

Fundamental Characteristics of Fluidable Material Dam Break
Flow with Finite Extent and Its Application

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Abstract

The fundamental study of flow characteristics is carried out by the approach of dam-break flow of finite extent model in this dissertation. Based on the finite extent dam-break flow model, the flow characteristics of viscous Newtonian and non-Newtonian fluids are investigated. Application of the study model to study the rheological properties of fresh concrete, which is treated as a kind of Bingham fluid is carried out.

The classical problem of dam-break flow of infinite extent is revisited to gain some insights to the problem of finite extent dam-break flow. The solution for the dam-break flow of infinite extent of ideal fluid which is available by Ritter's solution is however not applicable to the problem of finite extent dam-break flow due to the effect of the end wall at the upstream part of the finite extent dam. In the case of finite extent dam-break flow problem, as soon as the flow is initiated by the instantaneous release of lock gate, the front wave will propagate downstream, and a negative wave will propagate upstream. The first negative wave reflected by the end wall will travel downstream, trailing behind the front wave. The region ahead of this point is analogous to the infinite dam-break flow problem and Ritter's solution is valid in this region. However Ritter's solution is not valid in the region behind this position (the position of the first reflected negative wave). Therefore, in this study, an approximate solution for this region is derived using moving coordinate system based on the position of this first reflected negative wave. The propagation of the front wave and the attenuation of the depth at the origin derived for the problem of finite extent dam-break of ideal fluid are used as parameters to define the characteristic of the inertial phase.

In studying the viscous phase flow characteristic, the model of finite extent dam is used as well. It is assumed that in the viscous phase flow, self-similarity exists for the depth and velocity of flow. This assumption is the basis for the analytical derivation of the characteristic of viscous phase flow. Similarity solutions for the propagation of front wave and attenuation of depth at the origin are derived for the general power-law viscous fluid. Based on this general solution, the viscous phase characteristics of Newtonian and non-Newtonian fluid are obtained and later verified with numerical models. The inertial phase flow characteristic of inviscid fluid derived earlier is observable in the inertial phase flow of viscous fluid as well.

An integral model is developed to re-produce the characteristics of inertial and viscous phase flow. Through integration, the governing equation of shallow water flow is transformed into ordinary differential equation which can be easily solved with initial boundary condition. The existence of inertial and viscous phase flow is verified with the earlier findings. The integral model is capable of reproducing the inertial and viscous phase flow characteristics.

The application of dam-break flow of finite extent is extended to the simulation of slump flow test of fresh concrete where the rheological characteristics of Bingham fluid are investigated. A numerical model is developed based on the VOF-CIP method and verified with available experimental data. The inertial and viscous flow characteristics of Bingham fluid are also derived and verified with the VOF-CIP model.

Related to the problem of finite extent dam is the phenomenon of abrupt expansion flow. The abrupt expansion flow at the vicinity of the expansion is analogous to a series of finite extent dam-break flows at different phases with the origin of each dams situated along center longitudinal axis. The phenomena of abrupt expansion flow of supercritical flow are studied qualitatively and analytically. A three dimensional numerical model based on the VOF-CIP method is used to verify the analytical findings.

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Chapter 1

Introduction

1.1 Preliminaries

The phenomenon of intrusion of gravity current is ubiquitous in nature and human-made activities. The propagation of sea breeze front, thunderstorm outflows, the flow of estuaries effluent are a few examples of natural phenomena of gravity currents. Human activities such as the discharge of industrial waste into sea, rivers or lakes, and the release of gases from factory chutes into the atmosphere are also phenomena displaying the nature of gravity current. The study of gravity current has been very intensive, with some of the earliest studies dealing with experiments of constant inflow of gravity current where a constant flux of higher density fluid is introduced or injected into a less dense fluid environment. Fixed volume gravity current experiments were carried out either by the instantaneous release of fixed volume of fluid of higher density held behind a gate into of a much deeper fluid of lower density or the release of a fixed volume fluid of higher density into a lower density fluid where the higher density fluid spreads axis-symmetrically on horizontal surface.

Keulegan (1957), Britter and Simpson (1978), Huppert and Simpson (1980) carried out a series of experiments to study the motion of saline water in fresh water. In their experiments, a fixed volume of saline water is released into fresh water of much greater volume and heights in a rectangular channel. Although the focus of their studies is different, in general, they observed the changing of flow phase during the instantaneous release of the saline water. Rottman and Simpson (1983) also conducted a series of experiments on the release of fixed volume of salt water into fresh water in rectangular channel, but focusing on the inviscid motion of the current. Rottman and Simpson (1983) noted that the fluid passes through an initial adjustment phase, followed by initial self-similarity phase and finally reaches viscous-pressure phase. In the initial adjustment phase the fluid propagates almost at a constant speed and depth. At this phases, the propagation of the front current, L is proportional to time ($L \propto t$). This phase is analogous to the solution of dam break flow of ideal fluid by Ritter (1892) where in the case of ideal fluid, the speed of the front current is given by $U = 2\sqrt{gh_o}$ where h_o is the initial depth of the dam. Based on Ritter solution, the front current propagation is therefore, $L = 2\sqrt{gh_o}t$, if the dam is located at origin $x = 0$. Therefore, the initial stage of the front current is governed by the initial condi-

tion of the dam e.g, initial depth h_o . In the initial self-similar phase, the motion of the current is governed by the balance between the inertia and buoyancy of the current is dominates the flow. Rottman and Simpson (1983), Huppert and Simpson (1980), show that from their experiments and theoretical analysis, that the front of the current at this inertia-buoyancy phase is proportional to the time, to the power of $2/3$, ($L \propto t^{2/3}$). This analytical work are based on the shallow water equations for two inviscid and incompressible fluids, of slightly different densities and without mixing between them. The results are verified by saline gravity current experiments. Huppert and Simpson (1980) derived a similarity solution for the inertia-buoyancy phase which also shows that in the inertial-buoyancy phase of intruding gravity current into a less dense fluid, the front current propagates proportional to time to the power of $2/3$.

The viscous-buoyancy phase in gravity current will dominate the flow in case where the gravity current flow time is more than t_1 where t_1 is the time where the viscous force overwhelms the inertia force of the current. Didden and Maxworthy (1982) and Huppert (1982) derived the similarity solution based on the mathematical model of two fluids, one with higher density intruding into one of the lower density. For the case of two-dimensional geometry, Huppert found that for the case of fixed volume gravity current, the front of the current propagates proportionally to time to the power of $1/5$ in the viscous-buoyancy phase. Hosoda et al. (2000) also reached the same finding based on the assumption of self similarity of velocity and flow depth in the viscous-pressure phase flow in his analytical work.

Although the phenomenon of intruding gravity current is ubiquitous and have been intensively studied, the problem free surface fluid released from behind a lock gate, also widely known as the dam-break flow problem, equally received attentions and rigorous studies. The dam-break flow problem, as opposed to fixed gravity current problem, deals with free-surface flow where fluid behind a lock is instantaneously released into a horizontal channel. In the case where the fluid behind the lock gate is of infinite extent as shown in Fig. 1.1 (a), Ritter (1892) gave the analytical solution for the front wave and negative wave propagation and the flow profile for the case of ideal fluid. Martin and Moyce (1952) conducted one of the earliest experiment of dam break flow of finite extent as shown Fig.(1.1) (b), by the release of a water column on a

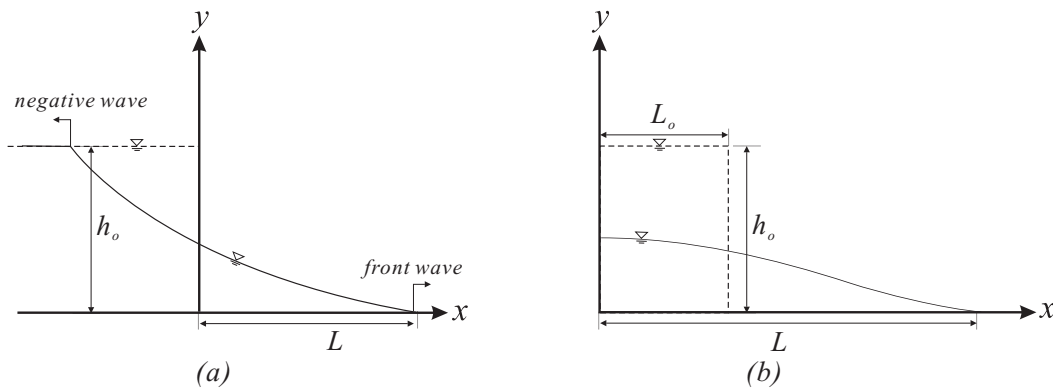


Figure 1.1: (a) Dam-break flow of infinite extent, (b) Dam-break flow of finite extent

rigid horizontal plane. Due to the upstream end-wall in the dam-break flow of finite extent, the solution by Ritter cannot be used to fully describe the flow in this case.

The application of dam break flow of finite extent ranges from the determination of rheological properties of fresh concrete in the work of Kokado et al. (2001), where the slump flow test of fresh is regarded as a kind of dam break flow of finite extent. Piau (2005) studied the behavior of power-law viscous fluids based using the dam-break flow of finite extent model. Dam-break flow of finite extent model was used by Ancey and Chochard (2008) in the study of Herschel-Bukley viscoplastic fluids down a steep flumes.

1.2 Objective and justification of Study

The main objective of this study is to firstly develop an approximate solution for the dam-break flow of finite extent problem, from the moment the flow is initiated by the release of the lock gate to sufficiently long time in the case of ideal fluid. As mentioned above, the Ritter solution for dam-break flow of infinite extent cannot be used to fully describe the flow in the case of dam-break flow of finite extent. This is due to fact that the negative wave, which initially propagates upstream towards the end-wall, before being reflected and travels downstream away from the end-wall causes disturbance to the flow trailing behind it, thus changing the nature of flow which is initially analogous to the flow in the case of infinite extent problem.

The propagation of the front current mentioned in the literature reviews has been the main focus of interest in most of the studies and has been used as parameter describing the flow characteristics. Since the study model is a finite extent dam-break model, it is useful to include the depth of flow at the origin in addition to the front current position as parameter to describe the flow characteristic. Even in the case of inviscid fluid, it is though that the characteristic of the attenuation of depth at the origin deserves attention, and maybe of pedagogic use to explain the whole dam-break flow of finite extent phenomena.

From the literature study, in the case of viscous fluid, the flow enters self-similarity viscous-pressure phase, shortly after the inertial flow phase. This study uses this self-similarity characteristic of the flow to derive a power-law solution for viscous-pressure phase flow. An integral model is developed to describe the characteristics phases of the flow and the transition from the inertial phase flow to viscous phase flow.

The study of flow characteristics is further extended to Bingham fluid through the study of slump flow test of fresh concrete by numerical model. A set of analytical solutions describing the characteristics of Bingham fluid is derived and verified with the numerical model. Finally, the phenomenon of abrupt expansion flow is studied analytically and qualitatively. A laboratory experiment and numerical simulation of abrupt expansion flow are carried out to verify the analytical findings.

In this study, hereafter, the term *inertial phase* will be used to represent the flow phase where the flow is governed by the inertia-pressure balance, while the term *viscous phase* represents the flow phase condition where the flow is governed by the balance of pressure force and viscous force.

1.3 Scope of study

This manuscript is divided into 8 chapters, including Introduction in Chapter 1, and Conclusion in Chapter 8.

Chapter 2 presents the solution of dam break flow of finite extent. Firstly, the derivation of the solution for dam break flow of finite-extent is explained. The approximate solution for the flow profile in the whole region from the moment the flow is initiated is derived. The solutions from the theoretical analysis are validated using numerical models. In the latter part of the chapter, the improvement for the approximate solution is presented. The similarity solution of the attenuation of the depth of flow at the origin is also derived in this chapter. The studies in this chapter provide an insight to the characteristic of inertial phase flow.

Chapter 3 deals with the derivation of viscous phase flow characteristics. Self-similarity is assumed for the flow depth and velocity in the viscous phase, and provided the basis for the theoretical analysis. Newtonian and non-Newtonian fluids are considered in this chapter. The theoretical findings are validated with numerical models.

Ensuing Chapter 2 and Chapter 3 findings is Chapter 4, which deals with the development of integral model to further verify the existence of inertial and viscous phase and the transition between the phases. The findings in Chapter 2 and 3 are used to validate the results from the integral model.

Chapter 5 explains the development of numerical model to be used in Chapter 6 and Chapter 7. A two-dimensional model based on cylindrical coordinate is developed for the simulation of slump for test of fresh-concrete in Chapter 6. On the other hand, a three-dimensional model is developed for the simulation of abrupt expansion flow in Chapter 7. Both numerical models are based on the VOF method (Hirt and Nicholas, 1981). Cubic Interpolated Propagation (CIP) scheme (Yabe and Aoki, 1991b) is used to solve the advection terms in the governing equations.

Chapter 6 is devoted for the application of dam-break flow of finite extent. In this chapter, numerical simulations of slump flow test of fresh concrete are carried out and the findings are validated with experiment data carried out by Kokado et al. (2001). The flow characteristics of Bingham fluid are dealt and later verified by the same numerical model.

Chapter 7 sees the phenomena of abrupt expansion flow as an analogy to the dam-break flow of finite extent model. The abrupt expansion flow phenomena is initially studied using the Method of Characteristics where the characteristics of the flow near the abrupt expansion e.g. expansion angle, disturbance wave angle are derived. The findings are verified with the results from the numerical simulation of abrupt expansion flow using a three-dimensional numerical model developed in Chapter 5 and also laboratory experiments.

Chapter 2

Inertial Flow Characteristics by Means of Dam-Break Flow of Finite Extent

2.1 Preliminaries

Dam-break flows have been studied intensively, analytically and experimentally as actual dam-break failure and associated effects such as debris flow and surging wave. The dam-break flow of finite-extent is equally vital and useful especially where it is easier to set up a finite extent dam-break model in the laboratory. The application of dam-break flow of finite extent in determining rheological properties of fresh-concrete can be seen in the work of Kokado et al. (2001), and Ancy and Chochard (2008) used it to study non-Newtonian fluid. The solution for dam-break flow of finite extent problem from the moment the flow is initiated to sufficiently long time is therefore vital and deserves a rigorous study.

Rottman and Simpson (1983), in his series of saline water intrusion experiments observed that during initial stage, immediately after the saline water current was initiated the front current travels proportionally to time ($L \propto t$). It is thought that this is analogous to the dam-break flow of infinite extent of ideal fluid where the front propagates proportionally with time, (Ritter, 1892). However, Rottman and Simpson (1983) later observed that the flow later enters a phase of inertial-bouyancy balance where the front current (front wave) propagates at a slower rate, ($t \propto t^{2/3}$). In the case of free surface flow, the inertial phase may not be of Rottman and Simpson (1983)'s observation. In fact, in the case of dam-break flow of free surface, the front wave is expected to propagate at a faster rate. A check on the front wave propagation of flow from a dam-break flow of finite extent is plotted on Fig 2.1. Based on the experiment carried out by Martin and Moyce (1952), it is thought that the experiment further confirms that in the case of free-surface flow, the front wave propagates almost proportionally to time $L \propto t$. Consequently, this result also represents the characteristic of inertial-pressure phase of the flow.

This chapter's objective is to derive an approximate solution for the problem of dam-break flow of finite-extent for ideal fluid. Based on the analytical solution, the inertial phase characteristic is reconfirmed by investigating the front wave propagation, as well as the attenuation of depth at the origin.

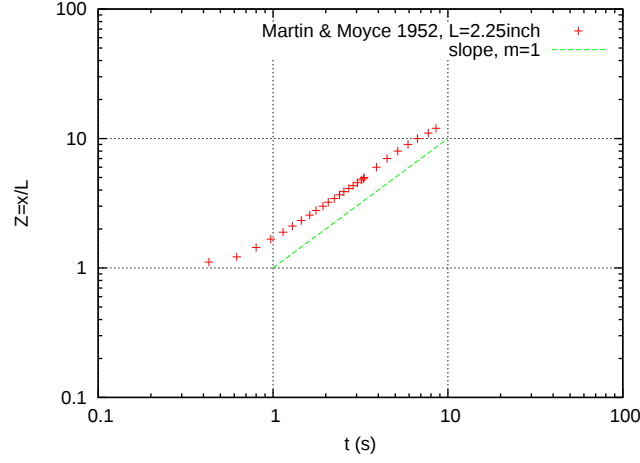


Figure 2.1: Propagation of front wave x , from experiment of water column by Martin and Moyce (1952) with initial column width, $L = 2.25 \text{ inch}$.

2.2 The approach to dam-break flow of finite extent problem

First of all, is worth to look at the dam-break flow of infinite extent model to gain some insights to study the dam-break flow of finite extent model. In the case of infinite extent dam-break model of ideal fluid, immediately after the flow is initiated, the front wave will propagate downstream with the speed of $\sqrt{2gh_o}$, while the a negative wave will propagate upstream with the speed of $\sqrt{gh_o}$, where h_o is the initial depth of the dam shown in Fig 2.2 (a). This solution is first given by Ritter (1892). Between this front wave and negative wave, the depth profile $h(x,t)$ and velocity $U(x,t)$ are given as follows, Ritter (1892):

$$h(x,t) = \frac{1}{9g} \left(2\sqrt{gh_o} - \frac{x}{t} \right)^2 \quad (2.1)$$

$$U(x,t) = \frac{2}{3} \left(\sqrt{gh_o} + \frac{x}{t} \right) \quad (2.2)$$

The front wave position of dam-break flow of infinite extent, L based on Ritter solution, can be therefore written as,

$$L(t) = 2\sqrt{gh_o}t \quad (2.3)$$

This solution can be plotted in hodograph form, as shown in Fig.2.2 (b).

However, in the case of dam-break flow of finite extent shown in Fig.2.3 (a), as soon as the flow is initiated by the instantaneous release of the fluid behind the lock gate, the front wave will travel downstream while a negative wave will travel upstream depressing the surface of the flow. Before the negative wave reaches the wall, the phenomena is exactly similar to the flow of dam-break flow of infinite extent. The negative wave propagating upstream will be reflected by the upstream wall before traveling downstream away from the upstream wall. The time for the negative wave to reach the upstream wall is defined as t_o , where it can be easily determined as,

$$t_o = \frac{L_o}{c_o} \quad (2.4)$$

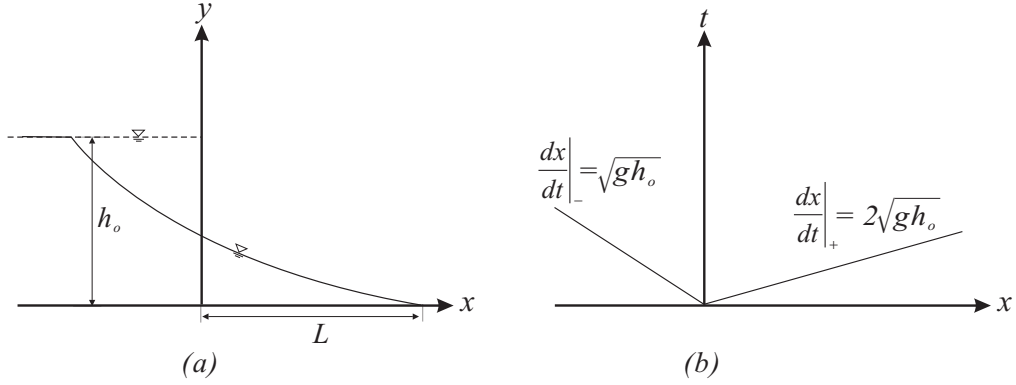


Figure 2.2: (a) Dam-break flow of infinite extent with initial depth h_o (b) hodograph of solution of dam-break flow of infinite extent

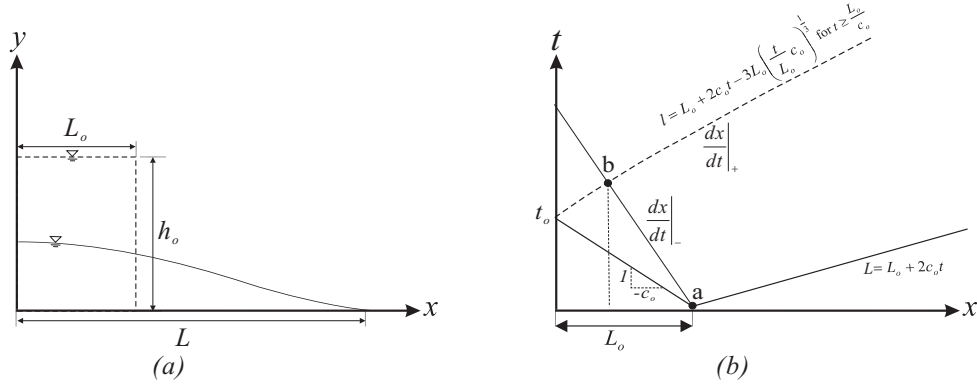


Figure 2.3: (a) Dam-break flow of finite extent with initial depth h_o (b) hodograph of solution of dam-break flow of finite extent

where c_o is the wave celerity defined as $\sqrt{gh_o}$. The position of the first negative wave reflected by the upstream wall (represented by dotted line in Fig.2.3 (b)) can be determined by solving $l(t)$, which is the position of the point b , by using characteristic line based on the Method of Characteristics (MOC) (Hogg and Pritchard, 2004). The solution for the position of this first negative wave by MOC is,

$$l(t) = L_o + 2c_o t - 3L_o \left(\frac{c_o t}{L_o} \right)^{\frac{1}{3}} \quad (2.5)$$

and the velocity at b , U_b can be derived as,

$$U_b(t) = 2c_o - c_o \left(\frac{c_o t}{L_o} \right)^{-\frac{2}{3}} \quad (2.6)$$

By comparing Eq.(2.7) and Eq.(2.5), it can be seen that the first negative wave reflected by the upstream wall always trail behind the front wave, to be exact by the amount of $-3L_o(c_o t/L_o)^{\frac{1}{3}}$. Hence, the approach to the problem of dam-break flow of finite extent is to divide the flow into two regions. The first region, named Region A hereafter, is the region preceding the first negative wave, undisturbed by the negative wave reflected from the upstream wall, traveling

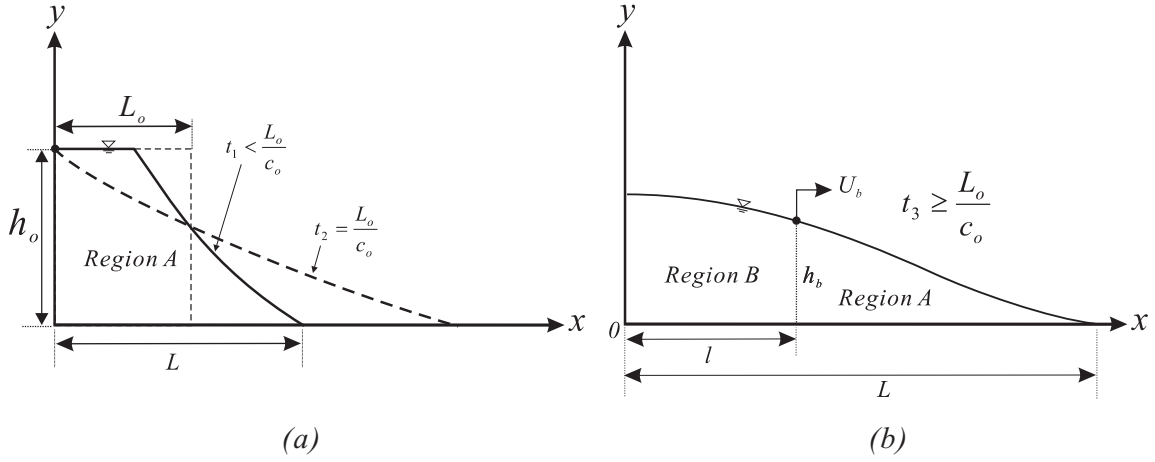


Figure 2.4: Dam break flow of finite extent: a) flow profile up to $t = L_o/c_o$ and b) flow profile for $t \geq L_o/c_o$

downstream, and remains analogous to the flow in dam-break flow of infinite extent. The second region, named Region B hereafter, is the region trailing behind the first negative wave. Region A is shown in Fig.2.4 (a) while both Region A and B are shown in Fig.2.4 (b). It is worth noting that Region B starts from $t > t_o = L_o/c_o$ and the area of Region B expands as point b travels downstream away from the wall with time. Since Region A is not influenced by the negative wave, it is analogous to the condition of dam break flow of infinite extent which solution is derived by Ritter (1892). With slight modification to consider the fact that the finite extent dam is placed at $x = L_o$, the front wave position and flow profile can be expressed as follows,

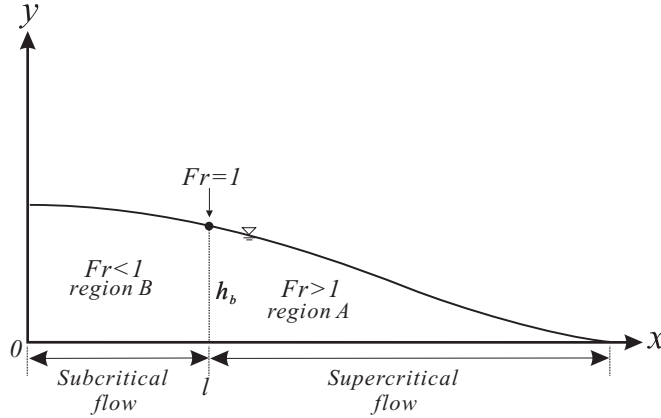
$$L(t) = L_o + 2\sqrt{gh_o}t \quad (2.7)$$

$$h(x,t) = \frac{1}{9g} \left(2\sqrt{gh_o} - \frac{x-L_o}{t} \right)^2 \quad (2.8)$$

It is worth to note that at the position of point b in Fig.2.4 (b) which is the position of the first reflected negative wave, the Froude number, Fr is equal to one, $Fr = 1$. The Froude number at point b can be determined as follows,

$$Fr = \frac{\frac{dl}{dt} - U_b}{c_o} = 1 \quad (2.9)$$

The importance of the position of point b can be seen here, as it divides the flow region into two, one defining super-critical flow, which is Region A and another defining region of sub-critical flow, which is Region B as shown in Fig 2.5. The existence of sub-critical flow in Region B means that analytical solution is obtainable by considering boundary conditions at one upstream point and one downstream point in Region B, which is used as the basis of the analytical formulation in this study.


 Figure 2.5: Sub-critical and super-critical regions are defined by the position of point b

2.3 Approximate analytical solution of dam-break flow of finite extent

2.3.1 Governing equations

In this section, the approximate solution for Region B will be derived. It is assumed that in the problem of sudden release of gravity current, the length scale of horizontal variation along the current greatly exceed the thickness, a depth averaged shallow water equation is adequate to describe the flow (Hoult, 1972). Therefore, the governing equations for the dam-break flow of finite extent problem can be expressed by the one-dimensional depth averaged continuity and momentum equations governing equations as follows,

$$\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x} = 0 \quad (2.10)$$

$$\frac{\partial hU}{\partial t} + \frac{\partial hU^2}{\partial x} + gh \frac{\partial h}{\partial x} = \frac{\tau_b}{\rho} \quad (2.11)$$

with h as the depth of flow, U as the flow velocity in x direction, ν as the kinematic viscosity, ρ as the fluid density, τ_b as the bottom shear stress and g as the gravity acceleration. Schematic of dam-break flow of finite extent used in this study is shown in Fig.2.6, where h_m is the depth at the origin, L is the wave front position from the origin, L_o and h_o are initial width and height of the dam respectively .

2.3.2 Temporal variation of depth of flow at the origin

The depth of flow h , and velocity U , are assumed to be expressible as a series of power. By using the Taylor's power series expansion, they can be written as in Eq. (2.12) and Eq. (2.13) respectively.

$$h(t, x) = h_m(t) + a_1(t) \left(\frac{x}{h_o} \right) + a_2(t) \left(\frac{x}{h_o} \right)^2 + a_3(t) \left(\frac{x}{h_o} \right)^3 + a_4(t) \left(\frac{x}{h_o} \right)^4 + \dots \quad (2.12)$$

$$U(t, x) = \sqrt{gh_o} \left[b_1(t) \left(\frac{x}{h_o} \right) + b_2(t) \left(\frac{x}{h_o} \right)^2 + b_3(t) \left(\frac{x}{h_o} \right)^3 + b_4(t) \left(\frac{x}{h_o} \right)^4 + \dots \right] \quad (2.13)$$

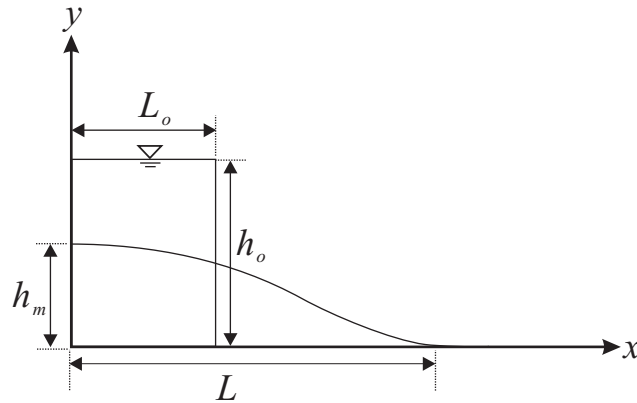


Figure 2.6: Dam break flow of finite extent.

where h_o = representative flow depth, x = distance from the origin, $a_1(t)$, $a_2(t)$, $a_3(t)$, $a_4(t)$, $b_1(t)$, $b_2(t)$, $b_3(t)$, and $b_4(t)$ are time dependent coefficients while other parameters are defined as in Fig. 2.6.

By substituting the power series of h and U into the continuity and momentum equations, we can rewrite both equations again by the order of x as in the following Eq. (2.14) to Eq. (2.22).

Continuity Equation:

0th order

$$\frac{dh_m}{dt} + \sqrt{gh_o} \frac{h_m b_1}{h_o} = 0 \quad (2.14)$$

1st order

$$\frac{1}{h_o} \frac{da_1}{dt} + \frac{\sqrt{gh_o}}{h_o^2} [2h_m b_2 + 2a_1 b_1] = 0 \quad (2.15)$$

2nd order

$$\frac{1}{h_o^2} \frac{da_2}{dt} + \frac{\sqrt{gh_o}}{h_o^3} [3h_m b_3 + 3a_1 b_2 + 3a_2 b_1] = 0 \quad (2.16)$$

3rd order

$$\frac{1}{h_o^3} \frac{da_3}{dt} + \frac{\sqrt{gh_o}}{h_o^4} [4h_m b_4 + 4a_1 b_3 + 4a_2 b_2 + 4a_3 b_1] = 0 \quad (2.17)$$

4th order

$$\frac{1}{h_o^4} \frac{da_4}{dt} + \frac{\sqrt{gh_o}}{h_o^5} [5a_1b_4 + 5a_2b_3 + 5a_3b_2 + 5a_4b_1] = 0 \quad (2.18)$$

Momentum equation:

0th order

$$\frac{gh_m^2 a_1}{h_o} = 0 \quad (2.19)$$

1st order

$$\frac{\sqrt{gh_o}}{h_o} h_m \frac{d}{dt} (h_m b_1) + \frac{g}{h_o} (2h_m^2 b_1^2) + \frac{g}{h_o^2} (2a_2 h_m^2 + 2h_m a_1^2) = -3v \sqrt{gh_o} \frac{b_1}{h_o} \quad (2.20)$$

2nd order

$$\begin{aligned} & \frac{\sqrt{gh_o}}{h_o^2} \left[h_m \frac{d}{dt} (h_m b_2 + a_1 b_1) + a_1 \frac{d}{dt} (h_m b_1) \right] + \frac{g}{h_o^2} (6b_1 b_2 h_m^2 + 5h_m a_1 b_2^2) \\ & + \frac{g}{h_o^3} [3a_3 h_m^2 + 4h_m a_1 a_2 + a_1 (a_1^2 + 2a_2 h_m)] = -3v\sqrt{gh_o} \frac{b_2}{h_o^2} \end{aligned} \quad (2.21)$$

3rd order

$$\begin{aligned} & \frac{\sqrt{gh_o}}{h_o^3} \left[h_m \frac{d}{dt} (h_m b_3 + a_1 b_2 + a_2 b_1) + \frac{a_1}{h_o^3} \frac{d}{dt} (h_m b_2 + a_1 b_1) + \frac{a_2}{h_o^3} \frac{d}{dt} (h_m b_1) \right] \\ & + \frac{g}{h_o^3} [8b_1 b_3 h_m^2 + 6h_m a_2 b_1^2 + 4b_2^2 h_m^2 + 8b_1 b_2 h_m + 6a_1 b_1 b_2 h_m + 3a_1 b_1^2] \\ & + \frac{g}{h_o^4} [4a_4 h_m^2 + 6h_m a_1 a_3 + 2a_2 (a_1^2 + 2a_2 h_m) + a_1 (2a_3 h_m + 2a_1 a_2)] = -3v\sqrt{gh_o} \frac{b_3}{h_o^3} \end{aligned} \quad (2.22)$$

We can deduce From Eq. (2.19) that $a_1 = 0$. Therefore from Eq. (2.15), we have $b_2 = 0$. Consequently, from Eq. (2.21) and Eq. (2.17), we can further deduce that $a_3 = 0$ and $b_4 = 0$. By using these results, the continuity and momentum equations can be further simplified as follows:

Continuity equation:

0th order

$$\frac{dh_m}{dt} + \sqrt{gh_o} \frac{h_m b_1}{h_o} = 0 \quad (2.23)$$

2nd order

$$\frac{1}{h_o^2} \frac{da_2}{dt} + \frac{\sqrt{gh_o}}{h_o^3} [3h_m b_3 + 3a_2 b_1] = 0 \quad (2.24)$$

4th order

$$\frac{1}{h_o^4} \frac{da_4}{dt} + \frac{\sqrt{gh_o}}{h_o^5} [5a_2 b_3 + 5a_4 b_1] = 0 \quad (2.25)$$

Momentum equation:

1st order

$$\frac{\sqrt{gh_o}}{h_o} h_m \frac{d}{dt} (h_m b_1) + \frac{g}{h_o} (2h_m^2 b_1^2) + \frac{g}{h_o^2} (2a_2 h_m^2) = -3v\sqrt{gh_o} \frac{b_1}{h_o} \quad (2.26)$$

3rd order

$$\begin{aligned} & \frac{\sqrt{gh_o}}{h_o^3} \left[h_m \frac{d}{dt} (h_m b_3 + a_2 b_1) + \frac{a_2}{h_o^3} \frac{d}{dt} (h_m b_1) \right] + \frac{g}{h_o^3} (8b_1 b_3 h_m^2 + 6h_m a_2 b_1^2) \\ & + \frac{g}{h_o^4} (4a_4 h_m^2 + 4a_2^2 h_m) = -3v\sqrt{gh_o} \frac{b_3}{h_o^3} \end{aligned} \quad (2.27)$$

The series of $h(t, x)$ and $U(t, x)$ are also simplified as follows,

$$h(t, x) = h_m(t) + a_2(t) \left(\frac{x}{h_o} \right)^2 + a_4(t) \left(\frac{x}{h_o} \right)^4 + \dots \quad (2.28)$$

$$U(t, x) = \sqrt{gh_o} \left[b_1(t) \left(\frac{x}{h_o} \right) + b_3(t) \left(\frac{x}{h_o} \right)^3 + \dots \right] \quad (2.29)$$

In order to non-dimensionalize the continuity and momentum equations, a set of dimensionless parameters for t , h_m , a_2 and a_4 are introduced as in Eq. (2.30)

$$t' = \frac{\sqrt{gh_o}}{h_o} t, \quad h'_m = \frac{h_m}{h_o}, \quad a'_2 = \frac{a_2}{h_o}, \quad a'_4 = \frac{a_4}{h_o} \quad (2.30)$$

We can therefore write the dimensionless form of continuity and momentum equations, as in Eq. (2.31) to Eq. (2.35). For inviscid fluid, $\nu = 0$ is assumed.

Continuity equation:

0th order

$$\frac{dh'_m}{dt'} + h'_m b_1 = 0 \quad (2.31)$$

2nd order

$$\frac{da'_2}{dt'} + 3h'_m b_3 + 3a'_2 b_1 = 0 \quad (2.32)$$

4th order

$$\frac{da'_4}{dt'} + 5a'_2 b_3 + 5a'_4 b_1 = 0 \quad (2.33)$$

Momentum equation:

1st order

$$h_m'^2 \frac{db_1}{dt'} + h'_m b_1 \frac{dh'_m}{dt'} + 2h_m'^2 b_1^2 + 2a_2' h_m'^2 = 0 \quad (2.34)$$

3rd order

$$\begin{aligned} & 2h'_m a'_2 \frac{db_1}{dt'} + h_m'^2 \frac{db_3}{dt'} + (a'_2 b_1 + b_3 h_m') \frac{dh'_m}{dt'} + (h'_m b_1) \frac{da'_2}{dt'} \\ & + 8b_1 b_3 h_m'^2 + 6h'_m a'_2 b_1^2 + (4a_4' h_m'^2 + 4a_2'^2 h_m') = 0 \end{aligned} \quad (2.35)$$

By assuming power law for the coefficients, we have,

$$h'_m = \hat{A} t'^a, \quad a'_2 = \hat{B} t'^b, \quad a'_4 = \hat{C} t'^c, \quad b_1 = \hat{D} t'^d, \quad b_3 = \hat{E} t'^e \quad (2.36)$$

and by substituting Eq. (2.36) into continuity and momentum equations, we can equate the coefficients of t' to obtain the following equations:

$$d = -1 \quad \text{from Eq. (2.31)} \quad (2.37)$$

$$a + e = b - 1 \quad \text{from Eq. (2.32)} \quad (2.38)$$

$$b + e = c - 1 \quad \text{from Eq. (2.33)} \quad (2.39)$$

$$b = -2 \quad \text{from Eq. (2.34)} \quad (2.40)$$

$$a + e = -3 \quad \text{from Eq. (2.35)} \quad (2.41)$$

$$c + a = -4 \quad \text{from Eq. (2.35)} \quad (2.42)$$

By utilizing the relationships between coefficients a, b, c, d and e in Eq. (2.37) to Eq. (2.42) and substituting them back into the dimensionless form of continuity and momentum equations in

Eq. (2.31) to Eq. (2.35), we can further obtain several relationships for $\hat{A}, \hat{B}, \hat{C}, \hat{D}$ and $\hat{E}$ as in the following equations, Eq. (2.43) to Eq. (2.47).

From Eq. (2.31)

$$\hat{D} = -a \quad (2.43)$$

From Eq. (2.32)

$$\hat{A}\hat{E} = \frac{1}{3}(2+3a)\hat{B} \quad (2.44)$$

From Eq. (2.33)

$$5\hat{B}\hat{E} = (6a+4)\hat{C} \quad (2.45)$$

From Eq. (2.34)

$$\hat{B} = -\frac{1}{2}a(a+1) \quad (2.46)$$

From Eq. (2.35)

$$\hat{A}\hat{C} = -\hat{B}^2 + \frac{1}{12}(9a^2 + 13a + 6)\hat{B} \quad (2.47)$$

By multiplying $\hat{A}$ on both sides of Eq. (2.45), we obtain,

$$5\hat{A}\hat{E}\hat{B} = (6a+4)\hat{A}\hat{C} \quad (2.48)$$

Therefore, by substituting Eq. (2.44), Eq. (2.46) and Eq. (2.47) into Eq. (2.48), we will obtain Eq. (2.49) that will lead to solving coefficient a .

$$a(a+1)(30a^3 + 56a^2 + 33a + 6) = 0 \quad (2.49)$$

The solutions for coefficient a are

$$a = -1 \quad \text{or} \quad a = -\frac{2}{3} \quad \text{or} \quad a = -\frac{1}{10}(-6 \pm \sqrt{6}) \quad \text{with } a \neq 0 \quad (2.50)$$

Therefore, the temporal variation of the depth at the origin h_m can be expressed as in Eq. (2.51) to Eq. (2.53).

$$a = -1, \quad \hat{B} = 0, \quad h'_m = \hat{A}t'^{-1} \rightarrow h_m \propto t^{-1} \quad (2.51)$$

$$a = -\frac{2}{3}, \quad \hat{B} = \frac{1}{9}, \quad h'_m = \hat{A}t'^{-\frac{2}{3}} \rightarrow h_m \propto t^{-\frac{2}{3}} \quad (2.52)$$

$$a = -\frac{1}{10}(-6 \pm \sqrt{6}), \quad \hat{B} = \frac{1}{100}(-9 \pm \sqrt{6}), \quad h'_m = \hat{A}t'^{-\frac{1}{10}(-6 \pm \sqrt{6})} \rightarrow h_m \propto t^{-\frac{1}{10}(-6 \pm \sqrt{6})} \quad (2.53)$$

2.3.3 Temporal variation of leading wave position

As explained earlier in Section 2.2, in the case of dam-break flow of finite extent of ideal fluid, the front wave position is always leading the flow, without the effect of the first reflected negative wave. Therefore, the solution of dam-break of infinite extent is applicable. Similar to Eq.(2.7) and Eq.(2.8), the front position of the front wave and the flow profile for Region A shown in Fig. 2.4 is presented again here for convenience,

$$L(t) = L_o + 2\sqrt{gh_o}t \quad (2.54)$$

$$h(x, t) = \frac{1}{9g} \left(2\sqrt{gh_o} - \frac{x - L_o}{t} \right)^2 \quad (2.55)$$

2.3.4 Validation of analytical findings with numerical models

Moving Particle Semi-Implicit or MPS (Koshizuka et al., 1998) is used as one of the numerical simulation models in this study. MPS is a grid-less particle method model. In this model, the interaction of a particle with its neighbors is taken into account through the introduction of a weight function with radius of interaction r_e . Meanwhile the number of particle per unit volume can be approximated in this model by introducing particle number density. The weight function and particle density number are defined as in Eq. (2.56) and Eq. (2.56)

Weight function:

$$w(r) = \begin{cases} \frac{r_e}{r} - 1 & \text{for } (0 \leq r < r_e) \\ 0 & \text{for } (r_e \leq) \end{cases}$$

Particle density number:

$$\langle n \rangle_i = \sum_{j \neq i} w(|\tilde{r}_j - \tilde{r}_i|) \quad (2.56)$$

The governing equations in this model are the conservation of mass and momentum equations as in Eq. (2.57) and Eq. (2.58).

$$\frac{D\rho}{Dt} = 0 \quad (2.57)$$

$$\frac{D\tilde{u}}{Dt} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \tilde{u} + \tilde{g} \quad (2.58)$$

where ρ = fluid density, P = pressure, ν = kinematic viscosity, $\tilde{u}$ = velocity vector and $\tilde{g}$ = gravity acceleration vector. The differential and divergence terms, as well as the Laplacian operator in the momentum equation are modified to accommodate the interaction of particles Koshizuka et al. (1998) as given in Eq. (2.59) and Eq. (2.60). In the case of inviscid fluid, the viscous term $\nu \nabla^2 \tilde{u}$ can be omitted since $\nu = 0$ for inviscid fluid.

