

Non-symplectic automorphisms of prime order on $K3$ surfaces

Shingo TAKI (Graduate School of Mathematics Nagoya University)

We study non-symplectic automorphisms of prime order on algebraic $K3$ surface X which act **trivially** on the Néron-Severi lattice S_X i.e. $\varphi^*\omega_X = \zeta\omega_X$. (ω_X : nowhere vanishing holomorphic 2-form, ζ : primitive p -th root of unity).

Main Theorem

Let r be the Picard number of X . We assume that $S_X^*/S_X \simeq (\mathbb{Z}/p\mathbb{Z})^s$.

(1) If $22 - r - (p-1)s \notin 2(p-1)\mathbb{Z}_{\geq 0} \Rightarrow \nexists \varphi$.

(2) If $22 - r - (p-1)s \in 2(p-1)\mathbb{Z}_{\geq 0} \Rightarrow \exists \varphi$.

$$X^\varphi = \begin{cases} \phi & \text{if } S_X = U(2) \oplus E_8(2) \\ C^{(1)} \amalg C^{(1)} & \text{if } S_X = U \oplus E_8(2) \\ \{P_1\} \amalg \cdots \amalg \{P_M\} & \text{if } p \neq 2, b_1(X^\varphi) = 0 \text{ and } \chi(X^\varphi) = M \\ \{P_1\} \amalg \cdots \amalg \{P_M\} \amalg C^{(g)} \amalg E_1 \amalg \cdots \amalg E_{N-1} & \text{otherwise} \end{cases}$$

– P_j is an isolated point.

– $C^{(g)}$ is a non-singular curve with genus g .

– E_k is a non-rational curve.

$$\chi(X^\varphi) = \frac{pr - (24 - 2p)}{p-1}, \quad b_1(X^\varphi) = \frac{22 - r - (p-1)s}{p-1},$$

$$g = \frac{22 - r - (p-1)s}{2(p-1)},$$

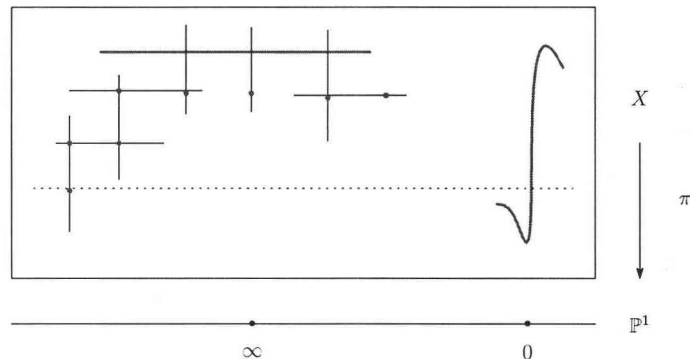
$$M = \begin{cases} 0 & p = 2 \\ \frac{(p-2)r + 22}{p-1} & p = 17, 19 \\ \frac{(p-2)r - 2}{p-1} & \text{otherwise} \end{cases}, \quad N = \begin{cases} \frac{r-s+2}{2} & p = 2 \\ \frac{2p+r-(p-1)s-24}{2(p-1)} & p = 17, 19 \\ \frac{2p+r-(p-1)s}{2(p-1)} & \text{otherwise} \end{cases}$$

Example

We give affine equations of elliptic $K3$ surfaces $X : y^2 = x^3 + x + t^7$. Then $S_X = U \oplus E_8$.

The elliptic fibration π has a singular fiber of type II^* over $t = \infty$ and 14 singular fibers of type I_1 over $t^{14} = -4/27$.

We put $\varphi(x, y, t) = (x, y, \zeta t)$. Then φ is a non-symplectic automorphisms of order 7.



Now $p = 7$, $r = 10$ and $s = 0 \rightsquigarrow X^\varphi = C^{(1)} \amalg \mathbb{P}^1 \amalg \{pt\} \times 8$.

Remark

We assume $22 - r - (p-1)s \in 2(p-1)\mathbb{Z}_{\geq 0}$. For each (r, s) , there exist a $K3$ surface which has a non-symplectic automorphism of order p acting trivially on S_X .

瀧 真語 (m04022x@math.nagoya-u.ac.jp)