mm]
N	=	number of fractal iterations (= 3 in this study)
n	=	porosity
N_c	=	number of data points
r	=	dimensionless pitch rate
Re_c	=	chord Reynolds number
Re_w	=	Reynolds number based on the wire diameter
R_L	=	a parameter describing the bar's length
R_t	=	a parameter describing the bar's thicknesses
S	=	projected area of a airfoil [mm ²]
T	=	total length of the fractal unit [mm]
T_p	=	pitching period [s]
t_j	=	the thicknesses of a bar at iteration j [mm]
U_0	=	freestream speed [m/s]
$\dot{\theta}$	=	pitch rate [rad/s]
ν	=	kinematic viscosity [m ² /s]
ρ	=	density [kg/m ³]
σ	=	blockage ratio

1 Introduction

In recent years, the miniaturization and enhancement of electronic devices have led to increased use of small unmanned aerial vehicles. These small vehicles are also called micro air vehicles (MAVs). Such aircraft can be employed in disaster sites, for observation, and in agriculture to reduce labor and time requirements, and are predicted to play an increasingly significant role as a solution to labor shortages. Moreover, Martian and planetary exploration have attracted considerable attention in recent years. To date, exploration has primarily been conducted through artificial satellites and ground rovers, but the use of fixed-wing aircraft for high-speed and extensive exploration is being considered [1]. The small size and low speed of MAVs, as well as the unique atmospheric composition of Mars, produce a low Reynolds number

around the airfoil, ranging from approximately 10^4 to 10^5 . Understanding the aerodynamic characteristics in the low-Reynolds-number regime is therefore crucial for enhancing the performance of MAVs and Martian aircraft. Moreover, understanding the unsteady aerodynamic characteristics is essential because small, lightweight MAVs are susceptible to wind effects.

The unique characteristics of the flow around the airfoil in the low-Reynolds-number regime include laminar separation and the formation of a laminar separation bubble (LSB) [2–4]. Unlike in high-Reynolds-number cases, most of the flow around the airfoil at low Reynolds numbers is laminar. Laminar flows have a smaller velocity gradient within the boundary layer than turbulent flows. This makes it more susceptible to reverse flow due to adverse pressure gradients when a lift force is generated, leading to early separation (laminar separation). Burgmann and Schröder [5] measured the flow field around an SD7003 airfoil in the low-Reynolds-number regime using particle image velocimetry. Their experiments demonstrated that the reattachment process following laminar separation is dominated by the Kelvin–Helmholtz instability, with vortex structures forming at the trailing edge of the separation bubble. Koca et al. [6] measured the flow field around a NACA4412 airfoil in the low-Reynolds-number regime using the smoke-wire method and a hot-film sensor. They showed that the characteristics of the wake behind the airfoil are influenced by laminar separation and that airfoil performance can be improved by removing the LSB. Through numerical analysis, Winslow et al. [7] demonstrated that thin airfoils have a higher lift-to-drag ratio than thick airfoils in the low-Reynolds-number regime. The sharp leading edge of a thin airfoil fixes the separation point at the leading edge, and the rapid transition to turbulent flow and reattachment improves the lift-to-drag ratio.

The exploration of unsteady aerodynamics, particularly in the context of pitching NACA0012 airfoils at low Reynolds numbers, serves as a crucial inquiry for improving the performance and predictability of small UAVs and other similar applications. Kurtulus et al. [8] examined the effects of sinusoidal pitching at a low Reynolds number, utilizing small amplitude oscillations at frequencies of 1 Hz and 4 Hz. The study revealed that lower frequencies maintain a more organized vortex-shedding pattern, whereas higher frequencies induce chaotic and complex vortex behaviors that significantly alter both lift and drag dynamics. Kay et al. [9] focused on the impact of turbulence on the unsteady aerodynamic performance of pitching airfoil, specifically how it influences stall characteristics and lift curves at low Reynolds numbers. Their findings suggest that increased turbulence not only raises the stall angle but also enhances the maximum lift coefficient. Ghamkhar et al. [10] employed wavelet analysis to investigate the transient flow dynamics surrounding a pitching NACA0012 airfoil. This study explores the aerodynamic behavior of the NACA0012 airfoil in pitching motion through wind tunnel experiments at a Reynolds number of 60,000 and reduced frequencies ranging from 0.05 to 0.3. The findings reveal that higher frequencies diminish the maximum lift coefficient due to “apparent mass” effects. Wavelet analysis of the surface pressures and wake velocities shows that the airfoil’s response is less marked at increased frequencies, as indicated by narrower wake widths and weaker trailing edge vortices.

To improve aerodynamic performance, it is important to control the laminar separation and LSB that occur on the airfoil surface. Techniques for controlling the flow on the airfoil surface include blowing air onto the surface and applying suction to the boundary layer. Kamari et al. [11] optimized the position, angle, and flow speed of slits (which either blow out or suck in air) on the SD7003 airfoil in the low-Reynolds-number regime using numerical analysis and genetic algorithms. They reported that blowing and suction improved the lift-to-drag ratio and increased the stall angle. Chng et al. [12] conducted wind tunnel experiments on a co-flow jet wing, which sucks in air on the downstream upper surface of the airfoil via a pump and blows it out near the leading edge. Their results confirmed the effectiveness of the co-flow jet concept at high angles of attack. However, these active boundary layer control techniques require additional energy input through a pump. In the case of MAVs, the weight of the pump relative to that of the aircraft could negatively affect flight performance. Therefore, it is desirable to develop a passive separation control method that does not require additional energy.

Research on passive separation control has considered airfoils with numerous holes through the upper and lower surfaces, effectively rendering the airfoil porous. Hanna et al. [13] carried out experiments on airfoils with minuscule holes covering their entire surface. The nonlinearity of the lift coefficient was improved, as was the lift-to-drag ratio at lower angles of attack. Similarly, Aldheeb et al. [14] experimented on porous airfoils with a slit, reporting a slight decrease in the drag coefficient at low porosity and a reduction in the lift coefficient and lift-to-drag ratio as the porosity increased. In our prior work [15], we utilized numerical computations to explore the flow characteristics and aerodynamics around partially porous airfoils. Our observations indicated that the magnitudes of both the drag and lift forces vary with changes in the internal structure of the porous medium. In addition, the LSB sizes display a degree of variability. However, previous studies have been limited to airfoils with homogeneous materials and uniform transmissivity in the steady state, which restricts the ability to control separation bubbles on the upper surface and hinders the achievement of high aerodynamic performance.

Fractals are geometric shapes in which parts of the shape resemble a reduced-size copy of the whole. Fluid systems that flow past fractal or multiscale objects are widespread in the fields of geology, environmental science, and biology [16–18]. Previous studies [19–26] have shown that the turbulence generated by fractal grids exhibits unique characteristics, including a wider range of length scales, higher intensity, and higher Reynolds numbers compared with the turbulence generated by regular grids. Based on the characteristics of the flow passing through a fractal grid, there has been an increased focus on its utilization in aviation applications. Nedic et al. [27] explored the use of fractal grids in low-noise aircraft spoilers, and reported noise reduction of up to 4 dB with no significant impact on the lift and drag characteristics. Particle image velocimetry measurements have demonstrated the potential for improved spoiler performance through minor design alterations, making fractal spoilers a promising avenue for aircraft noise reduction. However, it is not yet clear how transforming the void part of the airfoil into a fractal shape, as seen in porous airfoils, will affect the flow

characteristics and aerodynamics. Therefore, this study aims to assess the characteristics of airfoils with fractal structures in steady and unsteady conditions through wind tunnel experiments.

In this paper, we assess the aerodynamic performance of airfoils with fractal parts (“fractal airfoils”) in comparison with airfoils containing conventional slits (“slit airfoils”). The assessment focuses on airfoils with a symmetric and flat geometry, as used in previous studies [14, 15]. We present the results of experiments conducted under fixed angles of attack and pitching motions. Aerodynamic measurements allow us to analyze the effects of the shape and porosity of the permeable part (i.e., fractal or slit part) on the airfoils. Flow visualization is achieved using the smoke-wire method.

The remainder of this paper is organized as follows. Section 2 presents a definition of the fractal structure, an introduction to the experimental equipment and models, and verification of the measurements. We investigate each model’s aerodynamic characteristics and visualization results in Sec. 3. Section 4 summarizes the main results of this study.