Divergence model

$$\langle \nabla \cdot u \rangle = \frac{d}{dn} \sum_{j \neq i} \frac{(\tilde{u}_j - \tilde{u}_i) \cdot (\tilde{r}_j - \tilde{r}_i)}{|\tilde{r}_j - \tilde{r}_i|} \quad (2.59)$$

Laplacian model

$$\langle \nabla^2 \phi \rangle = \sum_{j \neq i} (\phi_j - \phi_i) w(\tilde{\phi}_j - \tilde{\phi}_i)$$

$$\text{where } \lambda = \frac{\int_{\text{volume}} w(r) r^2 dv}{\int_{\text{volume}} w(r) dv} \quad (2.60)$$

MPS is chosen due to its simplicity grid-less pre-simulation set up and the ability to define complicated free-surface flow (Gotoh et al., 2005). The second model that is used for the numerical analysis is a depth averaged model with Harten's TVD (Total Variation Diminishing) scheme being used.

The simulation of dam-break flow is carried out for inviscid fluid in an infinitely long, dry, prismatic rectangular channel. The initial condition of the dam reservoir is set to a finite size of 0.5m in depth and 0.5m in length in both MPS and depth averaged models. The initial conditions of the simulation are shown in Table 2.1.

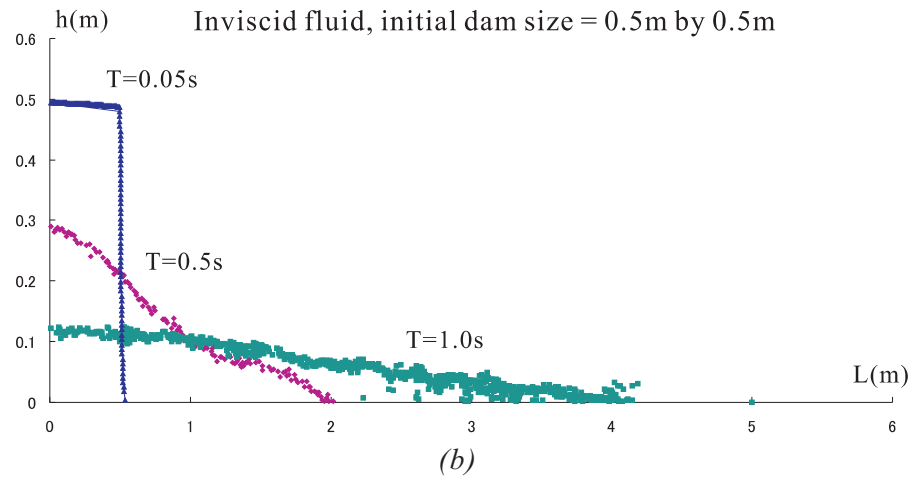
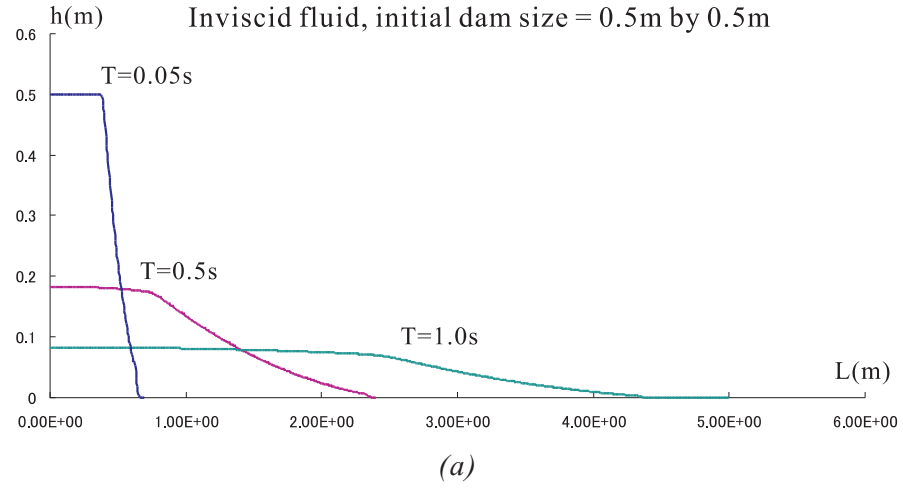


Figure 2.7: Profile of dam-break flow of inviscid fluid using (a) depth averaged model, (b) MPS model, with initial dam size of $h_0 = 0.5m$ and $L_0 = 0.5m$.

2. INERTIAL FLOW CHARACTERISTICS BY MEANS OF DAM-BREAK FLOW OF FINITE EXTENT

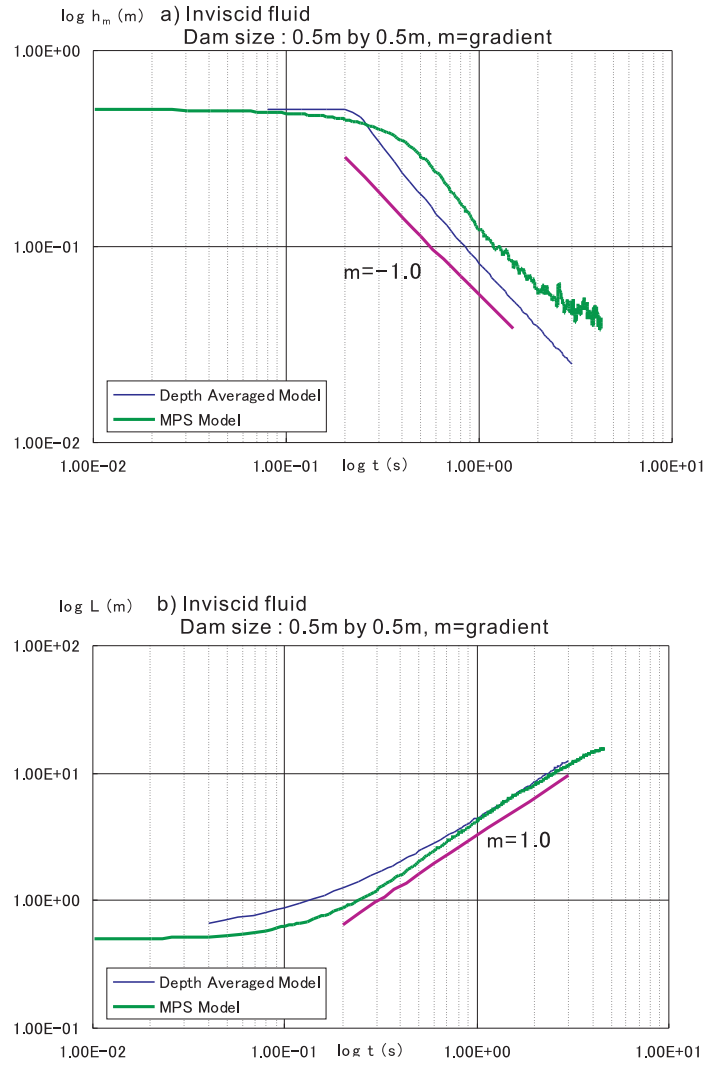


Figure 2.8: (a) temporal variation of depth at the origin h_m , for inviscid fluid, (b) temporal variation of front wave propagation L , for inviscid fluid.

Model	$h_o(m)$	$L_o(m)$	$v(m^2s^{-1})$
MPS	0.5	0.5	0.0
Depth Averaged Model	0.5	0.5	0.0

Table 2.1: Initial conditions of simulation for inviscid fluid with MPS model and depth averaged model.

2.3.5 Discussion on findings

The temporal variation of depth at the origin h_m and the front wave position L for both models are plotted in Fig. 2.8, while the dam-break flow profiles for both models are shown in Fig. 2.7. The numerical results shown in Fig. 2.8 show that the temporal variation of L agrees satisfactorily with the analytical results ($L \propto t$). However, in the case of temporal variation of depth at the origin h_m , the analytical analysis yields two different results as in Eq. (2.51) and Eq. (2.52). The numerical analysis results satisfy the first relation of $h_m \propto t^{-1}$ when $a = -1$; therefore allowing us to write the temporal variation of h_m as in Eq. 2.62

$$\begin{aligned} a &= -1, \\ \therefore \hat{B} &= \hat{C} = \hat{D} = \hat{E} = 0 \end{aligned} \quad (2.61)$$

$$h(t, x) = \hat{A}t'^{-1}h_o \quad (2.62)$$

Eq. (2.62) implies a flat free-surface profile near the origin, which is observed at the profile near the origin in the numerical simulation. Since the time needed for the negative wave to reach the upstream boundary is $t_o = L_o/c_o$, coefficient $\hat{A}$ can be approximated as follows;

$$h(t, x) = h_o = \hat{A} \frac{h_o^2}{c_o} \left(\frac{c_o}{L_o} \right) \rightarrow \hat{A} = \frac{L_o}{c_o} \quad (2.63)$$

Therefore, the approximate solution for the depth of flow near the origin in the case of $a = -1$ can be written as follows;

$$h(t, x) = \frac{L_o h_o}{c_o} t^{-1} \quad (2.64)$$

As for the second and third relations derived for h_m , as in Eq. (2.65) and Eq. (2.53), they imply a concave free surface profile near the origin which was not observed in the numerical simulation. For $a = -\frac{2}{3}$

$$h_m \propto t^{-\frac{2}{3}}, \quad \hat{B} = \frac{1}{9}, \quad \hat{A} \neq \hat{D} \neq 0, \quad \hat{E} = \hat{D} = 0, \quad \therefore h(t, x) = \hat{A}h_o t'^{-\frac{2}{3}} + \frac{1}{9}h_o \left(\frac{x}{h_o} \right)^2 t'^{-2} \quad (2.65)$$

For $a = -\frac{1}{10} (6 \pm \sqrt{6})$

$$h_m \propto t^{-\frac{1}{10} (6 \pm \sqrt{6})}, \quad \hat{B} = \frac{1}{100} (9 \pm \sqrt{6}), \quad \hat{C} \neq \hat{D} \neq \hat{E} \neq 0, \quad (2.66)$$

$$\therefore h(t, x) = \hat{A}h_o t'^{-\frac{2}{3}} + \frac{1}{100} (9 \pm \sqrt{6}) h_o \left(\frac{x}{h_o} \right)^2 t'^{-2} + \hat{C}h_o \left(\frac{x}{h_o} \right)^4 t'^c + \dots$$

However, solution defining the surface near the origin in Eq.(2.62) is not adequate to define the flow profile in the earlier stage after the first negative wave started moving away from the upstream wall, for example at $t = 0.5s$ in Fig. 2.7 (a). Eq. (2.62) also implies that the flow profile is constant and independent of x which is unsatisfactory. Although the flow profile in Eq. (2.62) is not satisfactory, the attenuation of the depth at the origin can be used to represent the inertial phase flow characteristic in dam-break flow of finite extent. A different approach is used to improve the flow profile in the next section.

2.3.6 Approximate solution for the flow profile

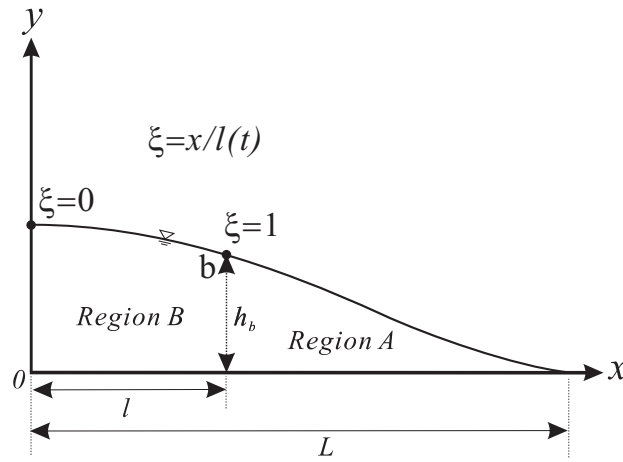


Figure 2.9: Moving coordinate system based on the position of the first negative wave reflected from the upstream wall, represented by point b

In this sub-section, an approximate solution for the depth profile for Region B shown in Fig.(2.9) will be derived to improve the earlier approximate solution of Eq.(2.62). From Section 2.3.2, the depth h and velocity U of flow can be expressed in the following form,

$$h(t, x) = h_m(t) + a_2(t) \left(\frac{x}{h_o} \right)^2 + a_4(t) \left(\frac{x}{h_o} \right)^4 + \dots \quad (2.67)$$

$$U(t, x) = \sqrt{gh_o} \left[b_1(t) \left(\frac{x}{h_o} \right) + b_3(t) \left(\frac{x}{h_o} \right)^3 + \dots \right] \quad (2.68)$$

Here, a new formulation based on a moving coordinate system is introduced. The moving coordinate system ξ , is based on the propagation of the first negative wave reflected from the upstream wall, hereafter, referred as point b as shown in Fig. 2.9,

$$\xi = \frac{x}{l(t)} \quad (2.69)$$

Continuity and momentum equations from Eq.(2.10) and Eq.(2.11) expressed in ξ coordinate systems are therefore:

$$\frac{\partial h}{\partial t} - \xi \frac{dl}{dt} \frac{\partial h}{\partial \xi} + \frac{h}{l} \frac{\partial U}{\partial \xi} + \frac{U}{l} \frac{\partial h}{\partial \xi} = 0 \quad (2.70)$$

$$\frac{\partial U}{\partial t} - \xi \frac{dl}{dt} \frac{\partial U}{\partial \xi} + \frac{U}{l} \frac{\partial U}{\partial \xi} + \frac{g}{l} \frac{\partial h}{\partial \xi} = 0 \quad (2.71)$$

Based on the results in Eq.(2.67) and Eq.(2.68), the depth of flow h , and velocity U , are assumed to be expressible as a series of power expanded from point b, applicable only in Region B. By using the Taylor's power series expansion, they can be written as in Eq.(2.72) and Eq.(2.73) respectively.

$$h(t, \xi) = a_0(t) + a_2(t)\xi^2 + a_4(t)\xi^4 + \dots \quad (2.72)$$

$$U(t, \xi) = b_1(t)\xi + b_3(t)\xi^3 + \dots \quad (2.73)$$

Rearranging the continuity and momentum based on the order of ξ ,
Continuity Equations :

0th order :

$$l \frac{da_0}{dt} + a_0 b_1 = 0 \quad (2.74)$$

1st order :

$$\text{no equation} \quad (2.75)$$

2nd order :

$$l \frac{da_2}{dt} - 2a_2 \frac{dl}{dt} + 2b_1 a_2 + 3a_0 b_3 = 0 \quad (2.76)$$

3rd order :

$$\text{no equation} \quad (2.77)$$

4th order :

$$l \frac{da_4}{dt} - 4a_4 \frac{dl}{dt} + 4b_1 a_4 + 2a_2 b_3 + a_4 b_1 + 3a_2 b_3 = 0 \quad (2.78)$$

Momentum Equations :

0th order :

$$\text{no equation} \quad (2.79)$$

1st order :

$$l \frac{db_1}{dt} - b_1 \frac{dl}{dt} + b_1^2 + 2ga_2 = 0 \quad (2.80)$$

2nd order :

$$\text{no equation} \quad (2.81)$$

3rd order :

$$l \frac{db_3}{dt} - 3b_3 \frac{dl}{dt} + b_1 b_3 + 3b_1 b_3 + 4ga_4 = 0 \quad (2.82)$$

4th order :

$$\text{no equation} \quad (2.83)$$

The coefficients of a_0 , a_2 , b_1 can be expressed as in the following series,

$$a_0(t) = a_{01} \left(\frac{t}{t_0} \right)^{-\frac{1}{3}} + a_{02} \left(\frac{t}{t_0} \right)^{-\frac{2}{3}} + a_{03} \left(\frac{t}{t_0} \right)^{-1} + a_{04} \left(\frac{t}{t_0} \right)^{-\frac{4}{3}} \quad (2.84)$$

$$a_2(t) = a_{20}(t) + a_{21} \left(\frac{t}{t_0} \right)^{-\frac{1}{3}} + a_{22} \left(\frac{t}{t_0} \right)^{-\frac{2}{3}} + a_{23} \left(\frac{t}{t_0} \right)^{-1} + a_{24} \left(\frac{t}{t_0} \right)^{-\frac{4}{3}} \quad (2.85)$$

$$b_1(t) = b_{10}(t) + b_{11} \left(\frac{t}{t_0} \right)^{-\frac{1}{3}} + b_{12} \left(\frac{t}{t_0} \right)^{-\frac{2}{3}} + b_{13} \left(\frac{t}{t_0} \right)^{-1} + b_{14} \left(\frac{t}{t_0} \right)^{-\frac{4}{3}} \quad (2.86)$$

As $t \rightarrow t_0$, $l \rightarrow 0$. Hence, $\xi \rightarrow \infty$. Therefore, the following conditions must be satisfied :

$$a_0(t_0) = \sum_{n=1}^4 a_{0n} = h_0 \quad (2.87)$$

$$a_2(t_0) = \sum_{n=0}^4 a_{2n} = 0 \quad (2.88)$$

$$b_1(t_0) = \sum_{n=0}^4 b_{1n} = 0 \quad (2.89)$$

$$b_3(t_0) = 0 \quad (2.90)$$

By substituting Eq.(2.84), Eq.(2.85) and Eq.(2.86) into h and U series, and thereafter, into continuity and momentum equations in Eq.(2.70) and Eq.(2.71), a set of equations grouped based on the order of $\left(\frac{t}{t_0} \right)$ can be obtained as follows :

From Eq.(2.74)

$-\frac{1}{3}$ order :

$$-\frac{2}{3}a_{01}c_0 + a_{01}b_{10} = 0 \quad (2.91)$$

$-\frac{2}{3}$ order :

$$-\frac{4}{3}a_{02}c_o + a_{01}b_{11} + a_{02}b_{10} = 0 \quad (2.92)$$

-1 order :

$$-2a_{03}c_o + a_{01}c_o + a_{01}b_{12} + a_{02}b_{11} + a_{03}b_{10} = 0 \quad (2.93)$$

$-\frac{4}{3}$ order :

$$-\frac{1}{3}a_{01}c_o - \frac{8}{3}a_{04}c_o + 2a_{02}c_o + a_{01}b_{13} + a_{02}b_{12} + a_{03}b_{11} + a_{04}b_{10} = 0 \quad (2.94)$$

From Eq.(2.80)

0 order :

$$-2c_o b_{01} + b_{10}^2 + 2ga_{20} = 0 \quad (2.95)$$

$-\frac{1}{3}$ order :

$$-\frac{2}{3}c_o b_{11} - 2c_o b_{11} + 2b_{10}b_{11} + 2ga_{21} = 0 \quad (2.96)$$

$-\frac{2}{3}$ order :

$$-\frac{4}{3}c_o b_{12} - 2c_o b_{12} + c_o b_{10} + b_{11}^2 + 2b_{10}b_{12} + 2ga_{22} = 0 \quad (2.97)$$

-1 order :

$$-2c_o b_{13} - c_o b_{11} - 2c_o b_{13} + c_o b_{11} + 2b_{10}b_{13} + 2b_{11}b_{12} + 2ga_{23} = 0 \quad (2.98)$$

$-\frac{4}{3}$ order :

$$-\frac{1}{3}c_o b_{11} - \frac{8}{3}c_o b_{14} + 2c_o b_{12} - 2c_o b_{14} + c_o b_{12} + b_{12}^2 + 2b_{10}b_{14} + 2b_{11}b_{13} + 2ga_{24} = 0 \quad (2.99)$$

From Eq.(2.76)

$-\frac{1}{3}$ order :

$$-\frac{2}{3}c_o a_{21} - 4c_o a_{21} + 2(a_{20}b_{11} + a_{21}b_{10}) = 0 \quad (2.100)$$

$-\frac{2}{3}$ order :

$$-\frac{4}{3}c_o a_{22} - 2c_o (2a_{22} - a_{20}) + 2(a_{20}b_{12} + a_{21}b_{11} + a_{22}b_{10}) + 6c_o a_{02} - 6a_{02}b_{10} - a_{01}b_{11} = 0 \quad (2.101)$$

−1 order :

$$\begin{aligned} & -2c_o a_{23} + c_o a_{21} - 2c_o (2a_{23} - a_{21}) + 2(a_{20}b_{13} + a_{21}b_{12} + a_{22}b_{11} + a_{23}b_{10}) \\ & + 3(2c_o a_{03} - a_{03}b_{10} - a_{02}b_{11} - a_{01}b_{12}) = 0 \end{aligned} \quad (2.102)$$

− $\frac{4}{3}$ order :

$$\begin{aligned} & -\frac{1}{3}c_o a_{21} - \frac{8}{3}c_o a_{24} - 2c_o a_{22} - 2c_o (2a_{24} - a_{22}) \\ & + 2(a_{20}b_{14} + a_{21}b_{13} + a_{22}b_{12} + a_{23}b_{11} + a_{24}b_{10}) \\ & + 3(2c_o a_{04} - a_{04}b_{10} - 2c_o a_{02} - a_{03}b_{11} - a_{02}b_{12} - a_{01}b_{13}) = 0 \end{aligned} \quad (2.103)$$

By solving simultaneous equations of Eq.(2.87) to Eq.(2.103), the following solutions are obtained:

Coefficient of $a_0(t)$

$$\begin{aligned} a_{01} &= a_{02} = a_{04} = 0 \\ a_{03} &= h_o, \\ \therefore a_0(t) &= h_o \left(\frac{t}{t_o} \right)^{-1} \end{aligned} \quad (2.104)$$

Coefficient of $a_2(t)$

$$\begin{aligned} a_{20} &= a_{21} = a_{22} = a_{23} = a_{24} = 0 \\ \therefore a_2(t) &= 0 \end{aligned} \quad (2.105)$$

Coefficient of $b_1(t)$

$$\begin{aligned} b_{11} &= b_{14} = 0 \\ b_{12} &= -3c_o, \quad b_{13} = c_o, \quad b_{10} = 2c_o \\ \therefore b_1(t) &= 2c_o - 3c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} + c_o \left(\frac{t}{t_o} \right)^{-1} \end{aligned} \quad (2.106)$$

At $x = l, \xi = 1,$

$$a_0(t) + a_2(t) + a_4(t) = h_b = h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \quad (2.107)$$

$$b_1(t) + b_3(t) = u_b = 2c_o - 2c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} \quad (2.108)$$

By utilizing Eq.(2.107) and Eq.(2.108), $a_4(t)$ and $b_3(t)$ can be obtained as follows:

$$a_4(t) = h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - h_o \left(\frac{t}{t_o} \right)^{-1} \quad (2.109)$$

$$b_3(t) = c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} - c_o \left(\frac{t}{t_o} \right)^{-1} \quad (2.110)$$

In order to distinguish the solution for Region B, the depth and velocity of flow in Region B is h_2 and U_2 respectively. Therefore, the depth and velocity profile for region B can be summarized as follows,

$$h_2(t, \xi) = a_0(t) + a_4(t)\xi^4 \quad (2.111)$$

$$\therefore h_2(t, \xi) = h_o \left(\frac{t}{t_o} \right)^{-1} + \left[h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - h_o \left(\frac{t}{t_o} \right)^{-1} \right] \xi^4 \quad (2.112)$$

$$U_2(t, \xi) = b_1(t)\xi + b_3(t)\xi^3 \quad (2.113)$$

$$\therefore U_2(t, \xi) = \left[2c_o - 3c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} + c_o \left(\frac{t}{t_o} \right)^{-1} \right] \xi + \left[c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} - c_o \left(\frac{t}{t_o} \right)^{-1} \right] \xi^3 \quad (2.114)$$

2.3.7 Validation of approximate solution of flow profile with numerical models

A depth averaged model with Harten's TVD (Total Variation Diminishing) scheme is used to solve the governing equations numerically. Two sets of spatial spacing in flow direction, dx and time step, dt are used in the model: $dx = 0.005m$ with $dt = 0.0002s$ and $dx = 0.001m$ with $dt = 0.0002s$. Depth of flow at time interval $t = 0.05s$, $t = 0.5s$ and $t = 1.0s$ are plotted.

Simulation Procedures

Table 2.2: Initial conditions of simulation for inviscid fluid with depth averaged model.

Model	$h_o(m)$	$L_o(m)$	$v(m^2s^{-1})$
Depth Averaged Model	0.5	0.5	0.0

The flow from dam-break of finite extent are simulated in a semi-infinitely long channel, with boundary wall at the origin, $x = 0$. The channel ahead of the fluid is initially dry. The size of the dam is set to $h_o = 0.5m$ by $L_o = 0.5m$. Flow is generated by instantaneously releasing the fluid in the dam. Viscosity is set to 0 for inviscid case. During the simulation, temporal variation of depth at the origin, h_m is observed. Initial conditions of the simulation are summarized in Table 2.2.

In order to validate the depth averaged numerical model, the propagation of the wave front is verified with the analytical solution provided in Eq. (2.7). The result is plotted in Fig.2.10 and shows good agreement between the depth averaged numerical model and the analytical solution in the case of the wave front propagation. The numerical model also shows satisfactory mass conservation, as shown in Fig.2.11.

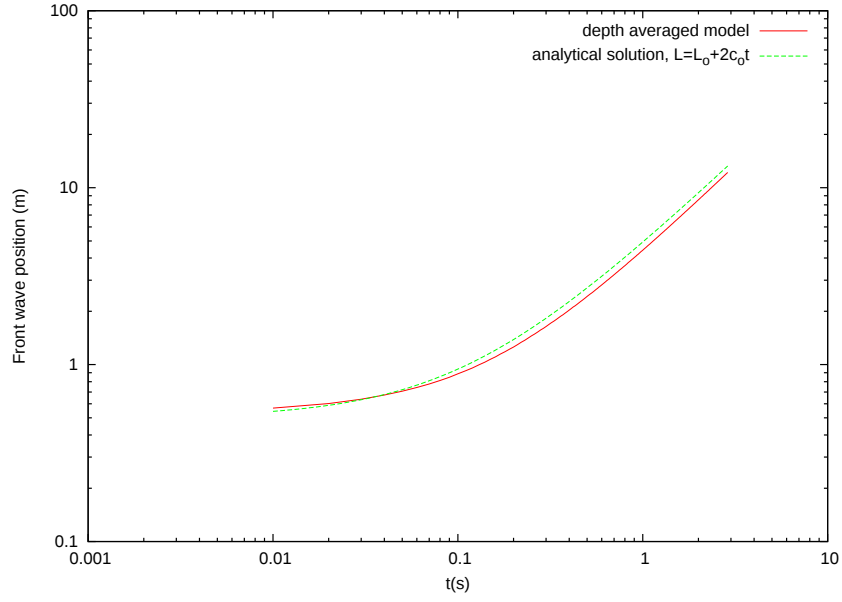


Figure 2.10: Temporal variation of wave front L_m for depth averaged model and analytical solution.

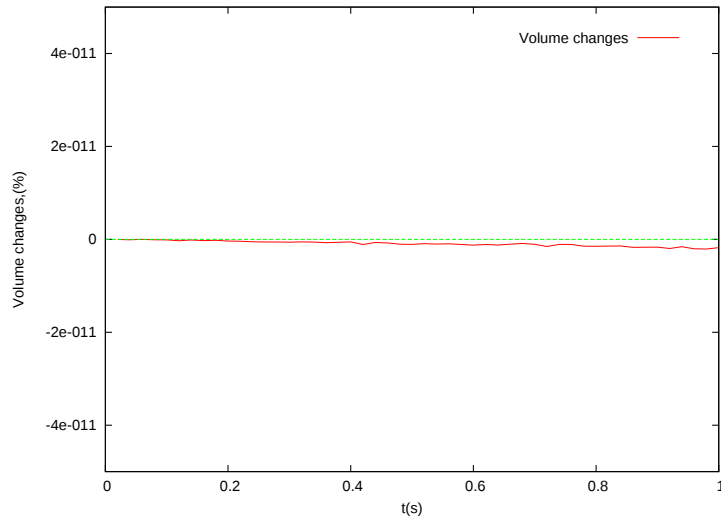


Figure 2.11: Volume changes with time for depth averaged model.

2.3. Approximate analytical solution of dam-break flow of finite extent

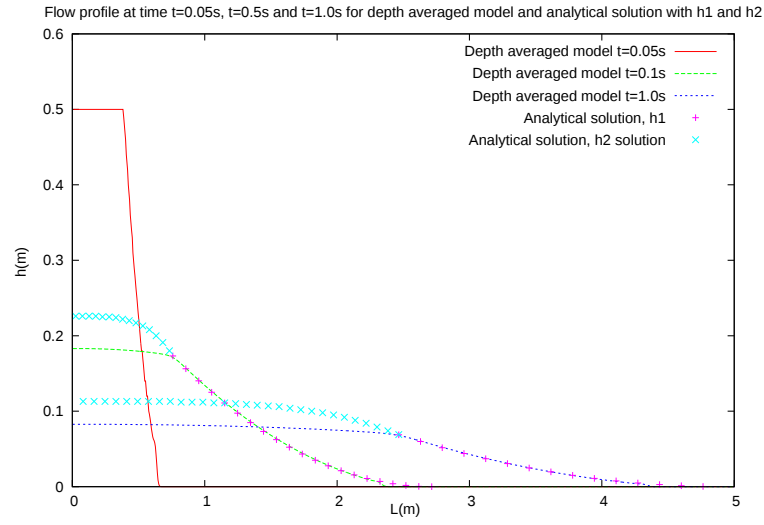


Figure 2.12: Profile of dam-break flow of inviscid fluid using depth averaged model and analytical solutions of $h_1(t)$ and $h_2(t)$

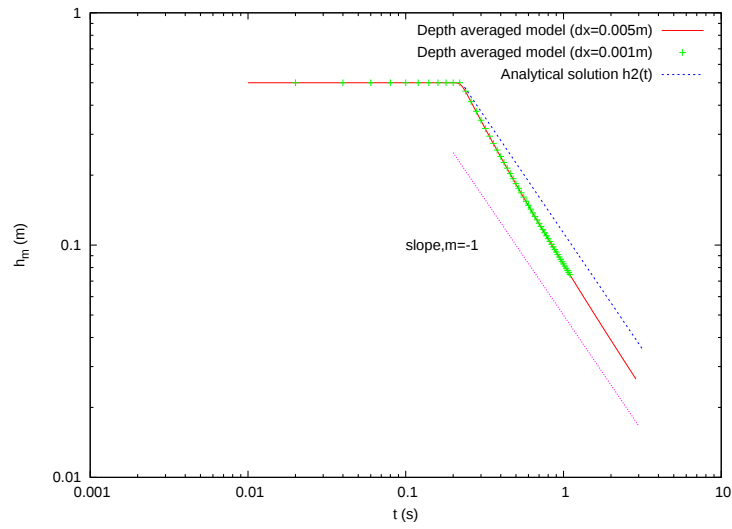


Figure 2.13: Temporal variation of depth at the origin h_m for depth averaged model and analytical solution.

2.3.8 Discussion on findings

The numerical results from depth averaged model and analytical solution for the flow are plotted in Fig.2.12 for $t = 0.5s$ and $t = 1.0s$. Analytical solution of h_1 which is valid only in Region A agrees satisfactorily with the profile obtained from the depth averaged model. This is because the depth averaged model is solving the same governing equations which the analytical solution of h_1 is derived. The results of the analytical solution of h_2 , however shows that the flow depth does not match the numerical model result accurately especially in the early stage. The discrepancy between the analytical solution h_2 and the numerical model reduces as the flow propagates further downstream. For the attenuation of depth at the origin, the result is plotted in Fig.2.13. In the author's previous work (Puay and Hosoda, 2007), the attenuation of depth at the origin is found to be inversely proportionate with time ($h_m \propto t^{-1}$). It can be seen from Fig.2.13 that the analytical solution $h_2(t)$ agrees quite satisfactorily with the slope, $m = -1$. The approximate solution derived here has overcome the problem of flat surface profile derived in previous sub-section 2.3.5. However, the flow profile and the attenuation of depth at the origin still shows some slight discrepancy which is dealt in the next section.

2.4 Improvement of approximate analytical solution

The result of the analytical solution shown in Fig.(2.13) shows discrepancy of the attenuation of the depth of flow at the origin. In the case of the analytical solution, the depth at the origin attenuate slower than the result of the depth averaged model. As the discrepancy increases with time, it is worth to re-examine the analytical solution.

2.4.1 Volume conservation of analytical solution

In order to investigate the volume conservation of the analytical solutions, the dam-break flow model is divided into two regions, as shown in Fig.(2.14). The solution for Region A is given previously as:

$$h_1(x, t) = \frac{1}{9g} \left(2\sqrt{gh_o} - \frac{x - L_o}{t} \right)^2 \quad (2.115)$$

$$L(t) = L_o + 2c_o t \quad (2.116)$$

with the flow depth for Region A is re-labeled as h_1 . The solution for Region B developed previously, is once again give here as:

$$h_2(\xi, t) = h_o \left(\frac{t}{t_o} \right)^{-1} + \left[h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - h_o \left(\frac{t}{t_o} \right)^{-1} \right] \xi^4 \quad (2.117)$$

The integration of volume of each regions is as follows,

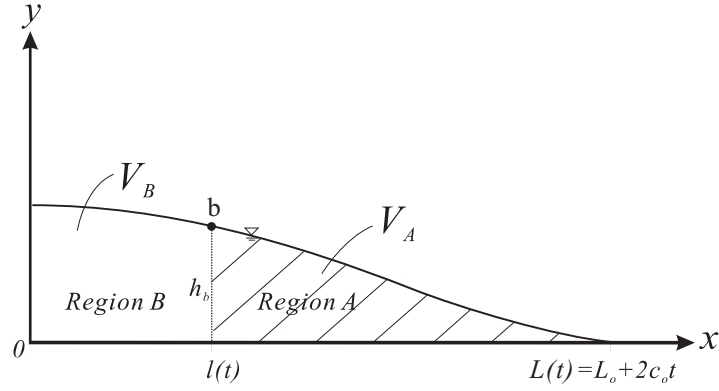


Figure 2.14: Dam break flow region divided into V_B for region behind $l(t)$ and V_A for region ahead of $l(t)$.

Volume of region A, V_A :

$$\begin{aligned}
 V_A &= \int_l^{L_o+2c_ot} \frac{1}{9g} \left(2\sqrt{gh_o} - \frac{x-L_o}{t} \right)^2 dx \\
 &= \int_l^{L_o+2c_ot} \frac{1}{9g} \left[4gh_o - 4\sqrt{gh_o} \frac{(x-L_o)}{t} + \frac{(x-L_o)^2}{t^2} \right] dx \\
 &= \int_l^{L_o+2c_ot} \frac{1}{9g} \left[4gh_o + 4\sqrt{gh_o} \frac{L_o}{t} - 4\sqrt{gh_o} \frac{1}{t} x + \frac{1}{t^2} x^2 - \frac{2L_o}{t^2} x + \frac{L_o^2}{t^2} \right] dx \\
 &= \frac{1}{9g} \left(4gh_o + 4\sqrt{gh_o} \frac{L_o}{t} + \frac{L_o^2}{t^2} \right) (L_o + 2c_ot - l) \\
 &\quad - \frac{1}{9g} \left(4\sqrt{gh_o} \frac{1}{t} + \frac{2L_o}{t^2} \right) \left[\frac{x^2}{2} \right]_l^{L_o+2c_ot} + \frac{1}{9g} \frac{1}{t^2} \left[\frac{x^3}{3} \right]_l^{L_o+2c_ot} \quad (2.118)
 \end{aligned}$$

Evaluating the terms $\left[\frac{x^2}{2} \right]_l^{L_o+2c_ot}$ and $\left[\frac{x^3}{3} \right]_l^{L_o+2c_ot}$:

$$\begin{aligned}
 \left[\frac{x^2}{2} \right]_l^{L_o+2c_ot} &= 6L_o (L_o + 2c_ot) \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} - 9L_o^2 \left(\frac{tc_o}{L_o} \right)^{\frac{2}{3}} \\
 \left[\frac{x^3}{3} \right]_l^{L_o+2c_ot} &= 9L_o (L_o + 2c_ot)^2 \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} - 27L_o^2 (L_o + 2c_ot) \left(\frac{tc_o}{L_o} \right)^{\frac{2}{3}} + 27L_o^3 \left(\frac{tc_o}{L_o} \right) \quad (2.119)
 \end{aligned}$$

Therefore, V_A is:

$$\begin{aligned}
 V_A &= \frac{1}{9g} \left(4gh_o + 4\sqrt{gh_o}L_o \frac{1}{t} + \frac{L_o^2}{t^2} \right) \left[L_o + 2c_o - \left(L_o + 2c_o - 3L_o \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} \right) \right] \\
 &\quad - \frac{1}{18g} \left(4\sqrt{gh_o} \frac{1}{t} + \frac{2L_o}{t^2} \right) \left[6L_o (L_o + 2c_o t) \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} - 9L_o^2 \left(\frac{tc_o}{L_o} \right)^{\frac{2}{3}} \right] \\
 &\quad + \frac{1}{27g} \frac{1}{t^2} \left[9L_o (L_o + 2c_o t)^2 \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} - 27L_o^2 (L_o + 2c_o t) \left(\frac{tc_o}{L_o} \right)^{\frac{2}{3}} + 27L_o^3 \left(\frac{tc_o}{L_o} \right) \right] \\
 &= \frac{12gh_o L_o}{9g} \left[1 + \frac{L_o}{tc_o} + \frac{1}{4} \left(\frac{L_o}{tc_o} \right)^2 \right] \left[\left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} \right] \\
 &\quad - \frac{c_o^2 L_o}{18g} \left[4 \frac{L_o}{tc_o} + 2 \left(\frac{L_o}{c_o t} \right)^2 \right] \left[6 \left(1 + \frac{2c_o t}{L_o} \right) \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} - 9 \left(\frac{tc_o}{L_o} \right)^{\frac{2}{3}} \right] \\
 &\quad + \frac{c_o^2 L_o}{27g} \left(\frac{L_o}{tc_o} \right)^2 \left[9 \left(1 + 2 \frac{c_o t}{L_o} \right)^2 \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} - 27 \left(1 + 2 \frac{c_o t}{L_o} \right) \left(\frac{tc_o}{L_o} \right)^{\frac{2}{3}} + 27 \frac{tc_o}{L_o} \right] \quad (2.120)
 \end{aligned}$$

Eq.(2.120) can be further reduced to the following form by introducing dimensionless parameter, $t' = tc_o/L_o$,

$$\begin{aligned}
 V_A &= \frac{12}{9} h_o L_o \left[1 + t' + \frac{1}{4} t'^{-2} \right] t'^{\frac{1}{3}} \\
 &\quad - \frac{1}{18} h_o L_o \left[4t'^{-1} + 2t'^{-2} \right] \left[6t'^{\frac{1}{3}} + 12t'^{\frac{4}{3}} - 9t'^{\frac{2}{3}} \right] \\
 &\quad + \frac{1}{27} h_o L_o t'^{-2} \left[9(1 + 4t' + 4t'^2) t'^{\frac{1}{3}} - 27(1 + 2t') t'^{\frac{2}{3}} + 27t' \right] \\
 &= \frac{12}{9} h_o L_o \left[t'^{\frac{1}{3}} + t'^{-\frac{2}{3}} + \frac{1}{4} t'^{-\frac{5}{3}} \right] \\
 &\quad - \frac{1}{18} h_o L_o \left[24t'^{-\frac{2}{3}} + 48t'^{\frac{1}{3}} - 36t'^{-\frac{1}{3}} + 12t'^{-\frac{5}{3}} + 24t'^{-\frac{2}{3}} - 18t'^{-\frac{4}{3}} \right] \\
 &\quad + \frac{1}{27} h_o L_o \left[9t'^{-\frac{5}{3}} + 36t'^{-\frac{2}{3}} + 36t'^{\frac{1}{3}} - 27t'^{-\frac{4}{3}} - 54t'^{-\frac{1}{3}} + 27t'^{-1} \right] \\
 &= h_o L_o \left[t'^{\frac{1}{3}} \left(\frac{12}{9} - \frac{48}{18} + \frac{36}{27} \right) + t'^{-\frac{1}{3}} \left(\frac{36}{18} - \frac{54}{27} \right) + t'^{-\frac{2}{3}} \left(\frac{12}{9} - \frac{24}{18} + \frac{36}{27} \right) \right. \\
 &\quad \left. + t'^{-1} \left(\frac{27}{27} \right) + t'^{-\frac{4}{3}} \left(\frac{18}{18} - \frac{27}{27} \right) + t'^{-\frac{5}{3}} \left(\frac{12}{9} - \frac{12}{18} + \frac{9}{27} \right) \right] \\
 &= h_o L_o \left[\frac{4}{3} t'^{-\frac{2}{3}} + t'^{-1} \right] \quad (2.121)
 \end{aligned}$$

It is worth to note that all the terms in V_A are time dependant, and $V_A \rightarrow 0$ when $t \rightarrow \infty$.

Volume of region B, V_B :

We could write $h_2(t, \xi)$, which is the depth distribution of flow in region B, based on the solution in Section. 2.3.6.

From Eq.(2.111),

$$h_2(t, \xi) = a_0(t) + a_4(t)\xi^4 \quad (2.122)$$

Therefore, volume of region B is calculated as follows,

$$\begin{aligned}
 V_B &= l \int_0^1 h_2(t, \xi) d\xi \\
 &= l \int_0^1 (a_0(t) + a_4(t)\xi^4) d\xi \\
 &= l \left(a_0(t) + \frac{1}{5}a_4(t) \right) \\
 &= \left\{ L_o + 2c_0t - 3L_o \left(\frac{tc_0}{L_o} \right)^{\frac{1}{3}} \right\} \left(a_0(t) + \frac{1}{5}a_4(t) \right)
 \end{aligned} \tag{2.123}$$

In order to satisfy the volume conservation condition, the volume in region B should satisfy the following condition:

$$\begin{aligned}
 V_B &= \text{Total Volume} - V_A \quad (\text{from Eq.(2.121)}) \\
 &= h_o L_o - h_o L_o \left(\frac{4}{3}t'^{-\frac{2}{3}} + t'^{-1} \right)
 \end{aligned} \tag{2.124}$$

It is worth to note that from Eq.(2.124), V_B should contain a constant term which is $h_o L_o$ if the volume of constrain is to be fulfilled, and when $t \rightarrow \infty$, $V_B \rightarrow h_o L_o$

2.4.2 Weakness in solution of h_2

A check of the volume of the previous solution of h_2 is as follows,

$$\begin{aligned}
 V_B = \int_0^l h_2 d\xi &= l \int_0^1 h_2 d\xi \\
 &= l \int_0^1 h_o \left(\frac{t}{t_o} \right)^{-1} + \left[h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right] \xi^4 d\xi \\
 &= h_o \left[L_o + 2c_0t - 3L_o \left(\frac{tc_0}{L_o} \right)^{\frac{1}{3}} \right] \left[\left(\frac{t}{t_o} \right)^{-1} + \frac{1}{5} \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - \frac{1}{5} \left(\frac{t}{t_o} \right)^{-1} \right] \\
 &= h_o L_o \left[1 + 2 \frac{c_0t}{L_o} - 3 \left(\frac{tc_0}{L_o} \right)^{\frac{1}{3}} \right] \left[\frac{4}{5} \left(\frac{t}{t_o} \right)^{-1} + \frac{1}{5} \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right]
 \end{aligned} \tag{2.125}$$

Substituting $t' = tc_0/L_o$,

$$\begin{aligned}
 V_B &= h_o L_o \left[1 + 2t' - 3t'^{\frac{1}{3}} \right] \left[\frac{4}{5}t'^{-1} + \frac{1}{5}t'^{-\frac{4}{3}} \right] \\
 &= h_o L_o \left[\frac{8}{5} + \frac{1}{5}t'^{-1} + \frac{12}{5}t'^{-\frac{2}{3}} + \frac{1}{5}t'^{-\frac{4}{3}} + \frac{2}{5}t'^{-\frac{1}{3}} \right]
 \end{aligned} \tag{2.126}$$

From Eq.(2.126), when $t \rightarrow \infty$, $V_B \rightarrow \frac{8}{5}h_o L_o$. This explains the reason that the flow profile plotted in Fig.2.12 using analytical solution of h_2 derived earlier in Eq.(2.112) is higher than the depth of flow of the depth averaged model.

