

Stokes' theorem, self-adjointness of the Laplacian and Hodge's theorem for hyperbolic 3-cone-manifolds

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§1. Introduction

By a hyperbolic 3-cone-manifold, we will mean an orientable (not necessarily volume-finite) riemannian 3-manifold C of constant sectional curvature -1 with cone-type singularity along a 1-dimensional graph Σ which consists of geodesic segments in C . The subset $M := C - \Sigma$ has a smooth, incomplete hyperbolic structure whose metric completion is identical to the singular hyperbolic structure on C . The hyperbolic 3-manifold M is incomplete near Σ .

In this paper, we will inform that Stokes' theorem for smooth L^2 -forms on the incomplete hyperbolic manifold M holds. The proof can be performed by following the argument described in Hodgson-Kerckhoff [5]. (In [5], Stokes' theorem in the case where each component of the singular locus Σ is homeomorphic to S^1 and the complement of an open tubular neighborhood of Σ is compact was shown.) Then from Stokes' theorem, by using a result of Gaffney [3], it is shown that there is a maximal extension of the Laplacian on M which is self-adjoint on its adequately defined domain. Thus, we have an extension of Hodge theory to hyperbolic 3-cone-manifolds whose singular loci are smooth 1-manifolds. Let E denote the flat vector bundle of local killing vector fields on the hyperbolic 3-manifold M . Then, if the singular locus Σ of the hyperbolic 3-cone-manifold C is a smooth 1-dimensional manifold, for any E -valued 1-form $\tilde{\omega}$ which represents an infinitesimal deformation of the hyperbolic structure on M around Σ and which satisfies some conditions related with the domain of the Laplacian ($\tilde{\omega}$ is called to be "in standard form"), there is a closed and co-closed E -valued 1-form ω which is equivalent to $\tilde{\omega}$ in the de Rham cohomology group $H^1(M; E)$. The 1-form ω is a representative with specific control on the asymptotic behavior near the singular locus.

§2. Stokes' theorem and self-adjointness of the Laplacian for hyperbolic 3-cone-manifolds

First we will give the definition of hyperbolic 3-cone-manifolds. Consider a smooth 3-dimensional manifold N , which has a path metric given by a gluing of the faces of finitely many geodesic polyhedra possibly with ideal vertices in the 3-dimensional hyperbolic space $\mathbf{H}^3$. The gluing is performed by orientation reversing isometries of $\mathbf{H}^3$. It is permitted that the polyhedra have “faces” on the sphere at infinity S_∞^2 which are not glued to another such “faces”. We assume that the link of a vertex is piecewise linear homeomorphic to a sphere and the link of an ideal vertex is piecewise homeomorphic to a torus, an open annulus or an open disk. We also assume that the path metric on N is complete. The manifold N with the metric above is called a hyperbolic 3-cone-manifold.

The singular locus Σ of a hyperbolic 3-cone-manifold consists of the points with no neighborhood isometric to a ball in $\mathbf{H}^3$. It is a union of totally geodesic closed simplices of dimension 1. At each point of Σ in an open 1-simplex, there is a cone angle which is the sum of dihedral angles of polyhedra containing the point. The subset $N - \Sigma$ has a smooth riemannian metric of constant curvature -1 , but this metric is incomplete near Σ if $\Sigma \neq \emptyset$.

Let C be a (not necessarily volume-finite) hyperbolic 3-cone-manifold with singular locus Σ . Let $M := C - \Sigma$ be a smooth (but incomplete) hyperbolic 3-manifold. A tubular neighborhood of a singular point of C , which is not a vertex, has the metric

$$dr^2 + \sinh^2 r d\theta^2 + \cosh^2 r dz^2,$$

by using the cylindrical coordinate. There are finitely many vertices of Σ .

We have a developing map of M from its universal covering space $\tilde{M}$,

$$\mathcal{D}_C : \tilde{M} \rightarrow \mathbf{H}^3,$$

and a holonomy representation,

$$\rho_C : \pi_1(M) \rightarrow \mathrm{PSL}_2(\mathbf{C}).$$

They are called a developing map and a holonomy representation of the cone-manifold C .