2 Experimental Setup

2.1 Fractal Structure

A square fractal grid consisting of self-similar fractal elements [20] is considered to constitute a fractal part of an airfoil. The geometric parameters of the square fractal part are illustrated in Fig. 1 and listed in Table 1. A characterization of the fractal grid is achieved through the number of fractal iterations N ; the lengths $L_j (= R_L^j L_0)$ and thicknesses $t_j (= R_t^j t_0)$ of the squares within the plane of the pattern at iteration j , where $j = 0, 1, 2, \dots, N-1$; and the count B^j ($j = 0, 1, 2, \dots, N-1$) of patterns at each iteration. B^j refers to the number of identical patterns generated at the j -th iteration of the fractal grid. This number dictates how many times the pattern is repeated at each iteration level. In other words, at each iteration level of the fractal grid, the pattern is repeated B^j times. Following the previous experiments on the fractal grid [20], we used $B = 4$ in all the fractal grids used here. In these interpretations, R_L symbolizes the ratio of successive square lengths, while R_t corresponds to the ratio of successive square thicknesses. L_0 and t_0 refer to the size and the thickness of the largest square, respectively. The scaling exponent, defining the fractal perimeter, is a fractal dimension given by

$$D_f = \frac{\log B}{\log(1/R_L)} \quad (1)$$

where $1 \leq D_f \leq 2$. The blockage ratio σ for the fractal grid is computed as the proportion of the solid area to the total area of the grid, T^2 . The solid area A of the fractal grid is the accumulated area A_j encompassed by each iteration j , with

$$A_j = L_j t_j B^j \quad (2)$$

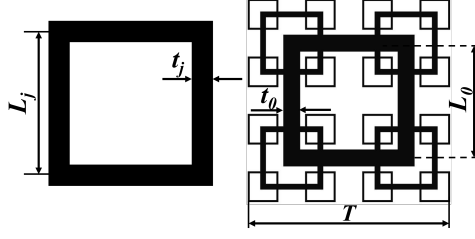


Fig. 1 Geometric parameters of the square fractal structure.

Table 1 Shape parameters.

n	0.3	0.6
N	3	3
D_f	2	2
L_0 (mm)	19.44	19.44
t_0 (mm)	3.24	1.8
R_L	0.5	0.5
R_t	0.6	0.6
T (mm)	36	36

We have

$$A = 4 \sum_{j=0}^{N-1} A_j = 4 \sum_{j=0}^{N-1} L_0 t_0 (R_t R_L B)^j = 4 L_0 t_0 \frac{(1 - (B R_t R_L)^N)}{(1 - B R_L R_t)} \quad (3)$$

and so the blockage ratio can be simplified as [20]

$$\sigma = \frac{4 L_0 t_0}{T^2} \frac{(1 - (B R_t R_L)^N)}{(1 - B R_L R_t)} \quad (4)$$

For consistency with previous research, we utilize the porosity

$$n = 1 - \sigma \quad (5)$$

to characterize the airfoil model. The porosity n , which indicates the ratio of the void to the total area, is set to 0.3 and 0.6.

2.2 Wind Tunnel and Airfoil Model

The experiments were conducted in the low-speed Göttingen wind tunnel at Nagoya University. The wind tunnel features an elliptical outlet with dimensions of 900×750 mm², and the test section is open. The maximum motor output is 22 kW, and the wind speed U_0 can be controlled smoothly in the range of 0–35 m/s through inverter control.

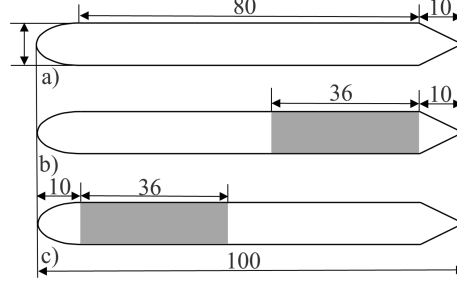


Fig. 2 Schematic of the airfoil profiles: a) flat airfoil, b) flat airfoil with a fractal or slit part located near the trailing edge, and c) flat airfoil with a fractal or slit part located near the leading edge. All units are in mm.

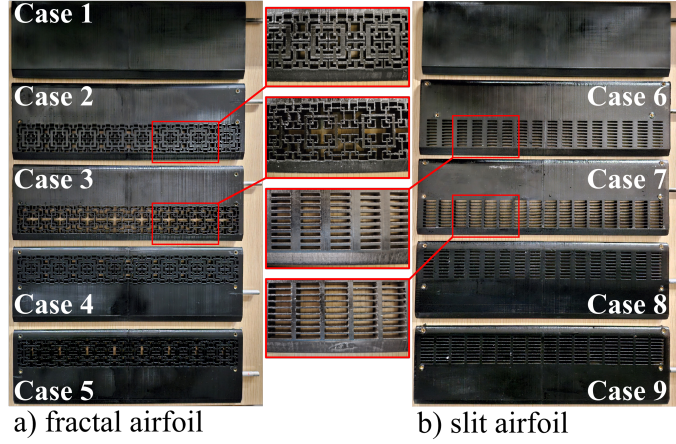
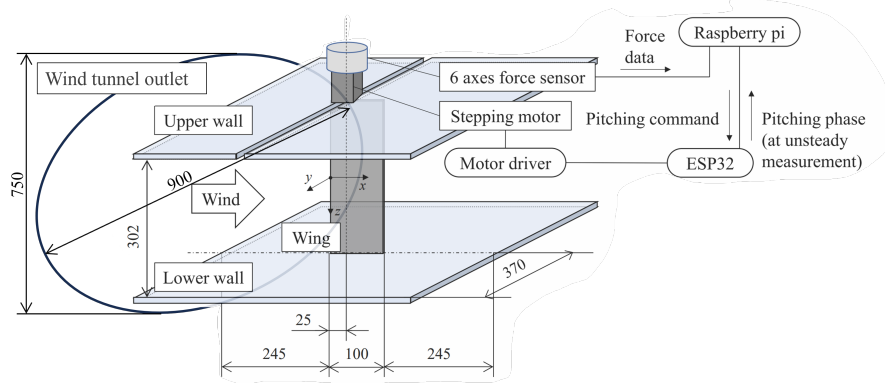


Fig. 3 Airfoil models with different permeable structures: a) fractal airfoil and b) slit airfoil. The flat airfoil without permeable structures is shown on the top of a) and b).

Experiments were conducted on symmetric flat airfoils with and without permeable structures. The shape of the airfoils is based on our previous computational model [15]. The permeable part consists of either fractal structures or slits. Figures 2 and 3 show the airfoil schematic and models used in the experiments. The experimental models are summarized in Table 2. All airfoils have a span of $b = 300$ mm, chord length of $c = 100$ mm, and aspect ratio of $AR = 3.0$, making them rectangular. The thickness is $h = 10$ mm. The leading edge is an ellipse with a major axis of 10 mm and a minor axis of 5 mm. The length of the triangular part of the trailing edge is 10 mm. We fabricated the airfoil model using PLA resin with a 3D printer (UltiMaker S3). Black filament was employed to suppress light reflection on the airfoil surface during visualization. As shown in Fig. 2, the permeable structure is located close to either the trailing edge or leading edge of the airfoil. Two types of permeable structures were utilized: the fractal structure shown in Fig. 3a and the slit configuration in Fig. 3b. The

Table 2 Experimental models.

Case	Airfoil	Location of the permeable part	n
1	Flat	–	–
2	Fractal	Trailing edge	0.3
3	Fractal	Trailing edge	0.6
4	Fractal	Leading edge	0.3
5	Fractal	Leading edge	0.6
6	Slit	Trailing edge	0.3
7	Slit	Trailing edge	0.6
8	Slit	Leading edge	0.3
9	Slit	Leading edge	0.6

**Fig. 4** Overview of the experimental apparatus. All units are in mm.

fractal part comprises eight $36 \times 36 \text{ mm}^2$ square structures, resulting in dimensions of $36 \times 288 \text{ mm}^2$.

2.3 Measurements

Figure 4 shows an overview of the experimental setup. To suppress the generation of wingtip vortices and enhance the two-dimensionality of the flow, acrylic plates were installed in parallel, with a spacing of 302 mm between the upper and lower sections of the flow path. The lower acrylic plate was transparent, while the upper acrylic plate had a black coating. The airfoil was placed vertically between the acrylic plates with a gap of 1 mm. The airfoil model was attached to a stepping motor (Oriental Motor, PKP246MD15) through a coupling, allowing it to rotate in the pitch direction. The stepping motor was driven by a motor driver (Pololu 36v4).

We utilized a strain gauge-type 6-axis force sensor (Reptronino, SFS080YA500U6) to measure the aerodynamic forces acting on the airfoil model. This sensor is equipped with an amplifier and an AD converter, allowing direct retrieval of values for translational forces and moments via serial communication. We connected the sensor to a

Raspberry Pi 3 Model B to measure the aerodynamics. The Raspberry Pi was connected to an ESP32-wrover-E motor driver, ensuring synchronization between motor rotation and measurement timing.