2.4.3 Improved new analytical solution of h_2

In Section 2.3.6, the following series is used to describe the depth of flow in region B :

$$h_2(t) = h_m(t) + a_1(t) \left(\frac{x}{h_o} \right) + a_2(t) \left(\frac{x}{h_o} \right)^2 + a_3(t) \left(\frac{x}{h_o} \right)^3 + a_4(t) \left(\frac{x}{h_o} \right)^4 + \dots \quad (2.127)$$

which is later reduced to the following form,

$$h(t, \xi) = a_0(t) + a_4(t) \xi^4 + \dots \quad (2.128)$$

where the coefficient $a_0(t)$ is assumed as follows,

$$a_0(t) = a_{01} \left(\frac{t}{t_o} \right)^{-\frac{1}{3}} + a_{02} \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} + a_{03} \left(\frac{t}{t_o} \right)^{-1} + a_{04} \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \quad (2.129)$$

The series of expansion of coefficient $a_0(t)$ assumed above is thought to be inadequate, and therefore contributes to the inaccuracy of the analytical solution derived earlier in Section 2.3.6. Based on the earlier derivation, $a_{01} = a_{02} = a_{04} = 0$, which is the the coefficients of the term $\left(\frac{t}{t_o} \right)^{-\frac{1}{3}}$, $\left(\frac{t}{t_o} \right)^{-\frac{2}{3}}$ and $\left(\frac{t}{t_o} \right)^{-\frac{4}{3}}$. It is therefore thought that an additional of extra term into the $a_0(t)$ series should be of the order of 2. Here, a new coefficient, a_{06} corresponding to the order of $\left(\frac{t}{t_o} \right)^2$ is added to series.

The new series of $a_0(t)$ can be written as :

$$a_0(t) = a_{03} \left(\frac{t}{t_o} \right)^{-1} + a_{06} \left(\frac{t}{t_o} \right)^{-2} \quad (2.130)$$

As one of the boundary conditions, explained in Eq.(2.107), the following relationship between $a_0(t)$ and $a_4(t)$ is derived,

$$a_0(t) + a_4(t) = h_b = h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \quad (2.131)$$

Using Eq.(2.131), the volume of region B, derived in Eq.(2.123) can be rewritten as

$$\begin{aligned} V_B &= \left\{ L_o + 2c_o t - 3L_o \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} \right\} \left(a_0(t) + \frac{1}{5} a_4(t) \right) \\ &= \left\{ L_o + 2c_o t - 3L_o \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} \right\} \left(\frac{4}{5} a_0(t) + \frac{1}{5} h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right) \\ &= L_o \left[a_0(t) + \frac{1}{5} a_4(t) \right] + 2c_o t \left[a_0(t) + \frac{1}{5} a_4(t) \right] - 3L_o \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} \left[a_0(t) + \frac{1}{5} a_4(t) \right] \\ &= L_o \left[\frac{4}{5} a_0(t) + \frac{1}{5} h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right] + 2c_o t \left[\frac{4}{5} a_0(t) + \frac{1}{5} h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right] \\ &\quad - 3L_o \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} \left[\frac{4}{5} a_0(t) + \frac{1}{5} h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right] \end{aligned} \quad (2.132)$$

By substituting the new series of $a_0(t)$ into the volume of region B in Eq.(2.132),

$$\begin{aligned} V_B = & L_o \left[\frac{4}{5} a_{03} \left(\frac{t}{t_o} \right)^{-1} + \frac{4}{5} a_{06} \left(\frac{t}{t_o} \right)^{-2} + \frac{1}{5} h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right] \\ & + 2c_o t \left[\frac{4}{5} a_{03} \left(\frac{t}{t_o} \right)^{-1} + \frac{4}{5} a_{06} \left(\frac{t}{t_o} \right)^{-2} + \frac{1}{5} h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right] \\ & - 3L_o \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} \left[\frac{4}{5} a_{03} \left(\frac{t}{t_o} \right)^{-1} + \frac{4}{5} a_{06} \left(\frac{t}{t_o} \right)^{-2} + \frac{1}{5} h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right] \end{aligned} \quad (2.133)$$

From Eq.(2.133), the only term which is constant is

$$2c_o t \left[\frac{4}{5} a_{03} \left(\frac{t}{t_o} \right)^{-1} \right] = \frac{8}{5} c_o t_o a_{03} \quad (2.134)$$

By comparing this new solution of V_B with the volume-conserved condition of V_B in Eq.(2.124), both constant terms should be equated,

$$\begin{aligned} \frac{8}{5} c_o t_o a_{03} &= h_o L_o \\ \frac{8}{5} c_o \left(\frac{L_o}{c_o} \right) a_{03} &= h_o L_o \end{aligned} \quad (2.135)$$

$$\therefore a_{03} = \frac{5}{8} h_o \quad (2.136)$$

Applying boundary condition that at $t = t_o$, $\xi = 0$ and $h(t_o, 0) = h_o$ to Eq.(2.128), the relationship between a_{03} and a_{06} is derived,

$$\begin{aligned} h(t, \xi) &= a_o(t) + a_4(t) \xi^4 \\ h(t = t_o, \xi = 0) &= a_0(t_o) = h_o \\ \therefore a_0(t) &= a_{03} \left(\frac{t}{t_o} \right)^{-1} + a_{06} \left(\frac{t}{t_o} \right)^{-2} \\ \therefore a_0(t_o) &= a_{03} + a_{06} = h_o \end{aligned} \quad (2.137)$$

Since $a_{03} = 5h_o/8$,

$$a_{06} = h_o - a_{03} = \frac{3}{8} h_o \quad (2.138)$$

Therefore, for the new solution, the coefficients $a_0(t)$ and $a_4(t)$ are as follows,

$$a_0(t) = a_{03} \left(\frac{t}{t_o} \right)^{-1} + a_{06} \left(\frac{t}{t_o} \right)^{-2} = \frac{5}{8} h_o \left(\frac{t}{t_o} \right)^{-1} + \frac{3}{8} h_o \left(\frac{t}{t_o} \right)^{-2} \quad (2.139)$$

$$a_4(t) = h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - a_o(t) = h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - \frac{5}{8} h_o \left(\frac{t}{t_o} \right)^{-1} - \frac{3}{8} h_o \left(\frac{t}{t_o} \right)^{-2} \quad (2.140)$$

The new analytical solution for the depth profile in region B, $h_2(t, \xi)$ can be therefore be expressed as,

$$\begin{aligned} h_2(t, \xi) &= a_o(t) + a_4(t) \xi^4 \\ &= \frac{5}{8} h_o \left(\frac{t}{t_o} \right)^{-1} + \frac{3}{8} h_o \left(\frac{t}{t_o} \right)^{-2} + \left[h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - \frac{5}{8} h_o \left(\frac{t}{t_o} \right)^{-1} - \frac{3}{8} h_o \left(\frac{t}{t_o} \right)^{-2} \right] \xi^4 \end{aligned} \quad (2.141)$$

A check of the volume of the new solution of h_2 is carried out as follows,

$$\begin{aligned}
 V_B &= lh_o \int_0^1 \frac{5}{8} \left(\frac{t}{t_o} \right)^{-1} + \frac{3}{8} \left(\frac{t}{t_o} \right)^{-2} + \left[\left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - \frac{5}{8} \left(\frac{t}{t_o} \right)^{-1} - \frac{3}{8} \left(\frac{t}{t_o} \right)^{-2} \right] \xi^4 d\xi \\
 &= h_o \left\{ L_o + 2c_o t - 3L_o \left(\frac{tc_o}{L_o} \right)^{\frac{1}{3}} \right\} \left[\frac{4}{8} \left(\frac{t}{t_o} \right)^{-1} + \frac{12}{40} \left(\frac{t}{t_o} \right)^{-2} + \frac{1}{5} \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} \right] \quad (2.142)
 \end{aligned}$$

By, substituting $t' = tc_o/L_o$,

$$\begin{aligned}
 V_B &= h_o L_o \left[1 + 2t' - 3t'^{\frac{1}{3}} \right] \left[\frac{4}{8} t'^{-1} + \frac{12}{40} t'^{-2} + \frac{1}{5} t'^{-\frac{4}{3}} \right] \\
 &= h_o L_o \left[1 + \frac{2}{5} t'^{-\frac{1}{3}} - \frac{12}{8} t'^{-\frac{2}{3}} + \frac{1}{2} t'^{-1} + \frac{1}{5} t'^{-\frac{4}{3}} - \frac{36}{40} t'^{-\frac{5}{3}} + \frac{12}{40} t'^{-2} \right] \quad (2.143)
 \end{aligned}$$

When $t \rightarrow \infty$, $V_B \rightarrow h_o L_o$, which satisfy the volume constraint. The results of the new solution of h_2 are shown in Fig.2.16 and Fig.2.15

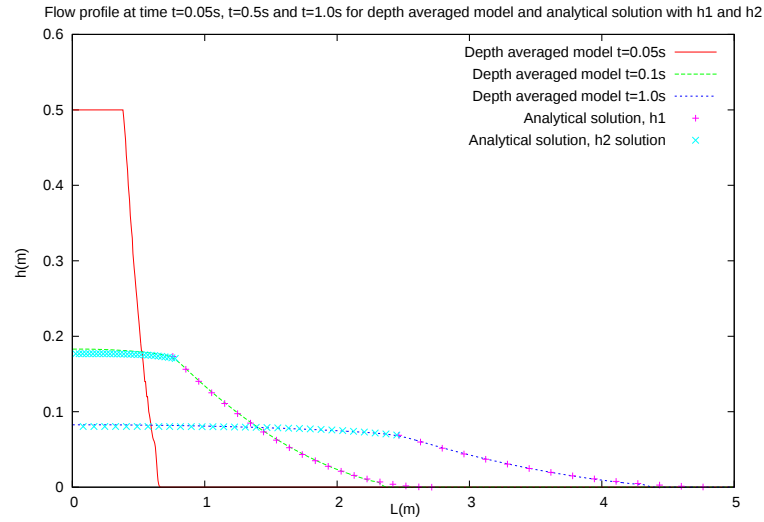


Figure 2.15: Profile of dam-break flow of inviscid fluid using depth averaged model and analytical solutions of $h_1(t)$ and new analytical solution of $h_2(t)$

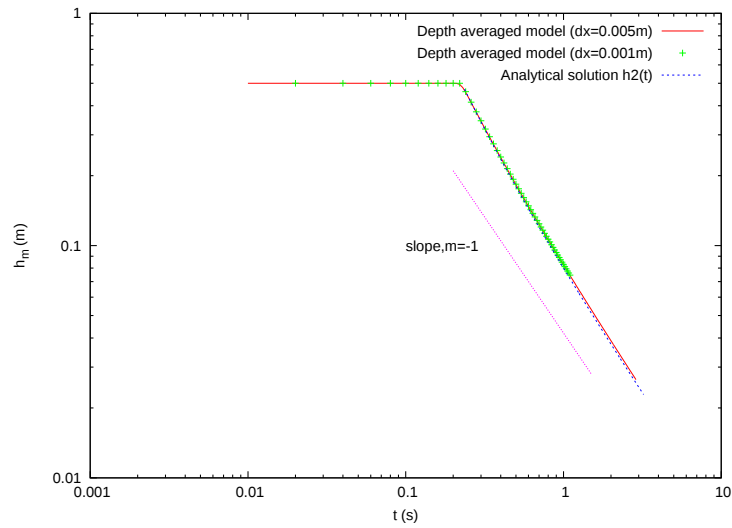


Figure 2.16: Temporal variation of depth at the origin h_m for depth averaged model and new analytical solution of h_2 .

2.5 Summary

In this chapter, a set of analytical solutions describing the dam-break flow of finite extent in the case of ideal fluid is derived. The dam-break flow model is divided into two regions, taking advantage that the front part of the flow, defined as Region A in the study is analogous to the problem of dam-break flow of infinite extent. At the region where the solution of dam-break flow of infinite extent is no longer applicable, an approximate solution is derived, improved and validated with depth-averaged model. The approximate analytical solution for the flow profile in Region B derived in the study is as follows,

$$h_2(t, \xi) = \frac{5}{8}h_o \left(\frac{t}{t_o}\right)^{-1} + \frac{3}{8}h_o \left(\frac{t}{t_o}\right)^{-2} + \left[h_o \left(\frac{t}{t_o}\right)^{-\frac{4}{3}} - \frac{5}{8}h_o \left(\frac{t}{t_o}\right)^{-1} - \frac{3}{8}h_o \left(\frac{t}{t_o}\right)^{-2} \right] \xi^4 \quad (2.144)$$

where $\xi = x/l(t)$, where $l(t)$ is the position of the first negative wave reflected by the upstream wall. $l(t)$ can be determined by MOC as in Eq.(2.5), and for reference convenience, is given again here as follows,

$$l(t) = L_o + 2c_o t - 3L_o \left(\frac{c_o t}{L_o}\right)^{\frac{1}{3}} \quad (2.145)$$

In this chapter, the attenuation of the depth at the origin, h_m is derived in the form of similarity solutions. It is found that h_m attenuates inversely proportional to time $h_m \propto t^{-1}$. Similar to the propagation of front wave L , the attenuation of the depth at the origin h_m can be used to represent the inertial phase flow characteristic in real fluid.

Chapter 3

Viscous Flow Characteristic by Means of Dam-Break Flow of Finite Extent

3.1 Preliminaries

When viscous fluid flows under the equilibrium of viscous force and pressure or buoyancy force, it is said to flow under the viscous-pressure phase or viscous-buoyancy phase in the case of gravity current of a higher density fluid intruding into a lower density fluid. Didden and Maxworthy (1982) carried out an experiment of axis-symmetric spreading of salt water into fresh water to investigate the characteristic of the viscous-pressure phase flow. The findings were further verified by Huppert (1982) who determined analytically the spreading of two-dimensional and axisymmetric gravity current over rigid horizontal surface supported by experiment of axis-symmetric oil spreading. However, Huppert only considered the case where the fluid are of Newtonian type.

This chapter will investigate the characteristic of viscous-pressure phase flow using the model of finite extent dam-break used in Chapter 2 by assuming similarity of flow depth and velocity in the viscous-pressure phase. A general power-law constitutive law which can be used to represent the constitutive relation of Newtonian and non-Newtonian fluids are used. The characteristic of viscous-pressure phase flow for Newtonian and non-Newtonian phase flow can be represented by the similarity solution of the front wave propagation and the attenuation of depth at the origin of the finite extent dam-break flow. In the thesis, the viscous-pressure phase flow is also referred as viscous phase flow.

3.2 Rheology of viscous fluid

The study on the relationship between shear-stresses and deformation of a viscous fluid or widely known as rheology study (Burliarello and Daily, 1961), is one of the essence in understanding the viscous-pressure phase flow characteristic. The relationship between the shear stress and rate of strain in a fluid is also known as the constitutive law or constitutive relations, which governs the behavior of deformation of the fluid.

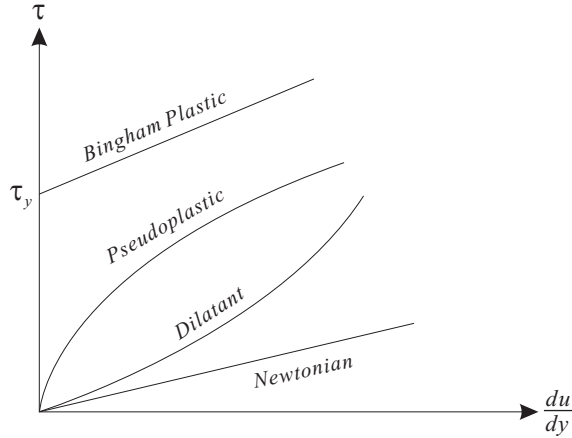


Figure 3.1: Relationship between shear-stress and rate of strain for different type of fluids

A simple classification of fluid rheology behavior based on the constitutive law is shown in Fig. 3.1. The simplest and most commonly used constitutive relation to describe common fluid such as water and air is the Newtonian constitutive relation. These fluids are often called Newtonian fluids. The Newtonian fluids behavior is described by the linear relationships between the shear stress, τ and the rate of strain, $\frac{du}{dy}$ as follows,

$$\tau = \mu \frac{du}{dy} \quad (3.1)$$

For fluids that do not obey this linear relation, they are termed non-Newtonian fluids. The simplest way to present the constitutive relations of non-Newtonian fluids is by the power-law constitutive model,

$$\tau = K \left[\frac{du}{dy} \right]^n \quad (3.2)$$

where K = viscosity coefficient or consistency, which has the dimension depending on n . For $n < 1$, the fluid is termed as shear thinning fluid (pseudoplastic) and for $n > 1$, it is known as shear thickening fluid (dilatant). For $n = 1$, the fluid is of Newtonian type, as mentioned earlier. Bingham fluid is characterized by the following constitutive relation,

$$\tau = \tau_y + \mu_s \frac{du}{dy} \quad (3.3)$$

where τ_y is the yield stress and μ_s is the viscosity which is also the slope of linear line Fig. Burliarello and Daily (1961) after $\tau > \tau_y$. Bingham fluid behaves like Newtonian fluid only after the stress exceeds the a threshold value, τ_y . Chapter 6 is dedicated for the study of Bingham fluid.

3.3 Viscous-pressure phase flow characteristics of Newtonian fluid

3.3.1 Governing equations

The general governing equations for viscous fluid can be written as in Eq.(3.4).

$$\frac{D\tilde{U}}{Dt} = -\frac{\nabla P}{\rho} + \tilde{g} + \frac{1}{\rho} \nabla \cdot \tau \quad (3.4)$$

where $\tilde{U}$ is the velocity vector, $\tilde{g}$ is the acceleration vector, P is pressure, ρ is the fluid density and τ is the deviatoric stress. For two dimensional case, the equation of motion in the x direction can be written as follows,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + g_x + \frac{1}{\rho} \left(\frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{xx}}{\partial x} \right) \quad (3.5)$$

By neglecting the inertia term through the assumption that in the viscous-pressure phase (viscous phase) the inertia term is negligible compared to the viscous and pressure term, we can reduce Eq. (3.5) to the following form:

$$\begin{aligned} \frac{\partial P}{\partial x} &= \frac{1}{\rho} \frac{\partial \tau_{yx}}{\partial y} \quad \text{as } g_x = 0 \quad \text{and} \\ \frac{\partial \tau_{xx}}{\partial x} &= 0 \quad \text{as } \frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y} \end{aligned} \quad (3.6)$$

Assuming static pressure distribution, $P = \rho gh$, and by integrating over the depth of flow, h ,

$$\begin{aligned} \tau_{yx} &= \int \frac{\partial P}{\partial x} dy \\ \tau_{yx} &= -\rho g \frac{dh}{dx} (y - h) \end{aligned} \quad (3.7)$$

By Power-Law model defining the general shear stress and rate of strain relation, with K = viscosity coefficient or consistency,

$$\tau = K \left(\frac{\partial u}{\partial y} \right)^n \quad (3.8)$$

$$K \left(\frac{\partial u}{\partial y} \right)^n = \rho g \frac{dh}{dx} (h - y) \quad (3.9)$$

$$\left(\frac{\partial u}{\partial y} \right)^n = R(h - y) \quad \text{with } R = \frac{\rho g}{K} \left(\frac{\Delta h}{L} \right) \quad (3.10)$$

$$\frac{\partial u}{\partial y} = R^{\frac{1}{n}} (h - y)^{\frac{1}{n}} \quad (3.11)$$

$$u = \frac{n}{n+1} R^{\frac{1}{n}} \left[(h)^{\frac{1}{n}+1} - (h-y)^{\frac{1}{n}+1} \right] \quad (3.12)$$

From velocity distribution derived in Eq. (3.12), we can therefore also derive the average velocity in x direction, $\bar{U}$.

$$\begin{aligned} \bar{U}h &= \int_0^h u dy \\ &= \frac{n}{2n+1} R^{\frac{1}{n}} h^{2+\frac{1}{n}} \\ \bar{U} &= \frac{n}{2n+1} R^{\frac{1}{n}} h^{1+\frac{1}{n}} \end{aligned} \quad (3.13)$$

This velocity distribution satisfies non-slip condition at the bottom boundary and vanishing shear stress at the free surface. The velocity distribution derived in Eq. (3.13) also agrees with the velocity distribution of viscous fluid derived by Ng and Mei (1994) and Huang and Garcia (1998), and is given in the following equation:

$$u = \bar{U} \frac{2n+1}{n+1} \left[1 - \left(1 - \frac{y}{h} \right)^{\frac{1}{n}+1} \right] \quad (3.14)$$

with $\bar{U}$ as the average velocity as in Eq. (3.13). The one dimensional depth averaged continuity and momentum equations in its general form is shown as in Eq. (3.15) and Eq. (3.16), with $\bar{U}$ as the depth averaged velocity,

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} (h\bar{U}) = 0 \quad (3.15)$$

$$\frac{\partial h\bar{U}}{\partial t} + \frac{\partial}{\partial x} (\hat{\beta} h\bar{U}^2) + gh \frac{\partial h}{\partial x} = -\frac{\tau_b}{\rho} \quad (3.16)$$

By referring to Eq. (3.7), we can write the bottom shear stress, τ_b as follows:

$$\tau_b = \rho g \frac{\Delta h}{L} h \quad (3.17)$$

By rearranging the equation Eq. (3.13) for average velocity, we can relate bottom shear stress τ_b with average velocity, $\bar{U}$ as in the following equations:

Averaged velocity,

$$\begin{aligned} \bar{U} &= \frac{n}{2n+1} R^{\frac{1}{n}} h^{1+\frac{1}{n}} \\ &= \frac{n}{2n+1} \left[\frac{\rho g}{K} \cdot \frac{\Delta h}{L} \cdot h \right]^{\frac{1}{n}} h \\ \bar{U} &= \frac{n}{2n+1} \left(\frac{1}{K} \right)^{\frac{1}{n}} (\tau_b)^{\frac{1}{n}} h \end{aligned} \quad (3.18)$$

Therefore,

$$\tau_b = K \left(\frac{2n+1}{n} \right)^n \left(\frac{\bar{U}}{h} \right)^n \quad (3.19)$$

The relation in Eq. (3.19) also agrees with the derivation made by Ng and Mei (1994), as given in Eq. (3.20)

$$\begin{aligned} \tau_b &= c_n \mu_n \left(\frac{\bar{U}}{h} \right)^n \text{ where } \mu_n = \text{viscosity coefficient,} \\ c_n &= \left(\frac{2n+1}{n} \right)^n \end{aligned} \quad (3.20)$$

By using the relation of bottom shear stress τ_b and the average velocity $\bar{U}$, we can therefore rewrite the one dimensional depth averaged equation of motion for general viscous fluid as follows:

Continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} (hV) = 0 \quad (3.21)$$

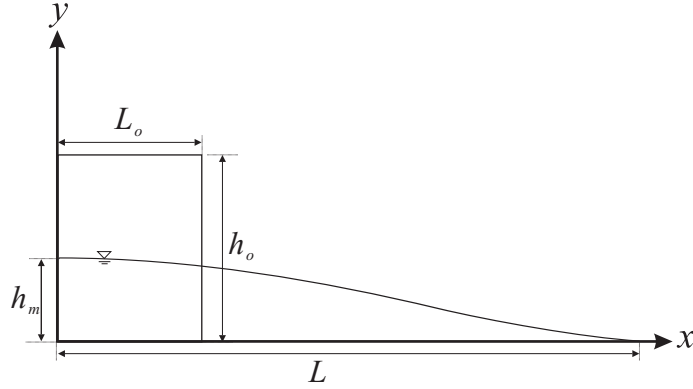


Figure 3.2: Dam break model with finite extent

Momentum equation

$$\begin{aligned} \frac{\partial hV}{\partial t} + \frac{\partial}{\partial x} (\beta hV^2) + gh \frac{\partial h}{\partial x} &= -\frac{\tau_b}{\rho} = -\frac{K}{\rho} \left(\frac{2n+1}{n} \right) \left(\frac{V}{h} \right)^n \\ &= -3\nu \left(\frac{V}{h} \right) \text{ for Newtonian fluid} \end{aligned} \quad (3.22)$$

In the case of viscous Newtonian fluid, $n=1$ and viscosity coefficient, K is normally written as μ . Therefore, the last term on the right hand side of Eq. (3.22) can be written as $-3\nu \left(\frac{V}{h} \right)$ for viscous Newtonian fluid.

3.3.2 Similarity function and derivation of flow characteristics

Similarity functions $p\left(\frac{x}{L}\right)$ and $q\left(\frac{x}{L}\right)$ are introduced for the depth of flow h and velocity of flow V respectively. Therefore, the depth of flow h and its respective velocity V at x can be expressed as in Eq. (3.23) and Eq. (3.24), where L is the front wave tip position of the flow measured from the origin as shown in Fig. 3.2.

$$h(t, x) = h_m(t) p\left(\frac{x}{L(t)}\right) \quad \alpha, \beta, \gamma \quad (3.23)$$

$$V(t, x) = V_m(t) q\left(\frac{x}{L(t)}\right) \quad (3.24)$$

As the front tip of the flow L is a function of time, parameter ξ is introduced, where ξ is defined as in the following equation:

$$\xi = \frac{x}{L(t)} \quad (3.25)$$

The boundary condition for the similarity functions, $p(\xi)$ and $q(\xi)$ can therefore be written as follows:

$$p(0) = 1, p(1) = 0, \text{ and } q(0) = 0 \quad (3.26)$$

Meanwhile, total volume of the flow can be expressed as the integration of the flow depth along the flow, as in the following equation:

$$\begin{aligned}
 \text{Volume of flow} &= \int_0^L h dx \\
 &= h_m \int_0^L p\left(\frac{x}{L}\right) dx \\
 &= h_m L \int_0^1 p(\xi) d\xi
 \end{aligned} \tag{3.27}$$

By substituting the depth of flow h and velocity of flow V , expressed in their similarity function $p(\xi)$ and $q(\xi)$, we can rewrite the governing equations as follows:

Continuity equation,

$$\frac{\partial}{\partial t} [h_m(t) p(\xi)] + \frac{\partial}{\partial x} [h_m(t) p(\xi) V_m(t) q(\xi)] = 0 \tag{3.28}$$

Rearranging Eq. (3.28)

$$p(\xi) \frac{\partial h_m}{\partial t} - \frac{h_m \xi}{L} \frac{\partial p(\xi)}{\partial \xi} \frac{\partial L}{\partial t} + \frac{h_m V_m}{L} p(\xi) \frac{\partial q(\xi)}{\partial \xi} + \frac{h_m V_m}{L} q(\xi) \frac{\partial p(\xi)}{\partial \xi} = 0 \tag{3.29}$$

Momentum equation,

$$\frac{\partial}{\partial t} [h_m p(\xi) V_m q(\xi)] + \frac{\partial}{\partial x} [\hat{\beta} h_m p(\xi) V_m^2 q^2(\xi)] + g h_m^2 \frac{p(\xi)}{L} \frac{\partial p(\xi)}{\partial \xi} = -3v \frac{V_m q(\xi)}{h_m p(\xi)} \tag{3.30}$$

Rearranging Eq. (3.30)

$$\begin{aligned}
 &h_m p(\xi) \left[q(\xi) \frac{\partial V_m}{\partial t} - V_m \frac{\xi}{L} \frac{\partial q}{\partial \xi} \frac{\partial \xi}{\partial t} \right] + V_m q(\xi) \left[p(\xi) \frac{\partial h_m}{\partial t} - h_m \frac{\xi}{L} \frac{\partial p}{\partial \xi} \frac{\partial \xi}{\partial t} \right] \\
 &+ \hat{\beta} \frac{V_m^2 h_m}{L} q^2(\xi) \frac{\partial p(\xi)}{\partial \xi} + 2\hat{\beta} \frac{V_m^2 h_m}{L} p(\xi) q(\xi) \frac{\partial q(\xi)}{\partial \xi} \\
 &+ g \frac{h_m^2}{L} p(\xi) \frac{\partial p(\xi)}{\partial \xi} = -3v \frac{V_m q(\xi)}{h_m p(\xi)}
 \end{aligned} \tag{3.31}$$

By assuming similarity solutions exist for h_m , V_m and L , Huppert (1982) we can introduce the following equations:

$$h_m = \alpha h_o \left(\sqrt{\frac{g}{h_o}} t \right)^a \tag{3.32}$$

$$V_m = \beta \sqrt{g h_o} \left(\sqrt{\frac{g}{h_o}} t \right)^b \tag{3.33}$$

$$L = \gamma L_o \left(\sqrt{\frac{g}{h_o}} t \right)^c \tag{3.34}$$

where h_o is the characteristic depth and L_o is the characteristic length. The dimensionless form of t is defined as follows:

$$t' = \sqrt{\frac{g}{h_o}} t \tag{3.35}$$

Therefore, by substituting the similarity form of h_m , V_m and L , as well as the dimensionless form of t , we can further write the governing equations as follows:

Continuity Equation

$$\begin{aligned} & \alpha h_o a t'^{a-1} \sqrt{\frac{g}{h_o}} p(\xi) - \alpha h_o a c t'^{a-1} \xi \sqrt{\frac{g}{h_o}} \frac{\partial p(\xi)}{\partial \xi} + \frac{\alpha \beta}{\gamma L_o} h_o \sqrt{g h_o} q(\xi) \frac{\partial p(\xi)}{\partial \xi} t'^{a+b-c} \\ & + \frac{\alpha \beta}{\gamma L_o} h_o \sqrt{g h_o} p(\xi) \frac{\partial q(\xi)}{\partial \xi} t'^{a+b-c} = 0 \end{aligned} \quad (3.36)$$

Momentum Equation

$$\begin{aligned} & \alpha \beta b g h_o p(\xi) q(\xi) t'^{a+b-1} - \alpha \beta c g h_o \xi p(\xi) \frac{\partial q(\xi)}{\partial \xi} t'^{a+b-1} + \alpha \beta a g h_o p(\xi) q(\xi) t'^{a+b-1} - \alpha \beta c g h_o \xi q(\xi) \\ & \frac{\partial p(\xi)}{\partial \xi} t'^{a+b-1} + \hat{\beta} \frac{\alpha \beta^2}{\gamma} \frac{g h_o^2}{L_o} q^2(\xi) \frac{\partial p(\xi)}{\partial \xi} t'^{a+2b-c} + 2 \hat{\beta} \frac{\alpha \beta^2}{\gamma} \frac{g h_o^2}{L_o} p(\xi) q(\xi) \frac{\partial q(\xi)}{\partial \xi} t'^{a+2b-c} \\ & + \frac{\alpha^2}{\gamma} \frac{g h_o^2}{L_o} p(\xi) \frac{\partial p(\xi)}{\partial \xi} t'^{2a-c} = -3 \nu \frac{\beta}{\alpha} \sqrt{\frac{g}{h_o}} \frac{q(\xi)}{p(\xi)} t'^{b-a} \end{aligned} \quad (3.37)$$

To satisfy dimensional homogeneity, we therefore equate the power of t' in Eq. (3.36), From continuity equation

$$a - 1 = a + b - c \rightarrow b - c = -1 \quad (3.38)$$

From volume of flow, as defined in Eq. (3.27)

$$\begin{aligned} V &= h_m L \int_0^1 p(\xi) d\xi \\ &= \alpha h_o \left(\sqrt{\frac{g}{h_o}} t \right)^a \gamma L_o \left(\sqrt{\frac{g}{h_o}} t \right)^c \int_0^1 p(\xi) d\xi \end{aligned} \quad (3.39)$$

As the volume of flow is assumed to be constant during the whole dam break duration;

$$a + c = 0 \quad (3.40)$$

In the case of flow of viscous fluid, it is assumed that in the viscous region, the flow propagates under the dynamic equilibrium of pressure and viscosity. Therefore, by equating the power of t' of the pressure term and viscous term in the momentum equation, Eq. (3.36), we can write the following relations between the coefficients a , b and c .

$$2a + c = b - a \quad (3.41)$$

By solving Eq. (3.38), Eq. (3.40) and Eq. (3.41) for coefficients a , b and c , we have,

$$a = -\frac{1}{5}, \quad b = -\frac{4}{5}, \quad c = \frac{1}{5} \quad (3.42)$$

Thus, we can write the following relations for depth of flow at the origin h_m and the front wave position L as follows:

$$h_m = \alpha h_o \left(\sqrt{\frac{g}{h_o}} t \right)^a \rightarrow h_m \propto t^{-\frac{1}{5}} \quad (3.43)$$

$$L = \gamma L_o \left(\sqrt{\frac{g}{h_o}} t \right)^c \rightarrow L \propto t^{\frac{1}{5}} \quad (3.44)$$

3.4 Viscous-pressure phase flow characteristics of non-Newtonian fluid

3.4.1 Governing equations and derivation of power-law

The one dimensional depth averaged continuity and momentum equations for general viscous fluid obeying the the Power Law model are derived as in Eq. (3.21) and Eq. (3.22). For simplicity, non-Newtonian fluid is studied in the case of $n > 1$ for shear thickening fluid and $n < 1$ for shear thinning fluid. The same method of introducing similarity functions of p and q , and assuming power law relating to time for h_m , V_m and L in the investigation of viscous Newtonian fluid is used here as well. Therefore, the continuity and momentum equations for non-Newtonian fluid can be written again as follows:

Continuity Equation:

$$\begin{aligned} & \alpha h_o a t'^{a-1} \sqrt{\frac{g}{h_o}} p(\xi) - \alpha h_o a c t'^{a-1} \xi \sqrt{\frac{g}{h_o}} \frac{dp(\xi)}{d\xi} + \frac{\alpha \beta}{\gamma L_o} h_o \sqrt{gh_o} q(\xi) \frac{dp(\xi)}{d\xi} t'^{a+b-c} \\ & + \frac{\alpha \beta}{\gamma L_o} h_o \sqrt{gh_o} p(\xi) \frac{dq(\xi)}{d\xi} t'^{a+b-c} = 0 \end{aligned} \quad (3.45)$$

Momentum Equation:

$$\begin{aligned} & \alpha \beta b g h_o p(\xi) q(\xi) t'^{a+b-1} - \alpha \beta c g h_o \xi p(\xi) \frac{dq(\xi)}{d\xi} t'^{a+b-1} + \alpha \beta a g h_o p(\xi) q(\xi) t'^{a+b-1} \\ & - \alpha \beta c g h_o \xi q(\xi) \frac{dp(\xi)}{d\xi} t'^{a+b-1} + \hat{\beta} \frac{\alpha \beta^2}{\gamma} \frac{gh_o^2}{L_o} q^2(\xi) \frac{dp(\xi)}{d\xi} t'^{a+2b-c} \\ & + 2\hat{\beta} \frac{\alpha \beta^2}{\gamma} \frac{gh_o^2}{L_o} p(\xi) q(\xi) \frac{dq(\xi)}{d\xi} t'^{a+2b-c} \text{inertial term} + \frac{\alpha^2}{\gamma} \frac{gh_o^2}{L_o} p(\xi) \frac{dp(\xi)}{d\xi} t'^{2a-c} \\ & = \frac{K}{\rho} \left(\frac{2n+1}{n} \right)^n \left(\frac{\beta}{\alpha} \sqrt{\frac{g}{h_o}} \frac{p(\xi)}{p(\xi)} t'^{b-a} \right)^n \end{aligned} \quad (3.46)$$

By equating the power of t' in the continuity equation, as in Eq. (3.45), the following relations of coefficients a b and c are obtained,

From continuity equation

$$a - 1 = a + b - c \quad \rightarrow \quad b - c = -1 \quad (3.47)$$

For constant volume of flow

$$\begin{aligned} V &= h_m L \int_0^1 p(\xi) d\xi \\ &= \alpha h_o \left(\sqrt{\frac{g}{h_o}} t \right)^a \gamma L_o \left(\sqrt{\frac{g}{h_o}} t \right)^c \int_0^1 p(\xi) d\xi \end{aligned} \quad (3.48)$$

Therefore

$$a + c = 0 \quad (3.49)$$

Similar to the procedure in the investigation of viscous Newtonian fluid, the dynamic equilibrium between viscous term and pressure term is assumed to exist in the viscous region of the flow.

Therefore, by equating the viscous term and pressure term in the equation of motion, we have,

$$2a - c = n(b - a) \quad (3.50)$$

Model	$h_o(m)$	$L_o(m)$	$\nu(m^2s^{-1})$
MPS	0.5	0.5	0.000001
MPS	0.5	0.5	0.00005
MPS	0.5	0.5	0.0005
MPS	0.5	0.5	0.001

Table 3.1: Initial conditions of simulations for viscous fluid with MPS model.

By solving Eq. (3.47), Eq. (3.49) and Eq. (3.50), solutions for coefficients a and c are obtained,

$$a = -\frac{n}{3+2n} \quad (3.51)$$

$$c = \frac{n}{3+2n} \quad (3.52)$$

Thus, the front wave position L and depth of flow at the origin h_m can be written as:

$$h_m = \alpha h_o \left(\sqrt{\frac{g}{h_o}} t \right)^a \rightarrow h_m \propto t^{-\frac{n}{3+2n}} \quad (3.53)$$

$$L = \gamma L_o \left(\sqrt{\frac{g}{h_o}} t \right)^c \rightarrow L \propto t^{\frac{n}{3+2n}} \quad (3.54)$$

The results in Eq. (3.53) and Eq. (3.61) can be used for any fluid obeying the constitutive law in the power law model. For Newtonian fluid ($n=1$), therefore the temporal propagation of leading front, L and temporal variation of depth at the origin, h_m for Newtonian fluid are as follows:

$$h_m \propto t^{-\frac{1}{5}} \quad (3.55)$$

$$L \propto t^{\frac{1}{5}} \quad (3.56)$$

which agree with the theoretical derivation made in Section 3.4.

3.5 Validation of analytical findings with numerical models

3.5.1 Simulation Procedures for Newtonian Fluid

The analytical results are validated with MPS model (Koshizuka et al., 1998). A dam break model with finite-extent, as in Fig.(3.2) is set up. The initial size of the dam break model is $L_o = 0.5m$ and $h_o = 0.5m$. The bottom floor and wall of the channel is set to non-slip condition. Table 3.1 shows the initial condition and kinematic viscosity, ν used for the simulation of Newtonian fluid.

3.5.2 Validation of analytical solution of Newtonian Fluid

Simulation results of viscous Newtonian fluid are shown in Fig.3.3 and Fig.3.4. The simulation results show good agreement with the results obtained from the theoretical analysis. Distinct regions can be observed for the temporal variation of depth at the origin h_m and front wave

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position of the flow L . These regions define the inertial and viscous phase of the flow. In the viscous region, the depth at the origin and the front wave position are proportional to time to the power of $-\frac{1}{5}$ and $\frac{1}{5}$ respectively. Huppert (1982) also observes the same results for the propagation of front wave L and depth of flow at the origin h_m in the viscous phase in the case of Newtonian fluid.

The initial region which defines a short moment immediately after the release of the mass of fluid in the dam shown in the numerical results represents the inertial phase flow of the fluid. The inertial phase flow characteristics as discussed in Chapter 2 can be observed in the initial region of the Fig.3.3 and Fig.3.4 where the front wave is propagating proportionally with time, $L \propto t^1$. The attenuation of the depth at the origin h_m also shows the characteristics of inertial phase flow where h_m is attenuating inversely proportional to time $h_m \propto t^{-1}$.

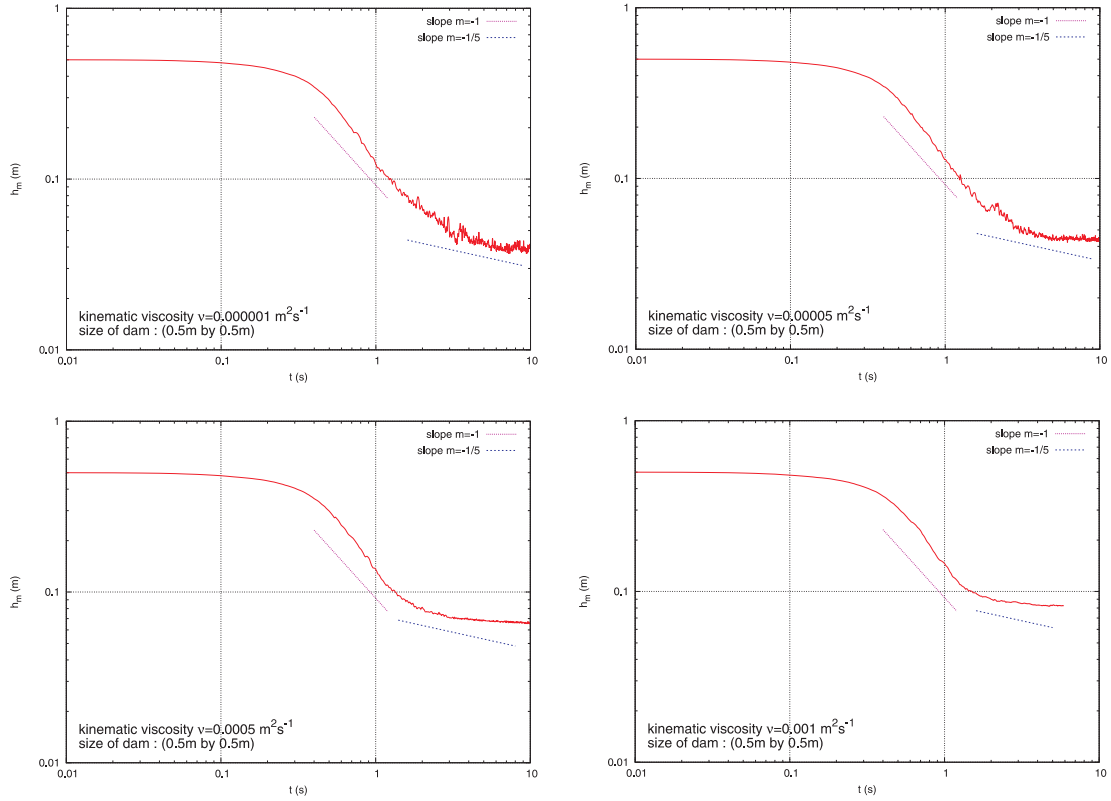


Figure 3.3: Temporal variation of depth at the origin, h_m for a) $v = 0.000001 \text{ m}^2 \text{ s}^{-1}$, b) $v = 0.00005 \text{ m}^2 \text{ s}^{-1}$, c) $v = 0.0005 \text{ m}^2 \text{ s}^{-1}$, d) $v = 0.001 \text{ m}^2 \text{ s}^{-1}$.