Let $\Omega^p(M)$ denote the space of smooth, real-valued p -forms of M and $\Omega^*(M)$ denote the space of smooth, real-valued forms on M . Let $\hat{d}$ be the usual exterior derivative of smooth real-valued forms on M :

$$\hat{d} : \Omega^p(M) \rightarrow \Omega^{p+1}(M).$$

Let $\hat{*}$ be the Hodge star operator defined by using the riemannian metric g on M :

$$g(\phi, \hat{*} \psi) dM = \phi \wedge \psi$$

for any real-valued p -form ϕ and $(3-p)$ -form ψ . Let $\hat{\delta}$ be the adjoint of $\hat{d}$:

$$\hat{\delta} : \Omega^p(M) \rightarrow \Omega^{p-1}(M).$$

Let $\hat{\Delta}$ be the Laplacian on smooth real-valued forms for the riemannian manifold M :

$$\hat{\Delta} = \hat{d}\hat{\delta} + \hat{\delta}\hat{d}.$$

We will use $\langle , \rangle$ to denote an L^2 inner product on real-valued forms:

$$\langle \xi, \eta \rangle = \int_M \xi \wedge \hat{*} \eta = \int_M g(\xi, \eta) dM.$$

It is seen that Stokes' theorem for smooth L^2 -forms on the incomplete hyperbolic manifold M can be proved as in Hodgson-Kerckhoff [5]. The proof is performed by using Cheeger's method in [1].

Theorem 1 (Stokes' theorem). *Let C be a hyperbolic 3-cone-manifold with singular locus Σ . Let $M := C - \Sigma$ be the smooth, incomplete hyperbolic 3-manifold. Then Stokes' theorem holds:*

$$\int_M \hat{d}\alpha \wedge \hat{*}\beta = \int_M \alpha \wedge \hat{*}\hat{\delta}\beta,$$

for smooth L^2 -forms α, β on M such that $\hat{d}\alpha, \hat{\delta}\beta$ are L^2 -forms on M .

If we define the domains of $\hat{d}$ and $\hat{\delta}$ by

$$\text{dom } \hat{d} = \{\alpha \in \Omega^*(M) ; \alpha \text{ and } \hat{d}\alpha \text{ are } L^2\},$$

$$\text{dom } \hat{\delta} = \{\beta \in \Omega^*(M) ; \beta \text{ and } \hat{\delta}\beta \text{ are } L^2\},$$

then Theorem 1 says that $\langle \hat{d}\alpha, \beta \rangle = \langle \alpha, \hat{\delta}\beta \rangle$ holds for all $\alpha \in \text{dom } \hat{d}, \beta \in \text{dom } \hat{\delta}$.

The strong closure $\overline{\hat{d}}$ of $\hat{d}$ is defined as follows (see [1]): $\overline{\hat{d}}\alpha = \eta$ means that α is an L^2 -form and there exist $\alpha_i \in \text{dom } \hat{d}$ ($i \in \mathbb{N}$) such that $\alpha_i \rightarrow \alpha, \hat{d}\alpha_i \rightarrow \eta$. The domain of $\overline{\hat{d}}$ is defined by

$$\text{dom } \overline{\hat{d}} = \{ \alpha ; \alpha \text{ and } \overline{\hat{d}}\alpha \text{ are } L^2\text{-forms on } M \}.$$

In the same manner, the strong closure $\overline{\hat{\delta}}$ of $\hat{\delta}$ and its domain $\text{dom } \overline{\hat{\delta}}$ are defined.

The theorem above means that the manifold M has a negligible boundary (see [3],[4]). Then, by the result of Gaffney [3], for our manifold M , the Hilbert space closure $\overline{\hat{\Delta}}$ of $\hat{\Delta}$ is self-adjoint.

Theorem 2 (self-adjointness of $\overline{\hat{\Delta}}$). *Let C be a hyperbolic 3-cone-manifold with singular locus Σ . Let $M := C - \Sigma$ be the smooth, incomplete hyperbolic 3-manifold. Let $\overline{\hat{\Delta}}$ be the*

Hilbert space closure of the Laplacian for the riemannian manifold M so that

$$\text{the domain of } \bar{\Delta} = \{\alpha \in \text{dom } \bar{d} \cap \text{dom } \bar{\delta} ; \bar{\delta}\alpha \in \text{dom } \bar{d}, \bar{d}\alpha \in \text{dom } \bar{\delta}\}.$$

Then $\bar{\Delta} = \bar{d} \bar{\delta} + \bar{\delta} \bar{d}$, and $\bar{\Delta}$ is a closed, non-negative, self-adjoint and elliptic operator.