For the steady-state measurements, the aerodynamic force acting on the airfoil was measured using a force sensor while the angle of attack (AOA) was held constant. Measurements were taken at the chord Reynolds number Re_c ($= cU_0/\nu$: ν is the kinematic viscosity) = 35,000 and 50,000 covering AOAs ranging from 0° to 28° in increments of 1° . Data was acquired at a sampling frequency of 1,200 Hz, with a total of 12,000 data points collected. The aerodynamic coefficients in the instantaneous state were calculated by making the measured translational forces in the x and y directions (F_x and F_y , respectively) dimensionless using the following relations:

$$C_l = \frac{2F_y}{\rho U_0^2 S}, \quad C_d = \frac{2F_x}{\rho U_0^2 S} \quad (6)$$

where C_l is the lift coefficient, C_d is the drag coefficient, ρ is the fluid density, and S is the projected area of the airfoil, respectively. In the case of small aircraft, weight (which essentially corresponds to the area of the solid part) becomes a significant factor. When contemplating the application of such aircraft, comparisons based on a per-unit weight basis are appropriate. For the context of this research, the area S is defined as the projected area of the solid portion of the airfoil. The measurements were conducted five times to reduce the statistical error.

For unsteady experiments, measurements were made using a force sensor with the airfoil pitching at a constant angular velocity. Measurements were conducted at $Re_c = 50,000$ for AOAs ranging from -24° to 24° . The pitch rate $\dot{\theta}$ [rad/s] was made dimensionless using the dimensionless pitch rate r as defined below, with measurements for $r = 0.0014, 0.0028, 0.0056$, and 0.0102 . The airfoil was pitched repeatedly at the angular velocity derived from the equation

$$r = \frac{\dot{\theta} c}{2U_0} \quad (7)$$

When the pitch rate is $\dot{\theta}$, the pitching period T_p is defined by

$$T_p = \frac{\dot{\theta}}{2\pi} \quad (8)$$

The number of data points N_c recorded in one pitching cycle is given by

$$N_c = 1200T_p \quad (9)$$

The phase-averaged lift coefficient is obtained at each phase. The results presented in this paper are the average values over five cycles.

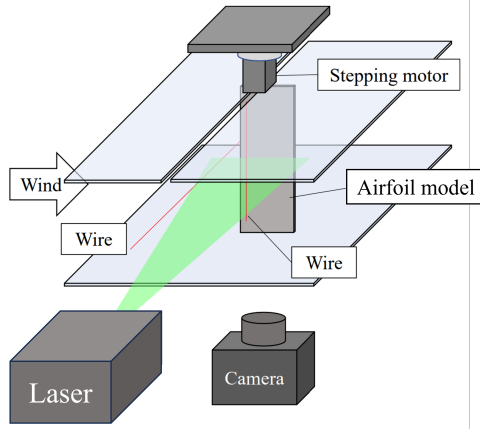


Fig. 5 Overview of the airfoil cross-sectional visualization experiment using the smoke-wire method.

2.4 Visualization

Figure 5 illustrates the visualization experiment in the airfoil cross-sectional direction using the smoke-wire method. A Nichrome wire was utilized as the heating wire. The turbulence behind the wire becomes negligible for the Reynolds number based on the wire diameter, Re_w , is smaller than 40 [28]. Therefore, Re_w was set to 35. A wire perpendicular to the span was positioned 85 mm upstream from the airfoil model. In addition, a wire parallel to the span was placed $0.01c$ away from the airfoil surface at either $x/c = 0.08$ on the upper side or $x/c = 0.08$ on the lower side or $x/c = 0.52$ on the lower side of the airfoil. As the wire expands when heated, a spring was attached to the end of the wire to prevent it from sagging. The turbulence caused by the wire in the direction parallel to the span does not affect the boundary layer. The Nichrome wire was heated by a stabilized power supply, and the timing of the heating was adjusted using an Arduino so that smoke from the two wires would appear simultaneously. Glycerin was used as the smoke agent and was manually applied to the wire for each shot. A continuous-light laser (Dantec Dynamics, Raypower2000, maximum output 2 W) was used as the light source. The laser passed through a cylindrical lens to create a laser sheet and was irradiated from a direction perpendicular to the span. A digital camera (Sony, $\alpha 7s$) with a single focus lens (Zhong Yi Optics, Creator; focal length 85 mm, F2.0) was installed facing upwards at the bottom of the flow path, and the images were recorded through an acrylic board.

2.5 Validation

To confirm the validity of the aerodynamic measurements, we compared our measurements with those reported by Anh et al. [29]. For this purpose, the measurements are conducted with a NACA0012 airfoil. The NACA0012 airfoil was fabricated with the 3D printer and has a span of $b = 300$ mm, chord length of $c = 100$ mm, and aspect ratio of $AR = 3.0$. Figure 6 shows the constant lift coefficient C_l and the phase-averaged lift coefficient when pitched up at a constant angular velocity with $Re_c = 50,000$. The unsteady results measured in this study were averaged from approximately 180

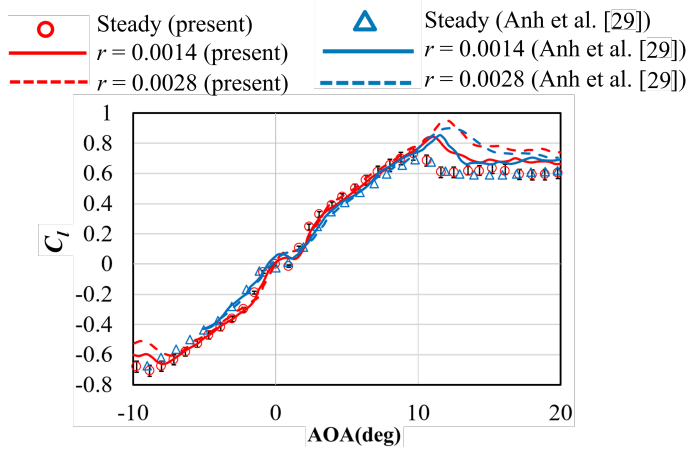


Fig. 6 Comparison of lift coefficient for the NACA0012 airfoil.

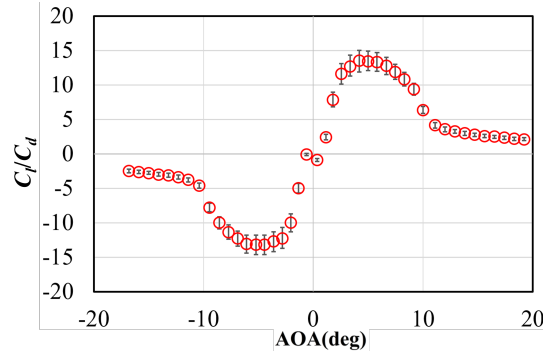


Fig. 7 The uncertainty estimatin of lift-to-drag ratio for the NACA0012 airfoil.

data points for each phase. The uncertainty ranges were shown by error bars for the steady case, which were calculated based on the errors in the time-averaged aerodynamic force, air density, main flow velocity, and airfoil's surface area. For both the steady and unsteady cases, the present results agree well with Anh et al. [29]. The small difference from the literature can be attributed to differences in the wind tunnel configurations. The match between the nonlinear lift slope at low AOAs and the stall characteristics indicates that the aerodynamic coefficients measured in this study can be considered valid. Figure 7 shows the lift-to-drag ratio for the steady case with the uncertainty ranges. The primary source of error in C_l/C_d is in C_d , which is due to the drag measured in this study being small compared to the force sensor's rating. With a sufficiently large number of measurements and minimal impact from variability in a single measurement, it is possible to evaluate the characteristics of changes in aerodynamic coefficients with respect to the angle of attack.

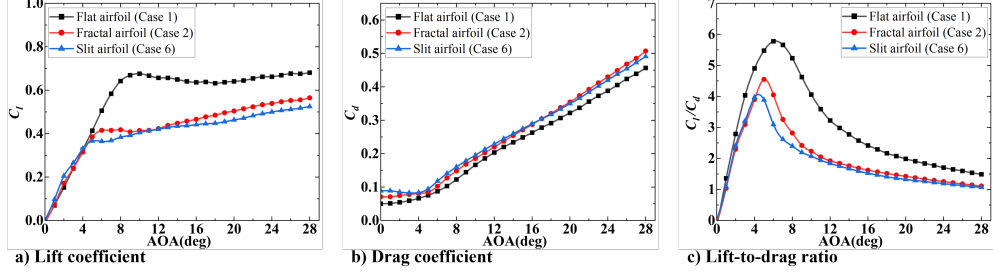


Fig. 8 Aerodynamic coefficients with the fractal or slit part located near the trailing edge at $Re_c = 35,000$ for $n = 0.3$.

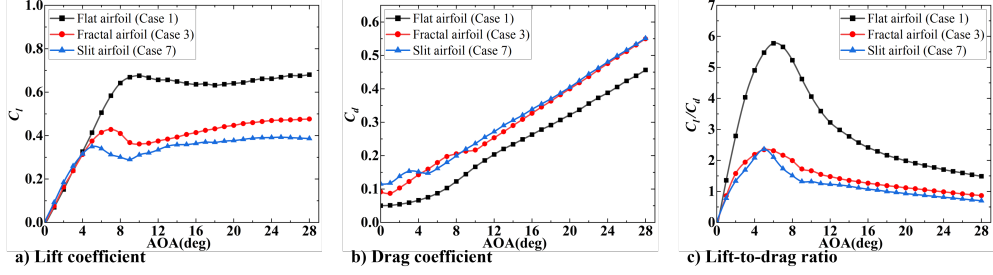


Fig. 9 Same as Fig. 8, but for $n = 0.6$.