3.5. Validation of analytical findings with numerical models

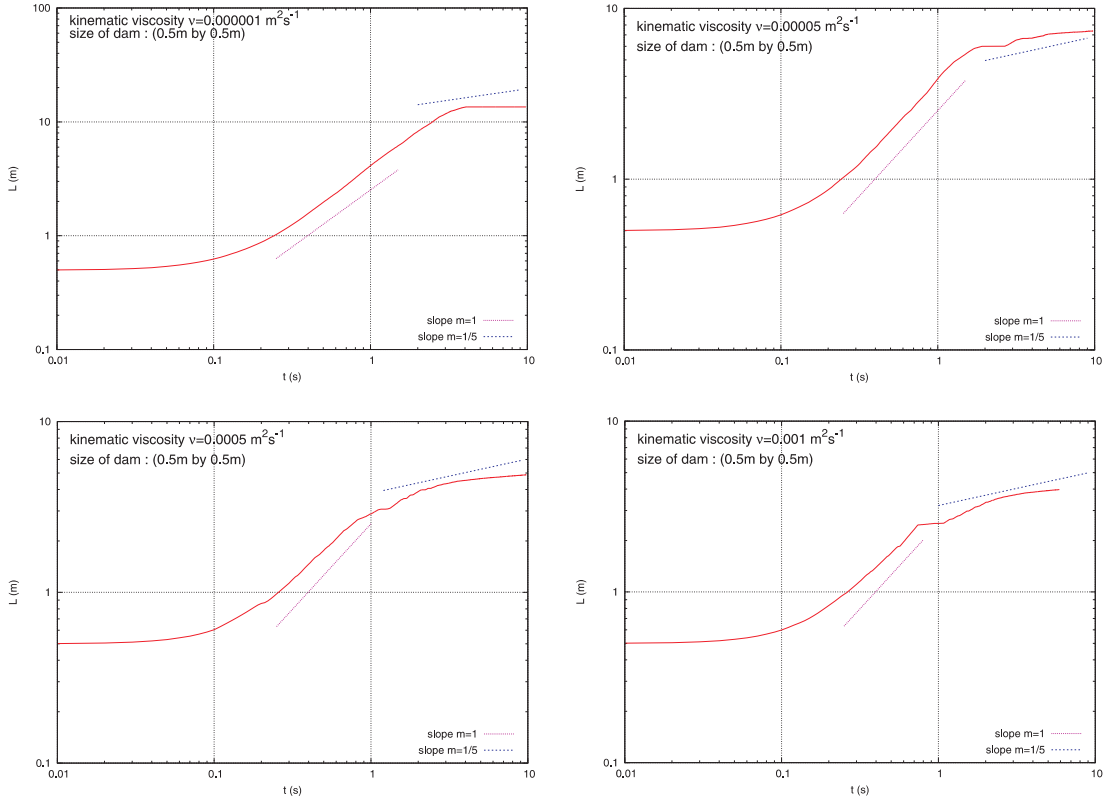


Figure 3.4: Temporal variation of front wave propagation, L for a) $\nu = 0.000001 \text{ m}^2 \text{ s}^{-1}$, b) $\nu = 0.00005 \text{ m}^2 \text{ s}^{-1}$, c) $\nu = 0.0005 \text{ m}^2 \text{ s}^{-1}$, d) $\nu = 0.001 \text{ m}^2 \text{ s}^{-1}$.

3.5.3 Simulation Procedures for non-Newtonian Fluid

Numerical simulation of non-Newtonian fluid is carried out using MPS model as well. But, the viscous term in the governing equation of MPS model has to be modified to take into account the constitutive law of non-Newtonian fluid. The viscous term is modified as follows:

$$\begin{aligned}
 \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_i} &= \frac{1}{\rho} \frac{\partial}{\partial x_j} \left[K \left(\frac{\partial u_i}{\partial x_j} \right)^n \right] \\
 &= \frac{1}{\rho} K n \left(\frac{\partial u_i}{\partial x_j} \right)^{n-1} \left(\frac{\partial^2 u_i}{\partial x_j^2} \right) \\
 &= \frac{1}{\rho} K n (\nabla u)^{n-1} (\nabla^2 u)
 \end{aligned} \tag{3.57}$$

where ∇u and $\nabla^2 u$ are the gradient and Laplacian form used in MPS model. The initial conditions of dam break flow simulation for non-Newtonian fluid are shown in Table 3.2, with $n=2.0$ assumed for shear thickening fluid and $n=0.5$ for shear thinning fluid.

Table 3.2: Initial conditions of simulation for non-Newtonian fluids with MPS model.

Non-Newtonian Fluids	$h_o(m)$	$L_o(m)$	K
Shear thickening			
$n = 2.0$	0.5	0.5	0.01
$n = 2.0$	0.5	0.5	0.1
$n = 2.0$	0.5	0.5	1.0
Shear thinning			
$n = 0.50$	0.5	0.5	0.01
$n = 0.50$	0.5	0.5	0.1
$n = 0.50$	0.5	0.5	1.0

3.5.4 Validation of analytical solution of non-Newtonian Fluid

The results of simulation of non-Newtonian fluid are shown in Fig.3.5 and Fig.3.6 for shear-thickening fluids ($n = 0.5$) and in Fig.3.7 and Fig.3.8 for shear-thinning fluid ($n = 2.0$). In both cases of shear thickening ($n = 2.0$) and shear thinning ($n = 0.5$) fluids, distinct inertia and viscous regions are observed. Both shear thickening and shear thinning fluids also show good agreement with the theoretical analysis in the viscous region, where in the case of shear thickening fluid ($n = 2.0$) and shear thinning fluid ($n = 0.5$), the depth at the origin h_m and propagation of front wave position is related to time as follows:

For $n=2.0$

$$\begin{aligned} h_m &\propto t^{-\frac{2}{7}} \\ L &\propto t^{\frac{2}{7}} \end{aligned} \quad (3.58)$$

For $n=0.5$

$$\begin{aligned} h_m &\propto t^{-\frac{1}{8}} \\ L &\propto t^{\frac{1}{8}} \end{aligned} \quad (3.59)$$

The viscous region becomes more dominant over the inertia region as consistency K is being increased for both shear thickening and shear thinning fluids.

3.5. Validation of analytical findings with numerical models

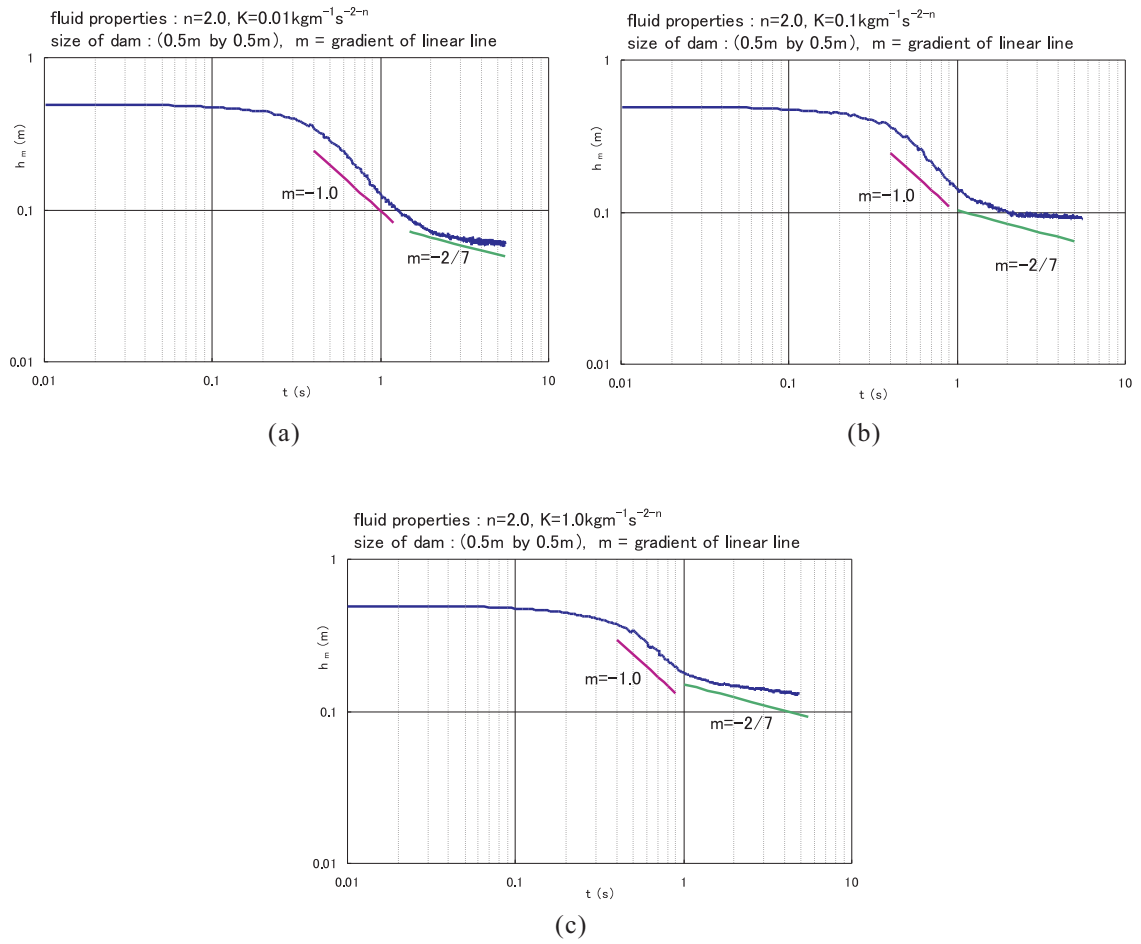


Figure 3.5: Temporal variation of depth at origin, h_m for shear-thickening fluid, $n = 2.0$ with (a) $K = 0.01\text{kgm}^{-1}\text{s}^{-4}$, (b) $K = 0.1\text{kgm}^{-1}\text{s}^{-4}$, (c) $K = 1.0\text{kgm}^{-1}\text{s}^{-4}$.

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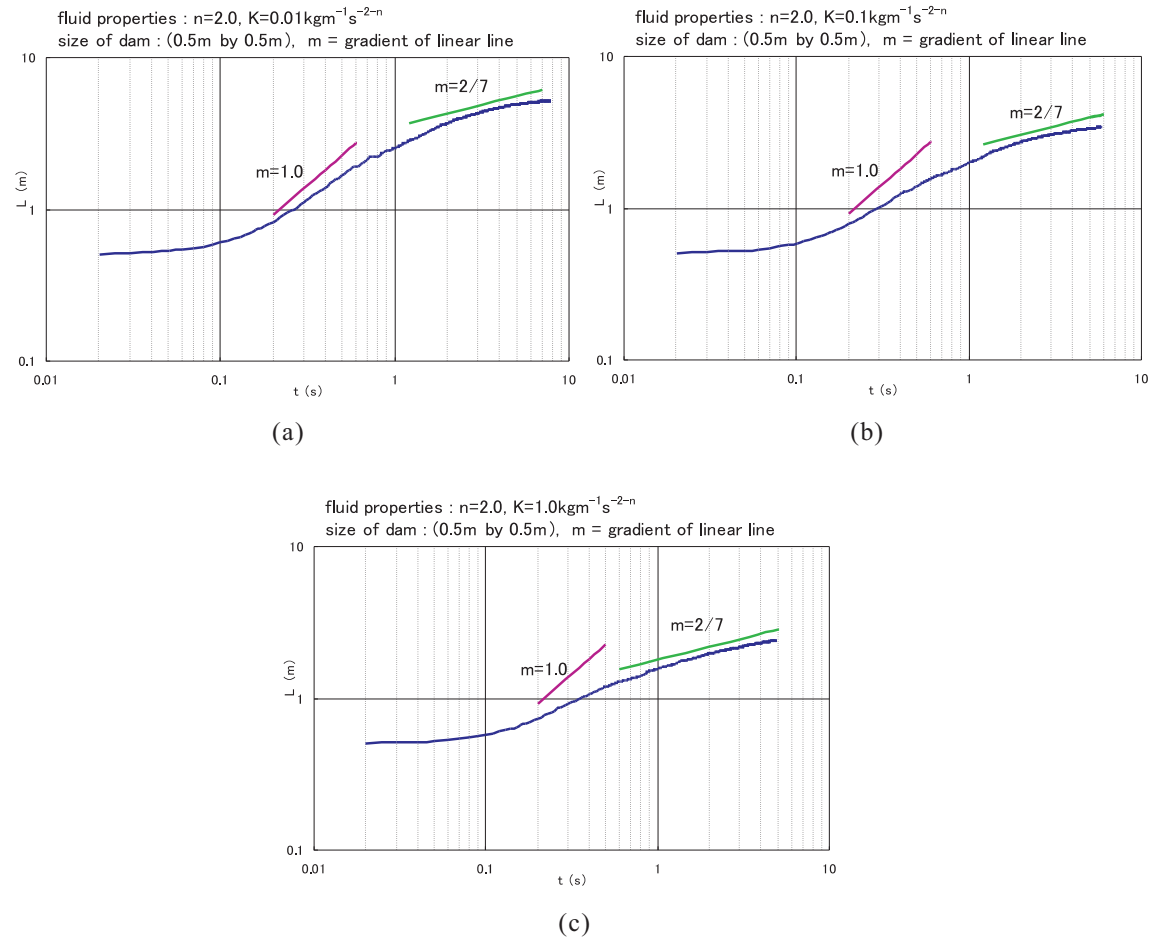


Figure 3.6: Temporal variation of front wave propagation, L for shear-thickening fluid, $n = 2.0$ with (a) $K = 0.01 \text{ kg m}^{-1} \text{ s}^{-4}$, (b) $K = 0.1 \text{ kg m}^{-1} \text{ s}^{-4}$, (c) $K = 1.0 \text{ kg m}^{-1} \text{ s}^{-4}$.

3.5. Validation of analytical findings with numerical models

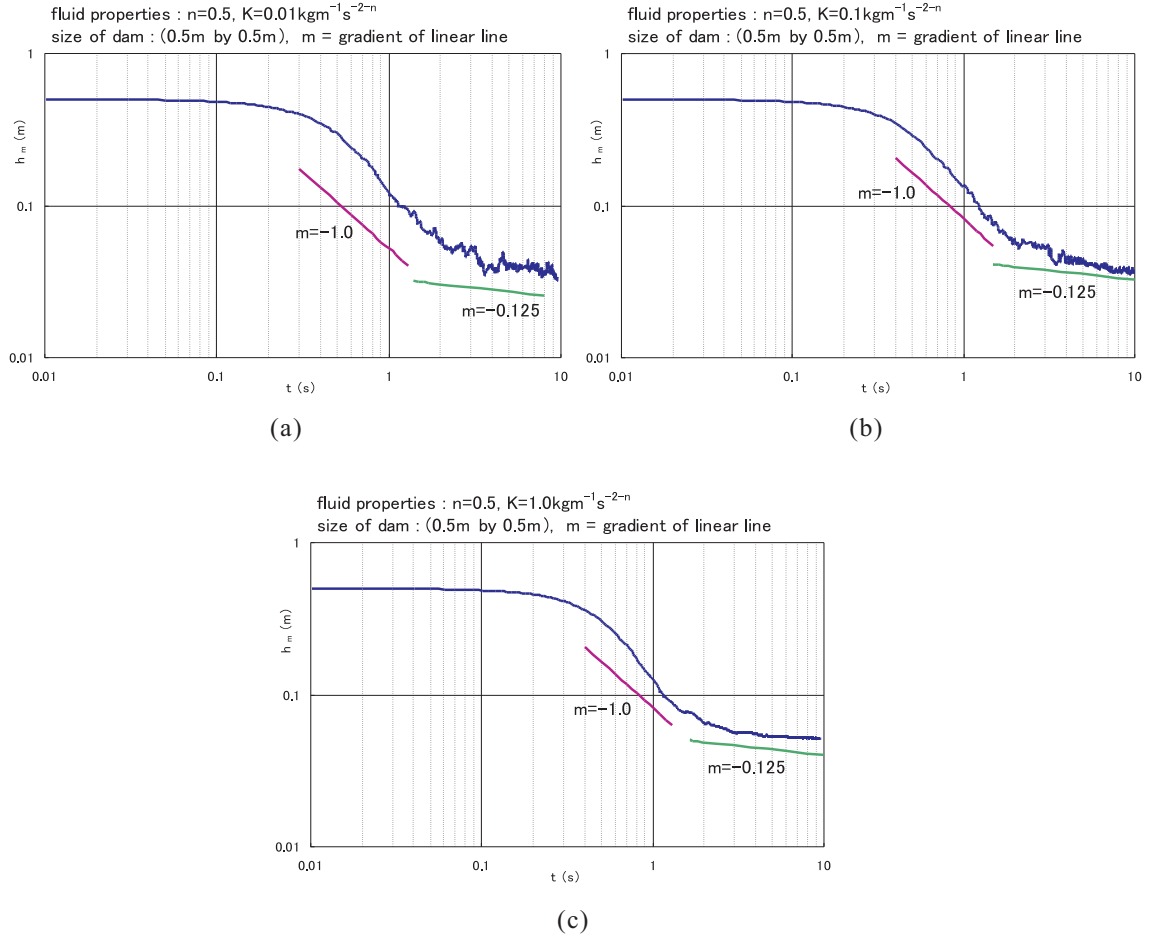


Figure 3.7: Temporal variation of depth at origin, h_m for shear-thinning fluid, $n = 0.5$ with (a) $K = 0.01 \text{ kgm}^{-1} \text{ s}^{-\frac{5}{2}}$, (b) $K = 0.1 \text{ kgm}^{-1} \text{ s}^{-\frac{5}{2}}$, (c) $K = 1.0 \text{ kgm}^{-1} \text{ s}^{-\frac{5}{2}}$.

3. VISCOUS FLOW CHARACTERISTIC BY MEANS OF DAM-BREAK FLOW OF FINITE EXTENT

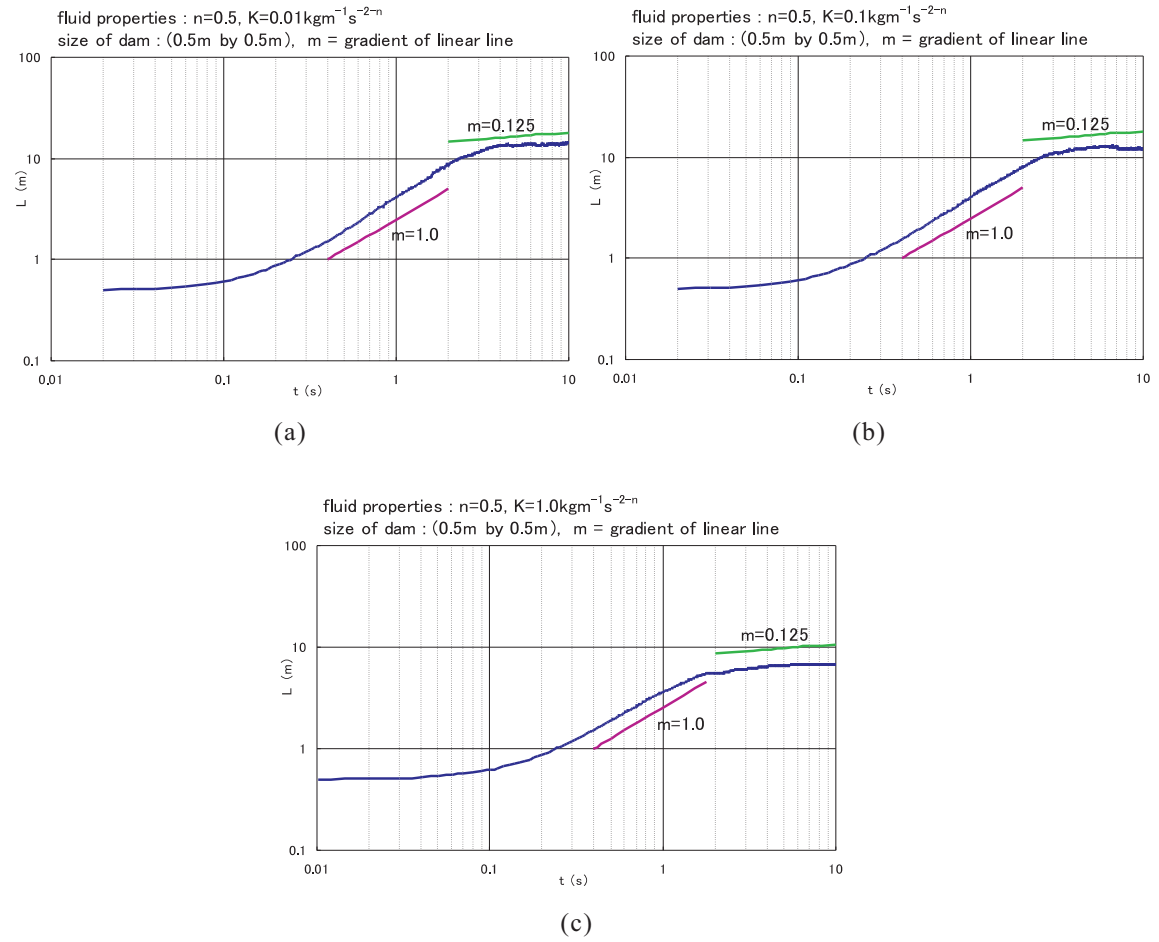


Figure 3.8: Temporal variation of front wave propagation, L for shear-thinning, $n = 0.5$ with (a) $K = 0.01 \text{ kg m}^{-1} \text{ s}^{-\frac{5}{2}}$, (b) $K = 0.1 \text{ kg m}^{-1} \text{ s}^{-\frac{5}{2}}$, (c) $K = 1.0 \text{ kg m}^{-1} \text{ s}^{-\frac{5}{2}}$.

3.6 Summary

In this chapter, the viscous-pressure phase (viscous phase) flow characteristics are investigated using the finite-extent dam break flow as study model. Based on the assumption that in the viscous-pressure phase (viscous phase) flow the flow enters a self-similar stage where the velocity and depth can be expressed in similarity function form. By using power-law constitutive law to represent Newtonian and non-Newtonian fluid, the characteristic of front wave propagation L and attenuation of depth at the origin h_m are derived. With $n = 1$ for Newtonian fluid, and $n > 1$ and $n < 1$ for non-Newtonian fluid, the characteristics of viscous-pressure phase (viscous phase) flow can be written in similarity form of front wave propagation and attenuation of depth at the origin, as follows,

$$h_m = \alpha h_o \left(\sqrt{\frac{g}{h_o}} t \right)^a \rightarrow h_m \propto t^{-\frac{n}{3+2n}} \quad (3.60)$$

$$L = \gamma L_o \left(\sqrt{\frac{g}{h_o}} t \right)^c \rightarrow L \propto t^{\frac{n}{3+2n}} \quad (3.61)$$

The characteristics of viscous-pressure phase (viscous phase) flow for Newtonian and non-Newtonian fluids are verified with numerical models with inertial phase flow in viscous fluid observable in the early stage of the flow. In the next Chapter, a integral model is developed to establish a continuous solution to which is capable of describing the transition from inertial to viscous phase flow.

Chapter 4

Integral Model

4.1 Preliminaries

The inertial and viscous phase flows are studied independently in Chapter 2 and Chapter 3. In this chapter, another mathematical approach is used to reproduce the inertial and viscous phase flow characteristics. By using integral transformation, the governing equations which are comprised of the one-dimensional depth averaged continuity and momentum equations, are transformed from partial differential equations (PDE) to ordinary differential equations (ODE). The ODE form of the governing equations are solved using Euler method.

4.2 Theoretical analysis

A one-dimensional depth averaged model representing the dam-break flow of finite volume shown in Fig. 4.1 can be expressed in the following form,
Continuity equation,

$$\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x} = 0 \quad (4.1)$$

Momentum equation,

$$\frac{\partial hU}{\partial t} + \frac{\partial \beta hU^2}{\partial x} + Gh \frac{\partial h}{\partial x} = -\frac{3v}{h}U \quad (4.2)$$

where G is gravity acceleration, v is the kinematic viscosity, β is the momentum coefficients and the other parameters are described in the schematic of dam break flow in Fig.4.1. The depth of flow h and velocity U are expressed in the following form,

$$h = h_m(t) f\left(\frac{x}{L}\right) = h_m(t) f(\xi) \quad (4.3)$$

$$U = U_m(t) g\left(\frac{x}{L}\right) = U_m(t) g(\xi) \quad (4.4)$$

where $U_m(t)$ is the representative velocity, $h_m(t)$ is the representative depth, which is also the depth at the origin. L is the position of the front wave from measured from the origin. $f(x/L)$

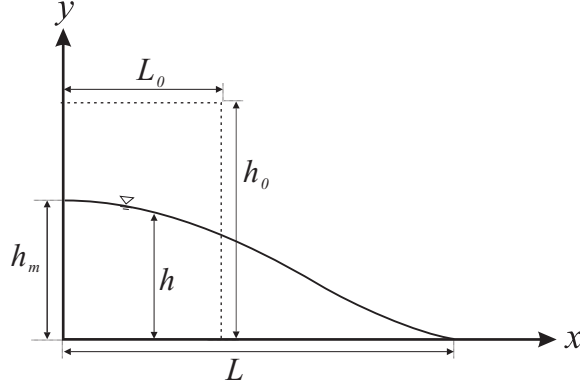


Figure 4.1: Schematic of dam break flow model

and $g(x/L)$ are shape functions for h and U respectively. ξ is defined as $\xi = x/L$. For simplicity, $h_m(t)$ and $U_m(t)$ are rewritten as h_m and U_m respectively, while $f(\xi)$ and $g(\xi)$ are rewritten as f and g . Therefore, Eq.(4.3) and Eq.(4.4) are rewritten as,

$$h = h_m(t) f\left(\frac{x}{L}\right) = h_m(t) f(\xi) = h_m f \quad (4.5)$$

$$U = U_m(t) g\left(\frac{x}{L}\right) = U_m(t) g(\xi) = U_m g \quad (4.6)$$

The energy equation representing the system can be written as follows,
Energy equation,

$$\frac{\partial}{\partial t} \left(\frac{hU^2}{2} \right) + \frac{\partial}{\partial x} \left(hU \frac{U^2}{2} \right) + U \frac{\partial}{\partial x} [(\beta - 1)hU^2] + G \frac{\partial}{\partial x} (h^2 U) + G \frac{\partial}{\partial t} \left(\frac{h^2}{2} \right) = -3v \frac{U^2}{h} \quad (4.7)$$

4.2.1 Integration of governing equations

The governing equations are integrated from $x = 0$ to $x = L$ as follows,

Integration of continuity equation,

$$\int_0^L \left(\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x} \right) dx = 0 \quad (4.8)$$

$$(4.9)$$

where

$$\int_0^L \frac{\partial h}{\partial t} dx = \frac{d}{dt} \int_0^L h dx - [h]_L \frac{dL}{dt} + [h]_0 \frac{d0}{dt} = \frac{d}{dt} \int_0^L h dx \quad (4.10)$$

and

$$\int_0^L \frac{\partial hU}{\partial t} dx = [hU]_L - [hU]_0 = 0 \quad (4.11)$$

therefore,

$$\begin{aligned} \frac{d}{dt} \int_0^L h dx &= 0 \\ \therefore \int_0^L h dx &= \text{volume} = h_0 L_0 = V \end{aligned} \quad (4.12)$$

h_0 and L_0 are the initial width and height of the dam as shown in Fig.4.1. By substituting the expression of $h = h_m f$ and $U = U_m g$,

$$\begin{aligned} \int_0^L h \, dx &= \int_0^1 h_m f L \, d\xi \quad \because d\xi = dx/L \\ h_m L \int_0^1 f \, d\xi &= h_0 L_0 \\ h_m L A_1 &= h_0 L_0 \end{aligned} \quad (4.13)$$

where

$$A_1 = \int_0^1 f \, d\xi \quad (4.14)$$

Integration of momentum equation with the substitution of $h = h_m f$ and $U = U_m g$ is as follows,
Integration of momentum equation

$$\int_0^L \frac{\partial h U}{\partial t} + \frac{\partial \beta h U^2}{\partial x} + G h \frac{\partial h}{\partial x} + \frac{3v}{h} U \, dx = 0 \quad (4.15)$$

$$\begin{aligned} \int_0^L \frac{\partial h U}{\partial t} \, dx &= \frac{d}{dt} \int_0^L h U \, dx - [h U]_L \frac{dL}{dx} + [h U]_0 \frac{d0}{dx} \\ &= \frac{d}{dt} \int_0^L h U \, dx \\ &= \frac{d}{dt} \int_0^L h_m f \cdot U_m g \, dx \\ &= \frac{d}{dt} (h_m U_m) \int_0^1 f g L \, d\xi \end{aligned} \quad (4.16)$$

$$\int_0^L \frac{\partial \beta h U^2}{\partial x} \, dx = [\beta h U^2]_0^L = 0 \quad (4.17)$$

$$\int_0^L G h \frac{\partial h}{\partial x} \, dx = G \int_0^L \frac{\partial}{\partial x} \left(\frac{h^2}{2} \right) \, dx = G \left[\frac{h^2}{2} \right]_0^L = -G \frac{h_m^2}{2} \quad (4.18)$$

$$\int_0^L 3v \frac{U}{h} \, dx = 3v \int_0^L \frac{U_m g}{h_m f} \, dx = 3v \int_0^1 \frac{U_m g}{h_m f} L \, d\xi = 3v \frac{U_m L}{h_m} \int_0^1 \frac{g}{f} \, d\xi \quad (4.19)$$

Therefore, the integral form of momentum equation can be written as follows,

$$\begin{aligned} \frac{d}{dt} (h_m U_m) \int_0^1 f g L \, d\xi - G \frac{h_m^2}{2} + 3v \frac{U_m L}{h_m} \int_0^1 \frac{g}{f} \, d\xi &= 0 \\ \frac{d}{dt} (h_m U_m L) A_2 - G \frac{h_m^2}{2} + 3v \frac{U_m L}{h_m} A_3 &= 0 \end{aligned} \quad (4.20)$$

where A_2 and A_3 are defined as follows,

$$A_2 = \int_0^1 f g \, d\xi \quad (4.21)$$

$$A_3 = \int_0^1 \frac{g}{f} \, d\xi \quad (4.22)$$

Integration of energy equation

The integration of energy equation is as follows,

$$\int_0^L \frac{\partial}{\partial t} \left(\frac{hU^2}{2} \right) + \frac{\partial}{\partial x} \left(hU \frac{U^2}{2} \right) + U \frac{\partial}{\partial x} [(\beta - 1)hU^2] + G \frac{\partial}{\partial x} (h^2U) + G \frac{\partial}{\partial t} \left(\frac{h^2}{2} \right) + 3v \frac{U^2}{h} dx = 0 \quad (4.23)$$

$$\begin{aligned} \int_0^L \frac{\partial}{\partial t} \left(\frac{hU^2}{2} \right) dx &= \frac{d}{dt} \int_0^L \frac{hU^2}{2} dx - \left[\frac{hU^2}{2} \right]_L \frac{dL}{dt} + \left[\frac{hU^2}{2} \right]_0 \frac{d0}{dt} \\ &= \frac{d}{dt} \int_0^1 \frac{1}{2} h_m f \cdot (U_m g)^2 L d\xi \\ &= \frac{d}{dt} \left(\frac{1}{2} h_m U_m^2 L \right) \int_0^1 f g^2 d\xi \end{aligned} \quad (4.24)$$

$$\int_0^L \frac{\partial}{\partial x} \left(hU \frac{U^2}{2} \right) dx = \left[hU \frac{U^2}{2} \right]_0^L = 0 \quad (4.25)$$

$$\int_0^L U \frac{\partial}{\partial x} [(\beta - 1)hU^2] dx = 0 \quad \text{assuming } \beta = 1 \quad (4.26)$$

$$\int_0^L G \frac{\partial}{\partial x} (h^2U) dx = G [h^2U]_0^L = 0 \quad (4.27)$$

$$\begin{aligned} \int_0^L G \frac{\partial}{\partial t} \left(\frac{h^2}{2} \right) dx &= G \left\{ \frac{d}{dt} \int_0^L \frac{h^2}{2} dx - \left[\frac{h^2}{2} \right]_L \frac{dL}{dt} + \left[\frac{h^2}{2} \right]_0 \frac{d0}{dt} \right\} \\ &= G \frac{d}{dt} \int_0^L \left(\frac{(h_m f)^2}{2} \right) dx \\ &= G \frac{d}{dt} \left(\frac{h_m^2 L}{2} \right) \int_0^1 f^2 d\xi \end{aligned} \quad (4.28)$$

$$\int_0^L 3v \frac{U^2}{h} dx = 3v \int_0^L \frac{(U_m g)^2}{h_m f} dx = 3v \int_0^1 \frac{(U_m g)^2 L}{h_m f} d\xi = 3v \frac{U_m^2 L}{h_m} \int_0^1 \frac{g^2}{f} d\xi \quad (4.29)$$

Therefore, the integral of energy equation can be summarized as follows,

$$\begin{aligned} \frac{d}{dt} \left(\frac{1}{2} h_m U_m^2 L \right) \int_0^1 f g^2 d\xi + G \frac{d}{dt} \left(\frac{h_m^2 L}{2} \right) \int_0^1 f^2 d\xi + 3v \frac{U_m^2 L}{h_m} \int_0^1 \frac{g^2}{f} d\xi &= 0 \\ \frac{d}{dt} \left(\frac{1}{2} h_m U_m^2 L \right) A_4 + G \frac{d}{dt} \left(\frac{h_m^2 L}{2} \right) A_5 + 3v \frac{U_m^2 L}{h_m} A_6 &= 0 \end{aligned} \quad (4.30)$$

where A_4, A_5 and A_6 are,

$$A_4 = \int_0^1 f g^2 d\xi \quad (4.31)$$

$$A_5 = \int_0^1 f^2 d\xi \quad (4.32)$$

$$A_6 = \int_0^1 \frac{g^2}{f} d\xi \quad (4.33)$$

4.3 Solution of integral model

The governing equations are reduced to ODE form after the transformation by integration. The reduced form of the governing equations are summarized as follows,
Continuity equation,

$$h_m L A_1 = h_0 L_0 \quad (4.34)$$

Momentum equation,

$$\frac{d}{dt} (h_m U_m L) A_2 - G \frac{h_m^2}{2} + 3\nu \frac{U_m L}{h_m} A_3 = 0 \quad (4.35)$$

Energy equation,

$$\frac{d}{dt} \left(\frac{1}{2} h_m U_m^2 L \right) A_4 + G \frac{d}{dt} \left(\frac{h_m^2 L}{2} \right) A_5 + 3\nu \frac{U_m^2 L}{h_m} A_6 = 0 \quad (4.36)$$

where, once again A_1 to A_6 are listed as follows,

$$\begin{aligned} A_1 &= \int_0^1 f \, d\xi & A_2 &= \int_0^1 f g \, d\xi & A_3 &= \int_0^1 \frac{g}{f} \, d\xi \\ A_4 &= \int_0^1 f g^2 \, d\xi & A_5 &= \int_0^1 f^2 \, d\xi & A_6 &= \int_0^1 \frac{g^2}{f} \, d\xi \end{aligned} \quad (4.37)$$

By using Eq.(4.34) , Eq.(4.35) and Eq.(4.36), h_m , U_m , and L can be solved using Euler method as follows,

From Eq.(4.34), we have

$$h_m L = \frac{h_0 L_0}{A_1} = \frac{\mathcal{V}}{A_1} \quad \text{where } \mathcal{V} = h_0 L_0 \quad (4.38)$$

By substituting Eq.(4.38) in the momentum equation, Eq.(4.35), we have,

$$\begin{aligned} \frac{d}{dt} (h_m L \cdot U_m) A_2 - G \frac{h_m^2}{2} + 3\nu \frac{U_m L}{h_m} A_3 &= 0 \\ \frac{d}{dt} \left(\frac{\mathcal{V}}{A_1} \cdot U_m \right) A_2 - G \frac{h_m^2}{2} + 3\nu \frac{U_m L}{h_m} A_3 &= 0 \\ \frac{\mathcal{V} A_2}{A_1} \frac{dU_m}{dt} - G \frac{h_m^2}{2} + 3\nu \frac{U_m L}{h_m} A_3 &= 0 \end{aligned} \quad (4.39)$$

$$\therefore \frac{dU_m}{dt} = \frac{A_1}{A_2 \mathcal{V}} \left(G \frac{h_m^2}{2} - 3\nu A_3 \frac{U_m L}{h_m} \right) \quad (4.40)$$

By using Eq.(4.38) and Eq.(4.40), the expression for $\frac{dh_m}{dt}$ is obtained from the energy equation, Eq.(4.36) as follows,

$$\begin{aligned} \frac{d}{dt} \left(\frac{1}{2} h_m L \cdot U_m^2 \right) A_4 + G \frac{d}{dt} \left(\frac{h_m L \cdot h_m}{2} \right) A_5 + 3v \frac{U_m^2 L}{h_m} A_6 &= 0 \\ \frac{d}{dt} \left(\frac{1}{2} \frac{V}{A_1} \cdot U_m^2 \right) A_4 + G \frac{d}{dt} \left(\frac{V}{2A_1} \cdot h_m \right) A_5 + 3v \frac{U_m^2 L}{h_m} A_6 &= 0 \\ \frac{VA_4}{2A_1} \frac{dU_m^2}{dt} + G \frac{VA_5}{2A_1} \frac{dh_m}{dt} + 3v \frac{U_m^2 L}{h_m} A_6 &= 0 \\ \frac{VA_4}{A_1} U_m \frac{dU_m}{dt} + G \frac{VA_5}{2A_1} \frac{dh_m}{dt} + 3v \frac{U_m^2 L}{h_m} A_6 &= 0 \end{aligned} \quad (4.41)$$

$$\therefore \frac{dh_m}{dt} = -\frac{2A_1}{VA_5G} \left[\frac{VA_4}{A_1} U_m \frac{dU_m}{dt} + 3v \frac{U_m^2 L}{h_m} A_6 \right] \quad (4.42)$$

If $[h_m]^n$, $[U_m]^n$ and $[L]^n$ are the values of h_m , U_m and L at time step n , then value at time step $n+1$ can be calculated from Eq.(4.38), Eq.(4.40) and Eq.(4.42) as follows,

From Eq.(4.40),

$$\left[\frac{dU_m}{dt} \right]^n = \frac{A_1}{A_2 V} \left\{ \frac{G}{2} [h_m^2]^n - 3vA_3 \left[\frac{U_m L}{h_m} \right]^n \right\} \quad (4.43)$$

$$[U_m]^{n+1} = [U_m]^n + dt \frac{A_1}{A_2 V} \left\{ \frac{G}{2} [h_m^2]^n - 3vA_3 \left[\frac{U_m L}{h_m} \right]^n \right\} \quad (4.44)$$

From Eq.(4.42),

$$[h_m]^{n+1} = [h_m]^n - dt \frac{2A_1}{VA_5G} \left\{ \frac{VA_4}{A_1} [U_m]^n \left[\frac{dU_m}{dt} \right]^n + 3v \left[\frac{U_m^2 L}{h_m} \right]^n A_6 \right\} = 0 \quad (4.45)$$

From Eq.(4.38),

$$[L]^{n+1} = \frac{V}{A_1 [h_m]^{n+1}} \quad (4.46)$$

where dt is the time increment in each n steps.