§3. Hodge theorem for hyperbolic 3-cone-manifolds

Let C be the hyperbolic 3-cone-manifold with singular locus Σ and $M = C - \Sigma$ be the hyperbolic 3-manifold considered in §2. Let G denote the group consisting of orientation preserving isometries of $\mathbf{H}^3$. The group G can be naturally identified with $\text{PSL}_2(\mathbf{C})$. Let $\mathcal{G}$ denote the Lie algebra of G and Ad the adjoint representation of G on $\mathcal{G}$. Associated to the hyperbolic structure ρ_C is a flat $\mathcal{G}$ vector bundle E over M :

$$E = \widetilde{M} \times_{Ad \circ \rho_C} \mathcal{G}.$$

Let $\Omega^p(M; E)$ denote the space consisting of smooth E -valued p -forms on M . Let d be a covariant exterior derivative

$$d : \Omega^p(M; E) \rightarrow \Omega^{p+1}(M; E),$$

which is given by the flat connection on E . Then the p th de Rham cohomology group $H^p(M; E)$ of M with coefficients in E is defined by d .

There is a natural metric on E as follows. For each $x \in M$, the fiber E_x of the bundle E decomposes as a direct sum $\mathcal{P} \oplus \mathcal{K}$, where $\mathcal{P}$ consists of the infinitesimal pure translations at x and $\mathcal{K}$ consists of the infinitesimal rotations at x . Since an infinitesimal pure translation at x corresponds to a tangent vector to M at x , $\mathcal{P}$ is identified with the tangent space $T_x M$ of M at x . Then we give $\mathcal{P}$ the metric induced from the riemannian metric on M . Similarly, since an element of $\mathcal{K}$ operates linearly and isometrically on the tangent space, a metric on $\mathcal{K}$ comes from identifying it with a subspace of $\mathfrak{o}(3)$ with its usual metric. In fact, $\mathcal{K}$ is identified with the total space $\mathfrak{o}(3)$. Then we give a metric on $\mathcal{P} \oplus \mathcal{K}$ by regarding the direct sum as an orthogonal direct sum. Let h denote the metric on E given as above.

Let $*$ denote the Hodge star operator on $\Omega^*(M; E)$ defined by using the riemannian metric h on E and the Hodge star operator $\hat{*}$ on $\Omega^*(M)$:

$$\alpha \wedge * \beta = (a\xi) \wedge (b\hat{*}\eta) = (ab) (\xi \wedge \hat{*}\eta) = h(a, b) g(\xi, \eta) dM,$$

for any $\alpha = a\xi$, $\beta = b\eta$ ($a, b \in \Omega^0(M; E)$, $\xi, \eta \in \Omega^*(M)$). For two forms $\alpha = a\xi$, $\beta = b\eta \in \Omega^*(M; E)$, put

$$(\alpha, \beta) = \int_M \alpha \wedge * \beta = \int_M h(a, b) g(\xi, \eta) dM.$$

This is an L^2 inner product on $\Omega^*(M; E)$. We define

$$\delta : \Omega^p(M; E) \rightarrow \Omega^{p-1}(M; E)$$

by putting

$$\delta\alpha = (-1)^{3(p+1)+1} * d * \alpha$$

for any $\alpha \in \Omega^p(M; E)$. Then the associated Laplacian Δ is defined by

$$\Delta := d\delta + \delta d.$$

Let ∇ denote the Levi-Civita connection on E with respect to the metric h , and D denote a covariant exterior derivative induced by the connection ∇ :

$$\begin{aligned} \nabla & : \Omega^0(M; E) \rightarrow \Omega^1(M; E), \\ D & : \Omega^p(M; E) \rightarrow \Omega^{p+1}(M; E). \end{aligned}$$

Put

$$D^*\alpha = (-1)^{3(p+1)+1} * D * \alpha,$$

for all $\alpha \in \Omega^p(M; E)$. Let $\{e_1, e_2, e_3\}$ be any orthonormal frame for TM and $\{\omega^1, \omega^2, \omega^3\}$ be the dual co-frame. Let $i(\cdot)$ denote the interior product on forms. Then D and D^* are described as in the following:

$$\begin{aligned} D &= \sum_{j=1}^3 \omega^j \wedge \nabla_{e_j}, \\ D^* &= -\sum_{j=1}^3 i(e_j) \nabla_{e_j}. \end{aligned}$$

