

Doctoral dissertation summary

Study on estimation of strain-rate fields through inversion analyses of geodetic data using basis function expansion

基底関数展開を用いた測地データの逆解析による

歪み速度場の推定に関する研究

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1. Introduction

In the Earth's surface layer, active crustal deformation is caused by relative motion between tectonic plates. Crustal deformation is directly related to seismic activity and the formation of geological structures. In a broad sense, these phenomena can be explained by the relative motion of rigid plates, the theory of plate tectonics. However, plate tectonics has limitations in explaining deformation in tectonically active regions, where continuous deformation is widely distributed (e.g., Molnar and Tapponier 1975). Therefore, more detailed investigations are required, and geodetic observations provide fundamental information for this purpose. In recent years, space geodetic techniques, such as Global Navigation Satellite System (GNSS) and Interferometric Synthetic Aperture Radar (InSAR), have been developed, and large amounts of data are accumulating.

Improved resolution of strain-rate fields will lead to advances in geosciences, such as better estimation of earthquake occurrence probabilities and the construction of more realistic rheological models for the crust and upper mantle. Therefore, it is essentially important to achieve higher-resolution estimation of spatially continuous strain-rate fields from spatially sporadic geodetic data. Based on these considerations, the thesis aims to apply and extend the method of basis function expansion to estimate strain-rate fields as accurately as possible.

2. Method

In Chapter 2, we introduce the method of basis function expansion, following the procedure of Yabuki and Matsu'ura (1992) and Okazaki et al. (2021). We consider the problem of estimating a spatially continuous deformation field from spatially sporadic velocity data. The velocity field is expressed by a linear combination of a set of basis functions. The observed velocity data are related to

a model velocity field by an observation equation, assuming that the residual (error) between the model and observed velocities follows the Gaussian distribution.

Next, following Yabuki and Matsu'ura (1992), we construct a Bayesian model that takes prior information into account. In this chapter, we explain the case of the L2 regularization, which will be used in Chapters 3 to 5. Because the velocity field is expected to change smoothly in space, we use smoothness constraints as a prior, where the L2-norm of the second derivatives of the velocity field should be small.

In Bayes' theorem, the posterior distribution of the model parameters for given velocity data is proportional to the product of the probability density functions of the observed data and the prior information. Maximizing the posterior distribution is identical to finding the model parameters that minimize the objective function. The objective function is composed of the residual sum of squares between the observed and estimated velocities and the smoothness constraint on the velocity field. The relative weight between these terms is controlled by the hyperparameters. The optimal values of the hyperparameters are objectively determined by Akaike's Bayesian Information Criterion (ABIC) (Akaike 1980).

Using the optimal hyperparameters that minimize ABIC, the optimal model parameters that minimize the objective function are obtained. By substituting the optimal model parameters into the equation of the basis function expansion, the velocity field is obtained. If smooth functions, including the cubic B-splines, are used as basis functions, the strain-rate field can be obtained by analytically differentiating the velocity field.

3. Importance of the choice of basis functions

Many functions, including boxcar functions, triangular functions, and cubic B-spline functions, can be used as basis functions. The choice of basis functions used to represent the model velocity field plays a crucial role in estimating deformation fields. In this study, for simplicity, we focus on the cases in which boxcars and cubic B-splines are used as basis functions. Boxcars are the easiest to implement and have been widely used as basis functions. However, because boxcars are discontinuous at the nodes, they are not suitable for accurately estimating velocity fields that are expected to basically change smoothly in space. In contrast, although the formulation of cubic B-splines is somewhat complex, cubic B-splines are smooth functions that are twice differentiable even at the nodes; therefore, a strain-rate field can be obtained through analytical differentiation of a velocity field. We quantitatively compare the performance of boxcars and cubic B-splines in estimating velocity fields, while systematically changing the node interval L .

For each basis function, we estimate velocity fields from GNSS velocity data over Japan for the period of 2006 to 2009. Regardless of the choice of the basis function, ABIC exhibits smooth variations with respect to the hyperparameter with a single minimum. By comparing eastward and northward velocity fields estimated for both basis functions by changing the value of L , we find that the estimated velocity fields vary significantly depending on the choice of the basis functions and the value of L .