4.4 Determination of shape functions

4.4.1 Shape function of inviscid fluid

In the case of inviscid fluid, the solution from Chapter 1 for the flow profile h_2 and flow velocity U , Eq.(2.143) and Eq.(2.114) are considered for the determination of shape function f and g . The profile h_2 and U are once again listed here as follows,

$$h_2(t, \xi) = \frac{5}{8} h_o \left(\frac{t}{t_o} \right)^{-1} + \frac{3}{8} h_o \left(\frac{t}{t_o} \right)^{-2} + \left[h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - \frac{5}{8} h_o \left(\frac{t}{t_o} \right)^{-1} - \frac{3}{8} h_o \left(\frac{t}{t_o} \right)^{-2} \right] \xi^4 \quad (4.47)$$

$$U(t, \xi) = \left[2c_o - 3c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} + c_o \left(\frac{t}{t_o} \right)^{-1} \right] \xi + \left[c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} - c_o \left(\frac{t}{t_o} \right)^{-1} \right] \xi^3 \quad (4.48)$$

Both the above equations are rearranged into the form of $h = h_m(t)f(\xi)$ and $U = U_m(t)g(\xi)$,

$$h_2(t, \xi) = \left[\frac{5}{8}h_o \left(\frac{t}{t_o} \right)^{-1} + \frac{3}{8}h_o \left(\frac{t}{t_o} \right)^{-2} \right] \left\{ 1 + \frac{h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - \frac{5}{8}h_o \left(\frac{t}{t_o} \right)^{-1} - \frac{3}{8}h_o \left(\frac{t}{t_o} \right)^{-2}}{\frac{5}{8}h_o \left(\frac{t}{t_o} \right)^{-1} + \frac{3}{8}h_o \left(\frac{t}{t_o} \right)^{-2}} \xi^4 \right\} \quad (4.49)$$

$$U(t, \xi) = \left[2c_o - 3c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} + c_o \left(\frac{t}{t_o} \right)^{-1} \right] \left\{ \xi + \frac{c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} - c_o \left(\frac{t}{t_o} \right)^{-1}}{2c_o - 3c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} + c_o \left(\frac{t}{t_o} \right)^{-1}} \xi^3 \right\} \quad (4.50)$$

From Eq.(4.49), the shape function $f(\xi)$ is approximated as,

$$f(\xi) = 1 + \frac{h_o \left(\frac{t}{t_o} \right)^{-\frac{4}{3}} - \frac{5}{8}h_o \left(\frac{t}{t_o} \right)^{-1} - \frac{3}{8}h_o \left(\frac{t}{t_o} \right)^{-2}}{\frac{5}{8}h_o \left(\frac{t}{t_o} \right)^{-1} + \frac{3}{8}h_o \left(\frac{t}{t_o} \right)^{-2}} \xi^4 \quad (4.51)$$

From Eq.(4.50), $g(\xi)$ is approximated as follows,

$$g(\xi) = \xi + \frac{c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} - c_o \left(\frac{t}{t_o} \right)^{-1}}{2c_o - 3c_o \left(\frac{t}{t_o} \right)^{-\frac{2}{3}} + c_o \left(\frac{t}{t_o} \right)^{-1}} \xi^3 \quad (4.52)$$

When $t \rightarrow \infty$, $g(x) \rightarrow \xi$, therefore for simplicity, $g(\xi)$ is approximated as,

$$g(\xi) = \xi \quad (4.53)$$

4.4.2 Shape function for viscous fluid

In Chapter 2, Section 3.3, the similarity solution for viscous Newtonian fluids are derived. In the viscous phase flow, it is assumed that pressure force and viscous force are in balance. Therefore, based on momentum equation on Eq.3.31 in Chapter 3 in Section 3.3, the pressure and viscous force in equilibrium can be written as follows,

$$g \frac{h_m^2}{L} p(\xi) \frac{\partial p(\xi)}{\partial \xi} = -3\nu \frac{V_m q(\xi)}{h_m p(\xi)} \quad (4.54)$$

The flow depth and velocity in the viscous phase flow which can be expressed in similarity form, are once again listed here for convenience as follows,

$$h_m = \alpha h_o \left(\sqrt{\frac{g}{h_o}} t \right)^a \quad (4.55)$$

$$V_m = \beta \sqrt{gh_o} \left(\sqrt{\frac{g}{h_o}} t \right)^b \quad (4.56)$$

$$L = \gamma L_o \left(\sqrt{\frac{g}{h_o}} t \right)^c \quad (4.57)$$

where h_o is the characteristic depth and L_o is the characteristic length. By substituting the similarity form of h_m , V_m and L into Eq.(4.54), and with, $a = -1/5$, $b = -4/5$ and $c = 1/5$ for viscous

Newtonian fluid, the t terms on both side are canceled off. The pressure-viscous equilibrium is therefore reduced to the following form,

$$\frac{\alpha^2 g h_o^2}{\gamma L_o} p(\xi) \frac{dp(\xi)}{d\xi} = -3\nu \frac{\beta}{\alpha} \sqrt{\frac{g}{h_o}} \frac{q(\xi)}{p(\xi)} \quad (4.58)$$

By rearranging the coefficients, and writing $p(\xi)$ as p and $q(\xi)$ as q , we can simply Eq.(4.58) as follows,

$$\begin{aligned} A p \frac{dp}{d\xi} &= -\frac{p}{q} \\ \frac{A}{3} \frac{d}{d\xi} (p^3) &= -q \end{aligned} \quad (4.59)$$

where A is

$$A = \frac{\alpha^3}{3\beta\gamma} \cdot \frac{h_o \sqrt{gh_o}}{\nu} \cdot \frac{h_o}{L_o} \quad (4.60)$$

Similarly, the continuity equation as in Eq.3.28 from Chapter 3, Section 3.3 is rewritten here for convenience.

$$\frac{\partial}{\partial t} [h_m(t) p(\xi)] + \frac{\partial}{\partial x} [h_m(t) p(\xi) V_m(t) q(\xi)] = 0 \quad (4.61)$$

By substituting the similarity form of h_m , V_m and L into Eq.(4.61), and canceling off the t terms with $a = -1/5$, $b = -4/5$ and $c = 1/5$ for viscous Newtonian fluid, the continuity equation is reduced to the following form,

$$\begin{aligned} -\frac{1}{5} \sqrt{\frac{g}{h_o}} p - \frac{1}{5} \xi \frac{dp}{d\xi} \sqrt{\frac{g}{h_o}} + \frac{\beta}{\gamma L_o} \sqrt{gh_o} q \frac{dp}{d\xi} + \frac{\beta}{\gamma L_o} \sqrt{gh_o} p \frac{dq}{d\xi} &= 0 \\ -\frac{1}{5} \sqrt{\frac{g}{h_o}} \left[p + \xi \frac{dp}{d\xi} \right] + \frac{\beta}{\gamma L_o} \sqrt{gh_o} \left[q \frac{dp}{d\xi} + p \frac{dq}{d\xi} \right] &= 0 \\ -\frac{1}{5} \sqrt{\frac{g}{h_o}} \left[\frac{d}{d\xi} (p\xi) \right] + \frac{\beta}{\gamma L_o} \left[\frac{d(pq)}{d\xi} \right] &= 0 \end{aligned} \quad (4.62)$$

Integrating Eq. (4.62) yields,

$$\begin{aligned} \int -\frac{1}{5} \sqrt{\frac{g}{h_o}} \left[\frac{d}{d\xi} (p\xi) \right] + \frac{\beta}{\gamma L_o} \left[\frac{d(pq)}{d\xi} \right] d\xi &= 0 \\ -\frac{1}{5} \sqrt{\frac{g}{h_o}} \cdot p\xi + \frac{\beta}{\gamma L_o} \sqrt{gh_o} pq + C &= 0 \end{aligned} \quad (4.63)$$

where C is a constant. Since at $\xi = 1$, $p = 0$, therefore, $C = 0$. Eq. (4.63) is further reduced to the following form,

$$\begin{aligned} -\frac{1}{5} \sqrt{\frac{g}{h_o}} \cdot p\xi + \frac{\beta}{\gamma L_o} \sqrt{gh_o} pq &= 0 \\ q &= E\xi \quad \text{where} \quad E = \frac{1}{5} \frac{\gamma L_o}{\beta h_o} \end{aligned} \quad (4.64)$$

By substituting Eq.(4.64) into Eq. (4.59), and integrate it from $\xi = 0$ to ξ , the shape function p can be determined as follows,

$$\begin{aligned}
 \frac{A}{3} \frac{d}{d\xi} (p^3) &= -q \\
 \int_0^\xi \frac{A}{3} \frac{d}{d\xi} (p^3) d\xi &= - \int_0^\xi E \xi d\xi \\
 \frac{A}{3} [p^3]_0^\xi &= -E \left[\frac{\xi^2}{2} \right]_0^\xi \\
 \frac{A}{3} [p^3(\xi) - p^3(0)] &= -\frac{E}{2} [\xi^2 - 0] \\
 p(\xi) &= \sqrt[3]{1 - \frac{3E}{2A} \xi^2}
 \end{aligned} \tag{4.65}$$

For simplicity, the value of A and E are determined from Eq.(4.60) and Eq. (4.64) by setting $\alpha = 1$, $\beta = 1$ and $\gamma = 1$.

4.5 Results and discussion

The solution from the integral model are plotted in the form of propagation of front wave L and the attenuation of depth at the origin h_m with the initial condition set as follows,

$$\begin{aligned}
 h_m &= h_o = 0.5m \\
 L &= L_o = 0.5m \\
 U_m &= 0.0ms^{-1}
 \end{aligned} \tag{4.66}$$

In the case of inviscid fluid, the shape function derived in Eq.(4.51) and Eq. (4.53) are used. The results are plotted in Fig. 4.3. The agreement between the results from the integral model and the characteristic of inertial phase flow is satisfactory, where the front wave is propagating proportionally with time $L \propto t$ and the depth at the origin is attenuating inversely proportional with time $h_m \propto t^{-1}$. In the case of viscous fluid, the shape function for viscous phase flow derived in Eq (4.65) and Eq. (4.64) are used. The results for viscous Newtonian fluid are plotted for different cases of kinematic viscosity, $\nu = 0.00005m^2s^{-1}$, $\nu = 0.0005m^2s^{-1}$, $\nu = 0.005m^2s^{-1}$ and $\nu = 0.05m^2s^{-1}$ in Fig. 4.5 , Fig. 4.7, Fig. 4.9 and Fig. 4.11 respectively. It can be seen that the characteristics of viscous phase flow is well-reproduced by the result from the integral model. In the initial stage of the flow, especially in low kinematic viscosity case in Fig. 4.5 and Fig. 4.7 the inertial phase flow can be re-produced as well, even though the shape functions based on the viscous phase flow are used. However, the viscous phase is not very clear in these low kinematic viscosity case. The dominance of viscous phase flow is clearer in the case of high kinematic viscosity case as in Fig. 4.9 and Fig. 4.11. In case $\nu = 0.05m^2s^{-1}$, it can be seen from Fig. 4.11 that for high viscosity fluid, the flow enters a short transition phase without entering the inertial flow phase before exhibiting the dominant viscous flow phase characteristic.

4.6 Summary

In this chapter, the characteristics of inertial and viscous phase flow are re-produced using the integral method to transform the partial different equations of the governing equations into ordinary differential equations which can be easily solved with initial conditions. However, the shape function for the flow profile and velocity are estimated based on the study in Chapter 2 for inertial phase flow and in Chapter 3 for viscous phase flow. Although a general shape function representing both inertial and viscous phase flow was not used, the results for viscous fluid are capable of showing the existence of both inertial and viscous phase flow. The viscous phase flow characteristic is not clear in the case of low kinematic viscosity phase, while the inertial phase flow characteristic is less dominant in high kinematic viscosity case. The results from the integral model also provides and insight to the transition of inertial phase flow to viscous phase flow in the case of viscous fluid.

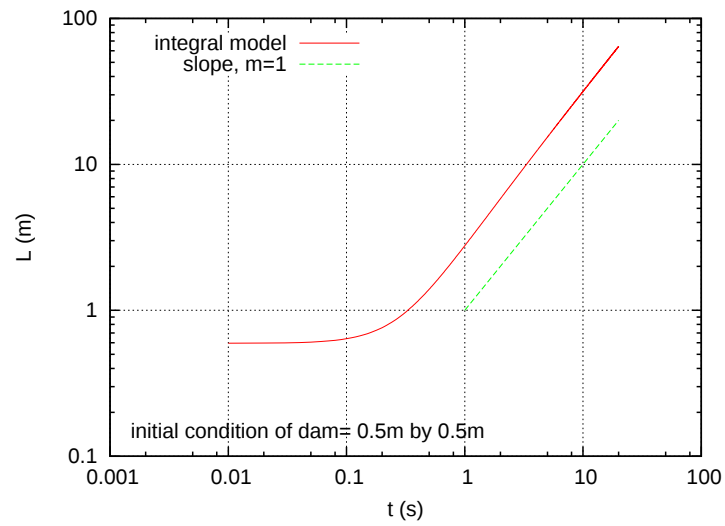


Figure 4.2: Temporal variation of front wave L using solution of integral model in the case of inviscid fluid

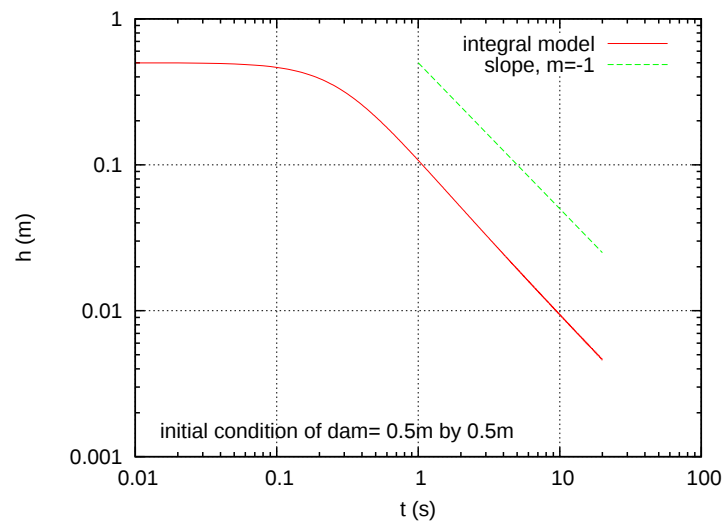


Figure 4.3: Temporal variation of depth at the origin h_m using solution of integral model in the case of inviscid fluid

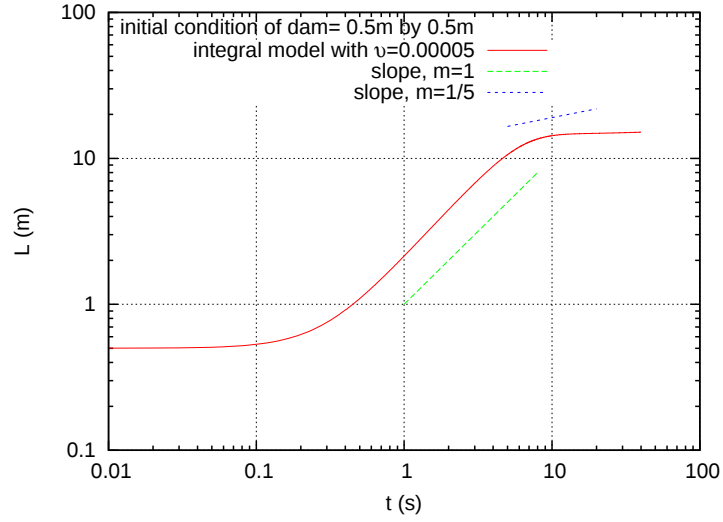


Figure 4.4: Temporal variation of front wave L using solution of integral model in the case of viscous fluid with kinematic viscosity $\nu = 0.00005 \text{ m}^2 \text{ s}^{-1}$

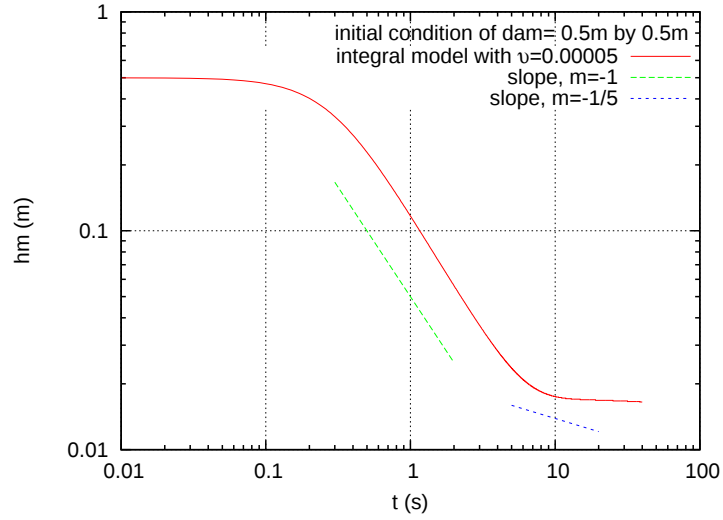


Figure 4.5: Temporal variation of depth at the origin h_m using solution of integral model in the case of viscous fluid with kinematic viscosity $\nu = 0.00005 \text{ m}^2 \text{ s}^{-1}$

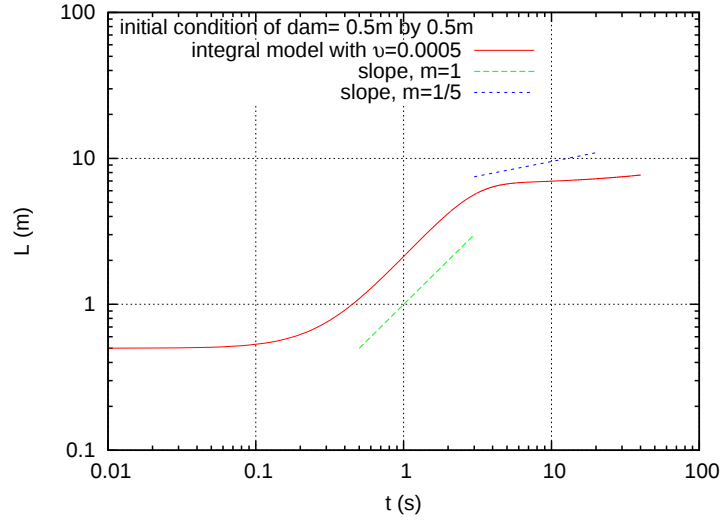


Figure 4.6: Temporal variation of front wave L using solution of integral model in the case of viscous fluid with kinematic viscosity $\nu = 0.0005 \text{ m}^2 \text{ s}^{-1}$

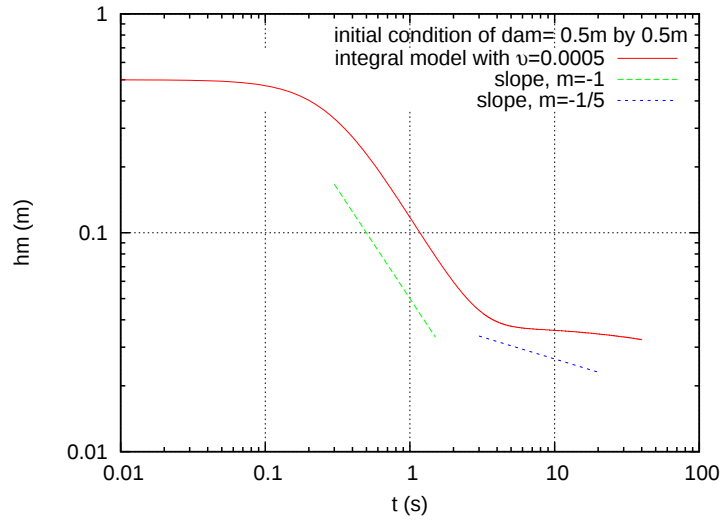


Figure 4.7: Temporal variation of depth at the origin h_m using solution of integral model in the case of viscous fluid with kinematic viscosity $\nu = 0.0005 \text{ m}^2 \text{ s}^{-1}$

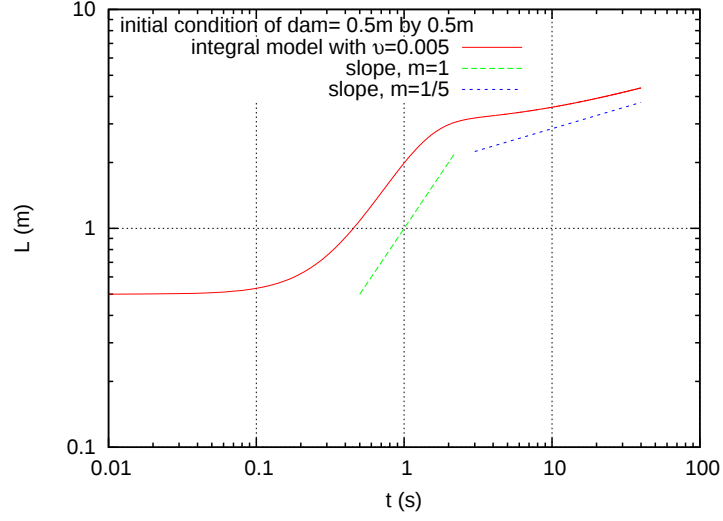


Figure 4.8: Temporal variation of front wave L using solution of integral model in the case of viscous fluid with kinematic viscosity $\nu = 0.005 \text{ m}^2 \text{ s}^{-1}$

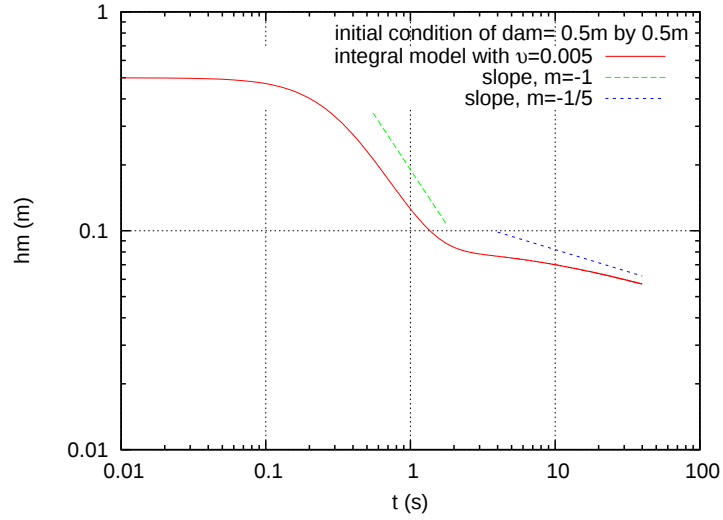


Figure 4.9: Temporal variation of depth at the origin h_m using solution of integral model in the case of viscous fluid with kinematic viscosity $\nu = 0.005 \text{ m}^2 \text{ s}^{-1}$

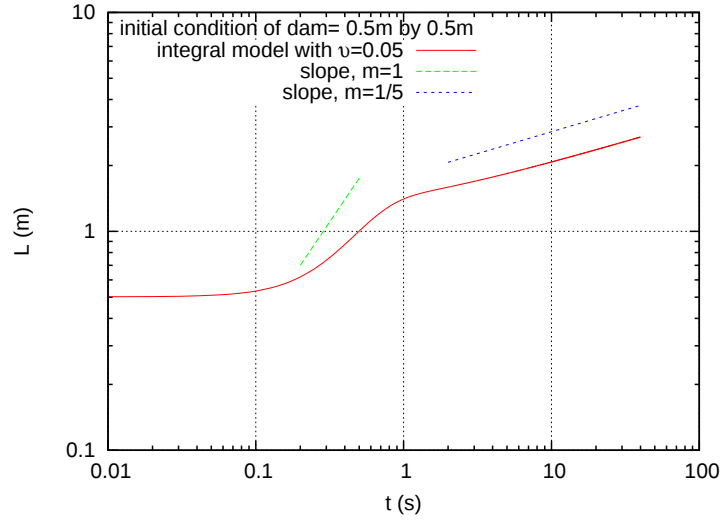


Figure 4.10: Temporal variation of front wave L using solution of integral model in the case of viscous fluid with kinematic viscosity $\nu = 0.05 \text{ m}^2 \text{ s}^{-1}$

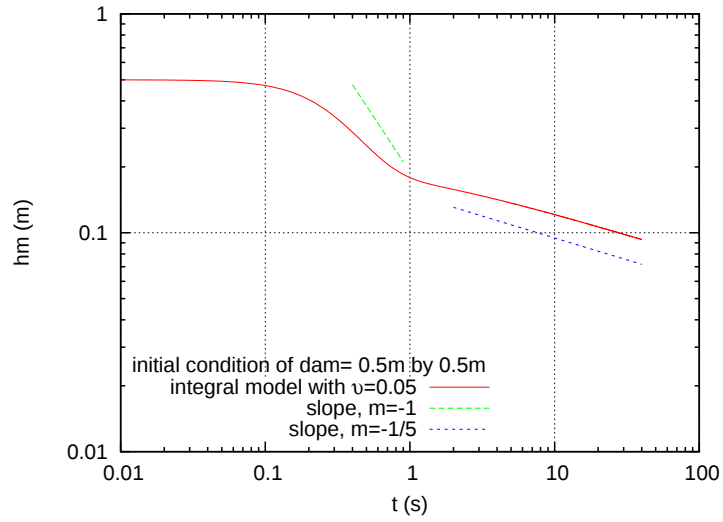


Figure 4.11: Temporal variation of depth at the origin h_m using solution of integral model in the case of viscous fluid with kinematic viscosity $\nu = 0.05 \text{ m}^2 \text{ s}^{-1}$

Chapter 5

Development of Numerical Model

5.1 Preliminaries

The earliest successful method to treat hydrodynamic problems is the marker-and-cell (MAC) method, which employed a distribution of marker particles to define fluid regions (Harlow and Welch, 1966). Surface is defined at the boundary between region with and without particles. Another method proposed by Hirt and Welch for free-surface flow problem is the Volume of Fluid method (VOF) (Hirt and Nicholas, 1981). In the VOF method, instead of using marker particles to define fluid region, a volume of fluid function, F is introduced. In cell without fluid, F is defined as zero, while in a fully filled cell, F is defined as unity. Cells having value between zero and unity are defined as semi-full cells where free surface exists.

This chapter presents the development numerical model which will be used to simulate the slump flow of fresh concrete in Chapter 6 and the phenomenon of abrupt expansion flow in Chapter 7. In the case of the simulation of slump flow test, the numerical model will be a two-dimensional model, set in a cylindrical coordinate system. On the other hand, the simulation of abrupt expansion flow will be in three dimensions and set in Cartesian coordinate system. Both models are free-surface models.

The models used in both simulations mentioned above will be based on the VOF method, with several improvements and modifications. Especially worth to mention is the solution of the advection terms in the governing equations using a higher order scheme CIP (Cubic-Interpolated Propagation) (Yabe and Takizawa, 2005). CIP method is also used to solve the advection of the volume of fluid function in both models.

5.2 Cubic Interpolated Propagation (CIP) Scheme

The CIP scheme is a higher-order and less diffusive scheme used to solve the hyperbolic equation of the following form (in the case of one dimensional) Yabe and Aoki (1991a).

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0 \quad (5.1)$$

In the case where u is constant, the advection of f in Eq. (5.1) can be represented by the trans-

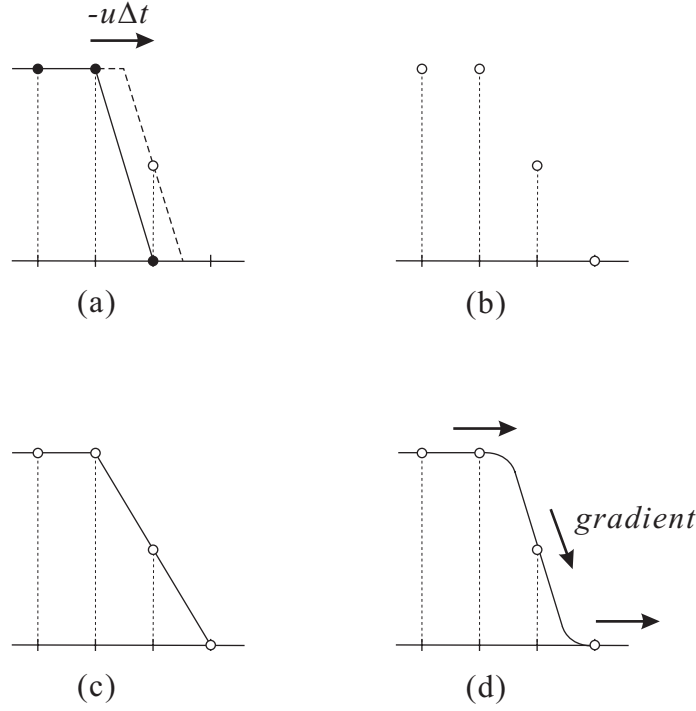


Figure 5.1: (a) initial profile with dash-line as exact solution after advection, (b) exact solution at each points of discretization, (c) interpolation of solution at new time step with linear line (upwind scheme), (c) interpolation of solution at new time step with CIP scheme

lation of solid line in Fig.5.1 (a) with velocity u in x -direction where the exact solution at time $t + \Delta t$, is represented by dash-line. If the profile of the exact solution of f at time $t + \Delta t$ in Fig.5.1 (a) is removed as in Fig.5.1 (b), the sub-grid information (the profile) is lost. If a typical first order upwind scheme is used, a linear line will be used to interpolate the profile of the solution at time $t + \Delta t$ as shown in Fig.5.1 (c). It can be seen that due to the loss of sub-grid information (the profile), numerical diffusion occurs. However, in CIP scheme, the spatial gradient of the profile is taken into account. For example, by differentiating Eq. (5.1) with x , the spatial gradient $g = \partial f / \partial x$ can be expressed as follows,

$$\frac{\partial g}{\partial t} + u \frac{\partial g}{\partial x} = - \frac{\partial u}{\partial x} g \quad (5.2)$$

In the case of constant velocity u , the term $\partial u / \partial x = 0$ and Eq. (5.2) is of the same form as Eq. (5.1) and represents the advection of the spatial derivative with velocity u which is represented by the arrow in Fig.5.1 (d). Therefore, by this constraint, the profile during the advection of f is maintained, thus reducing numerical diffusion during the advection of f . In the CIP scheme, the profile between two discretization points is approximated by a cubic polynomial as follows, (Yabe and Aoki, 1991a)

$$F(x) = ax^3 + bx^2 + cx + d \quad (5.3)$$

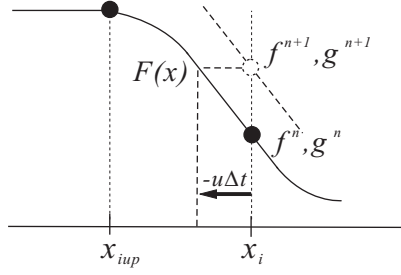


Figure 5.2: New value of f and g are obtained by shifting the profile by $-u\Delta t$.

In the case of constant velocity u , the new values of f and g at time step $t + \Delta t$ defined as f^{n+1} and g^{n+1} are obtained by shifting the profile by $u\Delta t$ as follows,

$$\begin{aligned} f^{n+1} &= F(x - u\Delta t) \\ g^{n+1} &= \frac{dF(x - u\Delta t)}{dx} \end{aligned} \quad (5.4)$$

Fig.5.2 shows the shifting of the profile to obtained the values of f^{n+1} and g^{n+1} . By utilizing the value of g and f at two discretization points, Yabe and Aoki (1991a) gives the solutions for polynomial in Eq. (5.3) as follows,

$$\begin{aligned} f_i^{n+1} &= a_i \xi_i^3 + b_i \xi_i^2 + g_i^n \xi_i + f_i^n \\ g_i^{n+1} &= 3a_i \xi_i^2 + 2b_i \xi_i + g_i^n \\ a_i &= \frac{g_i + g_{iup}}{\Delta x_i^2} + \frac{2(f_i - f_{iup})}{\Delta x_i^3} \\ b_i &= \frac{(f_{iup} - f_i)}{\Delta x_i^2} - \frac{2g_i + g_{iup}}{\Delta x_i} \\ \Delta x_i &= x_{iup} - x_i \quad iup = i - \text{sgn}(u_i) \end{aligned} \quad (5.5)$$

where $\text{sgn}(u)$ stands for the sign of u .

In the presence of non-advection term H such as follows,

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = H \quad (5.6)$$

$$\frac{\partial g}{\partial t} + u \frac{\partial g}{\partial x} = -\frac{\partial u}{\partial x} + \frac{\partial H}{\partial x} \quad (5.7)$$

Yabe and Aoki (1991a) proposed time-splitting technique, where the equations are solved in two phases : advection and non-advection phase. The time-splitting technique to solve Eq. (5.7) is as follows:

Non-advection Phase

The non-advection term is solved as follows,

$$f_i^* = f_i^n + H_i \Delta t \quad (5.8)$$

$$\frac{g_i^* - g_i^n}{\Delta t} = \frac{f_{i+1}^* - f_{i-1}^*}{2\Delta x \Delta t} - \frac{f_{i+1}^n - f_{i-1}^n}{2\Delta x \Delta t} - \left(\frac{\partial u}{\partial x} \right)_i g_i^n \quad (5.9)$$

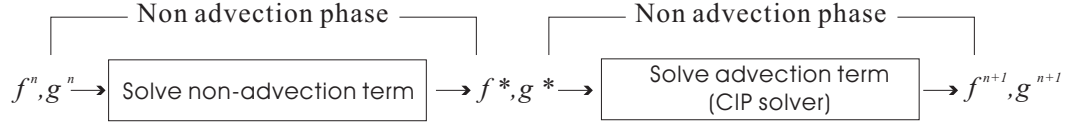


Figure 5.3: Time-splitting method to solve non-linear hyperbolic equation with CIP scheme

where n represents calculation step at time t , and the sign $*$ means the time after one time step $t + \Delta t$ in the non-advection phase. If the non-advection term H contains diffusion term and must be solved implicitly, the implicit solver can be applied at this phase.

Advection Phase

After f and g are advanced in the non-advection phase, the CIP solver in Eq. (5.5) is applied to the advection phase. The value f_i^* and g_i^* calculated from Eq. (5.9) are used in the CIP solver to obtain the value at calculation step $n + 1$.

$$f_i^{n+1} = a_i \xi_i^3 + b_i \xi_i^2 + g_i^n \xi_i + f_i^* \quad (5.10)$$

$$g_i^{n+1} = 3a_i \xi_i^2 + 2b_i \xi_i + g_i^* \quad (5.11)$$

A one time step calculation using the time-splitting method described here in show in Fig. 5.3.

The two-dimensional and three-dimensional hyperbolic equations such as in Eq. (5.12) and Eq. (5.13) can be solved directly using the two-dimensional and three-dimensional CIP solvers respectively (in the case where non-advection term H is zero). The two-dimensional and three-dimensional CIP solver are provided in Appendix A and Appendix B.

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} = H \quad (5.12)$$

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + w \frac{\partial f}{\partial z} = H \quad (5.13)$$

In the case where non-advection term H present in the two-dimensional or three-dimensional hyperbolic equation, the same procedure of time-splitting method for one-dimensional case explained earlier is applicable.

5.3 Numerical models

A two-dimensional in cylindrical coordinate system and a three dimensional numerical model in Cartesian coordinate system are developed in this dissertation. Both model are based on the free-surface Volume of Fluid method by Hirt and Nicholas (1981). The advection term in the momentum equations and the advection of volume of fluid function F are solved using the CIP scheme. Therefore, hereafter the two-dimensional model and three-dimensional model are referred as the 2D VOF-CIP and the 3D VOF-CIP model respectively.

5.3.1 Two-dimensional numerical model (2D VOF-CIP)

The governing equation used in the two-dimensional numerical model are the Navier-Stokes equations,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial r} + g_r + \nu \left[\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial y^2} + \left(\frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) \right] \quad (5.14)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + g_y + \nu \left[\frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{r} \frac{\partial v}{\partial r} \right] \quad (5.15)$$

where (u, v) are velocity in (r, y) direction in cylindrical coordinate. Body acceleration are denoted by (g_x, g_y) . ν is the kinematic viscosity, ρ is the fluid density and pressure is denoted by P . The two-dimensional numerical model is used to simulate the slump flow of fresh concrete in Chapter 6, and therefore by assuming incompressibility of the fresh concrete, the following incompressibility condition must be fulfilled,

$$\frac{\partial u}{\partial r} + \frac{\partial v}{\partial y} + \frac{u}{r} = 0 \quad (5.16)$$

Solving non-advection phase

Eq.(5.14) and Eq.(5.15) are solved with time-splitting method explained in Section 5.2, where CIP method is used to solve the advection term in the equations. The non-advection terms of Eq.(5.14) and Eq.(5.15) are discretized as follows,

$$u_{i,j}^* = u_{i,j}^n + \Delta t \left[-\left(p_{i,j}^{n+1} - p_{i-1,j}^{n+1} \right) / \Delta x + g_x + VISX \right] \quad (5.17)$$

$$v_{i,j}^* = v_{i,j}^n + \Delta t \left[-\left(p_{i,j}^{n+1} - p_{i,j-1}^{n+1} \right) / \Delta y + g_y + VISY \right] \quad (5.18)$$

where $u_{i,j}^*$ and $v_{i,j}^*$ are the value of $u_{i,j}$ and $v_{i,j}$ after solving non-advection phase. $VISX$ and $VISY$ are defined as follows,

$$VISX = \nu \left[\frac{u_{i+1,j}^n + u_{i-1,j}^n - 2u_{i,j}^n}{\Delta r^2} + \frac{u_{i,j+1}^n + u_{i,j-1}^n - 2u_{i,j}^n}{\Delta y^2} + \frac{1}{r_{ij}} \frac{u_{i+1,j}^n - u_{i-1,j}^n}{2\Delta r} - \frac{u_{i,j}^n}{r_{ij}^2} \right] \quad (5.19)$$

$$VISY = \nu \left[\frac{v_{i+1,j}^n + v_{i-1,j}^n - 2v_{i,j}^n}{\Delta r^2} + \frac{v_{i,j+1}^n + v_{i,j-1}^n - 2v_{i,j}^n}{\Delta y^2} + \frac{1}{r_{ij}} \frac{v_{i+1,j}^n - v_{i-1,j}^n}{2\Delta r} \right] \quad (5.20)$$

The location of the variables are defined in staggered mesh cell system shown in Fig. 5.4. r_{ij} is the radius measured from the origin and defined at the center of the cell. In this non-advection phase, the pressure is solved implicitly using HSMAC method where the continuity equation in Eq (5.16) satisfies the following condition,

$$D_{ij}^{n+1} = \frac{u_{i+1,j}^{n+1} - u_{i,j}^{n+1}}{\Delta r} + \frac{v_{i,j+1}^{n+1} - v_{i,j}^{n+1}}{\Delta y} + \frac{u_{i+1,j}^{n+1} + u_{i,j}^{n+1}}{2r_{ij}} = 0 \quad (5.21)$$

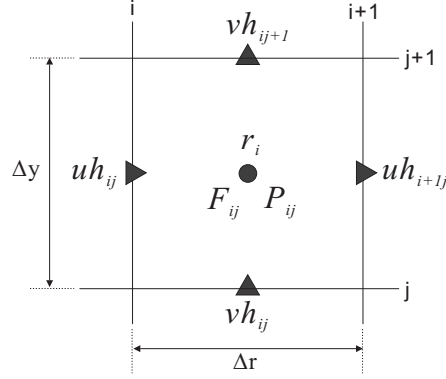


Figure 5.4: Variables defined in staggered cell system

The velocities and pressures are updated in each iterations until condition in Eq. (5.21) is satisfied as follows,

$$\begin{aligned}
 u_{i+1,j}^{k+1} &= u_{i+1,j}^k + \Delta t \delta P^k / \Delta x \\
 u_{i,j}^{k+1} &= u_{i,j}^k - \Delta t \delta P^k / \Delta x \\
 v_{i,j+1}^{k+1} &= v_{i,j+1}^k + \Delta t \delta P^k / \Delta y \\
 v_{i,j}^{k+1} &= v_{i,j}^k - \Delta t \delta P^k / \Delta y
 \end{aligned} \tag{5.22}$$

$$\begin{aligned}
 P_{i,j}^{k+1} &= P_{i,j}^k + \delta P^k \\
 \delta P^k &= \frac{-\omega D_{ij}^k}{2\Delta t \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right)}
 \end{aligned} \tag{5.23}$$

where k is the iteration steps and $\omega = 1.8$ is used in the numerical model.

Solving advection phase

After non-advection phase is solved, the value $u_{i,j}^*$ and $v_{i,j}^*$ are obtained. These values will be used to solve the advection phase using the two-dimensional CIP solver. The two-dimensional CIP solver is provided in Appendix A. The value of $u_{i,j}^{n+1}$ and $v_{i,j}^{n+1}$ are obtained after completing the non-advection and advection phases. The algorithm to solve the governing equations of Eq.5.14 and Eq.5.15 is shown in Fig. 5.5

Advection of volume of fluid function, F

The volume of fluid function, F in the 2D VOF-CIP set in cylindrical coordinate is governed by the following equations,

$$\frac{\partial F}{\partial t} + \frac{1}{r} \frac{\partial (ruF)}{\partial r} + \frac{\partial (vF)}{\partial y} = 0 \tag{5.24}$$

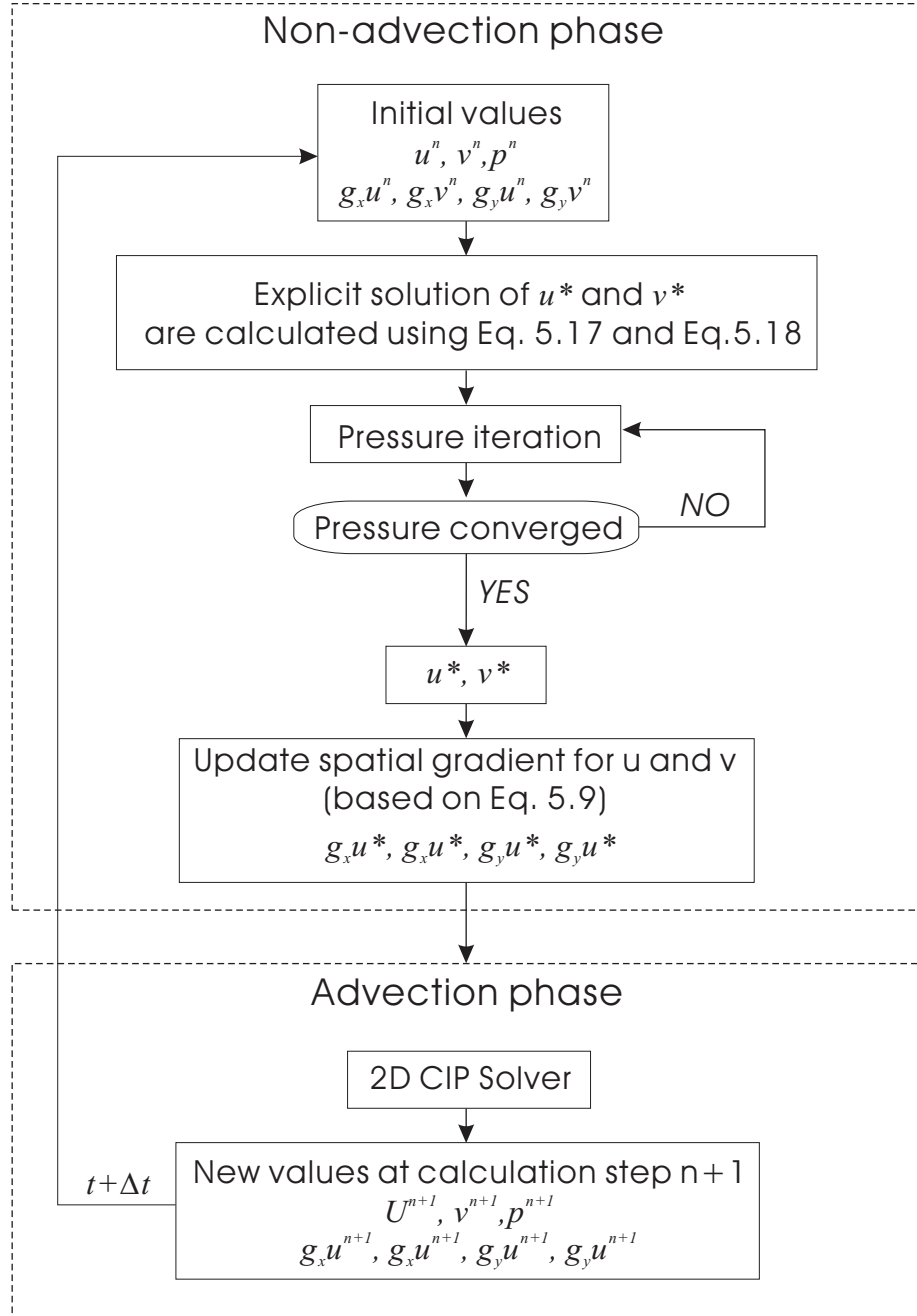


Figure 5.5: Algorithm of VOF-CIP numerical model

Eq.5.24 is re-arranged into the non-conservative form as follows,

$$\frac{\partial F}{\partial t} + u \frac{\partial F}{\partial r} + v \frac{\partial F}{\partial y} = -F \left(\frac{1}{r} \frac{(ru)}{\partial r} + \frac{\partial v}{\partial y} \right) \quad (5.25)$$

The left hand side of Eq. 5.25 is the advection term and the right hand side is the non-advection term. Therefore, Eq.5.25 can be solved using time-splitting method described in Section 5.2. In order to improve the performance of the advection of F function, especially in maintaining sharp surface, a digitizer function of the following form is used (Yamada et al., 1998),

$$h = \tan[0.85\pi(F - 0.5)] \quad (5.26)$$

Instead of directly using F value, h value is used in the CIP solver, and the new value of h after advection is inversed to obtain the new value of F as follows,

$$F = \frac{1}{0.85\pi} \tan^{-1} h + 0.5 \quad (5.27)$$

Performance of 2D VOF-CIP model

Verification 1 - 2D dam-break flow of finite volume in Cartesian coordinate system

Simulation of dam-break flow of finite extent (finite volume) is carried out to verify the performance of the 2D VOF-CIP model. The performance of the model is compared against the experimental results of release of water column by Martin and Moyce (1952). The initial condition of the dam break is shown in Fig. 5.6 with $L=1.0$ m and $v = 1.0 \times 10^{-6} m^2 s^{-1}$. The 2D VOF-CIP model is also compared against a typical 2D VOF-UPWIND model where upwind scheme is used in the discretization of advection terms in the Navier-Stokes equations. The volume of fluid function F in the 2D VOF-UPWIND is advected using CIP scheme. Both models are compared with the experimental result of the release of water column by Martin and Moyce (1952) in terms of the propagation of front position of the flow Z as shown in Fig. 5.6. In the experimental work by Martin and Moyce (1952), L is set to 2.25 inch. The flow vector profile of the simulation using 2D VOF-CIP model is shown in Fig. 5.8. The volume changes during the simulation using 2D VOF-CIP model is shown in Fig. 5.7, and calculated as follows,

$$Volume\ changes(\%) = \frac{present\ volume - initial\ volume}{initial\ volume} \times 100 \quad (5.28)$$

Verification 2 - 2D dam-break flow of finite volume in cylindrical coordinate system.