Put

$$\begin{aligned} T &:= \sum_{j=1}^3 \omega^j \wedge \text{ad}(E_j), \\ T^* &:= \sum_{j=1}^3 i(e_j) \text{ad}(E_j), \end{aligned}$$

where E_j is the element in the fiber over any point on M , which is the infinitesimal translation in the direction e_j at that point, and $\text{ad}(E_j)$ sends an element Y in the fiber to $[E_j, Y]$. Then we have

$$\begin{aligned} d &= D + T, \\ \delta &= D^* + T^*. \end{aligned}$$

This shows a relationship between the flat structure on E , which is defined by the hyperbolic structure on M , and the natural metric h on E , which is defined by using the local geometry on M . (See Matsushima-Murakami [8] for the formulation above.)

As described above, at each point $x \in M$, the fiber E_x is decomposed into the orthogonal direct sum $\mathcal{P} \oplus \mathcal{K}$. Then the vector bundle E is decomposed into an orthogonal direct sum of two sub-bundles which we also denote as $\mathcal{P}$ and $\mathcal{K}$:

$$E = \mathcal{P} \oplus \mathcal{K}.$$

This decomposition induces a decomposition:

$$\Omega^p(M; E) = \Omega^p(M; \mathcal{P}) \oplus \Omega^p(M; \mathcal{K}).$$

The bundle $\mathcal{P}$ is naturally identified with the tangent bundle TM of M . The Levi-Civita connection ∇ restricted to $\mathcal{P}$ -valued forms is the Levi-Civita connection on M . On $\mathcal{K} = o(3) \subset Hom(TM, TM)$, it is again the Levi-Civita connection induced by the one on TM . The operators D and D^* preserve the decomposition, while T and T^* map $\Omega^*(M; \mathcal{P})$ to $\Omega^*(M; \mathcal{K})$ and vice versa:

$$\begin{array}{ccc} \Omega^*(M; \mathcal{P}) \oplus \Omega^*(M; \mathcal{K}) & \Omega^*(M; \mathcal{P}) \oplus \Omega^*(M; \mathcal{K}) \\ \begin{array}{c} D, D^* \downarrow \end{array} & \begin{array}{c} \downarrow D, D^* \end{array} & \begin{array}{c} T, T^* \downarrow \end{array} & \begin{array}{c} \downarrow T, T^* \end{array} \\ \Omega^*(M; \mathcal{P}) \oplus \Omega^*(M; \mathcal{K}), & \Omega^*(M; \mathcal{K}) \oplus \Omega^*(M; \mathcal{P}). \end{array}$$

The Lie algebra $\mathcal{G} = sl_2(\mathbb{C})$ has a natural complex structure which is related to the decomposition $E = \mathcal{P} \oplus \mathcal{K}$ by $\mathcal{K} = i \mathcal{P}$. The multiplication by i in the Lie algebra induces a bundle isomorphism from $\mathcal{P}$ to $\mathcal{K}$, which respects the local geometry of M . For example, if t denotes an infinitesimal translation, then it is an infinitesimal rotation around the axis of t , and t and it are orthogonal. Now we will think of $\Omega^*(M; \mathcal{P})$ and $\Omega^*(M; \mathcal{K})$ as the real and imaginary parts of $\Omega^*(M; E)$:

$$\begin{aligned} \Omega^*(M; E) &= \text{Re } \Omega^*(M; E) \oplus \text{Im } \Omega^*(M; E) \\ &= \Omega^*(M; \mathcal{P}) \oplus \Omega^*(M; \mathcal{K}) \\ &= \Omega^*(M; \mathcal{P}) \oplus i \Omega^*(M; \mathcal{P}). \end{aligned}$$

