

On a characteristic property of the tent map

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As a generalization of a characteristic property of fundamental chaotic maps such as the tent map, a primitive chaos is defined by $(X, \{X_\lambda, \lambda \in \Lambda\}, \{f_{X_\lambda}, \lambda \in \Lambda\})$ which satisfies the following property, where X is a set, $\{X_\lambda, \lambda \in \Lambda\}$ is a family of subsets of X which contains no empty set, and $\{f_{X_\lambda}, \lambda \in \Lambda\}$ is a family of maps $f_{X_\lambda} : X_\lambda \rightarrow X$ [1].

(P) For any infinite sequence $\omega_0, \omega_1, \omega_2, \dots$ of sets, there exists an initial point $x_0 \in \omega_0$ such that $f_{\omega_0}(x_0) \in \omega_1, f_{\omega_1}(f_{\omega_0}(x_0)) \in \omega_2, \dots$, where $\omega_i \in \{X_\lambda, \lambda \in \Lambda\}$ for each i .

The primitive chaos is closely related to general problems about determinism, causality, free will, and irreversibility [1], which are vital problems in science (e.g., refs. [2-26]).

Then, the following propositions present sufficient conditions for guaranteeing the primitive chaos.

Proposition 1 [1]. *If X is a nondegenerate Peano continuum, for any $\varepsilon > 0$, there exist finitely many nondegenerate Peano subcontinua $X_1, \dots, X_n$ covering X such that $\text{dia } X_i < \varepsilon, i = 1, \dots, n$. Then, for each i , for any positive integer n^i , for any n^i points $x_1^i, \dots, x_{n^i}^i \in X_i$ and $y_1^i, \dots, y_{n^i}^i \in X$, there exists a continuous surjection $f_{X_i} : X_i \rightarrow X$ such that $f_{X_i}(x_1^i) = y_1^i, \dots, f_{X_i}(x_{n^i}^i) = y_{n^i}^i$, and they satisfy the property (P).*

Proposition 2 [27]. *If X is a Cantor set, for any positive integer n , there exist a partition $\{X_1, \dots, X_n\}$ of X and maps $f_{X_i} : X_i \rightarrow X$, $i = 1, \dots, n$, and they satisfy the property (P).*

Here, a nondegenerate space means the space that consists of more than one point, a Peano continuum is a locally connected continuum, and a continuum is a nonempty connected compact metric space. A Cantor set is any space that is homeomorphic to the Cantor middle-third set, and a space is a Cantor set if and only if it is a zero-dimensional, perfect compact metric space [28, Theorem 8.1].

Proposition 1 explains the reason why we are surrounded by diverse chaotic behaviors [29], and Proposition 2 implies the possibility of the Cantor set for a new recognition of natural phenomena.

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