The performance of 2D VOF-CIP model is verified with the simulation of dam-break flow of finite extent (finite volume) in cylindrical coordinate as shown in Fig.5.9. By setting slip condition on vertical axis at the origin, the flow profile is compared with analytical solution of axis-symmetry flow of fixed volume viscous fluid. Pattle (1959) gives the approximate solution for the flow profile of axis-symmetry flow from the release of fixed volume viscous fluid as follows valid in the region away from the vicinity of the origin,

$$y(z) = \left(\frac{3}{16} \right)^{\frac{1}{3}} (1 - z^2)^{\frac{1}{3}} \quad (5.29)$$

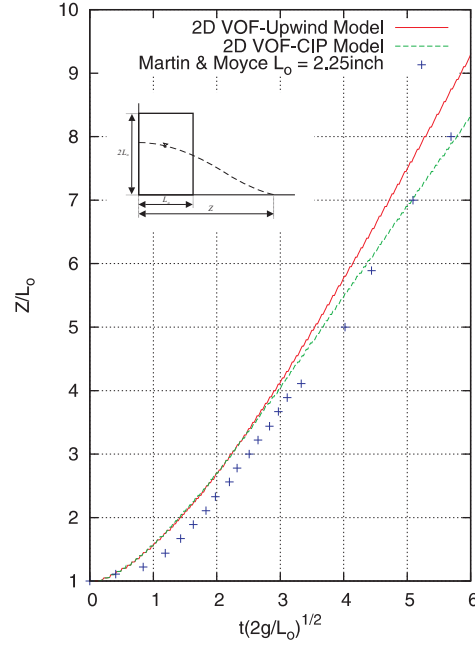


Figure 5.6: Comparison of propagation of front position of flow between 2D VOF-CIP, 2D VOF-UPWIND and experimental results by Martin and Moyce (1952).

where $z = \xi/\xi^N$ with ξ and ξ^N defined as follows,

$$\xi = \left(\frac{1}{3} g \frac{\mathbf{V}}{\nu} \right)^{-\frac{1}{8}} r t^{-\frac{1}{8}} \quad \text{and} \quad \xi^N = 0.894 \quad (5.30)$$

where g is gravity acceleration, ν is kinematic viscosity and $\mathbf{V}$ is the volume of the released fluid. Fig.5.9 shows the comparison of the release of fixed volume of viscous fluid with $\nu = 0.01 \text{ m}^2 \text{ s}^{-1}$ with analytical solution by Pattle (1959).

5.3.2 Three-dimensional numerical model (3D VOF-CIP)

The 3D VOF-CIP model is used to simulate the phenomena of abrupt expansion flow in Chapter 7. The governing equations used in the three-dimensional numerical model are the Navier-Stokes equations,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = g_x - \frac{\partial P}{\partial x} + \nu \nabla^2 u \quad (5.31)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = g_y - \frac{\partial P}{\partial y} + \nu \nabla^2 v \quad (5.32)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = g_z - \frac{\partial P}{\partial z} + \nu \nabla^2 w \quad (5.33)$$

The continuity equation used is as follows,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (5.34)$$

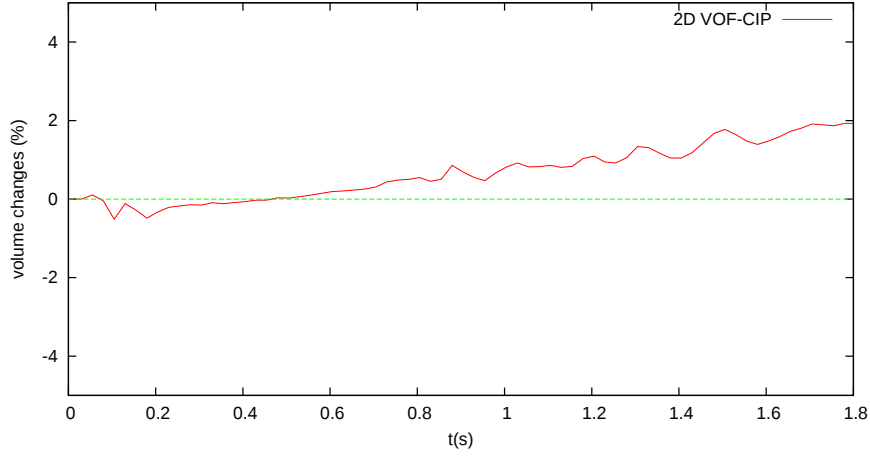


Figure 5.7: Volume changes during simulation of dam break flow with 2D VOF-CIP model

The algorithm used in this model is similar to the algorithm used in 2D VOF-CIP model described in Subsection 5.3.1 with direct extension from two-dimensions to three-dimensions. During the solution of advection phase with CIP scheme, the three-dimensional CIP solver is used. The three-dimensional CIP solver is provided in Appendix B. Similar to the 2D VOF-CIP model, the volume of fluid function F in the 3D VOF-CIP model is also solved using the CIP scheme with digitizer as described in Section 5.3.1. The advection equation of F in the 3D VOF-CIP model is as follows,

$$\frac{\partial F}{\partial t} + \frac{\partial (uF)}{\partial x} + \frac{\partial (vF)}{\partial y} + \frac{\partial (wF)}{\partial z} = 0 \quad (5.35)$$

Performance of 3D VOF-CIP model

The performance of 3D VOF-CIP model is verified by simulating the phenomena of dam-break flow of finite extent. The propagation of front position of the flow is compared with the 3D VOF-UPWIND numerical model and experimental results from the release of water column by Martin and Moyce (1952). The comparison of the 3D VOF-CIP model with the 3D VOF-UPWIND model and the experimental results by Martin and Moyce (1952) is shown in Fig. 5.10, where L is set as 1.0 m and v is set as $1.0 \times 10^{-6} m^2 s^{-1}$ in the 3D VOF-CIP model. In Fig. 5.11, the attenuation of depth at the origin for both numerical models and experimental results is compared. In the experimental work by Martin and Moyce (1952), L is set to 2.25 inch. Volume changes during the simulation is calculated based on Eq.(5.28) and shown in Fig 5.12

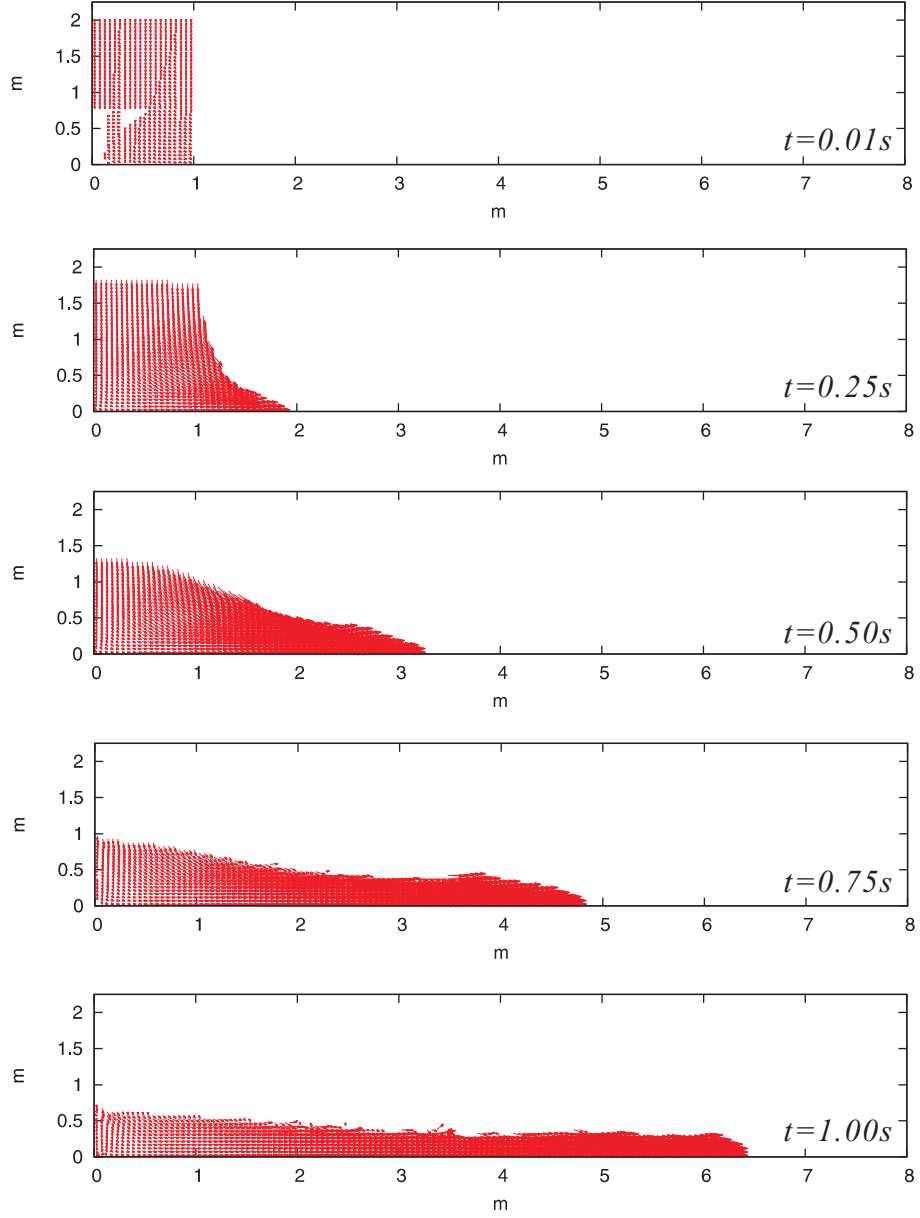


Figure 5.8: Simulation of dam-break flow of finite volume with 2D VOF-CIP model

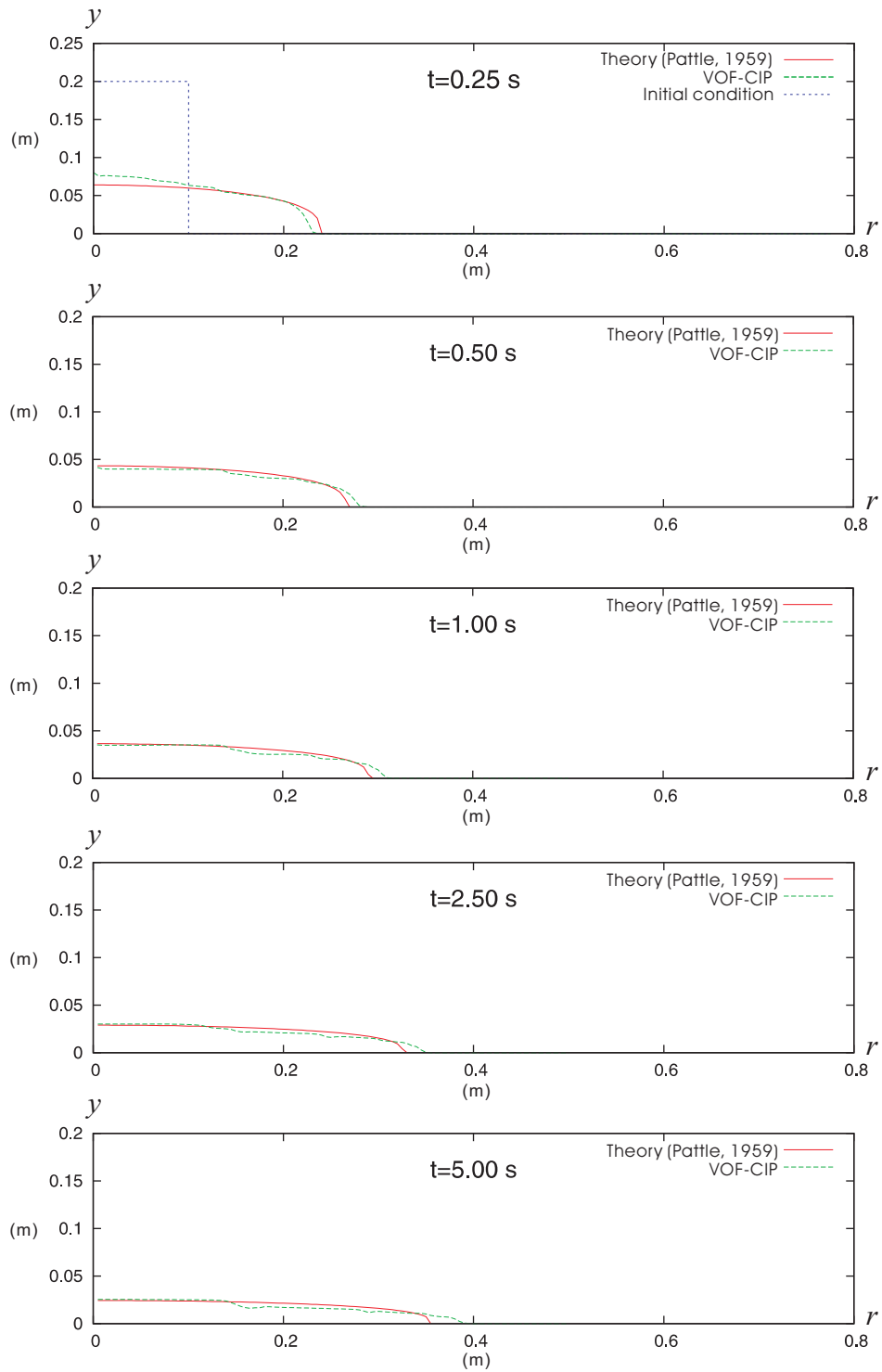


Figure 5.9: Comparison of flow profile of VOF-CIP model versus analytical solution in cylindrical coordinate system.

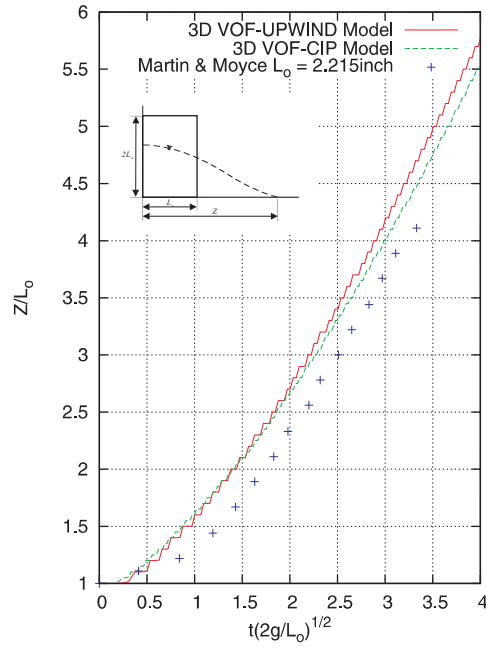


Figure 5.10: Comparison of propagation of front position of flow between 3D VOF-CIP, 3D VOF-UPWIND and experimental results by Martin and Moyce (1952)

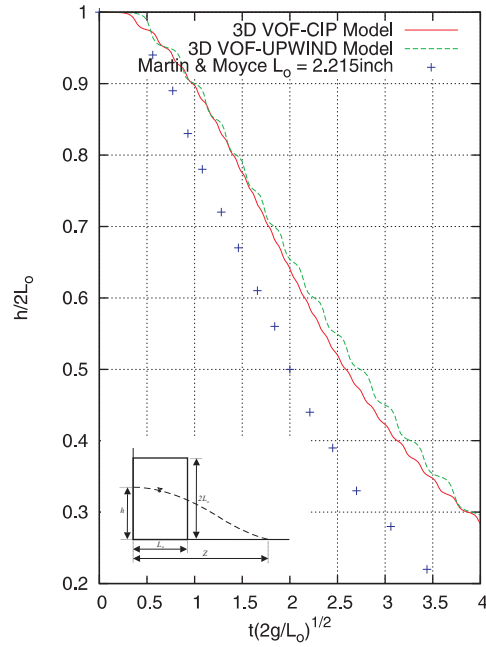


Figure 5.11: Comparison of the attenuation of depth at the origin between 3D VOF-CIP, 3D VOF-UPWIND and experimental results by Martin and Moyce (1952)

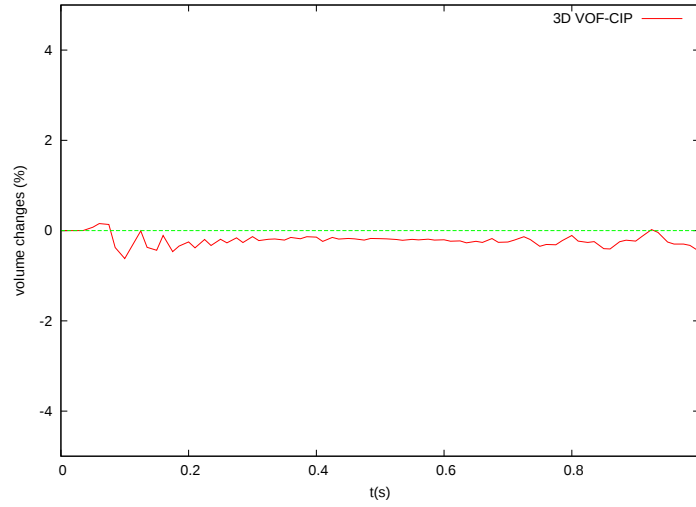


Figure 5.12: Volume changes during simulation of dam break flow with 3D VOF-CIP model

5.4 Conclusion

In this chapter, two numerical models are developed. Both models are based on the Volume of Fluid (VOF) method. A higher order scheme, CIP is used to solve the advection terms in the Navier-Stokes equations. The advection of volume of fluid function F is also solved with CIP scheme. The first model is a two-dimensional model, referred to as the 2D VOF-CIP model. This model is initially set in Cartesian coordinate system to simulate a dam-break flow problem. The performance of this model is verified with the experimental result from the release of water column by Martin and Moyce (1952). The 2D VOF-CIP model is later set in cylindrical coordinate where the release of viscous fluid is simulated in the form of dam-break flow. The flow profile of this simulation is compared with the approximate analytical solution of axis-symmetry flow of viscous fluid by Pattle (1959). The 2D VOF-CIP model performance fairly well in both simulations. The second model developed in this chapter is a three-dimensional model based on the VOF method and using the CIP scheme to solve the advection term in the Navier-Stokes equations and the advection of volume of fluid function F . The model is referred as 3D VOF-CIP model. The model is verified by comparing the result of dam-break flow simulation with the experimental result from the release of water column by Martin and Moyce (1952). The 3D VOF-CIP model performed satisfactorily. The 2D VOF-CIP model is used to simulate the slump flow test of fresh concrete in Chapter 6 while the 3D VOF-CIP model is used to simulate the abrupt expansion flow in Chapter 7.

Chapter 6

Bingham Fluid

6.1 Preliminaries

The application of dam-break flow of finite extent to determine rheological properties can be seen in the work of Ancely and Chochard (2008) and Balmforth and Craster (2001) where the behavior of non-Newtonian viscoplastic fluids are investigated using the model of finite extent dam break flow. Kokado et al. (2001) carried out numerical simulation of the phenomenon of slump flow test based on the rheological properties of high-flow concrete obtained from a series of slump flow tests and sphere drag test by Kokado and Miyagawa (1999). In the numerical simulation, a numerical model based on the Marker and Cell method (MAC) (Harlow and Welch, 1966) is used, and since the slump flow of fresh concrete is axis-symmetric, Hosoda treated the slump flow test as a kind of finite extent dam-break flow in his numerical simulation.

In this chapter, a numerical simulation model is used to simulate the slump flow test of fresh concrete to study the behavior of Bingham fluid. Measurement of rheological coefficients using rotational viscometers and sphere drag viscometers has shown that fresh concrete behavior can be represented by an idealized Bingham fluid model (Tattersall and Banfill, 1983). The numerical model is based on the VOF method by Hirt and Nicholas (1981) with the application of CIP scheme (Yabe and Aoki, 1991b). The propagation of the flow radius of the slump flow simulation is used as parameter to study the characteristics of Bingham fluid in the latter part of this chapter.

6.2 Constitutive relations of bingham fluid

High-flow fresh concrete can be treated or assumed to have properties of a Bingham fluid (Kokado and Miyagawa, 1999). The constitutive relations for Bingham fluid based on the extension by Hohenemser and Prager in arbitrary stress state can be written as in the following relations (Fung, 1994):

$$2\eta_{pl}e_{ij} = \begin{cases} 0 & \text{if } \sqrt{J'_2} \leq \tau_y \\ \left(1 - \frac{\tau_y}{\sqrt{J'_2}}\right) \tau'_{ij} & \text{if } \sqrt{J'_2} > \tau_y \end{cases} \quad (6.1)$$

where, τ'_{ij} is the stress-deviation tensor, η_{pl} is the plastic viscosity e_{ij} is the strain rate tensor, τ_y is the yield value. J'_2 is the second invariant of stress-deviation tensor defined as,

$$J'_2 = \frac{1}{2}(\tau'^2_{rr} + \tau'^2_{\theta\theta} + \tau'^2_{zz}) + \tau'^2_{r\theta} + \tau'^2_{\theta z} + \tau'^2_{zr} \quad (6.2)$$

The second invariant strain tensor I_2 is given by:

$$I_2 = \frac{1}{2}(e^2_{rr} + e^2_{\theta\theta} + e^2_{zz}) + e^2_{r\theta} + e^2_{\theta z} + e^2_{z\theta} \quad (6.3)$$

From Eq.(6.1), in the case of $\sqrt{J'_2} > \tau_y$, we can write the following relations,

$$\tau'_{ij} = 2\eta_{pl} \left(\frac{\sqrt{J'_2}}{\sqrt{J'_2} - \tau_y} \right) e_{ij} \quad \text{for } \sqrt{J'_2} > \tau_y \quad (6.4)$$

The second invariant stress deviation tensor J'_2 and second invariant strain tensor I_2 can be related as follows,

$$J'_2 = 4\eta_{pl} I_2 \quad (6.5)$$

Therefore, based on Eq.(6.5) and Eq.(6.4), the following equation relating J'_2 , I_2 , η_{pl} and τ_y can be written as follows,

$$\text{From Eq.(6.4)} \quad (\tau'_{ij})^2 = 4 \left(\eta_{pl} \frac{\sqrt{J'_2}}{\sqrt{J'_2} - \tau_y} \right)^2 e_{ij}^2 \quad \text{for } \sqrt{J'_2} > \tau_y \quad (6.6)$$

$$J'_2 = 4 \left(\eta_{pl} \frac{\sqrt{J'_2}}{\sqrt{J'_2} - \tau_y} \right)^2 I_2 \quad (6.7)$$

Eq.(6.7) can be further simplified as follows,

$$\sqrt{J'_2} = 2\eta_{pl} \sqrt{I_2} + \tau_y \quad (6.8)$$

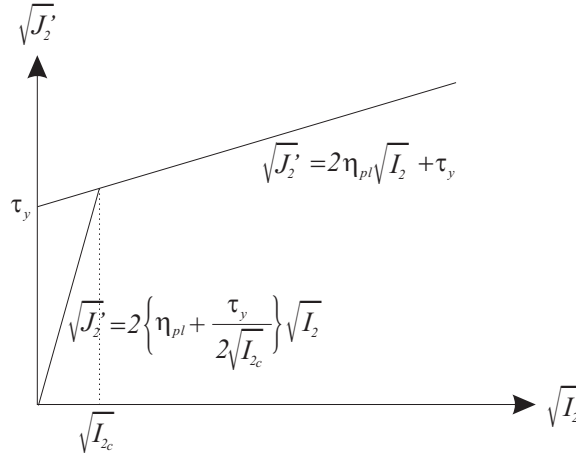
The relationship in Eq.(6.8) is shown in Fig.6.1 as a linear line passing through τ_y at y-axis with constant gradient of η_{pl} . Eq.(6.8) is substituted into Eq.(6.4) as follows,

$$\begin{aligned} \text{From Eq.(6.4)} \quad \tau'_{ij} &= 2\eta_{pl} \frac{\sqrt{J'_2} - \tau_y + \tau_y}{\sqrt{J'_2} - \tau_y} e_{ij} \\ &= 2 \left(\eta_{pl} + \frac{\tau_y}{2 \frac{\sqrt{J'_2} - \tau_y}{2\eta_{pl}}} \right) e_{ij} \end{aligned} \quad (6.9)$$

$$\begin{aligned} \text{From Eq.(6.8)} \quad \sqrt{I_2} &= \frac{\sqrt{J'_2} - \tau_y}{2\eta_{pl}} \\ \therefore \tau'_{ij} &= 2 \left(\eta_{pl} + \frac{\tau_y}{2\sqrt{I_2}} \right) e_{ij} \end{aligned} \quad (6.10)$$

In the case where $\sqrt{J'_2} < \tau_y$, we can see from Fig.6.1 that the value of $\sqrt{J'_2}$ cannot be determined. This poses problem in numerical analysis. Therefore, a second linear line is introduced to represent the relationships between $\sqrt{J'_2}$ and $\sqrt{I_2}$ in the case $\sqrt{J'_2} < \tau_y$ as follows,

$$\sqrt{J'_2} = 2 \left(\eta_{pl} + \frac{\tau_y}{2\sqrt{I_{2c}}} \right) \sqrt{I_2} \quad (6.11)$$


 Figure 6.1: Proposed bilinear model showing relationship between $\sqrt{J'_2}$ and $\sqrt{I_2}$

where $\sqrt{I_{2c}}$ is a critical value. Therefore, for $\sqrt{J'_2} < \tau_y$,

$$\tau'_{ij} = 2 \left(\eta_{pl} + \frac{\tau_y}{2\sqrt{I_{2c}}} \right) e_{ij} \quad (6.12)$$

To summarize,

$$\tau'_{ij} = 2 \left(\eta_{pl} + \frac{\tau_y}{2\sqrt{I_2}} \right) e_{ij} \quad \text{for } \sqrt{J'_2} \leq \tau_y \quad (6.13)$$

$$\tau'_{ij} = 2 \left(\eta_{pl} + \frac{\tau_y}{2\sqrt{I_{2c}}} \right) e_{ij} \quad \text{for } \sqrt{J'_2} > \tau_y \quad (6.14)$$

The total stress, σ_{ij} on a fluid body can be expressed as,

$$\begin{aligned} \sigma_{ij} &= -p\delta_{ij} + \tau'_{ij} \\ &= -p\delta_{ij} + 2\eta e_{ij} \end{aligned} \quad (6.15)$$

where p is pressure, δ_{ij} is the Kronecker delta and η is the viscosity. By comparing Eq.(6.15) with Eq.(6.13) and Eq.(6.14), we can expressed the viscosity, η as

$$\eta = \eta_{pl} + \frac{\tau_y}{2\sqrt{I_2}} \quad \text{for } (\sqrt{I_2} > \sqrt{I_{2c}}) \quad (6.16)$$

$$\eta = \eta_{pl} + \frac{\tau_y}{2\sqrt{I_{2c}}} \quad \text{for } (\sqrt{I_2} \leq \sqrt{I_{2c}}) \quad (6.17)$$

In the numerical model, the second invariant strain rate, I_2 is calculated using Eq.(6.3) and used to determine the viscosity expressed in Eq.(6.16) and Eq.(6.17).

6.3 Numerical simulation of slump flow test of fresh concrete

The slump flow test of fresh concrete can be regarded as a type of dam-break flow with finite extent. This is because in an ideal slump flow test, the flow is axis-symmetric. Therefore, the

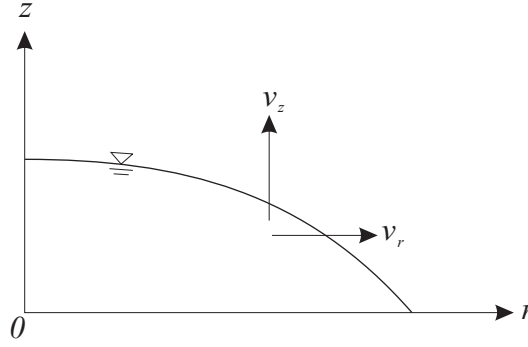


Figure 6.2: Slump flow model in cylindrical coordinate system

motion of the fresh concrete in a slump flow test can be reduced into two dimensional as shown in Fig. 6.2 and can be adequately described in the following equation of motion, Continuity Equation:

$$\frac{1}{r} \frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0 \quad (6.18)$$

Momentum Equation:

$$\frac{\partial v_r}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(rv_r^2) + \frac{\partial}{\partial z}(v_r v_z) = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\eta}{\rho} \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r}(rv_r) \right) + \frac{\partial^2 v_r}{\partial z^2} \right] + g_r \quad (6.19)$$

$$\frac{\partial v_z}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(rv_r v_z) + \frac{\partial}{\partial z}(v_z^2) = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\eta}{\rho} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r}(v_z) \right) + \frac{\partial^2 v_z}{\partial z^2} \right] + g_z \quad (6.20)$$

where v_r is the velocity in r-direction, v_z is the velocity in z-direction, r is the radius measured from the origin, ρ is the density of the fluid, η is the viscosity of the fluid and g_r and g_z are gravity acceleration in r and z directions respectively.

6.3.1 Numerical model

The numerical model used to simulate the slump flow test of fresh concrete is based on Volume of Fluid (VOF) method, (Hirt and Nicholas, 1981) explained in Chapter 5. The governing equations for the numerical model are the continuity and momentum equations in cylindrical coordinate as in Eq.(6.18), Eq.(6.19) and Eq.(6.20).

Solving the advection term in momentum equations with CIP scheme

The advection term in both Eq.(6.19) and Eq.(6.20) are solved using CIP as follows,
From Eq.(6.19), the advection term of momentum equation in r direction is,

$$\begin{aligned}
 \frac{\partial v_r}{\partial t} + \frac{1}{r} \frac{\partial(rv_r^2)}{\partial r} + \frac{\partial(v_r v_z)}{\partial z} &= \frac{\partial v_r}{\partial t} + \frac{1}{r} \left[\frac{rv_r \partial v_r}{\partial r} + v_r \frac{\partial(rv_r)}{\partial r} \right] + \frac{v_r \partial v_z}{\partial z} + \frac{v_z \partial v_r}{\partial z} \\
 &= \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_r}{r} \frac{\partial(rv_r)}{\partial r} + v_r \frac{\partial v_z}{\partial z} + v_z \frac{\partial v_r}{\partial z} \\
 &= \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} + v_r \left[\frac{1}{r} \frac{\partial(rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} \right] \\
 &= \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z}
 \end{aligned} \tag{6.21}$$

The advection term in Eq.(6.21) is reduced to the form where it can be solved by using the CIP scheme directly. However, since non-advection term exists on the right hand side of Eq.(6.19), Eq.(6.19) has to be solved in 2 phases: advection phase and non-advection phase, as explained in Chapter 5. The same procedure applies to the momentum equation in z direction, Eq.(6.20) as well. If $\vec{f}$ represents the vector to be advected in advection phase and $\vec{G}$ represents the vector of the non-advection terms to be solved in the non-advection phase, then, the momentum equation in Eq.(6.19) and Eq. (6.20) can be represented in advection and non-advection vectors as follows,

$$\vec{f} = (v_r, v_z) \tag{6.22}$$

$$\vec{G} = \begin{pmatrix} -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\eta}{\rho} \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(rv_r)}{\partial r} \right) + \frac{\partial^2 v_r}{\partial z^2} \right] \\ -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\eta}{\rho} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(v_z)}{\partial r} \right) + \frac{\partial^2 v_z}{\partial z^2} \right] + g \end{pmatrix} \tag{6.23}$$

Solving the advection of density function, F with CIP scheme

The advection equation for the density function in cylindrical coordinate can be written as,

$$\frac{\partial f}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (ru f) + \frac{\partial}{\partial z} (vf) = 0 \tag{6.24}$$

The advection form of F in Eq. (6.24) can be re-written in the following form so that it can be readily solved with CIP scheme,

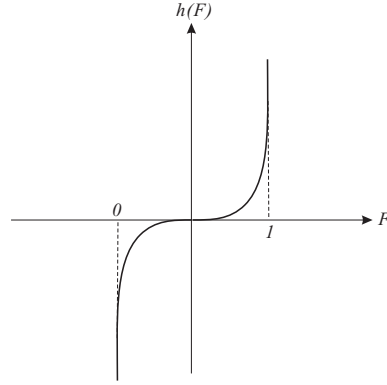
$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial r} + v \frac{\partial f}{\partial z} = -f \left(\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial v}{\partial z} \right) \tag{6.25}$$

Eq.(6.25) is solved using CIP scheme, where the advection and non-advection terms are,

$$\begin{aligned}
 \vec{f} &= f \\
 \vec{G} &= -f \left(\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial v}{\partial z} \right)
 \end{aligned} \tag{6.26}$$

Since F function is a step-function, advection of F with CIP method will results in smearing at surface boundary. To overcome this problem, Yabe and Xiao (1995) suggested the use of a digitizer function of the following form,

$$h = \tan[\alpha\pi(F - 0.5)] \tag{6.27}$$

Figure 6.3: Tangent function $H(f)$

Because $\tan \frac{\pi}{2} = \infty$, coefficient α is introduced to avoid numerically error due to the evaluation of $\tan \frac{\pi}{2}$. The value of α is initially suggested as 0.99 (Yabe and Xiao, 1995), but Yamada et al. (1998) found that $\alpha = 0.85$ is more suitable to maintain sharp interface of F . Eq.(6.27) is solved with CIP scheme, and the value of F after advection is calculated from the inversion of the new h value.

$$F = \frac{1}{0.85\pi} \tan^{-1}(h) + 0.5 \quad (6.28)$$

Due to the nature of the function, when $F = 1.0$, or $F = 0.0$, function h will have extreme value and therefore, function F will have value bounded within the limit of 0 and 1 by using the digitizer function, h as shown in Fig.6.3 .

6.3.2 Numerical simulation conditions

Numerical model setup

The cell size in the radial direction, r and vertical direction z are set as 5mm. The initial shape of the simulation of the slump flow of fresh concrete is shown in Fig.(6.4). Time increment is set as $\Delta t = 1.0 \times 10^{-5}s$ and adjusted based on the following criteria (Hirt and Nicholas, 1981),

$$\Delta t < \min \left(\frac{\Delta r_i}{|v_{rij}|}, \frac{\Delta z_j}{|v_{zij}|} \right) \quad (6.29)$$

$$\Delta t \eta < \frac{1}{2} \frac{\Delta r_i^2 \Delta z_j^2}{(\Delta r_i^2 + \Delta z_j^2)} \quad (6.30)$$

Boundary conditions

The vertical axis at the origin which represents the center line of axis-symmetric fresh concrete slump flow test is set as slip wall condition. The bottom floor is set as non-slip condition. The slump test cone wall, is placed in the position shown in Fig.(6.4) and the rate of pulling of the cone wall is set as 40mm/s based on the experimental work by Kokado et al. (2001). Non-slip condition is set for the cone wall.

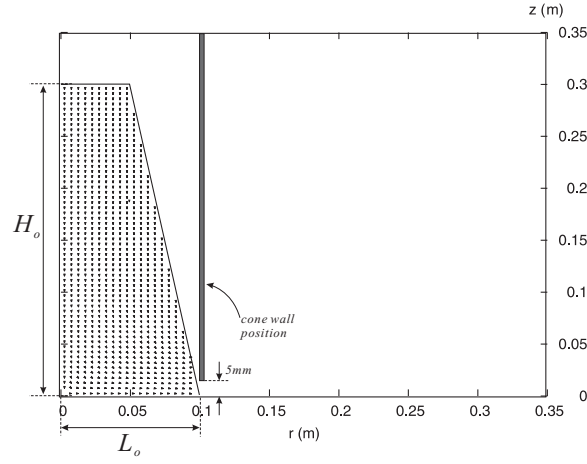


Figure 6.4: Initial condition of slump flow simulation

Rheological parameters for simulations

In order to compare with the result from the numerical work carried out by Kokado et al. (2001), similar rheological properties of fresh concrete are used for the numerical simulation of fresh concrete slump flow test in this study. The rheological properties of the fresh concrete are summarized in Table 6.1 where dimensionless parameter $R = \frac{\tau_y \rho}{\mu^2} H_o L_o$. H_o and L_o are initial depth and width of the fresh concrete slump flow test shown in Fig. 6.4. The value of the square root of the critical value of second invariant strain rate tensor $\sqrt{I_{2c}}$ is fixed as 0.03/s based on the simulation by Kokado et al. (2001).

Consideration of slump test cone

The effect of slump test cone is important to be included in the numerical simulation (Kokado et al., 2001). As the slump test cone is lifted up in actual slump flow test, the whole area of the fresh concrete is detached from the cone and as the process of slumping begins the fresh concrete will be in contact with the cone wall again. Therefore, it is adequate to simulate this detaching phenomena during the lifting of the cone by setting up a vertical cone wall as shown in Fig. 6.4. Based on the experiments by Kokado and Miyagawa (1999), and numerical simulation by Kokado et al. (2001), the flow radius is measured when it reaches 110mm in the numerical model overcome the discrepancy between the numerical model and experiment. The discrepancy is due to the fact that the numerical simulation starts with the cone wall 5mm pulled up while in the experiments by Kokada the measurement of flow radius starts as soon as the cone is pulled up.

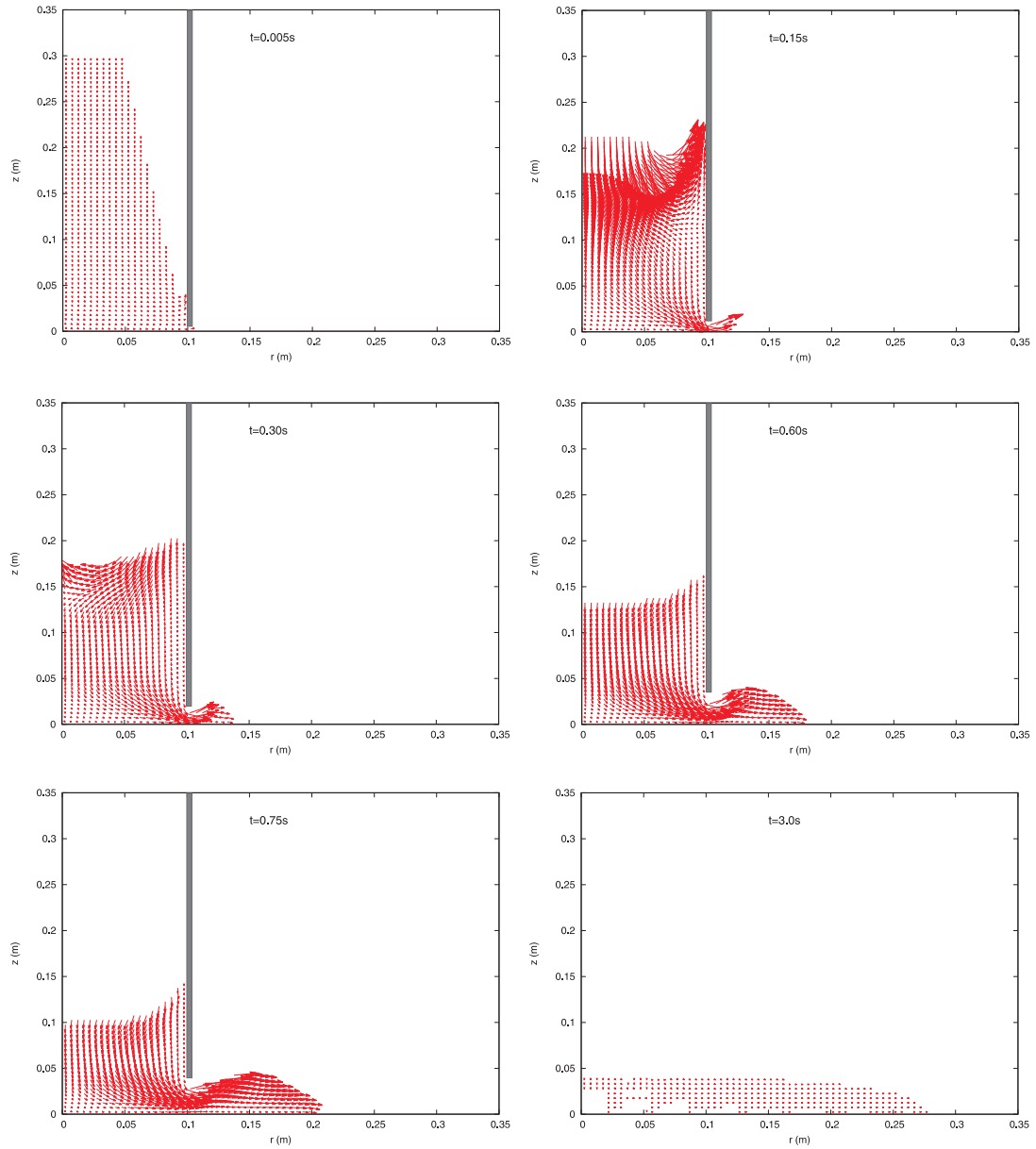


Figure 6.5: Simulation of slump flow test for case M35-7 with VOF-CIP model

Table 6.1: Rheological properties of fresh concrete for numerical simulation cases

Exp / Sim no	ρ (kgm^{-3})	τ_y (Pa)	η_{pl} (Pa.s)	R
M025-1	2187	112	44	3.796
M025-2	2187	72	43	2.555
M025-3	2187	38	32	2.435
M025-4	2187	30	29	2.340
M025-5	2187	33	29	2.574
M025-6	2187	14	24	1.595
M025-7	2187	12	21	1.785
M050-1	2187	57	76	0.647
M050-2	2187	80	74	0.959
M050-3	2187	53	61	0.935
M050-4	2187	35	53	0.817
M050-5	2187	28	45	0.907
M050-6	2187	24	42	0.893
M050-7	2187	11	33	0.663
M30-1	2237	121	60	2.256
M30-2	2237	83	41	3.314
M30-3	2237	45	37	2.206
M30-4	2237	34	28	2.910
M30-5	2237	19	23	2.410
M30-6	2237	13	22	1.803
M30-7	2237	10	21	1.522
M30-8	2237	2.8	14	0.959
M35-1	2187	102	33	6.145
M35-2	2187	106	18	21.465
M35-3	2187	44	17	9.989
M35-4	2187	32	15	9.331
M35-5	2187	19	13	7.376
M35-6	2187	15	11	8.133
M35-7	2187	11	9.8	7.515
M40-1	2140	19	6.8	26.380
M40-2	2140	17	6.8	23.603
M40-3	2140	10	4.7	29.063

6.4 Results and performance of numerical model

6.4.1 The simulation of slump flow test

The simulation of fresh concrete slump flow test for one of the simulation cases is shown in Fig. 6.5. It can be seen that as soon as the cone is pulled up, the fresh concrete flow radius starts to propagate. The effect of the cone pulling can be seen in Fig.6.5 especially when simulation is at $t = 0.60s$ and $t = 0.75s$, where the fresh concrete near to the cone wall is pulled up due to the adhesion to the cone wall.