3 Results and Discussion

3.1 $Re_c = 35,000$

In this section, we examine the effects of the fractal structure in comparison with the slit structure at $Re_c = 35,000$. Figures 8 and 9 show the results for the fractal or slit part located near the trailing edge for $n = 0.3$ and $n = 0.6$, respectively. Conversely, Figs. 10 and 11 represent configurations where the fractal or slit part is located near the leading edge. As illustrated in Fig. 8a, for AOAs less than 4° , the lift coefficient C_l of the slit airfoil exceeds those of the flat and fractal airfoils with a pronounced nonlinearity. As the AOA exceeds 4° , C_l continues to increase for the flat airfoil, reaching its peak at 9° , whereafter a mild stall occurs. Subsequently, C_l increases slightly from 18° to 28° . For the slit airfoil, C_l ceases its rapid increase from $AOA = 5^\circ$ onwards, maintaining a slow-growth trend up to 28° with no evident stall. For the fractal airfoil, C_l increases swiftly until $AOA = 6^\circ$, then slowly rises from $AOA = 10^\circ$, and consistently exceeds that of the slit airfoil. Figure 8b presents the drag coefficient C_d , indicating that both the fractal and slit airfoils consistently exhibit higher values than the flat airfoil across all AOAs. Between $AOA = 0^\circ$ and 16° , C_d is slightly higher for the slit airfoil than the fractal airfoil, but the trend reverses thereafter. Notably, C_d for the slit airfoil exhibits a decreasing trend from $AOA = 0^\circ$ to 4° . The lift-to-drag ratio results in Fig. 8c indicate that both the fractal and slit airfoils produce

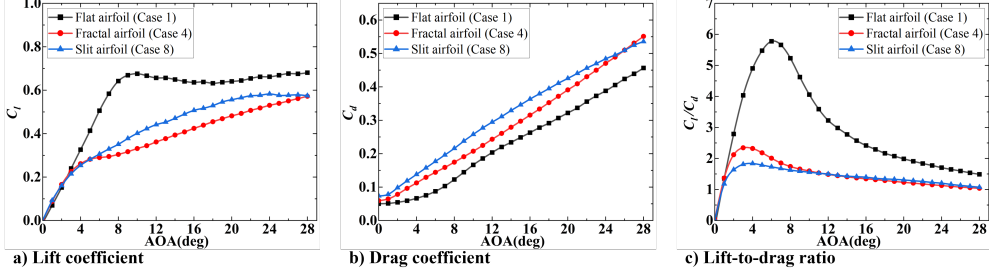


Fig. 10 Aerodynamic coefficients with the fractal or slit part located near the leading edge at $Re_c = 35,000$ for $n = 0.3$.

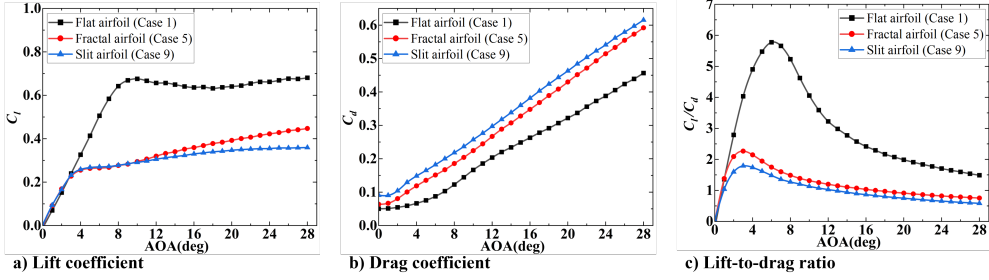


Fig. 11 Same as Fig. 10, but for $n = 0.6$.

consistently lower values than the flat airfoil across all AOAs. Between $AOA = 0^\circ$ and 4° , C_l/C_d is generally equivalent for the fractal and slit airfoils. The slit result peaks at $AOA = 4^\circ$, while the fractal result continues to grow until $AOA = 5^\circ$ and exhibits a higher peak value compared with the slit airfoil. As the AOA increases further, C_l/C_d for the fractal airfoil consistently surpasses that of the slit airfoil.

When the porosity n increases to 0.6, Fig. 9a reveals that C_l is greater in the slit airfoil than in the flat and fractal airfoils for AOAs ranging from 0° to 4° . Compared with the case of $n = 0.3$, the nonlinearity of C_l for the slit airfoil diminishes within this AOA range. The slit airfoil exhibits a stall in the AOA range of 5° – 9° , after which C_l gradually increases until $AOA = 24^\circ$, then slowly decreases toward $AOA = 28^\circ$. Compared with the case of $n = 0.3$, C_l is lower for all AOAs. For the fractal airfoil, C_l grows until $AOA = 7^\circ$. The general trend in C_l is similar to that for $n = 0.3$ for all airfoils, but the difference between the fractal and slit airfoils is more significant for $n = 0.6$. Figure 9b shows that the difference in C_d between the flat and fractal or slit airfoils is greater than in the $n = 0.3$ case. This results in a reduction in C_l/C_d , as shown in Fig. 9c. For all AOAs, C_l/C_d is consistently higher for the fractal airfoil than for the slit airfoil, as in the $n = 0.3$ case.

Figure 10a presents the results for C_l when the fractal or slit part is located near the leading edge. For $n = 0.3$, C_l is consistent in both the fractal and slit airfoils between $AOA = 0^\circ$ and 5° . Subsequently, within the AOA range from 5° to 22° , C_l

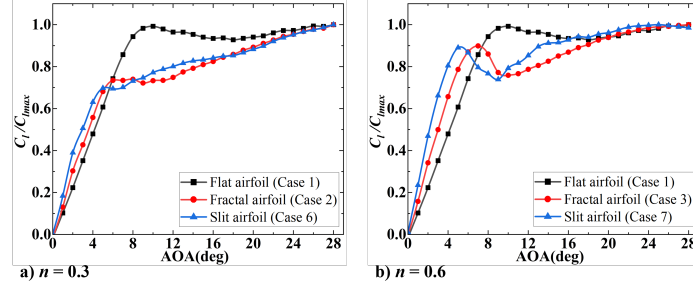


Fig. 12 Characteristics of the normalized lift coefficient with the fractal or slit part located near the trailing edge at $Re_c = 35,000$.

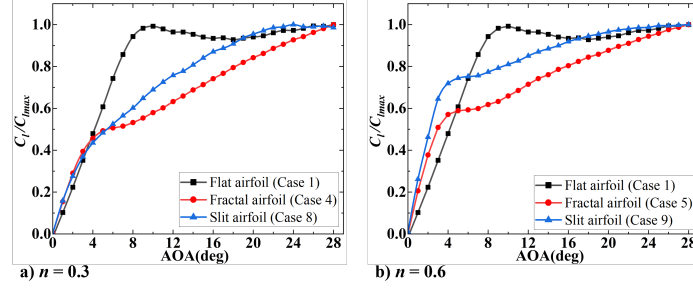


Fig. 13 Characteristics of the normalized lift coefficient with the fractal or slit part located near the leading edge at $Re_c = 35,000$.

increases at an accelerated rate in the slit airfoil, resulting in a higher value than for the fractal airfoil. A mild stall event occurs at $AOA = 22^\circ$, leading to a slight decrease in C_l . For the fractal airfoil, the growth of C_l decelerates between $AOA = 5^\circ$ and 8° . Thereafter, from 8° to 28° , C_l exhibits a rapid linear increase. No stall is observed at any AOA for the fractal airfoil. The results for C_d in Fig. 10b indicate that the slit airfoil consistently exhibits higher values from 0° to 26° . This results in the fractal airfoil having a larger C_l/C_d between 0° and 10° , as shown in Fig. 10c.

When the porosity $n = 0.6$, the fractal and slit airfoils have nearly identical C_l values from $AOA = 0^\circ$ to 10° , as shown in Fig. 11a. In both cases, the rate of increase in C_l starts to slow after $AOA = 3^\circ$. The slit airfoil subsequently maintains this gradual growth trend, whereas the fractal airfoil sees a slight acceleration in its growth rate from $AOA = 10^\circ$, eventually displaying a linear increase. The results for C_d in Fig. 11b reveal that the slit airfoil produces consistently higher values across all AOAs. Consequently, C_l/C_d is smaller for the slit airfoil than the fractal airfoil, as depicted in Fig. 11c.