We investigate the relation between the values of L and the minimum values of ABIC for different basis functions. The minimum values of ABIC increase monotonically with increasing values of L for both functions, but exhibit smaller variations for the cubic B-splines than for the boxcars. This suggests that the estimated velocity field converges to an appropriate solution when the cubic B-splines with smaller values of L are used.

To compare the results more quantitatively, we introduce two additional evaluation measures: RMSR and RMSD15. The former is the root mean square of the residuals between the observed and estimated velocities at the GNSS stations, while the latter is the root mean squared difference between the velocity field estimated for each basis function with a value of L and that estimated for the cubic B-splines with $L = 15$ km. RMSR and RMSD15 decrease monotonically with decreasing values of L for both basis functions. However, if we compare the boxcars and the cubic B-splines that have nearly equal values of RMSR, the latter has an about 2.5 times larger L than the former, and the latter also has a much smaller value of RMSD15. This result suggests that about $2.5^2 = 6.25$ times as many boxcars as cubic B-splines are at least required to obtain an equivalent result, because we are now estimating a strain-rate field in the two dimensions. Because the computation of the inverse matrix, which is the most time-consuming task in this inversion analysis, is proportional to the cube of the number of model parameters, this difference implies that the computational cost of the inverse matrix for the cubic B-splines is reduced by a factor of about $1/2.5^3 \cong 1/15.6$ compared to that for the boxcars. Although the actual time required to compute a value of ABIC for a given hyperparameter differed by roughly a factor of about 100, cubic B-splines are clearly superior to boxcars in terms of computational efficiency.

Because of these reasons, in the subsequent parts of the thesis, we use the cubic B-splines as basis functions.

4. Spatiotemporal variation of strain-rate fields

A strain-rate field can change not only in space but also in time. Monitoring spatiotemporal variation of strain rates is beneficial for assessing seismic hazards. Such temporal variations can

modify the stress state of the crust, affecting crustal activities including seismicity and fluid migration. As an example, we estimate strain-rate fields of central Japan before and after the 2011 Tohoku-oki earthquake, using the basis function expansion method.

On 1 January 2024, an M7.6 earthquake severely hit the Noto Peninsula in central Japan. Before the earthquake, the Noto Peninsula had swarm activities that had become active since December 2020. The swarm activities are considered to be triggered by upward fluid migration from the lower crust based on significant crustal deformation (Nishimura et al. 2023). However, the reason for the upward fluid migration at this timing has been an open question. In this chapter, to tackle this problem, we estimate temporal changes of strain-rate fields in the Noto Peninsula using the basis function expansion.

We invert GNSS velocity data of central Japan to estimate strain-rate fields in the periods before (2003–2006 and 2008–2010) and after (2012–2015 and 2016–2019) the 2011 Tohoku-oki earthquake. Because of the postseismic deformation of the 2011 Tohoku-oki earthquake, the estimated strain-rate fields exhibit a drastic change before and after the earthquake. Before the earthquake, EW contraction with negative dilatation dominated central Japan, which is consistent with the trilateration and triangulation measurements for more than 120 years (e.g., Ishikawa and Hashimoto 1999) and the long-term deformation estimated from geological data (e.g., Huzita 1980). On the other hand, after the 2011 Tohoku-oki earthquake, extension oriented from EW to NE–SW direction with positive dilatation rates are estimated in a wide region of central Japan. This postseismic extension is interpreted as a consequence of viscoelastic relaxation. The estimated strain-rate fields are further converted into stress-rate fields under the assumption of an isotropic elastic medium. The obtained stress-rate fields also show a reversal from EW compression before the earthquake to EW tension after the earthquake.