6.4.2 Comparison of VOF-CIP model results to MAC model results

The performance of the VOF-CIP model is compared to the results from the simulation by MAC model carried out by Kokado et al. (2001) in Fig. 6.6. Three special cases are picked : 1) case M35-6 where the VOF-CIP model result is better than the MAC model, 2) case M025-2 where both cases do not fare well, and 3) case M050-6 where the MAC model result is better than the VOF-CIP result. Although a general conclusion cannot be made based on this result, it can be seen that the VOF-CIP model performed much better in the case of where the ratio of plastic viscosity to yield stress is less than one $\frac{\eta}{\tau_y} < 1.0$. Due to the lack of data from the simulation by MAC model, it is hard to tell whether the MAC model performed better than the VOF-CIP model.

6.4.3 Comparison of VOF-CIP model results to experimental results

The comparison between the numerical results by VOF-CIP model and experimental results of slump flow test of fresh concrete are shown in Fig. 6.7 for M050-(1 to 7) cases, in Fig. 6.8 for M025-(1 to 7) cases, in Fig. 6.9 for M030-(1 to 8) cases, in Fig.6.10 for M35-(1 to 7) cases and in Fig. 6.11 for M40-(1 to 3) cases. The overall performance of the VOF-CIP model is satisfactory based on the comparison with the experimental data. Based on the comparison of all the results, the VOF-CIP performed best in the cases where the ratio of plastic viscosity and yield stress is less than one, $\frac{\eta}{\tau_y} < 1$. In cases where the ratio $\frac{\eta}{\tau_y} > 1$, the flow radius deviates from the experimental data with the increase of time. However, the VOF-CIP model performed satisfactorily in the initial stage of the flow in almost all the simulation cases.

To further investigate the performance of the VOF-CIP model, the time for the flow radius to reach 200mm plotted against $\frac{\eta}{\tau_y}$ in Fig.6.12. It can be seen that the flow radius reaches the 200mm point faster in the experiments compared to VOF-CIP model in almost all cases. As reported by Kokado et al. (2001), simulation cases with yield stress, $\tau_y > 60\text{Pa}$, the time to reach 200mm in the numerical model is slower than the experimental data (slower by 0.5s to 2.0s). Based on Fig. 6.12, it can be seen that the VOF-CIP model does not face this problem as in most simulation cases, the time for flow radius to reach 200mm fall within $\pm 1.0\text{s}$, and with almost half of the cases fall within $\pm 0.5\text{s}$ regardless of whether τ_y is less or larger than 60Pa

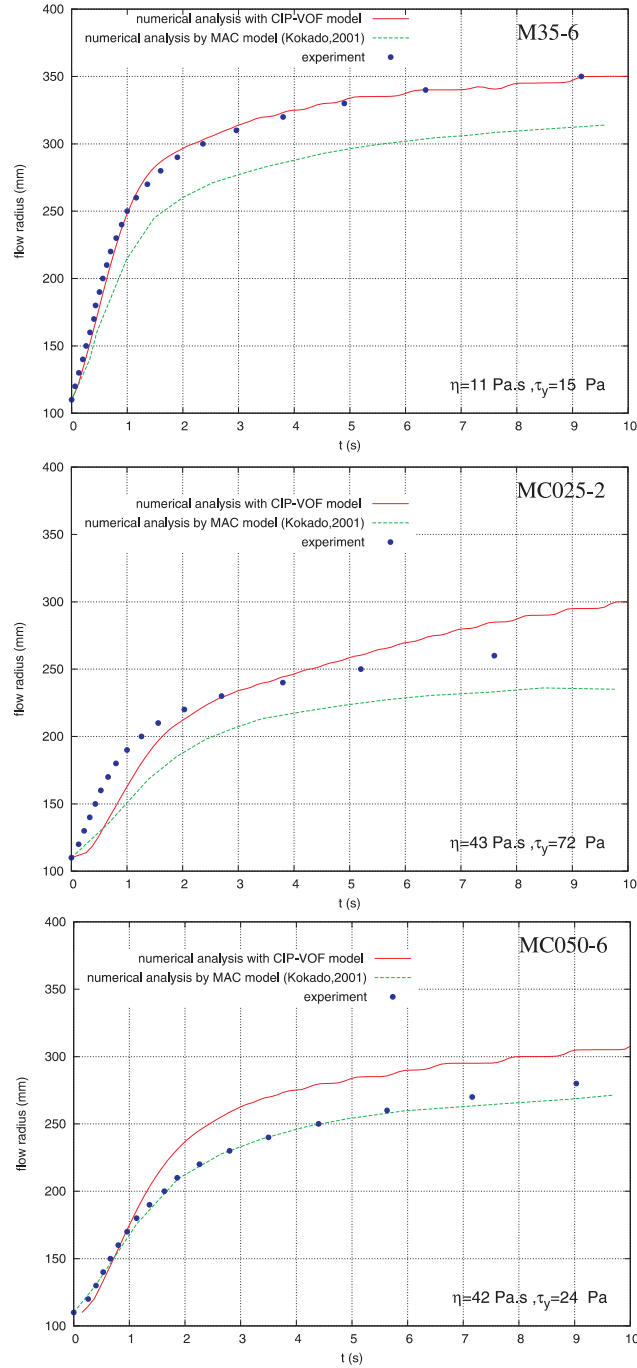


Figure 6.6: Comparison of flow radius propagation for case M025-2, M035-6 and M050-6 with VOF-CIP model and MAC model by Kokado et al. (2001)

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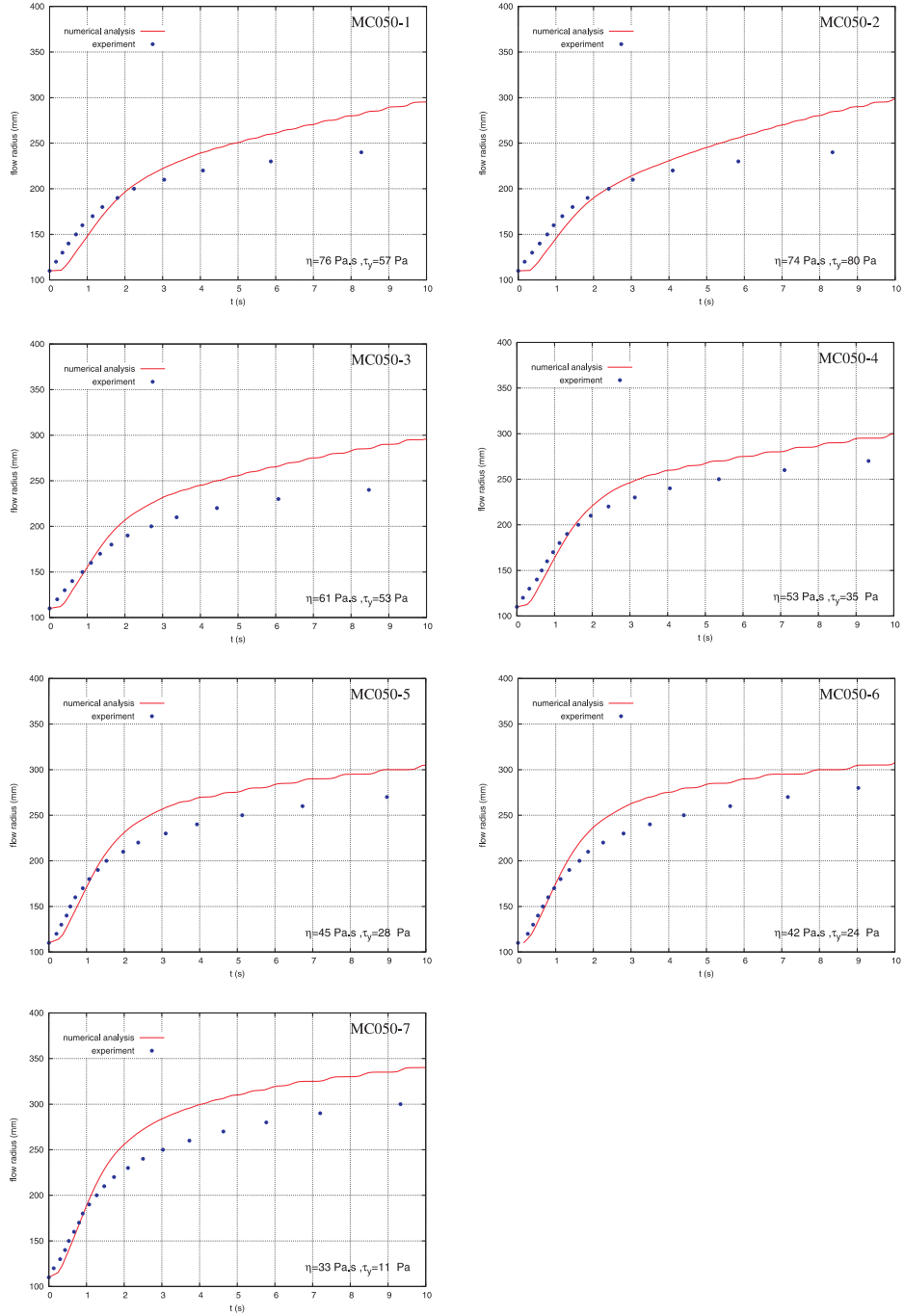


Figure 6.7: Flow radius propagation for case M050-1 to M050-7

6.4. Results and performance of numerical model

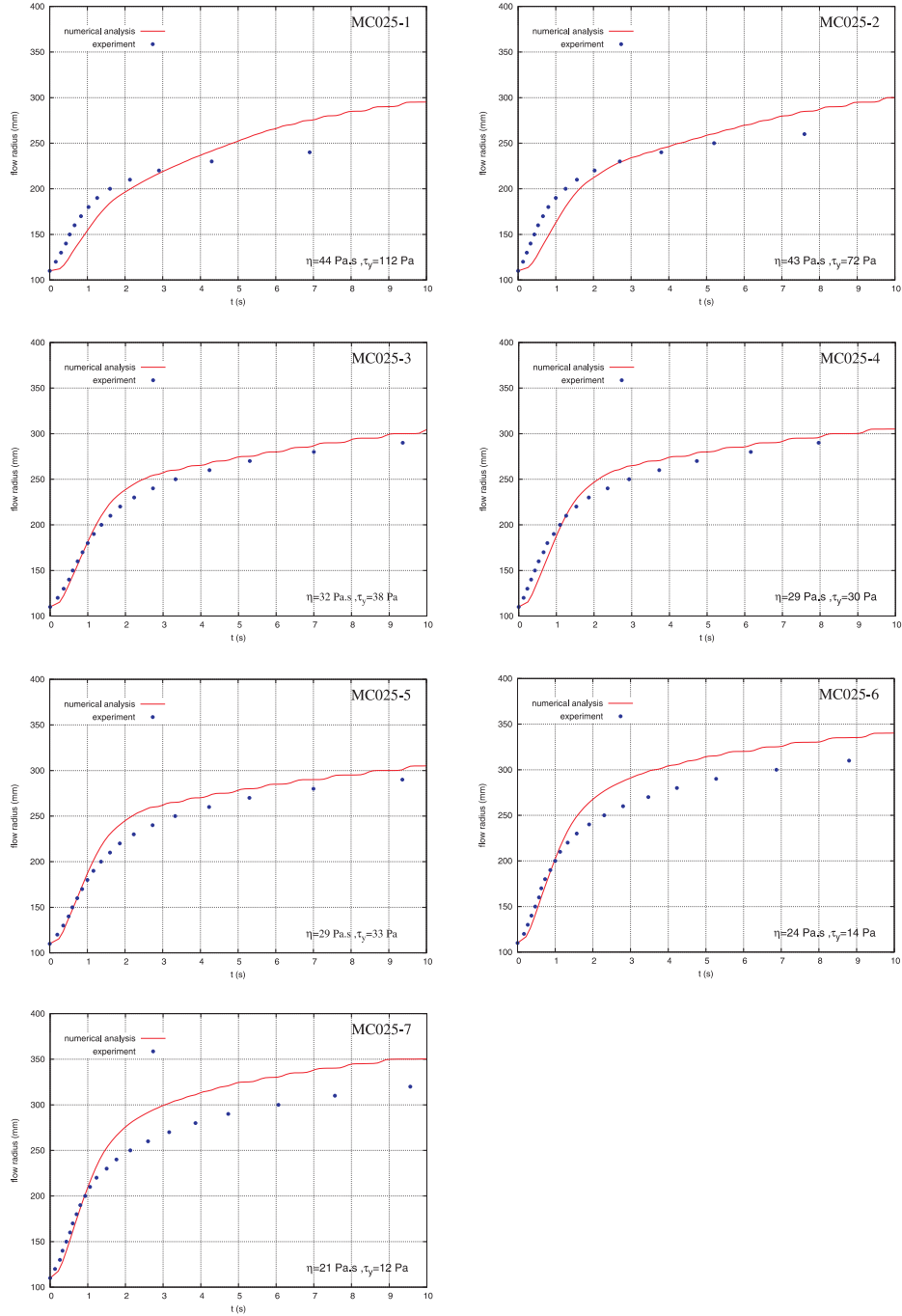


Figure 6.8: Flow radius propagation for case M025-1 to M025-7

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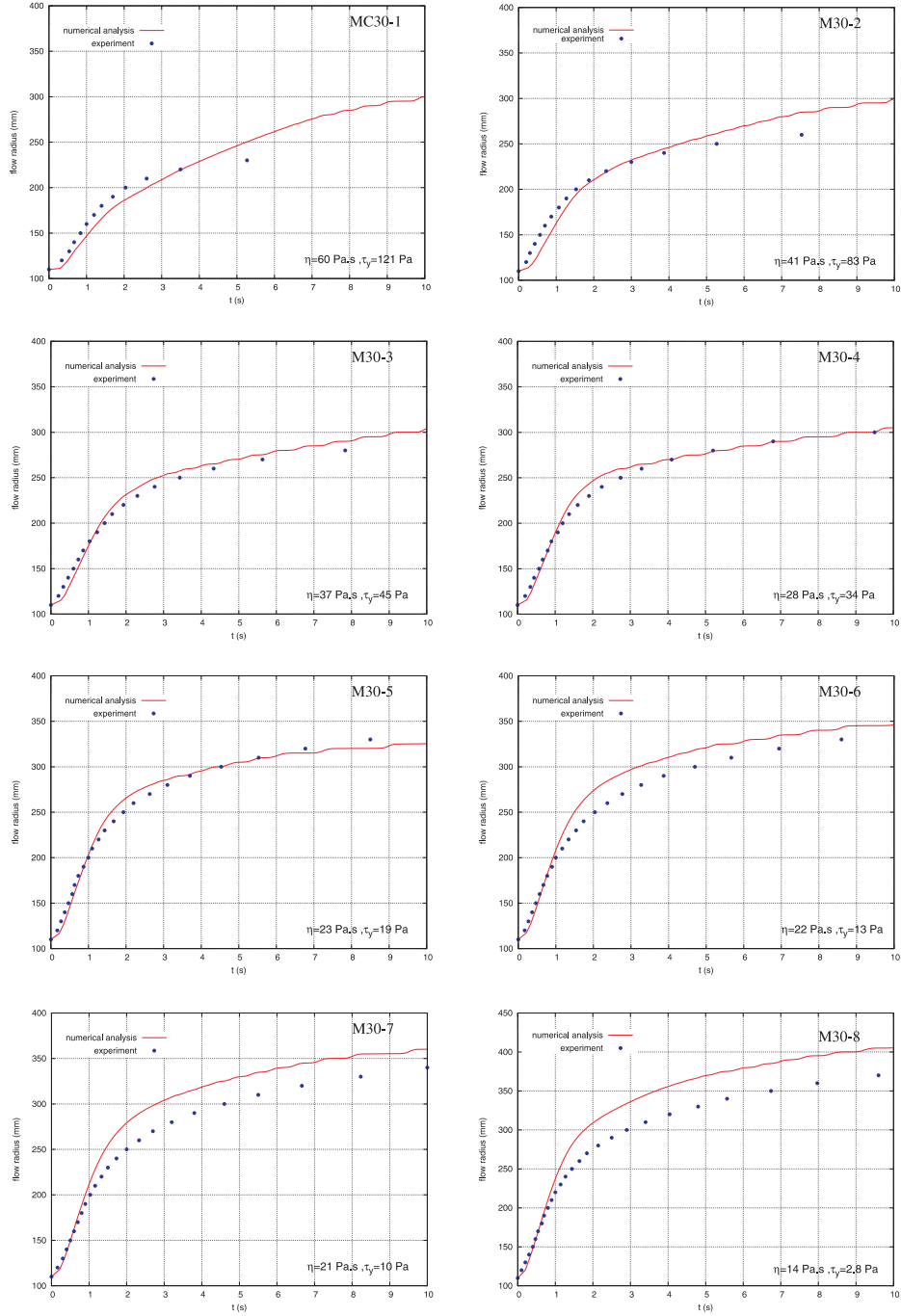


Figure 6.9: Flow radius propagation for case M30-1 to M30-8

6.4. Results and performance of numerical model

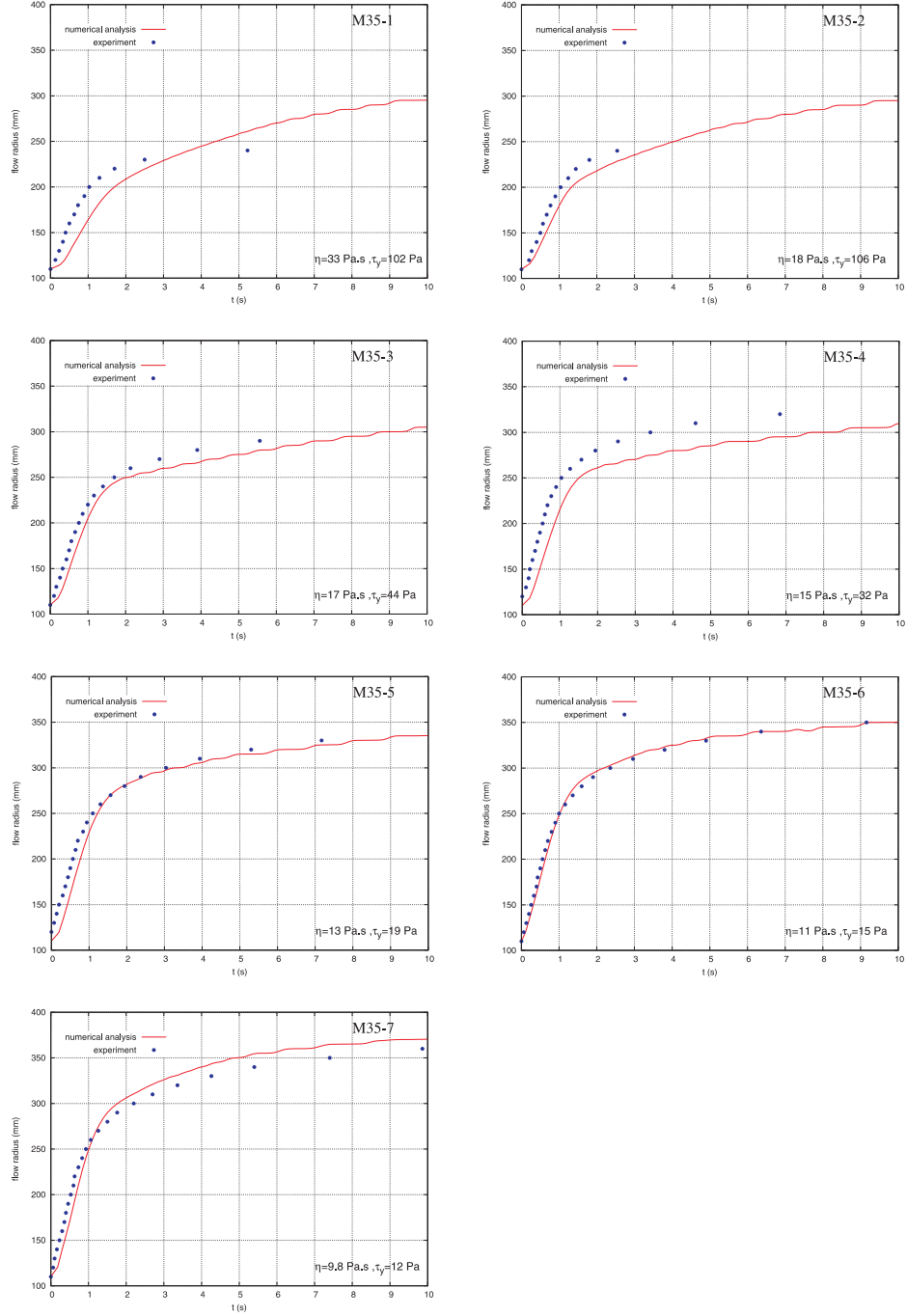


Figure 6.10: Flow radius propagation for case M35-1 to M35-7

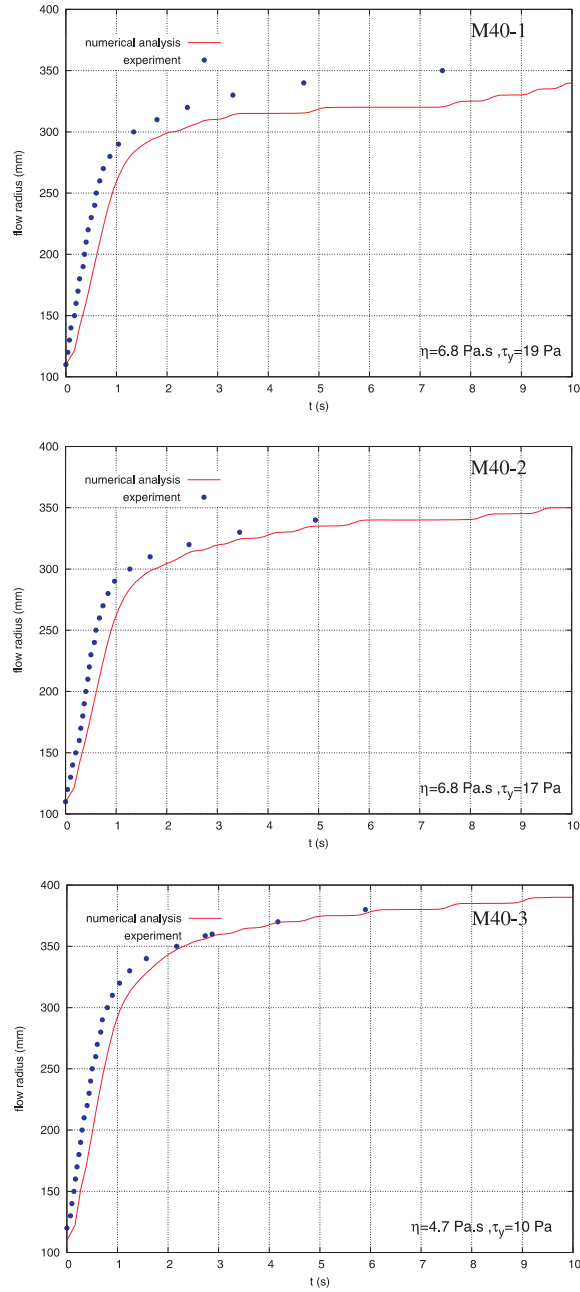


Figure 6.11: Flow radius propagation for case M40-1 to M40-3

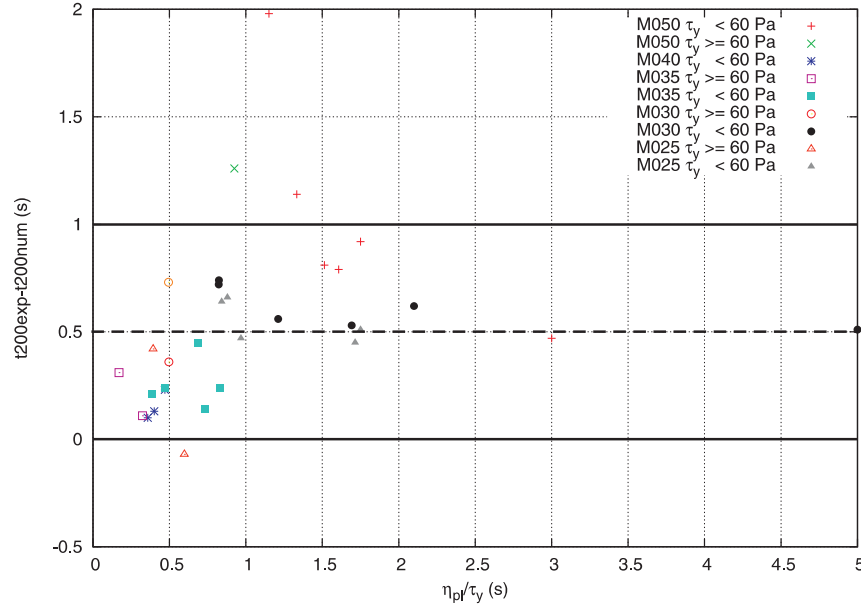


Figure 6.12: Difference of time for flow radius to reach 200mm in experiment (t_{200exp}) and numerical model (t_{200num}), $t_{200exp} - t_{200num}$ versus ratio of η_{pl}/τ_y

6.5 Bingham fluid flow characteristics

In Chapter 3, the viscous phase flow characteristics are derived on the assumption of self-similarity. In order to investigate the flow characteristics using the flow radius propagation of slump flow test, similar technique used in Chapter 3 is used by considering the dam break flow of finite-extent model in cylindrical coordinate system.

6.5.1 Governing equations

The governing equations to describe the slump flow test can be adequately represented by the following depth averaged continuity and momentum equations in cylindrical coordinate system,

$$\frac{\partial h}{\partial t} + \frac{1}{r} \frac{\partial (rhU_r)}{\partial r} = 0 \quad (6.31)$$

$$\frac{\partial (hU_r)}{\partial t} + \frac{1}{r} \frac{\partial (r\hat{\beta}hU_r^2)}{\partial r} + gh \frac{\partial h}{\partial r} = -\frac{\tau_b}{\rho} = -\frac{3\nu}{h} u_r \quad (6.32)$$

with h as the depth of flow, U_r as the depth averaged flow velocity in r direction, ν as the kinematic viscosity, ρ as the fluid density, $\hat{\beta}$ is the coefficient of momentum τ_b as the bottom shear stress and g as the gravity acceleration.

6.5.2 Similarity functions and derivation of flow characteristics

Similarity functions p and q are used to express the depth $h(r,t)$ and flow velocity $U_r(r,t)$ as follows,

$$h = h_m(t)p\left(\frac{r}{L}\right) = h_m(t)p(\xi) \quad (6.33)$$

$$U_r = u_m(t)q\left(\frac{r}{L}\right) = u_m(t)q(\xi) \quad (6.34)$$

where ξ is defined as follows,

$$\xi = \frac{r}{L(t)} \quad (6.35)$$

and the boundary condition of the similarity function p and q are as follows,

$$p(0) = 1, \quad p(1) = 0 \quad \text{and} \quad q(0) = 0 \quad (6.36)$$

The total volume of the finite extent dam in cylindrical coordinate is,

$$\begin{aligned} \text{Vol} &= \int_0^{2\pi} \int_0^L hr \, d\theta \, dr \\ &= 2\pi h_m(t)L(t)^2 \int_0^1 p(\xi)\xi \, d\xi \end{aligned} \quad (6.37)$$

By assuming that self-similarity exists in the viscous phase, the function h_m and u_m and front position, L can be expressed as follows,

$$h_m(t) = \alpha h_o \left(\sqrt{\frac{g}{h_o}} \right)^a \quad (6.38)$$

$$u_m(t) = \beta \sqrt{gh_o} \left(\sqrt{\frac{g}{h_o}} \right)^b \quad (6.39)$$

$$L(t) = \gamma L_o \left(\sqrt{\frac{g}{h_o}} \right)^c \quad (6.40)$$

where α, β, γ are coefficients, h_o and L_o are the initial depth and width of the dam respectively. By substituting Eq. (6.38), Eq.(6.39) and Eq. (6.40) into Eq.(6.33) and Eq. (6.34) and thereafter into the governing equation in Eq. (6.32) and Eq. (6.31), the continuity and momentum equations can be written as follows with time t substituted by its dimensionless form t' , where $t' = \sqrt{g/h_o}t$, Continuity equation

$$\begin{aligned} &\alpha h_o a t'^{a-1} \sqrt{\frac{g}{h_o}} p(\xi) - \alpha h_o a t'^{a-1} \xi c \sqrt{\frac{g}{h_o}} \frac{dp(\xi)}{d\xi} + \frac{\alpha \beta}{\gamma L_o \xi} h_o \sqrt{gh_o} q \\ &+ \frac{\alpha \beta}{\gamma L_o} h_o \sqrt{gh_o} q(\xi) \frac{dp(\xi)}{d\xi} t'^{a+b-c} + \frac{\alpha \beta}{\gamma L_o} h_o \sqrt{gh_o} p(\xi) \frac{dq(\xi)}{d\xi} t'^{a+b-c} = 0 \end{aligned} \quad (6.41)$$

Momentum equation

$$\begin{aligned} & \alpha\beta b g h_o p(\xi) q(\xi) t'^{a+b-1} - \alpha\beta c g h_o p(\xi) \frac{dq(\xi)}{d\xi} t'^{a+b-1} + \\ & \alpha\beta a g h_o p(\xi) q(\xi) t'^{a+b-1} - \alpha\beta c g h_o p(\xi) \frac{dq(\xi)}{d\xi} t'^{a+b-1} + \\ & \frac{\hat{\beta}\alpha\beta^2}{\xi\gamma L_o} g h_o^2 p(\xi) q(\xi)^2 + \hat{\beta} \frac{\alpha+\beta^2}{\gamma} \frac{g h_o^2}{L_o} q(\xi)^2 \frac{dp(\xi)}{d\xi} t'^{a+2b-c} + 2\hat{\beta} \frac{\alpha\beta^2}{\gamma} \frac{g h_o^2}{L_o} p(\xi) q(\xi) \frac{dq(\xi)}{d\xi} t'^{a+2b-c} \\ & + \frac{\alpha^2}{\gamma} \frac{g h_o^2}{L_o} p(\xi) \frac{dp(\xi)}{d\xi} t'^{2a-c} = -3\nu \frac{\beta}{\alpha} \sqrt{\frac{g}{h_o}} \frac{q(\xi)}{p(\xi)} t'^{b-a} \end{aligned}$$

By satisfying the dimension of t' ,

From volume conservation in Eq.(6.37)

$$a + 2c = 0 \quad (6.42)$$

From continuity equation,

$$a - 1 = a + b - c \quad \therefore b - c = -1 \quad (6.43)$$

Assuming that for inertial phase flow the inertia and pressure terms are in equilibrium,

From momentum equation, by equating the inertia-pressure terms,

$$a + b - 1 - a + 2b - c = 2a - c \quad (6.44)$$

And for inertial phase flow the viscous and pressure terms are in equilibrium,

From momentum equation, by equating the viscous-pressure terms,

$$2a - c = b - a \quad (6.45)$$

By using Eq.(6.42), Eq.(6.43) and Eq.(6.44), the solution for the inertial phase flow is as follows,

$$a = -1, \quad b = -\frac{1}{2}, \quad c = \frac{1}{2} \quad (6.46)$$

And by using Eq.(6.42), Eq.(6.43) and Eq.(6.45), the solution for the viscous phase flow is as follows,

$$a = -\frac{1}{4}, \quad b = -\frac{7}{8}, \quad c = \frac{1}{8} \quad (6.47)$$

Based on the results from Eq.(6.46) and Eq.(6.47) the flow radius characteristics can be written as follows,

In inertial phase,

$$L \propto t^{\frac{1}{2}} \quad (6.48)$$

In viscous phase,

$$L \propto t^{\frac{1}{8}} \quad (6.49)$$

6.5.3 Validation of characteristics of inertial and viscous phase

The flow radius propagation is plotted in Fig. 6.13 in 3 extreme cases where the yield stress is relatively low, mild and high. In each cases, two distinct region can be observed. In the inertial phase flow, the characteristic of inertial flow can be observed in case where the yield stress is low and mild. In the case of high yield stress, this inertial characteristic is not distinct. It is thought that in the case of high yield stress, energy is initially used to overcome the yield stress and therefore the existence of inertial flow is less dominant. In the case of viscous phase characteristic which is shown in the latter part of the flow, the characteristics of $L \propto t^{1/8}$ is most distinct in the case of high yield stress. In both low and mild yield stress, the flow radius plot have gradient less than 1/8. In the case of low yield stress, the slope is approaching to zero with the increase of time.

In each cases of low, mild and high yield stress, different cases of plastic viscosity are plotted. For easier comparison, the dimensionless parameter R is used to represents the combination of effects of yield stress, plastic viscosity and density. It can be clearly seen that in a group of fluids same order of yield stress, flow radius propagation is higher for fluid with higher R value.

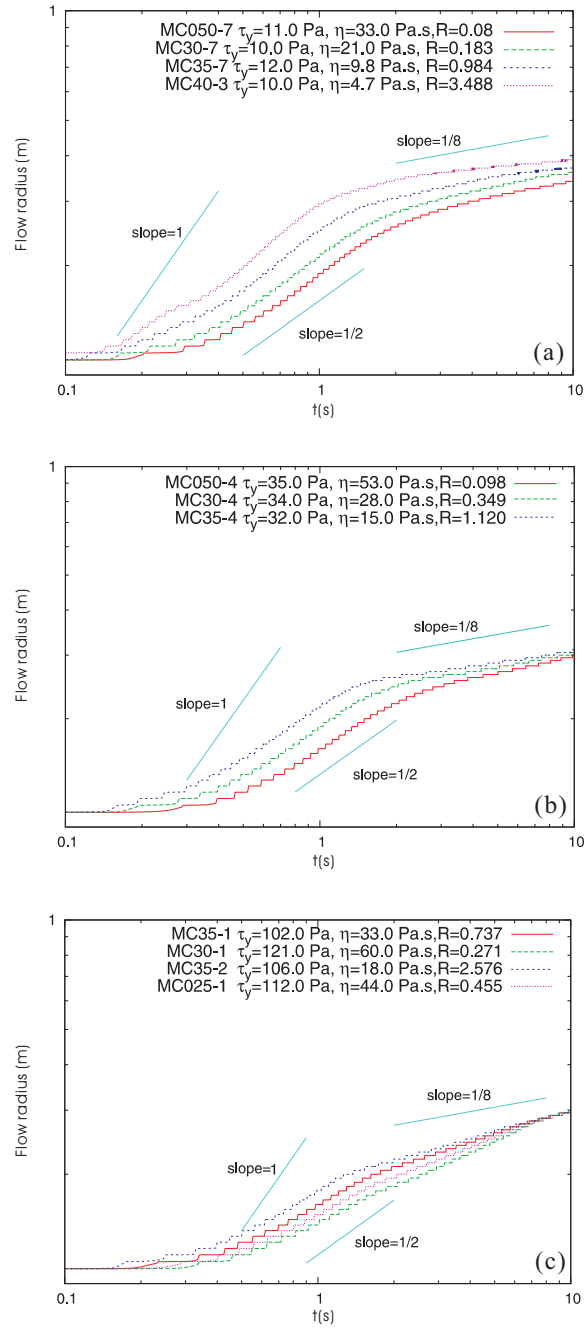


Figure 6.13: Characteristic regions shown for front propagation for relatively low (a), mild (b) and high (c) yield stress cases

6.6 Summary

In this chapter, the slump flow test of fresh concrete is simulated by treating it as dam break flow of finite-extent model. The simulation results are verified with experimental data and the performance of the numerical is satisfactory except in some cases where the ratio of plastic viscosity to yield stress, η/τ_y is high. Although the inertial and viscous phase flow are derived based on the Newtonian fluid constitutive relation, the results are shown to be applicable to describe the inertial and viscous phase flow of the flow radius of fresh concrete in the slump flow test. This is due to the fact that behavior of Bingham fluid is similar or analogous to the behavior of Newtonian fluid after the yield stress is overcome by the stress in the flow body. The flow characteristics for the inertial and viscous phase flow for cylindrical coordinate system are summarized here,

In inertial phase,

$$L \propto t^{\frac{1}{2}} \quad (6.50)$$

In viscous phase,

$$L \propto t^{\frac{1}{3}} \quad (6.51)$$

In the case of low yield stress, the simulation results show that the flow radius is slowing down to a constant value. This is due to the fact that as the flow slows down, the shear stress in the body reduces and when the value of shear stress is less than the yield stress value, the rate of strain is approaching zero ideally. Therefore there will be no deformation and the fluid body reaches a static state of pressure-yield stress balance.

Chapter 7

Abrupt Expansion of High-Velocity Flow

7.1 Preliminaries

Abrupt expansion flow is an important phenomena in open-channel flow. It occurs when flow is subjected to sudden transition due to sudden change in the geometry of the channel. The abrupt expansion phenomena can be observed in the transition structure in a spillway and at the outlet where a smaller channel is discharging into a wider channel. The understanding of the characteristics of the flow from an abrupt expansion such as the flow expansion angle the formation of shock waves is vital in improving the design of hydraulics structures. Early studies (Ippen, 1951), (Hosoda and Yoneyama, 1994), (Hager, 1992) have shown that the phenomenon of abrupt expansion depends on the approach width b , the approach depth h_o and the approach Froude number Fr . In the case of supercritical flow the relation between Froude number and the change of flow direction was developed.

7.1.1 Abrupt expansion flow

The characteristics of supercritical flow of abrupt expansion can be described qualitatively as in Fig. 7.1. When the approach flow passes through the section of abrupt expansion, the flow direction is subjected to changes due to the abrupt changes in the flow cross section. However there exists a zone close to the abrupt expansion (zone-ABC) where the flow velocity and depth are constant and equivalent to the approach flow velocity, V_o and depth, h_o . The flow impinges the boundary of the channel at point D and F. The line AD and CF form the zero-depth line, defining the zone of between the expansion of the flow and the dry corner.

Beyond the the impingement points, the flow is again subjected to flow direction changes, where the flow will be aligned paralelly to the boundary wall. The changes of flow direction causes the formation of disturbance wave which propagates towards the opposite boundary wall, indicated by line DE and FE. The disturbance waves are again subjected to change in direction when they meet at point E. The flow passing under line DE and FE is parallel to the wall beyond impingement points D and E. In this particular example, since the wall deflects the flow inwards ,the disturbance wave is a positive disturbance in the form of a surge wave that causes the rise in the water surface between line DE (or FE) and the boundary walls. The zone ADEFC

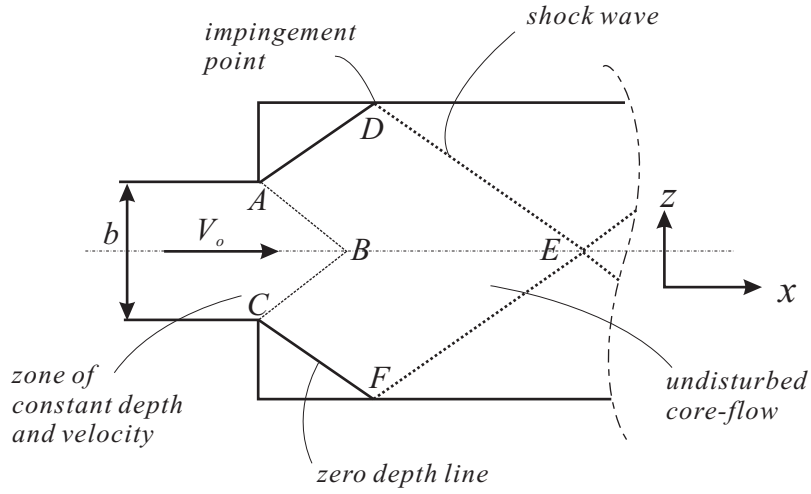


Figure 7.1: Abrupt expansion flow showing zero-depth-line and shock waves

is therefore the core-flow where the flow is undisturbed by the disturbance wave (Hager, 1992).

7.2 Theoretical study on abrupt expansion flow

The flow from a abrupt expansion is shown schematically in Fig.(7.2) and Fig.(7.3), and the steady state of the flow can be described with a depth-averaged model as follows;

Continuity equation:

$$\frac{\partial hu}{\partial x} + \frac{\partial hw}{\partial z} = 0 \quad (7.1)$$

Momentum equation:

$$\frac{\partial hu^2}{\partial x} + \frac{\partial huw}{\partial z} = -gh \frac{\partial h}{\partial x} + gh \sin \alpha - \frac{\tau_{bx}}{\rho} \quad (7.2)$$

$$\frac{\partial huw}{\partial x} + \frac{\partial hw^2}{\partial z} = -gh \frac{\partial h}{\partial z} - \frac{\tau_{bz}}{\rho} \quad (7.3)$$

where u and w are the depth-averaged velocity, h is the flow depth, τ_{bx} and τ_{bz} are the bottom shear stress in x and z direction respectively, α is the slope of the channel and g is the gravity acceleration.