Figures 12 and 13 illustrate the variations in the lift coefficients normalized by their maximum values, C_l/C_{lmax} , with respect to the AOA. A crucial point on the curve is where $C_l/C_{lmax} = 1$, signifying the AOA at which maximum lift is achieved. From the results for the flat airfoil, it is evident that C_l initially increases linearly. The rate of increase decelerates as it approaches the maximum lift before a minor decrease occurs. This trend is followed by a slight increase around $AOA = 18^\circ$. Figure 12a shows that

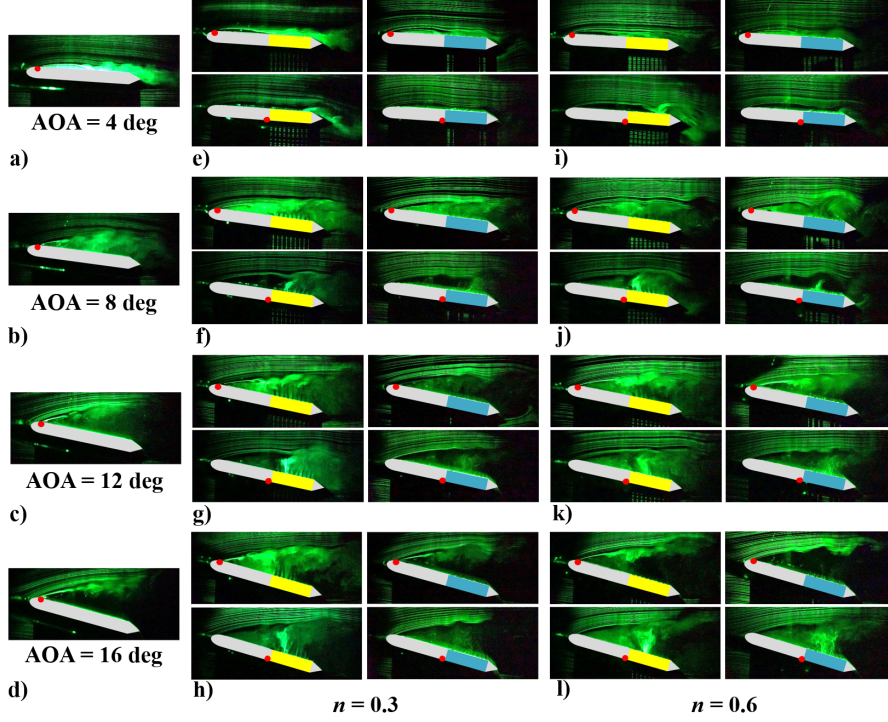


Fig. 14 Visualization results for different AOAs with the fractal or slit part located near the trailing edge at $Re_c = 35,000$. Panels a), b), c), and d) represent the case for the flat airfoil without permeable structures at AOAs = 4° , 8° , 12° , and 16° , respectively; Panels e), f), g), and h) represent the case for $n = 0.3$ at AOAs = 4° , 8° , 12° , and 16° , respectively; panels i), j), k), and l) represent the case for $n = 0.6$ at AOAs = 4° , 8° , 12° , and 16° , respectively. Yellow indicates the slit part and blue represents the fractal part.

the variations in C_l/C_{lmax} are similar for the fractal and slit airfoils. Notably, there is a minor alteration between $AOA = 4^\circ$ and 8° , and then the coefficient steadily ascends to achieve the maximum lift at $AOA = 28^\circ$. As demonstrated in Fig. 12b for $n = 0.6$, both the fractal and slit airfoils exhibit a swift decrease after reaching $C_l/C_{lmax} = 0.9$. This is followed by a continuation of the increase from $AOA = 8^\circ$. When the fractal or slit part is located near the leading edge, the slope of the curves tends to decelerate around $AOA = 4^\circ$ for both airfoils, as depicted in Figs. 13a and b. Then, both the fractal and slit airfoils exhibit the constant increase in C_l/C_{lmax} toward their maxima at around $24 \sim 28^\circ$.

Figures 14 and 15 show the visualization results. The red dots indicate the position of the wire on the airfoil surface, from which the smoke is generated. Note that the wire is placed $0.01c$ away from the airfoil surface. Figure 14 presents the results when the fractal or slit part is located near the trailing edge. At $AOA = 4^\circ$, a separation bubble appears on the flat airfoil (Fig. 14a). The separation occurs at the leading edge and then reattachment occurs at $x/c = 0.57$. The results for $n = 0.3$ in Fig. 14e show that both the fractal and slit airfoils undergo separation at the leading edge and reattachment just upstream of the fractal or slit part. However, with the wire placed

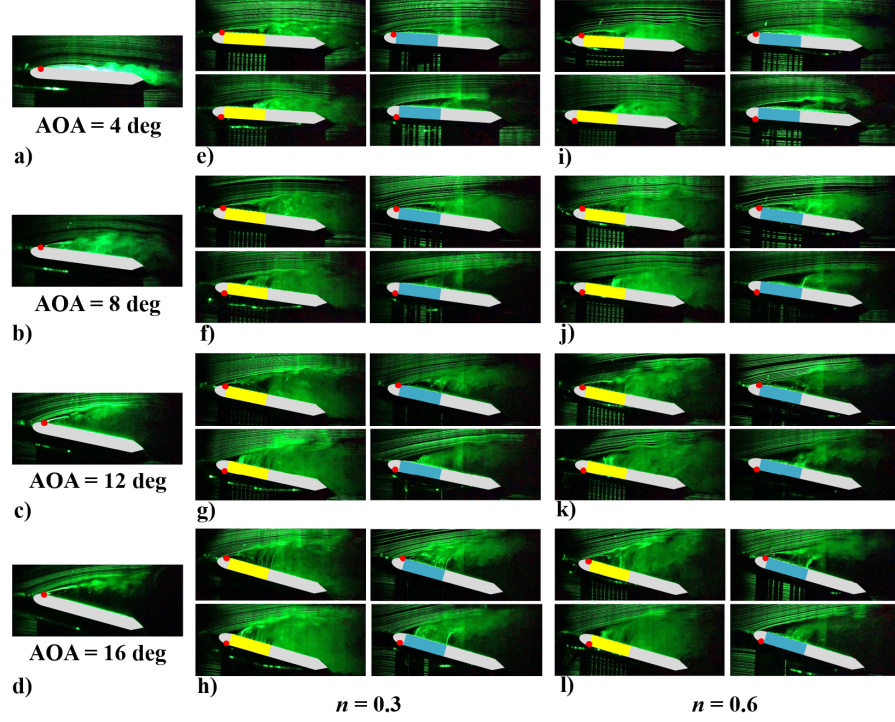


Fig. 15 Visualization results for different AOAs with the fractal or slit part located near the leading edge at $Re_c = 35,000$. Panels a), b), c), and d) represent the case for the flat airfoil without permeable structures at AOAs = 4° , 8° , 12° , and 16° , respectively; Panels e), f), g), and h) represent the case for $n = 0.3$ at AOAs = 4° , 8° , 12° , and 16° , respectively; panels i), j), k), and l) represent the case for $n = 0.6$ at AOAs = 4° , 8° , 12° , and 16° , respectively. Yellow indicates the slit part and blue represents the fractal part.

under the airfoil, the flow from the lower surface appears at the slit part, disrupting the flow at the trailing edge. No such flow appears in the fractal airfoil results. Note that the laser also illuminates the flow just under the fractal or slit part. For $n = 0.6$ in Fig. 14i, more flow from the lower surface can be seen near the trailing edge of the slit part. A weak flow occurs from the lower side of the fractal airfoil. At $AOA = 8^\circ$, significant separation occurs at the leading edge of the flat airfoil, and no reattachment can be confirmed. Observing the results for $n = 0.3$ in Fig. 14f, there is a strong flow from the lower side, with a clear slit pattern of the smoke on the upper side of the slit airfoil. Only a relatively weak flow can be observed on the fractal airfoil. One possible reason for the smaller C_l is that the flow may accelerate on the lower surface of the slit airfoil, causing a reduction in pressure. As the flow disturbance is more extensive in the slit airfoil, this could be the reason for the larger C_d . Examining the results for $n = 0.6$ in Fig. 14j, the slit-shaped flow appears indistinct for the slit airfoil. The unique fractal geometry gives rise to a distinct flow pattern for the fractal airfoil, with a pronounced vortex evident at the trailing edge. For $AOA = 12^\circ$ and 16° , large-scale flow separation occurs in all cases, and similar behaviors can be confirmed for the same airfoil.

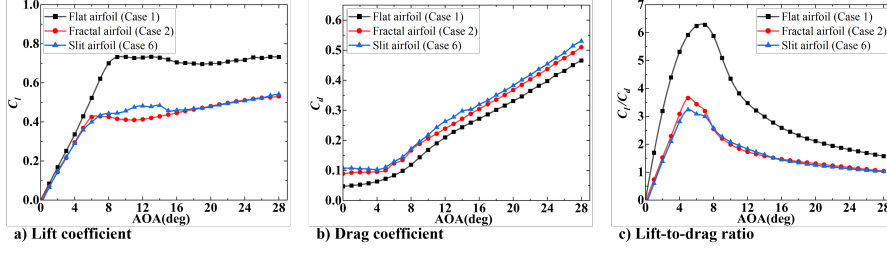


Fig. 16 Aerodynamic coefficients with the fractal or slit part located near the trailing edge at $Re_c = 50,000$ for $n = 0.3$.

