

Summary of thesis: Theoretical study of unconventional finite-momentum superconductivity

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In 1964, it was theoretically predicted that Cooper pairs with finite center-of-mass momentum could be stabilized in superconductors in a strong magnetic field. This state is known as the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state and is the first example of finite-momentum superconducting states. Since then, it has been revealed that finite-momentum superconductivity can be achieved in a variety of mechanisms and systems and, even sixty years after the first prediction, it continues to be intensively investigated both experimentally and theoretically. In particular, following the discovery of its relation to the superconducting diode effect, it has become one of the central topics in condensed matter physics and has attracted much attention for potential applications.

In order to clarify the diverse phenomena associated with finite-momentum superconductivity, we perform theoretical analyses of two types of unconventional finite-momentum superconductivity which, unlike conventional finite-momentum superconductivity such as the FFLO state, do not arise from the Pauli paramagnetic effect. The first is the 'anapole superconductivity,' which can be realized when the superconducting order parameter spontaneously breaks inversion and time-reversal symmetry due to the parity mixing of Cooper pairs. In this superconducting state, Cooper pairs can acquire finite center-of-mass momentum even in the absence of an external magnetic field, due to the asymmetric Bogoliubov spectrum or the quantum geometric effect. Using a minimal model of UTe_2 , one of the candidates for anapole superconductors, we numerically calculate Cooper pairs' momentum, the superconducting piezoelectric effect, and the superconducting diode effect. We reveal their properties and relation to the microscopic origin of Cooper pairs' momentum based on results obtained from several parameter sets with different origins. In particular, we confirm a close relation between the superconducting piezoelectric effect and Cooper pairs' momentum, independent of its origin. In addition, we investigate the 'monopole superconductivity,' in which Cooper pairs have zero center-of-mass momentum due to the symmetry of the superconducting order parameter. We compare its properties with those of the anapole superconductivity to

highlight essential differences between zero- and finite-momentum superconducting states.

The second is the 'orbital FFLO superconductivity.' In contrast to the conventional FFLO state induced by the Pauli paramagnetic effect, finite-momentum Cooper pairs are stabilized by the orbital effect in an external magnetic field. Recent reports have demonstrated the realization of this superconducting state in various transition metal dichalcogenide (TMD) systems, including atomically thin films and bulk materials. A notable example is the misfit compound $(\text{PbSe})_{1.14}(\text{NbSe}_2)_3$, consisting of alternating monolayer PbSe and trilayer 2H-type NbSe₂. A recent experiment reveals that its superconducting state is two-dimensional and suggests the realization of the orbital FFLO state in the individual trilayer NbSe₂ blocks. Motivated by this experimental result, we theoretically investigate the superconducting phase diagram of trilayer NbSe₂. Using a minimal model for trilayer NbSe₂, we numerically solve the Bogoliubov-de Gennes equation and predict the multiple finite-momentum superconducting phases: the 'orbital FFLO phase' and the 'layer-selective FFLO phase.' In the former phase, the finite-momentum Cooper pairs are stabilized only in the outer layers, while a Larkin-Ovchinnikov-type superconducting order parameter arises in the middle layer due to the proximity effect. On the other hand, in the latter phase, zero-momentum Cooper pairs corresponding to the uniform Ising superconductivity are realized in the middle layer in addition to the above finite-momentum Cooper pairs. Furthermore, we confirm that there is a large residual local density of states in the low energy region of the orbital FFLO phase, while the layer-selective FFLO phase shows a V-shaped local density of states. Since the mechanism realizing these phases is ubiquitous in multilayer TMDs, our prediction paves the way for further research of the exotic superconductivity in these materials.