The basic equations can be expressed in the matrix form as :

$$A_1 \frac{\partial U}{\partial x} + A_2 \frac{\partial U}{\partial z} = B \quad (7.4)$$

where U, A_1, A_2 and B are defined as :

$$U = \begin{bmatrix} h \\ u \\ w \end{bmatrix}, A_1 = \begin{bmatrix} u & h & 0 \\ g & u & 0 \\ 0 & 0 & u \end{bmatrix}, A_2 = \begin{bmatrix} w & 0 & h \\ 0 & w & 0 \\ g & 0 & w \end{bmatrix}, B = \begin{bmatrix} 0 \\ g \sin \alpha - \frac{\tau_{bx}}{\rho h} \\ -\frac{\tau_{bz}}{\rho h} \end{bmatrix} \quad (7.5)$$

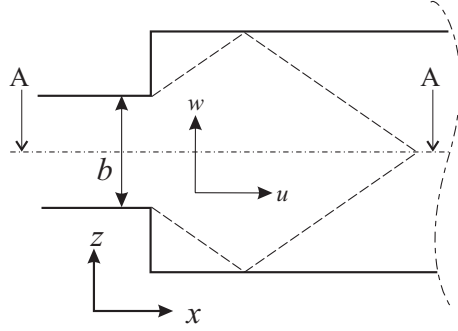


Figure 7.2: Abrupt expansion flow with b as width of the upstream approach channel

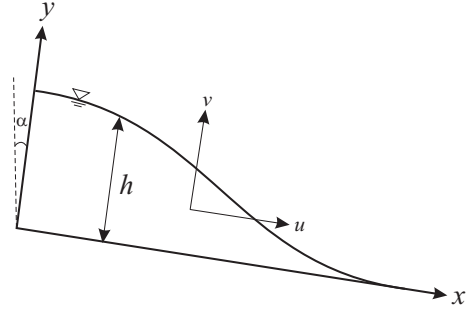


Figure 7.3: Schematic of section A-A.

Employing the method of characteristics, the characteristic lines λ_1, λ_2 and λ_3 are derived as the eigen values of $A_1^{-1}A_2$

$$\lambda_1 = \frac{dz}{dx} = \frac{w}{u} \quad (7.6)$$

$$\lambda_2 = \frac{dz}{dx} = \frac{uw + \sqrt{gh(u^2 + w^2 - gh)}}{u^2 - gh} \quad (7.7)$$

$$\lambda_3 = \frac{dz}{dx} = \frac{uw - \sqrt{gh(u^2 + w^2 - gh)}}{u^2 - gh} \quad (7.8)$$

The eigen vector corresponding to the above eigen values are as follows:

For λ_1

$$\mu_1 = [1 \quad u/g \quad w/g] \quad (7.9)$$

For λ_2

$$\mu_2 = \left[\sqrt{gh(u^2 + w^2 - gh)}/(uh) \quad -w/u \quad 1 \right] \quad (7.10)$$

For λ_3

$$\mu_3 = \left[-\sqrt{gh(u^2 + w^2 - gh)}/(uh) \quad -w/u \quad 1 \right] \quad (7.11)$$

From Eq. (7.6), we can see that the characteristic line λ_1 is a stream line. Eq.(7.7) and Eq.(7.8) also indicate that if the flow is super-critical, or $\sqrt{u^2 + w^2}/\sqrt{gh}$ is over unity, then Eq.(7.1) to Eq.(7.3) are classified as hyperbolic type. By multiplying the basic equations in Eq.(7.4) by eigenvector μ_i ($i = 1 \sim 3$) of $A_1^{-1}A_2$, the equations that satisfied on the characteristic line are derived as follows:

$$\mu_i \left(\frac{\partial U}{\partial x} + A_1^{-1}A_2 \frac{\partial U}{\partial z} \right) = \mu_i \left(\frac{\partial U}{\partial x} + \lambda_i \frac{\partial U}{\partial z} \right) = \mu_i A_1^{-1}B, \quad (7.12)$$

where (i=1,2,3), or

$$\frac{\partial}{\partial x} \left(h + \frac{u^2 + w^2}{2g} \right) + \frac{w}{u} \frac{\partial}{\partial z} \left(h + \frac{u^2 + w^2}{2g} \right) = \sin \alpha - \frac{\tau_{bx}}{\rho gh} - \frac{\tau_{bz}}{\rho gh} \frac{w}{u} \quad (7.13)$$

$$\begin{aligned} & \frac{\sqrt{gh(u^2 + w^2)}}{hu} \left(\frac{\partial h}{\partial x} + \lambda_2 \frac{\partial h}{\partial z} \right) - \frac{w}{u} \left(\frac{\partial u}{\partial x} + \lambda_2 \frac{\partial u}{\partial z} \right) - \left(\frac{\partial w}{\partial x} + \lambda_2 \frac{\partial w}{\partial z} \right) \\ & = \left(g \sin \alpha - \frac{\tau_{bx}}{\rho h} \right) \frac{uw + \sqrt{gh(u^2 + w^2 - gh)}}{u(gh - u^2)} - \frac{\tau_{bz}}{\rho hu} \end{aligned} \quad (7.14)$$

If the stress of the channel slope and the bottom shear stresses are neglected, Eq.(7.13) and Eq.(7.14) are reduced to

$$\frac{V^2}{2g} + h = H_o = \text{constant} \quad (7.15)$$

$$\frac{1}{V} \frac{dV}{d\theta} = \frac{\sqrt{H_o - V^2/2g}}{\sqrt{3V^2/2g - H_o}} \quad (7.16)$$

where V and θ are defined as,

$$V \equiv \sqrt{u^2 + w^2} \quad (7.17)$$

$$\tan \theta = w/u \quad (7.18)$$

By introducing $V' = V/\sqrt{2gH_o}$, Eq.(7.15) and Eq.(7.16) can be written in its dimensionless form as follows,

$$V'^2 + \frac{h}{H_o} = 1 \quad (7.19)$$

$$\frac{1}{V'} \frac{dV'}{d\theta} = \frac{\sqrt{1 - V'^2}}{\sqrt{3V'^2 - 1}} \quad (7.20)$$

The solution to the integration of Eq.(7.20) is given as follows, (Ippen, 1951)

$$\theta = \sqrt{3} \tan^{-1} \sqrt{\frac{\frac{3h}{2H_o}}{1 - \frac{3h}{2H_o}}} - \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) \sqrt{\frac{\frac{3h}{2H_o}}{1 - \frac{3h}{2H_o}}} - \theta_1 \quad (7.21)$$

θ_1 is the integral constant defined by the condition that for $\theta = 0$ the initial depth (before the flow changes its direction), $h = h_1$. By using Eq.(7.19), the relation between Froude number, Fr and h/H can be defined as follows,

$$\begin{aligned} \frac{h}{H} &= 1 - V'^2 = 1 - \frac{h}{H} \frac{1}{2} Fr^2 \\ \frac{h}{H} &= \frac{2}{2 + Fr^2} \end{aligned} \quad (7.22)$$

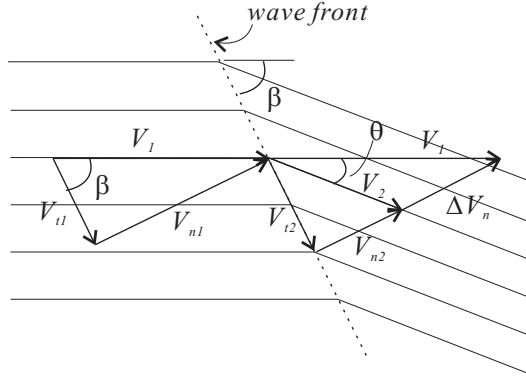


Figure 7.4: Flow crossing a wave front

By substituting Eq.(7.22) into the solution of θ in Eq.(7.21), we can express the solution of θ in Froude number, Fr . The solution in Eq.(7.22) is valid for Froude number ≥ 1 . The relations between $(\theta + \theta_1) \leftrightarrow Fr$ and $(\theta + \theta_1) \leftrightarrow h/H$ are plotted in Fig.(7.5) and Fig.(7.6) respectively.

$$\theta = \sqrt{3} \tan^{-1} \frac{\sqrt{3}}{\sqrt{Fr^2 - 1}} - \tan^{-1} \frac{1}{\sqrt{Fr^2 - 1}} - \theta_1 \quad (7.23)$$

It is worth to notice that from Eq.(7.20), the solution in Eq.(7.21) and Eq.(7.23) are valid under maximum and minimum conditions below;

$$V' = 1, \quad \theta = 0^\circ, \quad \frac{h}{H} = 0, \quad Fr = \infty \quad (7.24)$$

$$V' = \frac{1}{\sqrt{3}}, \quad \theta = 65^\circ 53', \quad \frac{h}{H} = \frac{2}{3}, \quad Fr = 1.0 \quad (7.25)$$

The flow near the abrupt expansion can be regarded as flow subjected continuous disturbances. In the case where the depth difference across the disturbance wave is small, the angle of the disturbance wave front β , which causes the change of flow direction θ is shown in Fig.(7.4), where the relation between Froude number Fr and β is given as follows (Ippen, 1951),

$$\sin \beta = \frac{1}{Fr} \quad (7.26)$$

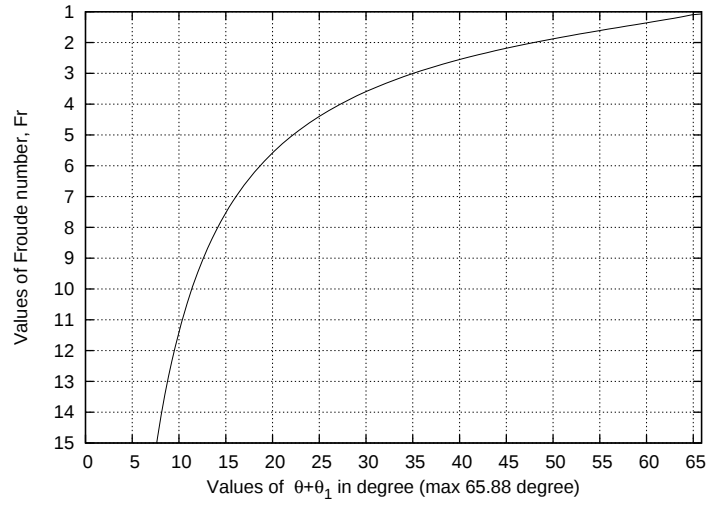


Figure 7.5: Value of $\theta + \theta_1$ corresponding to Froude number, Fr .

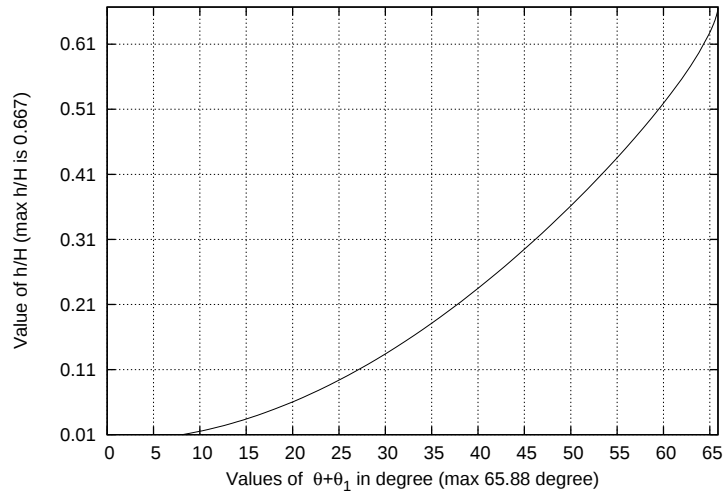


Figure 7.6: Value of $\theta + \theta_1$ corresponding to h/H .

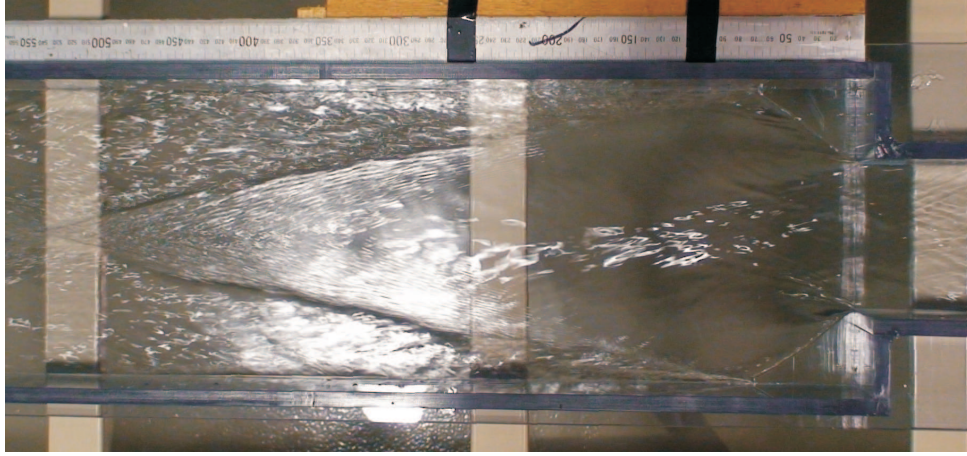


Figure 7.7: Laboratory experiment setup

7.3 Validation of characteristics of abrupt expansion flow

7.3.1 Experiment Setup

Laboratory experiment setup consists of a perspex channel of length of 4m, transverse width of 0.2m, height of 0.1m and approach width of 0.1m. Slope of the channel is adjusted to obtain desirable approach depth, h_o and Froude number. The depth of the flow is recorded in the interval of 10 mm from the upstream point until the point where the surge waves meet again at the end of the core flow. Fig.(7.7) shows the laboratory experiment of abrupt expansion flow, with approach depth of 16 mm and Froude number 2.78.

7.3.2 Experiment results

The laboratory experiment result is shown in Fig.(7.8), and the surface-contour is plotted in Fig.(7.9). The depth-contour of the laboratory experiment result is plotted in Fig.(7.10). The result shows the phenomenon of abrupt expansion flow clearly with the formation of core-flow, zero depth line as well as the shock wave. The flow is seen to spread almost linearly near the expansion before impinging the boundary wall. Formation of shock wave propagating from the point of impingement to the center of the flow can be seen clearly as well.

By referring to Fig.(7.10) the angle of flow expansion in the laboratory experiment is ($\theta = 41.0^\circ$). However, in the laboratory experiment, the angle of β is hard to observed. This is due to the size of the experiment setup and low Fr number.

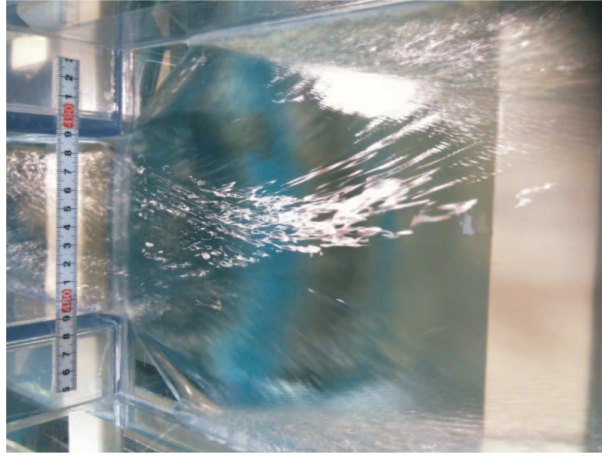


Figure 7.8: Laboratory experiment results at steady state with approach depth of 16mm and $Fr=2.78$

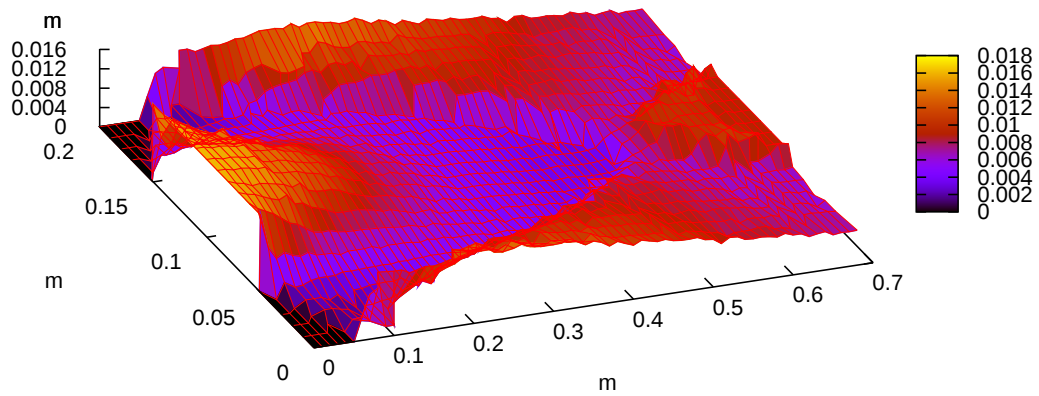
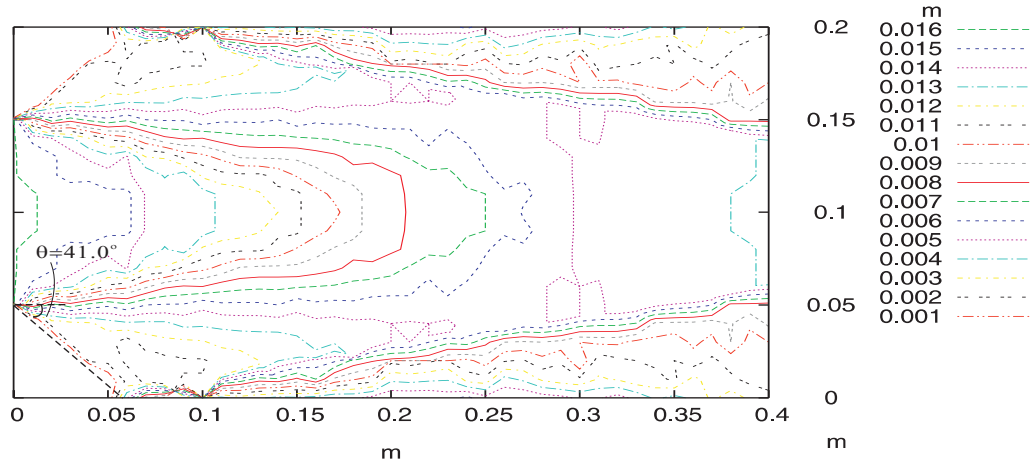


Figure 7.9: Surface contour of laboratory experiment results at steady state with approach depth of 16mm and Froude number 2.78


 Figure 7.10: Flow depth contour from laboratory experiment with $Fr=2.78$

7.3.3 Numerical model and simulation setup

The phenomenon of abrupt expansion flow is simulated numerically using a free-surface three-dimensional model based on the SOLA-VOF (Volume of Fluid) algorithm (Hirt and Nicholas, 1981). The governing equations of the numerical model are;

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (7.27)$$

Momentum equation

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} &= g_x - \frac{\partial p}{\partial x} + \nu \nabla^2 u \\ \frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} &= g_y - \frac{\partial p}{\partial y} + \nu \nabla^2 v \\ \frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} &= g_z - \frac{\partial p}{\partial z} + \nu \nabla^2 w \end{aligned} \quad (7.28)$$

The advection terms in the governing equation are solved using CIP method (Yabe and Aoki, 1991b). SOR (Successive Over Relaxation) method is used in the pressure iteration where pressure is adjusted iteratively to satisfy the continuity equation. The VOF density function, F is advected in its conservative form, Eq.(7.29) using CIP method.

$$\frac{\partial F}{\partial t} + \frac{\partial uF}{\partial x} + \frac{\partial vF}{\partial y} + \frac{\partial wF}{\partial z} = 0 \quad (7.29)$$

Eq.(7.28) can be written in the following form,

$$\frac{\partial \vec{f}}{\partial t} + (\vec{u} \cdot \nabla) \vec{f} = \vec{G} \quad (7.30)$$

where,

$$\vec{f} = (u, v, w) \quad \text{and} \quad \vec{G} = \begin{pmatrix} g_x - \frac{\partial p}{\partial x} + \nu \nabla^2 u, \\ g_y - \frac{\partial p}{\partial y} + \nu \nabla^2 v, \\ g_z - \frac{\partial p}{\partial z} + \nu \nabla^2 w \end{pmatrix} \quad (7.31)$$

Eq.7.30 is solved with the method of splitting it into 2 phases: the non-advection and advection phase explained in Chapter 5. For the advection of VOF density function F ,

$$\vec{f} = (F) \quad (7.32)$$

$$\vec{G} = \left(-F \frac{\partial u}{\partial x} - F \frac{\partial v}{\partial y} - F \frac{\partial w}{\partial z} \right) \quad (7.33)$$

In order to avoid the smearing of function F , which is the source of surface definition loss, digitizer in the following tangential function is used (Yamada et al., 1998).

$$\phi(F) = \tan[0.85\pi(F - 0.5)] \quad (7.34)$$

The phenomenon of abrupt expansion flow in a rectangular channel as shown in Fig.(7.2) is simulated numerically. The length in lateral x -direction is set to 800mm, in transverse y -direction 200mm, and in vertical z -direction, 40mm. All the boundaries are set to slip-condition in the case of inviscid fluid simulation, and non-slip condition in the case of viscous fluid. Inflow boundary condition is created in the upstream of the channel and atmospheric pressure flow is used to set the continuous outflow at the downstream of the channel. The time increment, Δt is set to 5/100000s and adjusted to satisfied the Courant ratio of 0.25.

The simulation is divided into two parts. The first simulation is set up to the laboratory experiment condition. The approach depth flow h_o is set to 16mm and approach velocity V_o is calculated based on the Froude number obtained from the laboratory experiment. The Froude number used is 2.78. Non-slip condition is applied on all boundary walls in the numerical model. Kinematic viscosity value is set to the value of water's, approximately $1.0 \times 10^{-6} m^2 s^{-1}$. In the second simulation, the condition of inviscid fluid is used. Kinematic viscosity is set to zero, while slip condition is applied on all boundary walls. The Froude number is increased to $Fr = 4$ and the approach depth is increased to 30mm. The simulation conditions are summarized in Table 7.1.

7.3.4 Numerical simulation results

The numerical simulation results are plotted in the form of depth-contour in Fig.7.11, Fig.7.12, Fig.7.13 and Fig.7.14. In the case of the numerical simulation, the angle of expansion θ is determined by drawing a line connecting the point where the expansion first initiated to the point of impingement at the boundary wall. This value of θ represent the angle calculated theoretically in Eq.(7.21) or Eq.(7.23). The value of β is determined by drawing a line connecting the upstream approach flow to the point at the center of the flow where the depth is equivalent

Table 7.1: Numerical simulation conditions

Experiment number	Fr2	Fr4
Froude number, Fr	2.78	4.0
Boundary conditions	non-slip	slip
Kinematic viscosity, ν	$1.0 \times 10^{-6} m^2 s^{-1}$	0.0
Approach depth, h_o	16 mm	30 mm
Approach width, b	100 mm	100 mm
Cell size, Δx	2 mm	5 mm
Cell size, Δy	2 mm	5 mm
Cell size, Δz	2 mm	5 mm

Table 7.2: Comparison of θ and β values of analytical, experimental and numerical results

Simulation Conditions	$Fr = 2.78$		$Fr = 4.00$	
Angles in degree	θ	β	θ	β
Analytical values	37.0	21.1	20.0	14.5
3D VOF CIP model	34.0	21.7	18.8	17.8
2D Depth averaged model	22.3	31.3	14.8	26.4
Experimental	41.0	undetectable	-	-

to the approach depth, h_o . Due to the limitation of the contour plot, the line of depth-contour representing the depth of flow nearest to the approach depth value is considered instead. The value of β is calculated theoretically in Eq.(7.26) and it represents the angle of flow disturbance line.

In the case of the first simulation with $Fr=2.78$, the flow expansion angle θ of the numerical simulation ($\theta = 34.0^\circ$) and theoretical analysis ($\theta = 37.0^\circ$) agrees with each other satisfactorily. The angle of β reproduced by the numerical model, ($\beta = 21.7^\circ$) also agrees satisfactorily with the theoretical value, ($\beta = 21.1^\circ$).

In the case of the second simulation with $Fr=4.0$, the numerical simulation result also agrees well with the theoretical analysis. The numerical model reproduced $\theta = 18.8^\circ$ while the theoretical analysis value of θ is 20.0° . The reproduction of angle β by the numerical model also agrees

fairly satisfactory with the theoretical analysis value. The numerical model reproduced $\beta = 17.8^\circ$ while the theoretical value is $\beta = 14.5^\circ$. A 2-D (two dimensional) depth averaged model is used to simulate the same condition as in the 3D VOF CIP model. The results from the 2-D depth averaged model are less satisfactory compared to the 3D VOF CIP results. The comparison of angle θ and β from the results of theoretical analysis, numerical simulation, and laboratory experiment are summarized in Table 7.3.3.

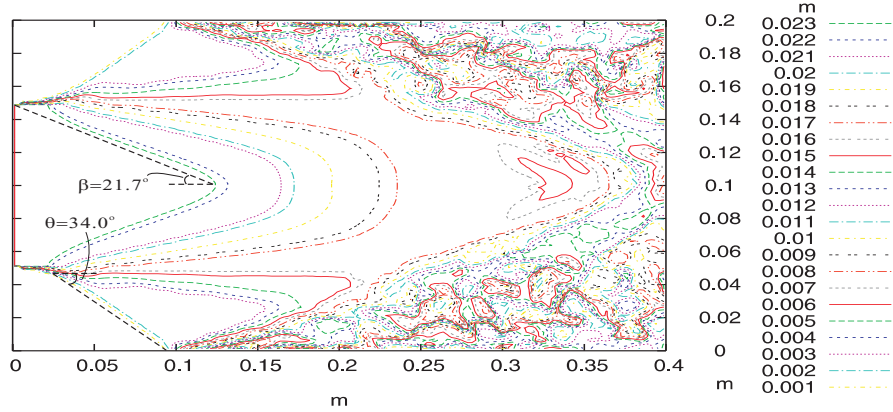


Figure 7.11: Flow depth contour with $Fr=2.78$ using 3D-VOF-CIP model

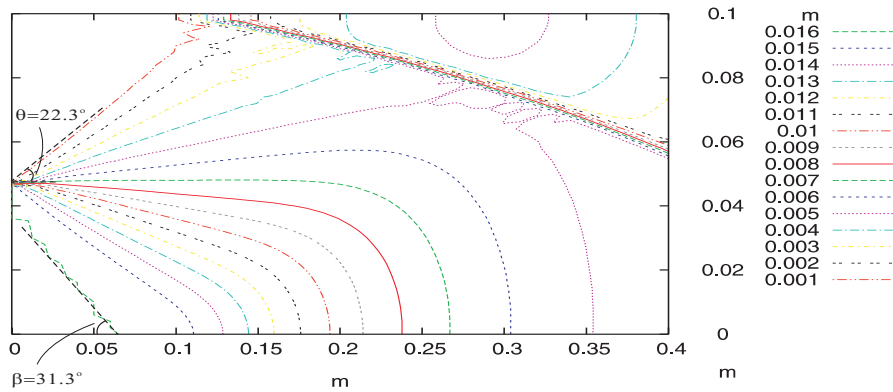
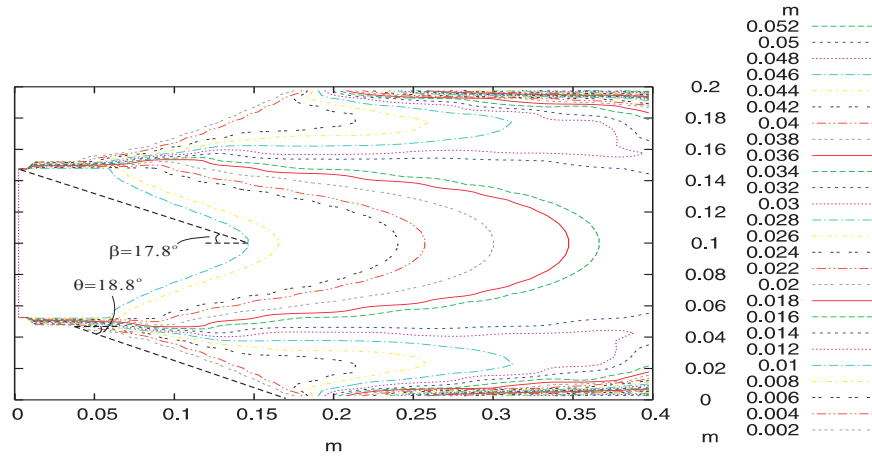
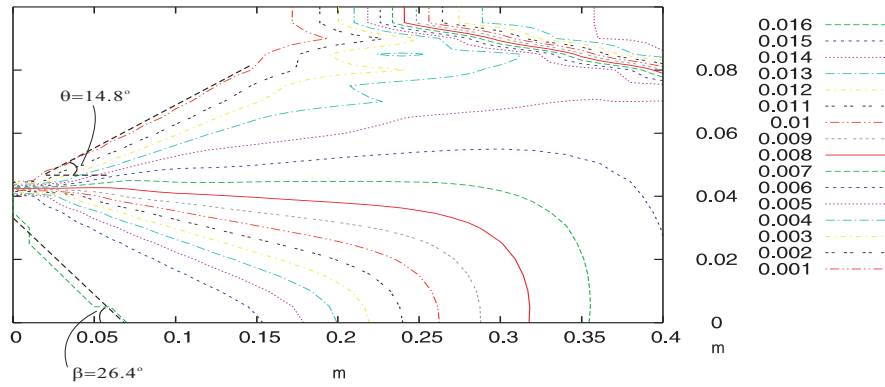


Figure 7.12: Flow depth contour with $Fr=2.78$ using 2-D depth averaged model

Figure 7.13: Flow depth contour with $Fr=4.0$ using 3D-VOF-CIP modelFigure 7.14: Flow depth contour with $Fr=4.0$ using 2-D depth averaged model

7.4 Summary

In this chapter, the fundamental study of abrupt expansion flow in the case of supercritical flow is presented. The characteristics of the flow near the abrupt expansion is analyzed qualitatively and quantitatively through laboratory experiments, numerical simulations and theoretical analysis. The formation of zero-depth-line, core-flow and shock waves can be clearly seen in the laboratory experiments and reproduced well in the numerical simulations. The theoretical analysis using the method of characteristics shows that the angle of flow expansion as well as the angle of disturbance are related to the Froude number of the flow.

Chapter 8

Conclusion

8.1 Summary

A fundamental study of flow characteristics was carried out by means of dam-break flow of finite extent in this dissertation. The results from the study of the dam-break flow of finite extent model were applied in the determination of inertial and viscous phase flow characteristics for viscous fluid, comprising of Newtonian, non-Newtonian and Bingham fluid.

Firstly, the classical dam-break flow of infinite extent solution, also known as the Ritter solution was reviewed. Based on the problem of dam-break flow of infinite extent, a method was devised to derive the approximate analytical solution for the problem of dam-break flow of finite extent from the moment the flow is initiated by the instantaneous release of lock gate to sufficiently long time. Taking advantage that Ritter solution can be used to describe the flow in the region ahead of the first reflected negative wave, the task to derive a full analytical solution for the problem was partly done. The region trailing behind the first reflected negative wave was shown to be obtainable if there is sufficient boundary condition at any two points in that region. It is due to the fact that the region is of sub-critical flow. The approximate solution was derived and verified with numerical models. By solving the problem of finite-extent dam break flow of ideal fluid, two parameters which are the propagation of front wave and the attenuation of depth at the origin were used to represent the characteristic of inertial phase flow.

In the succeeding chapters, the viscous phase flow characteristics were investigated based on the same dam-break flow of finite extent model. The analytical study framework was based on the assumption that self-similarity of depth and velocity of flow exist during the viscous phase flow. Based on these assumptions, a general solution for power-law fluid, was derived for the propagation of front wave and attenuation of depth at the origin. By using this general solution for power-law fluid, the flow characteristics for Newtonian and non-Newtonian fluid could be easily derived. The characteristics of the viscous phase flow were verified using Moving Particle Semi-Implicit (MPS) model. The numerical results were found to be in good agreement with the analytical findings on the characteristics of viscous phase flow.

In order to re-confirm the characteristic of inertial and viscous phase flow, another analytical method was used. Integral transformation method was used to solve the governing equation of

dam-break flow of finite extent. The information from the study of the flow characteristics in the earlier part of this dissertation contributed to the development of the integral model. Solutions from the integral model showed good agreement with the findings from the earlier chapters in this dissertation. The integral model also showed the transitions of inertial phase to viscous phase flow qualitatively.

The application of dam-break flow of finite extent was carried out in the simulation of slump flow test of fresh concrete. The rheological properties of fresh concrete which was treated as a kind of Bingham fluid, was investigated in the study. The characteristics of the inertial and viscous phase flow for Bingham fluid were derived to describe the flow behavior of fresh concrete. A numerical model to simulate the slump flow test was developed using VOF method with higher order scheme (CIP scheme) to solve the advection term in the momentum equations as well as the advection of volume of fluid function F . The numerical model was verified with a set of experimental data of slump flow test. This model was used in the latter part to validate the analytical findings.

In the final chapter, the phenomenon of abrupt expansion flow was investigated qualitatively and analytically. A three-dimensional numerical model based on the VOF-CIP model was developed to verify the characteristic of the phenomenon, such as the angle of expansion and the angle of wave front generated by the sudden change of flow direction. Laboratory experiments were carried out to verify the numerical model as well as the analytical findings.

8.2 Recommendation

The work in the derivation of viscous phase flow can be extended to include the determination of flow profile and velocity profile which can be used to describe the whole flow phases : inertial and viscous phase. This same improvement is applicable for non-Newtonian and Bingham fluid as well. By deriving the shape function for the flow depth and velocity, the flow characteristic of viscous fluid comprising of Newtonian, non-Newtonian and Bingham fluids can be represented more precisely. The determination of the point of transition of flow phase analytically is also recommended.

In the development of numerical model, especially in the two-dimensional model used to simulate the slump flow test of fresh concrete, the discrepancy of simulation results is to be reduced by re-examining the weakness of the numerical model, especially the way the second invariant strain rate is calculated in the model. The author also plans to improve the model by using other Bingham constitutive law model.

Appendix A

CIP 2-dimensional solver

The two-dimensional CIP solver is as follows (Yabe and Aoki, 1991a) with input variables: u , v , f , $g_x = \partial f / \partial x$, $g_y = \partial f / \partial y$, Δx , Δy and Δt . The output of this routine is f^{n+1} , g_x^{n+1} and g_y^{n+1}

$$XX = -u\Delta t$$

$$YY = -v\Delta t$$

$$isgn = sgn(u)$$

$$jsgn = sgn(v)$$

$$X1 = isgn \times \Delta x$$

$$Y1 = jsgn \times \Delta y$$

$$iup = i - isgn$$

$$jup = j - jsgn$$

$$A8 = f_{i,j} - f_{iup,j} - f_{i,jup} + f_{iup,jup}$$

$$TMP = GY_{iup,j} - GY_{i,j}$$

$$A1 = [(GX_{iup,j} + GX_{i,j}) \times X1 - 2(f_{i,j} - f_{iup,j})] / X1^3$$

$$A2 = [-A8 - (GX_{i,jup} - GX_{i,j}) \times X1] / (X1^2 \times Y1)$$

$$A3 = [3(f_{iup,j} - f_{i,j}) + (GX_{iup,j} + 2GX_{i,j}) \times X1] / X1^2$$

$$A4 = (A2 \times X1^2 - TMP) / X1$$

$$A5 = [-2(f_{i,j} - f_{i,jup}) + (GY_{i,jup} + GY_{i,j}) \times Y1] / Y1^3$$

$$A6 = (-A8 - TMP \times Y1) / (X1 \times Y1^2)$$

$$A7 = [3(f_{i,jup} - f_{i,j}) + (GY_{i,jup} + 2GY_{i,j}) \times Y1] / Y1^2$$

$$f_{i,j}^{n+1} = ((A1 \times XX + A2 \times YY + A3)XX + A4 \times YY + GX_{i,j})XX + ((A5 \times YY + A6 \times XX + A7) \times YY + GY_{i,j})YY + f_{i,j} \quad (A.1)$$

$$GX_{i,j}^{n+1} = (3A1 \times XX + 2(A2 \times YY + A3))XX + (A4 + A6 \times YY)YY + GX_{i,j} \quad (A.2)$$

$$GY_{i,j}^{n+1} = (3A5 \times YY + 2(A6 \times XX + A7))YY + (A4 + A2 \times XX)XX + GY_{i,j} \quad (A.3)$$

where $GX_{i,j}$ and $GY_{i,j}$ represent $g_{xi,j}$ and $g_{yi,j}$ respectively.

Appendix B

CIP 3-dimensional solver

The three-dimensional CIP solver is as follows (Yabe and Aoki, 1991a) with input variables: u , v , w , f , $g_x = \partial f / \partial x$, $g_y = \partial f / \partial y$, $g_z = \partial f / \partial z$, Δx , Δy , Δz and Δt . The output of this routine is f^{n+1} , g_x^{n+1} , g_y^{n+1} and g_z^{n+1} .

$$\begin{aligned}
 XX &= -u\Delta t \\
 YY &= -v\Delta t \\
 ZZ &= -w\Delta t \\
 isgn &= \text{sgn}(u) \\
 jsgn &= \text{sgn}(v) \\
 ksgn &= \text{sgn}(w) \\
 X1 &= isgn \times \Delta x \\
 Y1 &= jsgn \times \Delta y \\
 Z1 &= ksgn \times \Delta z \\
 iup &= i - isgn \\
 jup &= j - jsgn \\
 kup &= k - ksgn
 \end{aligned}$$

$$\begin{aligned}
 A01 &= [(GX_{i,j,k} + GX_{iup,j,k}) \times X1 + 2(f_{i,j,k} - f_{iup,j,k})] / X1^3 \\
 A11 &= [3(f_{iup,j,k} + f_{i,j,k}) - (GX_{iup,j,k} + 2GX_{i,j,k}) \times X1] / X1^2 \\
 A02 &= [(GY_{i,j,k} + GY_{i,jup,k}) \times Y1 + 2(f_{i,j,k} - f_{i,jup,k})] / Y1^3 \\
 A12 &= [3(f_{i,jup,k} + f_{i,j,k}) - (GY_{i,jup,k} + 2GY_{i,j,k}) \times Y1] / Y1^2 \\
 A03 &= [(GZ_{i,j,k} + GZ_{i,j,kup}) \times Z1 + 2(f_{i,j,k} - f_{i,j,kup})] / Z1^3 \\
 A13 &= [3(f_{i,j,kup} + f_{i,j,k}) - (GZ_{i,j,kup} + 2GZ_{i,j,k}) \times Z1] / Z1^2
 \end{aligned}$$

$$\begin{aligned}
 B1 &= f_{i,j,k} - f_{iup,j,k} - f_{i,jup,k} - f_{iup,jup,k} \\
 B2 &= f_{i,j,k} - f_{i,jup,k} - f_{i,j,kup} - f_{i,jup,kup} \\
 B3 &= f_{i,j,k} - f_{iup,j,k} - f_{i,j,kup} - f_{iup,j,kup} \\
 A05 &= [B1 - (-GY_{i,j,k} + GY_{iup,j,k}) \times Y1] / (X1 \times Y1^2) \\
 A04 &= [B1 - (-GX_{i,j,k} + GY_{i,jup,k}) \times X1] / (X1^2 \times Y1) \\
 A14 &= [-B1 + (-GX_{i,j,k} + GX_{i,jup,k}) \times X1 + (-GY_{i,j,k} + GY_{iup,j,k}) \times Y1] / \\
 &\quad (X1 \times Y1) \\
 A09 &= [B2 - (-GZ_{i,j,k} + GZ_{i,jup,k}) \times Z1] / (Y1 \times Z1^2) \\
 A08 &= [B2 - (-GY_{i,j,k} + GY_{i,j,kup}) \times Y1] / (Y1^2 \times Z1) \\
 A15 &= [-B2 + (-GY_{i,j,k} + GY_{i,j,kup}) \times Y1 + (-GZ_{i,j,k} + GZ_{i,jup,k}) \times Z1] / (Y1 \times Z1) \\
 A07 &= [B3 - (-GX_{i,j,k} + GX_{i,j,kup}) \times X1] / (X1^2 \times Z1) \\
 A06 &= [B3 - (-GZ_{i,j,k} + GZ_{iup,j,k}) \times Z1] / (X1 \times Z1^2) \\
 A16 &= [-B3 (-GZ_{i,j,k} + GZ_{iup,j,k}) \times Z1 + (-GZ_{i,j,k} + GZ_{i,j,kup}) \times X1] / (X1 \times Z1) \\
 A10 &= [-f_{i,j,k} + (f_{iup,j,k} + f_{i,jup,k} + f_{i,j,kup}) - (f_{iup,jup,k} + f_{iup,j,kup} + f_{i,jup,kup}) + f_{iup,jup,kup}] / \\
 &\quad (X1 \times Y1 \times Z1)
 \end{aligned}$$

$$\begin{aligned}
 f_{i,j,k}^{n+1} &= [(A01 \times XX + A04 \times YY + A07 \times ZZ + A11)XX + A14 \times YY + GX_{i,j,k}]XX \\
 &\quad [(A02 \times YY + A05 \times XX + A08 \times ZZ + A12)YY + A15 \times ZZ + GY_{i,j,k}]YY \\
 &\quad [(A03 \times ZZ + A06 \times XX + A09 \times YY + A12)ZZ + A16 \times XX + GZ_{i,j,k}]ZZ \\
 &\quad + A10 \times XX \times YY \times ZZ + f_{i,j,k}
 \end{aligned} \tag{B.1}$$

$$\begin{aligned}
 GX_{i,j,k}^{n+1} &= [3A01 \times XX + 2(A04 \times YY + A07 \times ZZ + A11)]XX \\
 &\quad (A05 \times YY + A10 \times ZZ + A14)YY \\
 &\quad (A06 \times ZZ + A16)ZZ + GX_{i,j,k}
 \end{aligned} \tag{B.2}$$

$$\begin{aligned}
 GY_{i,j,k}^{n+1} &= [3A02 \times YY + 2(A05 \times XX + A08 \times ZZ + A12)]YY \\
 &\quad (A09 \times ZZ + A10 \times XX + A15)ZZ \\
 &\quad (A04 \times XX + A14)XX + GY_{i,j,k}
 \end{aligned} \tag{B.3}$$

$$\begin{aligned}
 GZ_{i,j,k}^{n+1} &= [3A03 \times ZZ + 2(A06 \times XX + A09 \times YY + A13)]ZZ \\
 &\quad (A07 \times XX + A10 \times YY + A16)XX \\
 &\quad (A08 \times YY + A15)YY + GZ_{i,j,k}
 \end{aligned} \tag{B.4}$$

where $GX_{i,j,k}$, $GY_{i,j,k}$ and $GZ_{i,j,k}$ represent $g_{xi,j,k}$, $g_{yi,j,k}$ and $g_{zi,j,k}$ respectively

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