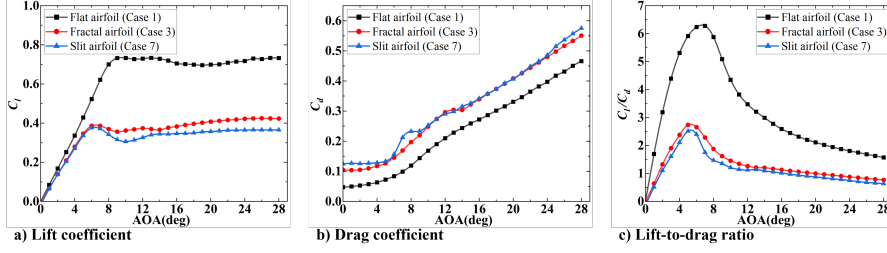


Fig. 17 Same as Fig. 16, but for $n = 0.6$.

Figure 15 presents the results with the fractal or slit part located near the leading edge. At $AOA = 4^\circ$, Fig. 15e indicates that the fluid flows from the lower side to the upper side of the airfoil through the permeable section, and reattachment does not occur after the separation at the leading edge. The flow passing through the slit is notably stronger than that in the fractal part, leading to more pronounced flow separation on the upper surface of the slit airfoil. The flow for $n = 0.6$ is similar to that for $n = 0.3$, as shown in Fig. 15i. For $AOA = 8^\circ, 12^\circ$, and 16° , a jet flow is observed on the upper surface of the slit airfoil, and this is more pronounced than for the fractal airfoil. A portion of the flow from the lower surface enters the slit near the leading edge, causing the velocity behind the slit to decrease on the lower surface and the pressure to rise. This phenomenon may result in a lift coefficient curve that does not exhibit stall, but instead continues to rise.

3.2 $Re_c = 50,000$

This section presents the results at $Re_c = 50,000$. Figures 16 and 17 show the results with the fractal or slit part located near the trailing edge for $n = 0.3$ and $n = 0.6$, respectively. Figures 18 and 19 show the results for the fractal or slit part located near the leading edge.

As illustrated in Fig. 16a, an increase in the Reynolds number increases C_l for the flat airfoil at higher AOAs. Compared with the results for $Re_c = 35,000$, the overall shape of the curve remains unchanged. At the higher Re_c , the values of C_l for the fractal and slit airfoils are nearly identical at lower AOAs (from 0° to 5°), and are lower than for the flat airfoil. Between $AOA = 6^\circ$ and 14° , the C_l curve for the slit airfoil continues to ascend, experiencing a stall from $AOA = 15^\circ$ onwards. For the fractal

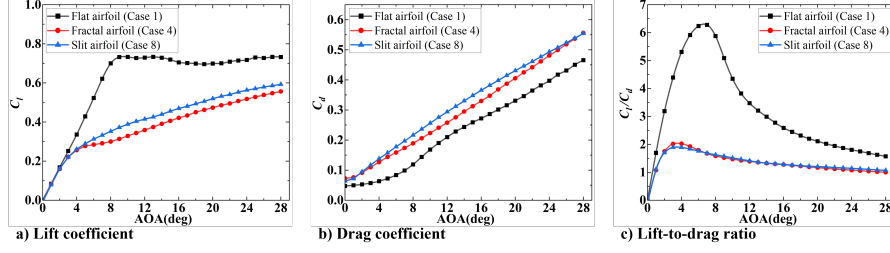


Fig. 18 Aerodynamic coefficients with the fractal or slit part located near the leading edge at $Re_c = 50,000$ for $n = 0.3$.

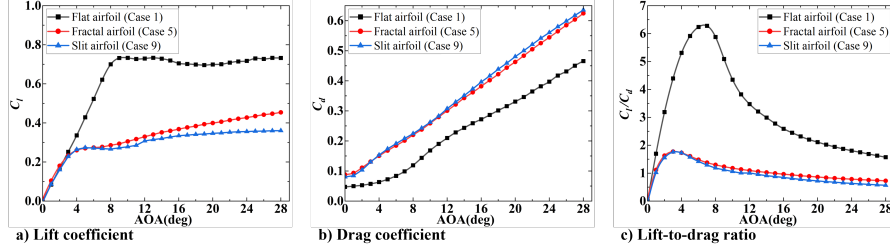


Fig. 19 Same as Fig. 18, but for $n = 0.6$.

airfoil, C_l shows a slight decrease between $AOA = 5^\circ$ and 10° , subsequently exhibiting a linear increase. This results in the slit airfoil having a higher C_l between $AOA = 5^\circ$ and 18° , with the values for the fractal and slit airfoils being nearly identical at other AOAs. Figure 16b shows that C_d for the slit airfoil is consistently larger across all AOAs. Consequently, as depicted in Fig. 16c, C_l/C_d is larger for the fractal airfoil from $AOA = 0^\circ$ to 8° . Compared with the results at $Re_c = 35,000$, the decrease in C_l and C_l/C_d for the fractal and slit airfoils relative to the flat airfoil becomes more significant.

For $n = 0.6$, as depicted in Fig. 17a, the fractal and slit airfoils exhibit similar C_l values for $AOA = 0^\circ$ – 5° , and these values are lower than that of the flat airfoil, as for $n = 0.3$. At $AOA = 6^\circ$, both the fractal and slit airfoils experience a stall, though the stall intensity is less pronounced for the fractal airfoil. The C_l value for the fractal airfoil resumes its increase at $AOA = 8^\circ$, while the slit airfoil only begins its ascent at $AOA = 10^\circ$. Consequently, for $AOA > 5^\circ$, the fractal airfoil has a consistently higher C_l value. Compared with the $n = 0.3$ case, both the fractal and slit airfoils exhibit a decrease in C_l . However, the decrease is more pronounced in the slit airfoil, for which there is a significant change in the shape of the C_l curve. Figure 17b indicates that C_d is larger in the slit airfoil for $AOA = 6^\circ$ – 10° , resulting in a large reduction in C_l/C_d in this AOA range, as shown in Fig. 17c. When the fractal or slit part is located at the leading edge, as presented in Fig. 18a, C_l exhibits a similar trend as for $Re_c = 35,000$. For $AOA > 4^\circ$, the slit airfoil exhibits greater values for $n = 0.3$. Moreover, neither the fractal nor slit airfoil exhibits any signs of stall. As shown in Fig. 18b, the slit airfoil produces larger values of C_d . Regarding C_l/C_d for $n = 0.3$, the fractal airfoil

is slightly better for $\text{AOA} = 2^\circ\text{--}6^\circ$, while the slit airfoil is superior at higher AOA values, as depicted in Fig. 18c.

For $n = 0.6$ in Fig. 19a, the trends in C_l are similar to those observed at $Re_c = 35,000$. Notably, the fractal airfoil has higher values of C_l than the slit airfoil. It is important to highlight that the fractal airfoil does not exhibit any stall, whereas the slit airfoil experiences a slight decrease in C_l from $\text{AOA} = 6^\circ$ to 8° . Compared with the $n = 0.3$ case, C_l decreases more significantly in the slit airfoil. The C_d results in Fig. 19b show that the fractal and slit airfoils exhibit nearly identical behavior from $\text{AOA} = 0^\circ$ to 10° . Beyond $\text{AOA} = 10^\circ$, the slit airfoil produces marginally higher values. Consequently, as presented in Fig. 19c, C_l/C_d is similar in the fractal and slit airfoils for $\text{AOA} = 0^\circ\text{--}6^\circ$. The fractal airfoil gives slightly higher values at higher AOAs. Compared with $Re_c = 35,000$ case, the fractal and slit airfoils produce similar values of C_l/C_d . The visualization results for $Re_c = 50,000$ are not shown because the qualitative difference in the flow between the fractal and slit airfoils is minor.

3.3 Unsteady Case

This section discusses the aerodynamic characteristics of airfoils undergoing unsteady motion. Figures 20 and 21 present the results for the fractal and slit airfoils when the fractal or slit part is located near the trailing edge. Figures 22 and 23 are the results when the fractal or slit part is located near the leading edge.

Figure 20 compares C_l for the flat airfoil and the fractal airfoil. Here, the solid lines represent the results for the flat airfoil, while the dashed lines correspond to the fractal airfoil. The black line denotes the results for the steady condition, while different colors indicate different dimensionless pitch rates r . As indicated by the gray arrows, the airfoil motion encompasses a pitching-up motion from $\text{AOA} = -24^\circ$ to 24° and a pitching-down motion from $\text{AOA} = 24^\circ$ to -24° . Upon observing the results for the flat airfoil during the pitching-up motion, it is evident that C_l tends to increase as the pitch rate increases. As AOA increases, a significant stall is observed, and this is more pronounced than in the steady case. The stall AOA increases with the pitch rate. This is presumably due to the increased pressure near the leading edge caused by the pitching motion, which alleviates the adverse pressure gradient and delays the flow separation. Furthermore, fluctuations in C_l can be observed at the low pitch rate, although these diminish as the pitch rate increases. When the airfoil undergoes a pitching-down motion, increasing the pitch rate produces a reduction in C_l . This phenomenon is termed a dynamic stall. Such a change in C_l gives rise to the occurrence of a hysteresis loop. The area enclosed by the loop enlarges as the pitch rate increases.