Based on these results, we propose a possible scenario of the upward fluid migration. In the compressive stress field before the 2011 Tohoku-oki earthquake, it would not be easy for dehydrated fluids from the Pacific slab to move upward through the upper mantle into the crust; they can only slowly ascend basically keeping the pressure equilibrium. After the 2011 Tohoku-oki earthquake, however, the temporal stress change became positive in a wide region of central Japan. This reduction in compressive stress is likely to have facilitated upward fluid migration in the lower crust, thereby triggering the swarm activity observed in the Noto Peninsula.

5. Joint inversion of GNSS and InSAR data

Southeastern Tibet is a tectonically active region characterized by numerous active faults, high seismicity, and significant crustal deformation. Although GNSS provides three-dimensional

velocities, observation networks in most regions of the world, including southeastern Tibet, are much sparser than those in the Japanese Islands, where dense GNSS networks are available. As a result, localized deformation may be underestimated due to limited data coverage. On the other hand, InSAR provides nominally continuous spatial coverage, although it measures displacement only in the line-of-sight (LOS) direction. By combining GNSS and InSAR data, it is possible to estimate a deformation field with higher resolution than by using either data set alone. Therefore, we develop a joint inversion method combining GNSS and InSAR data to estimate a strain-rate field by using the basis function expansion.

We use GNSS velocity data compiled by Elliott et al. (2025) within 97°E–104°E and 28°N–35°N. The horizontal GNSS velocities exhibit an abrupt change across the Xianshuihe fault (XSHF), a NW-trending left-lateral strike-slip fault. We also use InSAR velocity data from four ascending and four descending frames acquired by the Sentinel-1 satellite, which has been in operation since 2014. The data are provided by the Centre for the Observation and Modelling of Earthquakes, Volcanoes and Tectonics (COMET) in the United Kingdom. To suppress the computational cost, the InSAR data are downsampled from about 1 km to about 5 km grids in the following analyses.

We extend the L2 regularization method based on the basis function expansion with ABIC to the joint inversion of GNSS and InSAR data. InSAR observations provide LOS velocities, which include information on displacements in the vertical direction as well as horizontal directions. Therefore, vertical velocities need to be considered in this inversion, and LOS velocities are expressed as linear combinations of horizontal and vertical components. Orbital errors included in InSAR velocities are corrected by applying a quadratic model. The relative weights of the data are set according to the magnitudes of observational errors. For InSAR data, off-diagonal components of the variance-covariance matrix are considered to account for spatially correlated errors between neighboring pixels, unlike GNSS data. The model parameters that express crustal deformation in the three dimensions (EW, NS, UD) and the parameters to correct InSAR orbital errors are simultaneously estimated by minimizing the objective function, in which three hyperparameters that control the weights of the smoothness constraint on velocities in each direction are determined by minimizing ABIC. By substituting the optimal model parameters into the equation of the basis function expansion, the velocity field is obtained; by analytically differentiating it, the strain-rate field can also be estimated.

The obtained velocity field shows clear discrepancies across XSHF, reflecting its very active motion. The strain-rate field also shows localization of the maximum shear strain rates along XSHF, with peak values exceeding 300×10^{-9} strain/yr, which are significantly larger than those estimated from GNSS data only, and about two or three times larger than those of Fang et al. (2024). The principal

axes of strain rates exhibit shear deformation in the NW–SE direction, which coincides with the strike of XSHF. The dilatation rate field exhibits irregular short-wavelength patterns regardless of the distance from XSHF, although most of which fall within the range of estimation uncertainty. One important advantage of the basis function expansion method is that the uncertainties can also be estimated, enabling a quantitative assessment of the reliability of the solution.

The developed joint inversion method has a rigorous framework for achieving high-resolution estimation. It is expected that this method can be widely applied to tectonic zones worldwide, enabling robust and high-resolution estimation of strain-rate fields.

6. Development of an inversion method to estimate a strain-rate field by introducing sparse modeling

The joint inversion method developed in Chapter 5 achieves high-resolution estimation of a strain-rate field. However, in regions with dense vegetation, including the Japanese Islands, it is difficult to obtain reliable interseismic velocity data from InSAR. Although GNSS stations are densely distributed across the Japanese Islands, the spacing between the stations is comparable to the width of strain concentration zones. This may result in an underestimation of the localized deformation. To address these limitations, it is also valuable to develop a new inversion method that avoids the underestimation of localized deformation even when only GNSS data are available. In this chapter, we incorporate sparse modeling into the basis function expansion framework to achieve high-resolution estimation of a strain-rate field.