An E -valued p -form α is a pair of a real part α_{real} and a imaginary part α_{imag} . The real part α_{real} is a $\mathcal{P}$ -valued p -form on M . If v is a $\mathcal{P}$ -valued 0-form (namely a tangent vector field) on M , then $(dv)_{real}$ is $Dv \in \Omega^1(M; \mathcal{P}) (= \Omega^1(M; TM) = Hom(TM, TM))$, which is also equal to ∇v , and $(dv)_{imag}$ is $Tv \in \Omega^1(M; \mathcal{K}) (= i \Omega^1(M; \mathcal{P}) = i \Omega^1(M; TM) = i Hom(TM, TM))$. By using the orthonormal frame $\{e_k, e_l, e_j\}$ and the dual co-frame $\{\omega^k, \omega^l, \omega^j\}$, we can describe a canonical isomorphism between skew-symmetric elements of $Hom(TM, TM)$ and vector fields:

$$Hom(TM, TM)_{skew} \ni e_l \otimes \omega^j - e_j \otimes \omega^l \rightarrow e_k \in \Omega^0(M; TM).$$

If v is a tangent vector field on M , Dv is an element of $\text{Hom}(TM, TM)$. The skew-symmetric part $(Dv)_{\text{skew}}$ of Dv is called the curl of v , and is denoted by $\text{curl } v$. By the isomorphism above, $\text{curl } v$ is regarded as a vector field on M . Note that this vector field is the half of the usual curl considered in elementary vector calculus. The trace of Dv is called the divergence of v , and is denoted by $\text{div } v$. The traceless, symmetric part of Dv is called the strain of v , and is denoted by $\text{str } v$.

If v is a locally defined tangent vector field on M , then we can consider a local section of the bundle E , which is defined by $s_v = v - i \text{curl } v$. Call it the canonical lift of v .

Let σ be any closed smooth E -valued 1-form on M . Choosing a point $x \in M$, we can locally define a section $\int_x \sigma$ of the bundle E by integrating σ along paths beginning at x , which is called the associated local section. Note that we are using the flat connection on E to identify the fibers at different points along the path in order to do the integration. Since σ is closed, the value of the integral depends only on the homotopy class of the path; a well-defined section is determined on any simply connected subset of M . Then $d \int_x \sigma = \sigma$ on such a subset. In general, the section will not extend to a global section on M .

In the rest of the paper, we assume that the singular locus Σ of the cone-manifold C is a smooth 1-manifold:

$$\Sigma \approx \mathbf{R} \sqcup \dots \sqcup \mathbf{R} \sqcup S^1 \sqcup \dots \sqcup S^1.$$

Some examples of hyperbolic 3-cone-manifolds with infinite volume, whose singular loci are homeomorphic to $\mathbf{R}$, are illustrated in [9].

In a tubular neighborhood U_k of each component Σ_k of Σ , we use cylindrical coordinates, (r, θ, z) . Then the hyperbolic metric on U_k is $dr^2 + \sinh^2 r d\theta^2 + \cosh^2 r dz^2$. We will use the orthonormal frame $\{e_1, e_2, e_3\}$ of TM adapted to this coordinate system:

$$e_1 := \frac{\partial}{\partial r}, \quad e_2 := \frac{1}{\sinh r} \frac{\partial}{\partial \theta}, \quad e_3 := \frac{1}{\cosh r} \frac{\partial}{\partial z}.$$

Then the dual co-frame $\{\omega^1, \omega^2, \omega^3\}$ is

$$\omega^1 = dr, \quad \omega^2 = \sinh r \, d\theta, \quad \omega^3 = \cosh r \, dz.$$

An E -valued 1-form can be interpreted as a complex-valued section of $\mathcal{P} \otimes T^*M \cong TM \otimes T^*M \cong \text{Hom}(TM, TM)$. Then an E -valued 1-form can be described as a matrix in $M_3(\mathbf{C})$ whose (i, j) entry is the coefficient of $e_i \otimes \omega^j$.

The form in (1) below is a closed and co-closed form which represents an infinitesimal deformation which does not change the real part of the complex length of an element of

the fundamental group of U_k which is so called the meridian of U_k . The meridian is the class of the fundamental group which wraps around Σ_k once and bounds a singular disk with cone angle equal to that of Σ_k . The infinitesimal deformation preserves the property that the meridian is elliptic. Then it gives a small deformation of the cone-manifold U_k to a cone-manifold. The infinitesimal deformation also has the remarkable property that it decreases the cone angle.

$$\tilde{\omega}_{(1)} = \begin{pmatrix} \frac{-1}{\cosh^2 r \sinh^2 r} & 0 & 0 \\ 0 & \frac{1}{\sinh^2 r} & \frac{-i}{\cosh r \sinh r} \\ 0 & \frac{-i}{\cosh r \sinh r} & \frac{-1}{\cosh^2 r} \end{pmatrix} \quad (1)$$