The results for the fractal airfoil in Fig. 20a show that, during the pitching-up motion, increasing the pitch rate tends to increase C_l . However, the range of this increase is smaller than for the flat airfoil. Nonetheless, C_l reaches the same level as for the flat airfoil during the pitching-down motion at a pitch rate of $r = 0.0102$. A dynamic stall is also observed, but the hysteresis loop becomes smaller, leading to a faster recovery in C_l . At lower pitch rates, the C_l curves for the pitching-up and -down motions almost coincide. Furthermore, oscillations in C_l appear across all pitch rates, although increasing the pitch rate reduces these oscillations. As shown in Fig. 20b, with a larger degree of porosity, the variation in C_l during pitching motion becomes smaller,

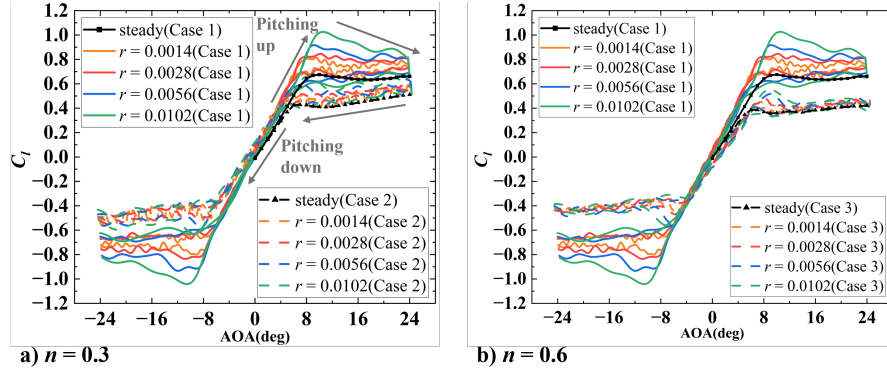


Fig. 20 Lift coefficients with the fractal part located near the trailing edge.

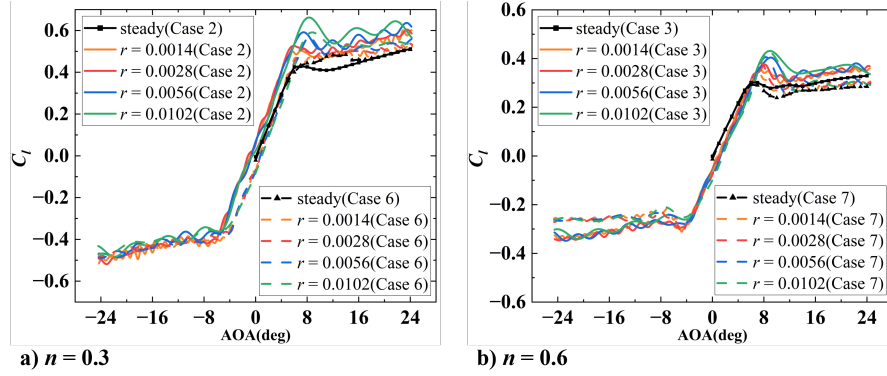


Fig. 21 Lift coefficients with the fractal or slit part located near the trailing edge.

producing results that are closer to the steady-state C_l . Moreover, the difference in C_l during reciprocating motion decreases, and the hysteresis loop almost disappears for lower pitch rates.

Next, we compare the results for the fractal and slit airfoils in Fig. 21. The solid and dotted lines represent the results for the fractal and slit airfoils, respectively. As the results are similar, only those for the pitching-up motion are shown. From the results with $n = 0.3$ in Fig. 21a, it can be observed that for $\text{AOA} > -5^\circ$, the fractal airfoil produces a larger C_l at the same pitch rate. For $\text{AOA} < -5^\circ$, the C_l values of the fractal and slit airfoils are almost the same. For higher AOAs, the oscillations in C_l are smaller for the slit airfoil. With $n = 0.6$ (Fig. 21b), there is little difference between the fractal and slit airfoils up to the stall angle and the point of maximum C_l for the same pitch rate. For $\text{AOA} < 0^\circ$, C_l for the slit airfoil remains almost constant and exhibits small oscillations.

We compare C_l for the flat and fractal airfoils with the fractal part located near the leading edge in Fig. 22. From the results for the fractal airfoil in Fig. 22a, it can be observed that under a pitching motion, the variation in C_l is almost the same as in the steady case. No stall occurs, even at higher pitch rates, and C_l increases with

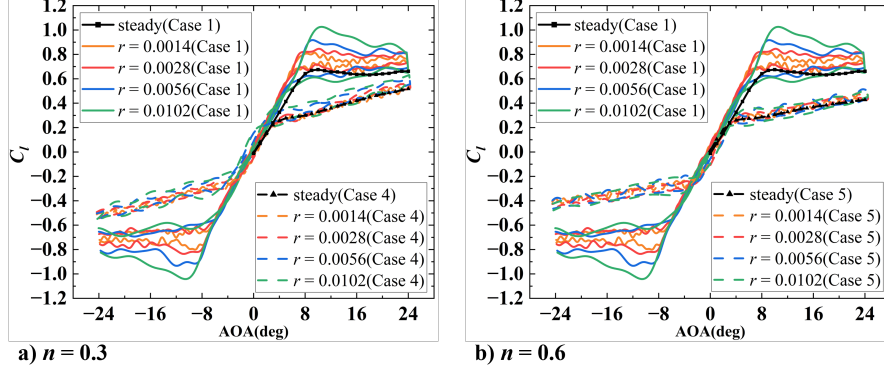


Fig. 22 Lift coefficients with the fractal part located near the leading edge.

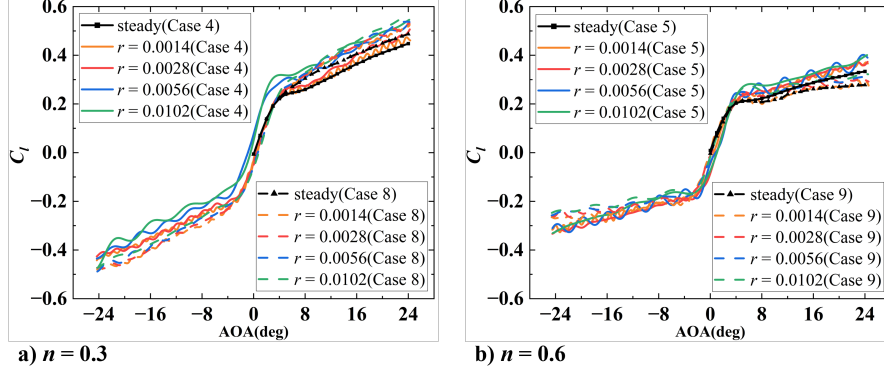


Fig. 23 Lift coefficients with the fractal or slit part located near the leading edge.

increasing AOA. Furthermore, the area enclosed by the hysteresis loop is small. Upon examining the results with $n = 0.6$ in Fig. 22b, C_l for the fractal airfoil is similar to that for $n = 0.3$, albeit with slightly smaller values. Additionally, the oscillations in C_l are smaller at higher pitch rates.

Finally, we compare the results for the fractal and slit airfoils in Fig. 23. From the results for $n = 0.3$ in Fig. 23a, we can observe that C_l is larger for the fractal airfoil. For $n = 0.6$ in Fig. 23b, the difference in C_l is small. Additionally, the oscillations in C_l for the slit airfoil are reduced.

4 Conclusion

We conducted a detailed investigation into the aerodynamic features of airfoils equipped with fractal structures in the low-Reynolds-number regime, i.e., $Re_c = 35,000$ and $50,000$. Experiments were carried out under both steady and unsteady conditions using a wind tunnel. To further aid our understanding and provide visual representations, the smoke-wire method was employed.

A major finding from this research is the powerful impact that fractal structures have on the flow dynamics surrounding the airfoil, especially when compared with airfoils containing simple slits, as used in previous studies. The benefit is further highlighted by the improved lift-to-drag ratio produced by the fractal airfoil, surpassing that of the slit airfoil. In certain conditions, the fractal structures induce gentle stalling characteristics. Moreover, when the fractal part is located near the leading edge, stall does not occur and C_l continues to increase.

In unsteady conditions, the fractal airfoils have a significant ability to reduce the occurrence of dynamic stall compared with a flat airfoil, providing a more stable aerodynamic response, as seen in the smoother C_l curve. The hysteresis loop also becomes smaller. When the fractal part is located near the leading edge, C_l is consistent with that under steady conditions. The slit airfoil performs similarly to the fractal airfoil in unsteady conditions, although the fractal airfoil exhibits a higher C_l during pitching-up phases.

