

Refinable maps and cohomological dimension

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The notion of refinable maps was introduced by Jo Ford and Rogers [2] as follows: A map $r: X \longrightarrow Y$ between compacta is *refinable* provided that for every $\varepsilon > 0$, there exists an ε -map $f_\varepsilon: X \longrightarrow Y$ such that $d(r, f_\varepsilon) < \varepsilon$. It is known that it plays an interesting part in dimension theory. Namely, let $r: X \longrightarrow Y$ be a refinable map between compacta. Then we have:

- (1) $\dim X = \dim Y$ (see [4] and [6]),
- (2) if X is weakly infinite-dimensional, then so is Y (see [5]),
- (3) if X has the property C , then Y also has the property C (see [1] and [3]),
- (4) $c\text{-dim}_G X \geq c\text{-dim}_G Y$ for every abelian group G ([6]).

Here a compactum X has *cohomological dimension* $\leq n$ with respect to G , written $c\text{-dim}_G X \leq n$, provided that every map $\alpha: A \longrightarrow K(G, n)$ of a closed subset A of X to an Eilenberg-MacLane space $K(G, n)$ of type (G, n) has a continuous extension $\bar{\alpha}: X \longrightarrow K(G, n)$ of α .

In [6], we posed the problem *whether the converse inequality of (4) hold or not*. In this note we will announce a partial answer.

Namely, in the case of $G = \mathbb{Z}$ or $\mathbb{Z}_p$, the converse one is valid. For the purpose, we introduce a new dimension-like function, $a\text{-dim}_G$, called *approximable dimension with respect to an abelian group G* (see [7]).

Definition 1. Let K be an ANR and let n be a natural number.

Let $\varepsilon > 0$ be a positive number. A map $\psi: Q \longrightarrow P$ between compact polyhedra is (K, n, ε) -*approximable* provided that there exists a triangulation L of P such that for any triangulation M of Q , there is a map $\psi': |M^{(n)}| \longrightarrow |L^{(n)}|$ satisfying the following conditions:

- 1) $d(\psi', \psi|_{|M^{(n)}|}) \leq \varepsilon$,
- 2) for any map $\alpha: |L^{(n)}| \longrightarrow K$, the map $\alpha \circ \psi': |M^{(n)}| \longrightarrow K$ admits a continuous extension over Q .

Definition 2. Let K be an ANR. A compactum X has *approximable dimension with respect to K less than n* , written $a\text{-dim}_K X \leq n$, provided that for every compact polyhedron P , every map $f: X \longrightarrow P$ and every positive number $\varepsilon > 0$, there exists a compact polyhedron Q and maps $\varphi: X \longrightarrow Q$, $\psi: Q \longrightarrow P$ such that

- 3) $d(f, \psi \circ \varphi) \leq \varepsilon$,
- 4) ψ is (K, n, ε) -approximable.

Specially, let G be an abelian group and let consider $K = K(G, n)$ in the above definitions. Then we use the terminology (G, n, ε) -*approximability* and *approximable dimension with respect to G* instead of $(K(G, n), n, \varepsilon)$ -approximability and approximable dimension with respect to $K(G, n)$, respectively. And we denote

by $\text{a-dim}_G X \leq n$ instead of $\text{a-dim}_{K(G,n)} X \leq n$.

Concerning the relation between approximable dimension and ordinal and cohomological dimension, we have the following:

Lemma ([7]). *Let X be a compactum. Then we have*

- (i) $\dim X \leq n$ if and only if $\text{a-dim}_{S^n} X \leq n$,
- (ii) in the case of $G = \mathbb{Z}$ or $\mathbb{Z}_p$, $\text{c-dim}_G X \leq n$ if and only if $\text{a-dim}_G X \leq n$.

Note that approximable dimension does not coincide with cohomological dimension with respect to the group $\mathbb{Q}$ of all rational numbers (see [8]).

In order to have the converse inequality of (4), we have shown the following:

Theorem. *Let K be an ANR and let $r: X \longrightarrow Y$ be a refinable map between compacta. Then $\text{a-dim}_K X \leq n$ if and only if $\text{a-dim}_K Y \leq n$.*

As its consequence, we have the following:

Corollary. *Let $r: X \longrightarrow Y$ be a refinable map between compacta. Then we have:*

- (i) $\dim X = \dim Y$,
- (ii) $\text{c-dim}_{\mathbb{Z}} X = \text{c-dim}_{\mathbb{Z}} Y$,
- (iii) $\text{c-dim}_{\mathbb{Z}_p} X = \text{c-dim}_{\mathbb{Z}_p} Y$.

Note that some of our results can be generalized to compact

Hausdorff spaces or to non-compact normal spaces. The detail will be appeared elsewhere.

References

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