The form in (2) below is a closed and co-closed form which represents an infinitesimal deformation which leaves the holonomy of the meridian (hence the cone angle) unchanged. If Σ_k is homeomorphic to S^1 , this deformation stretches the length of Σ_k .

$$\tilde{\omega}_{(2)} = \begin{pmatrix} \frac{-1}{\cosh^2 r} & 0 & 0 \\ 0 & -1 & \frac{-i \sinh r}{\cosh r} \\ 0 & \frac{-i \sinh r}{\cosh r} & \frac{\cosh^2 r + 1}{\cosh^2 r} \end{pmatrix} \quad (2)$$

Definition (in standard form). Let $\tilde{\omega}$ be a smooth, closed, E -valued 1-form on M such that $\delta\tilde{\omega}, d(\delta\tilde{\omega}), \delta d(\delta\tilde{\omega})$ are L^2 . We say that the 1-form $\tilde{\omega}$ is in standard form if the following conditions are satisfied:

- The associated local section $\int_x \tilde{\omega}$ is the canonical lift of its real part:

$$\int_x \tilde{\omega} = \left(\int_x \tilde{\omega} \right)_{\text{real}} - i \operatorname{curl} \left(\int_x \tilde{\omega} \right)_{\text{real}}, \text{ for any } x \in M.$$

- In a tubular neighborhood U_k of a component Σ_k of the singular locus Σ ,

$$\tilde{\omega} = h_1 \tilde{\omega}_{(1)} + h_2 \tilde{\omega}_{(2)} \text{ for some } h_1, h_2 \in \mathbb{C}.$$

Theorem 3 (Hodge theorem for hyperbolic 3-cone-manifolds). *Let C be a hyperbolic 3-cone-manifold with singular locus Σ . Let $M := C - \Sigma$ be the smooth, incomplete hyperbolic 3-manifold. Assume that Σ is a disjoint union of smooth 1-manifolds; $\Sigma \approx \mathbf{R} \sqcup \dots \sqcup \mathbf{R} \sqcup S^1 \sqcup \dots \sqcup S^1$. Let $\tilde{\omega} \in \Omega^1(M; E)$ be a smooth, E -valued 1-form which is in standard form. Then there exists a smooth, closed and co-closed E -valued 1-form ω , which is cohomologous to $\tilde{\omega}$ and whose associated local section $\int_x \omega$ is the canonical lift of a divergence-free, harmonic vector field. Moreover, there is a unique such form satisfying the condition that $\tilde{\omega} - \omega = ds$ where s is a globally defined L^2 section of E .*

Outline of the proof. We want to solve the equation $\Delta s = \delta\tilde{\omega}$ for a globally defined section s of E . Since the associated local section $\int_x \tilde{\omega}$ is the canonical lift of its real part, $\delta\tilde{\omega}$ is also the canonical lift of its real part. Thus, it suffices to solve $\Delta v = (\delta\tilde{\omega})_{real}$ for a globally defined vector field v on M . Let $\zeta \in \Omega^1(M)$ be a smooth, real-valued 1-form which is the dual to the vector field $(\delta\tilde{\omega})_{real}$. Then, by using a Weitzenböck formula, we can see that it suffices to solve

$$(\hat{\Delta} + 4)\tau = \zeta,$$

for a smooth, real-valued 1-form $\tau \in \Omega^1(M)$. Now we apply the self-adjointness of the closure $\overline{\hat{\Delta}}$ of the Laplacian $\hat{\Delta}$ on $\Omega^*(M)$. Since ζ is in the domain of $\overline{\hat{\Delta} + 4}$, then by Theorem 2, there is a unique solution $\tau \in$ the domain of $\overline{\hat{\Delta} + 4}$. Since ζ is smooth, then, by the usually regularity theory for elliptic operators, τ is also smooth. Therefore, we can find a globally defined smooth section s of E which satisfies $\Delta s = \delta\tilde{\omega}$. Then put $\omega := \tilde{\omega} - ds$. It is easy to see that ω and s satisfy the condition described in the theorem. $\square$

If each component Σ_k of the singular locus Σ is homeomorphic to S^1 and $M - \sqcup_k U_k$ is compact, each cohomology class has a representative in standard form (see Lemma 3.3 in [5]).

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