Strain-rate fields are generally expected to change smoothly in most areas, but can exhibit abrupt changes in a narrow zone such as a fault zone. To reflect these characteristics, we introduce the sparsity constraint on strain-rate changes as well as the smoothness constraint on velocities. These prior constraints are formulated using the L1-norm and the L2-norm of the second derivatives of the velocity field, respectively. The objective function to be minimized consists of the residual sum of squares (RSS) between observed and estimated velocities, the sparsity constraint, and the smoothness constraint. The relative weights of the sparsity constraint and the smoothness constraint with respect to the RSS are controlled by two hyperparameters, whose optimal values are determined using the leave-one-out cross-validation.

To confirm the validity and limitations of the developed method, we conduct numerical experiments on the anti-plane problem. By changing the values of the locking depth, observation interval, and observation error, we examine how well the estimated strain-rate fields reproduce the true strain-rate field and compare the performance of the proposed method with that of the L2 regularization

method. As a result, we obtain the following findings. (1) The developed method achieves almost equivalent or higher accuracy than the L2 regularization method for both wide and narrow strain concentration zones, corresponding to deep and shallow locking depths, respectively. (2) The advantage of the developed method over the L2 regularization method becomes clearer under less favorable conditions, where fewer observation stations are available (i.e., larger observation intervals). (3) The developed method is effectively applicable to velocity data with small observation errors, but it does not work well for data with large errors.

We apply the developed method to GNSS velocity data across the Arima-Takatsuki fault zone (ATFZ), a group of right-lateral strike-slip faults located in central Japan. The developed method estimates strain rates around the fault zone that are about 10×10^{-9} strain/yr (20–30%) larger than those estimated by the L2 regularization method. The inferred locking depth is 11 km, which is shallower and more realistic than the value obtained by the L2 regularization, given that the depth above which 90% of earthquakes occur (D90) is 12–14 km around ATFZ (e.g., Ito and Nakamura 1998; Omuralieva et al. 2012). These results suggest that the developed method performs well in estimating localized strain rates.

7. Discussion, conclusions, and future works

The basis function expansion has several advantages over many estimation methods used previously: it provides (1) a straightforward and robust estimation of deformation fields and (2) objective determination of hyperparameters that control the strength of prior constraints, and it enables (3) accurate strain-rate field estimation by analytically differentiating the velocity field and (4) reasonable uncertainty estimation. Throughout the thesis, the basis function expansion has achieved high-precision estimation of strain-rate fields. In Chapters 5 and 6, we further extended the applicability of the basis function expansion method. To further improve the basis function expansion method, we propose the following directions for future work: (1) estimating the uncertainties of the maximum shear strain rate, (2) replacing the inverse of a variance-covariance matrix with a precision matrix in inversion analyses of InSAR data, and (3) extending the inversion method of the sparse modeling to two-dimensional problems.

Strain-rate fields are essential for advancing earthquake forecast models and long-term deformation models. In recent years, improvements in both the quality and the spatial coverage of geodetic data have led to more favorable conditions for accurately estimating strain-rate fields. Nevertheless, the development of accurate estimation methods remains crucial; such methodological improvements have a similar impact on estimation accuracy as an increase in the number of

observation stations. Once a robust methodology for accurately estimating strain-rate fields is established, it can be applied to regions worldwide. This broad applicability represents a major advantage of achieving a high-resolution strain-rate field estimation method.

In summary, the thesis has an important contribution to geosciences in achieving more accurate and reliable estimation of the strain-rate fields through geodetic data inversion using the basis function expansion, which includes several methodological extensions. This achievement makes it possible to use strain-rate fields for more reliable and advanced geoscientific applications and provides a foundation for future developments of geodetic data inversion methods.

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