In summary, adding fractal structures to airfoil designs is a promising direction of aerodynamic research. Their clear advantage over slit airfoils, particularly in overcoming challenges like separated flow and dynamic stall, highlights their potential role in advancing future developments in the aviation field. As efforts continue to build more efficient and dependable aircraft, this research strongly indicates promising future innovations utilizing fractal structures.

Acknowledgements. The authors acknowledge Professor Taku Nonomura (Nagoya University) for his valuable comments on this study. They also acknowledge Mr. Kosuke Shimizu (Nagoya University) for his great help in setting and conducting the experiments. This study was supported by JSPS KAKENHI through grant number JP22K18767. J.L. was financially supported by the DII collaborative Graduate Program for Accelerating Innovation in Future Electronics and the THERS Interdisciplinary Frontier Next Generation Researcher, Nagoya University.

References

- [1] Sharma, M., Gupta, A., Gupta, S.K., Alsamhi, S.H., Shvetsov, A.V.: Survey on unmanned aerial vehicle for Mars exploration: Deployment use case. *Drones* **6**(1) (2022) <https://doi.org/10.3390/drones6010004>
- [2] Tani, I.: Low-speed flows involving bubble separations. *Progress in Aerospace Sciences* **5**, 70–103 (1964) [https://doi.org/10.1016/0376-0421\(64\)90004-1](https://doi.org/10.1016/0376-0421(64)90004-1)
- [3] Liang, Y., Yang, Y., Shan, X., Wang, Z.: Effect of airfoil dimple on low-Reynolds-number differing laminar separation behavior via multi-objective optimization. *Journal of Aircraft* **59**(5), 1243–1256 (2022) <https://doi.org/10.2514/1.C036558>
- [4] Counsil, J.N.N., Goni Boulama, K.: Low-Reynolds-number aerodynamic performances of the NACA 0012 and Selig–Donovan 7003 airfoils. *Journal of Aircraft* **50**(1), 204–216 (2013) <https://doi.org/10.2514/1.C031856>

- [5] Burgmann, S., Schröder, W.: Investigation of the vortex induced unsteadiness of a separation bubble via time-resolved and scanning PIV measurements. *Experiments in Fluids* **45**(4), 675–691 (2008) <https://doi.org/10.1007/s00348-008-0548-7>
- [6] Koca, K., Genç, M.S., Açikel, H.H., Çağdaş, M., Bodur, T.M.: Identification of flow phenomena over NACA 4412 wind turbine airfoil at low Reynolds numbers and role of laminar separation bubble on flow evolution. *Energy* **144**, 750–764 (2018) <https://doi.org/10.1016/j.energy.2017.12.045>
- [7] Winslow, J., Otsuka, H., Govindarajan, B., Chopra, I.: Basic understanding of airfoil characteristics at low Reynolds numbers (10^4 – 10^5). *Journal of Aircraft* **55**(3), 1050–1061 (2018) <https://doi.org/10.2514/1.C034415>
- [8] Kurtulus, D.F.: Unsteady aerodynamics of a pitching NACA 0012 airfoil at low Reynolds number. *International Journal of Micro Air Vehicles* **11**, 1756829319890609 (2019) <https://doi.org/10.1177/1756829319890609>
- [9] Kay, N.J., Richards, P.J., Sharma, R.N.: Effect of turbulence and sinusoidal pitching on low-Reynolds-number lift. *AIAA Journal* **58**(6), 2377–2387 (2020) <https://doi.org/10.2514/1.J059169>
- [10] Ghamkhar, K., Ebrahimi, A., Shariloo, K.: Wavelet analysis of the flow field around an oscillating airfoil undergoing pure pitching motion at low Reynolds number. *Physics of Fluids* **35**(6), 063607 (2023) <https://doi.org/10.1063/5.0154125>
- [11] Kamari, D., Tadjfar, M., Madadi, A.: Optimization of SD7003 airfoil performance using TBL and CBL at low Reynolds numbers. *Aerospace Science and Technology* **79**, 199–211 (2018) <https://doi.org/10.1016/j.ast.2018.05.049>
- [12] Chng, T.L., Rachman, A., Tsai, H.M., Zha, G.-C.: Flow control of an airfoil via injection and suction. *Journal of Aircraft* **46**(1), 291–300 (2009) <https://doi.org/10.2514/1.38394>
- [13] Hanna, Y.G., Spedding, G.R.: Aerodynamic performance improvements due to porosity in wings at moderate Re. *AIAA Aviation 2019 Forum* (2019) <https://doi.org/10.2514/6.2019-3584>
- [14] Aldheeb, M., Asrar, W., Sulaeman, E., Omar, A.A.: Aerodynamics of porous airfoils and wings. *Acta Mechanica* **229**(9), 3915–3933 (2018) <https://doi.org/10.1007/s00707-018-2203-6>
- [15] Li, J., Nagata, K., Watanabe, T.: Large-eddy simulation of low-Reynolds-number flow around partially porous airfoils. *Journal of Aircraft* **60**(6), 1998–2009 (2023) <https://doi.org/10.2514/1.C037253>

- [16] Atangana, A.: Chapter 11 - Groundwater flow within a fracture, matrix rock and leaky aquifers: Fractal geometry. In: Atangana, A. (ed.) *Fractional Operators with Constant and Variable Order with Application to Geo-Hydrology*, Academic Press, pp. 263–297 (2018). <https://doi.org/10.1016/B978-0-12-809670-3.00011-4>
- [17] Atta, M., Hussain, S.M., Hussain, F., Shah, H.H., Shah, H., Ro, J.-S.: Fractals flow simulation for groundwater flow with varying apertures by using analytic element method. *Fractal and Fractional* **6**(10) (2022) <https://doi.org/10.3390/fractalfract6100573>
- [18] Li, P., Pan, Q., Jiang, S., Yan, M., Yan, J., Ning, G.: Development of novel fractal method for characterizing the distribution of blood flow in multi-scale vascular tree. *Frontiers in Physiology* **12** (2021) <https://doi.org/10.3389/fphys.2021.711247>
- [19] Staicu, A., Mazzi, B., Vassilicos, J.C., Water, W.: Turbulent wakes of fractal objects. *Phys. Rev. E* **67**, 066306 (2003) <https://doi.org/10.1103/PhysRevE.67.066306>
- [20] Hurst, D., Vassilicos, J.C.: Scalings and decay of fractal-generated turbulence. *Physics of Fluids* **19**(3), 035103 (2007) <https://doi.org/10.1063/1.2676448>
- [21] Seoud, R.E., Vassilicos, J.C.: Dissipation and decay of fractal-generated turbulence. *Physics of Fluids* **19**(10), 105108 (2007) <https://doi.org/10.1063/1.2795211>
- [22] Nagata, K., Sakai, Y., Inaba, T., Suzuki, H., Terashima, O., Suzuki, H.: Turbulence structure and turbulence kinetic energy transport in multiscale/fractal-generated turbulence. *Physics of Fluids* **25**(6), 065102 (2013) <https://doi.org/10.1063/1.4811402>
- [23] Hearst, R.J., Lavoie, P.: Decay of turbulence generated by a square-fractal-element grid. *Journal of Fluid Mechanics* **741**, 567–584 (2014) <https://doi.org/10.1017/jfm.2013.684>
- [24] Suzuki, H., Nagata, K., Sakai, Y., Ukai, R.: High-Schmidt-number scalar transfer in regular and fractal grid turbulence. *Physica Scripta* **2010**(T142), 014069 (2010) <https://doi.org/10.1088/0031-8949/2010/T142/014069>
- [25] Suzuki, H., Nagata, K., Sakai, Y., Hasegawa, Y.: Fractal analysis of turbulent mixing in fractal-generated turbulence by planar laser-induced fluorescence. *Physica Scripta* **2013**(T155), 014062 (2013) <https://doi.org/10.1088/0031-8949/2013/T155/014062>
- [26] Nicolleau, F.C.G.A., Salim, S.M.M., Nowakowski, A.F.: Experimental study of a turbulent pipe flow through a fractal plate. *Journal of Turbulence* **12**, 44 (2011) <https://doi.org/10.1080/14685248.2011.637046>

- [27] Nedic, J., Ganapathisubramani, B., Vassilicos, J.C., Borée, J., Brizzi, L.E., Spohn, A.: Aeroacoustic performance of fractal spoilers. *AIAA Journal* **50**(12), 2695–2710 (2012) <https://doi.org/10.2514/1.J051387>
- [28] Batill, S.M., Mueller, T.J.: Visualization of transition in the flow over an airfoil using the smoke-wire technique. *AIAA Journal* **19**(3), 340–345 (1981) <https://doi.org/10.2514/3.50953>
- [29] Anh, N., Mizoguchi, M., Itoh, H.: Unsteady flow field around NACA0012 airfoil undergoing constant pitch rates at low Reynolds numbers. *AIAA AVIATION 2020 FORUM* (2020) <https://doi.org/10.2514/6.2020-3041>