

Exact WKB analysis of anharmonic oscillators.

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Anharmonic oscillators have attracted much attention of physicists, particularly because of their relevance to the ϕ^4 -model in quantum field theory.

Here we show how to apply exact WKB analysis to the analysis of their eigenvalue problems, namely,

$$\left(-\frac{d^2}{dx^2} + \frac{1}{4}x^2(1 + \lambda x^{2N})\right)\psi = E(\lambda)\psi \quad (\lambda > 0),$$

$$\lim_{|x| \rightarrow \infty} \psi(x) = 0.$$

For $\lambda \ll 1$, an eigenvalue $E(\lambda)$ has a formal expansion with respect to λ (the so-called Rayleigh-Shrödinger perturbation series) ;

$$E^K(\lambda) = K + \frac{1}{2} + \sum_{n=1}^{\infty} A_n^K \lambda^n \quad \text{where } K = 0, 1, 2, \dots.$$

Our purpose is to determine the asymptotic behavior of A_n^K for arbitrary N . The result is as follows :

$$A_n^K = \frac{(-1)^{n+1} N}{K!(2\pi^3)^{1/2}} 4^{(K+\frac{1}{2})/N} \left(\frac{B(\frac{3}{2}, \frac{1}{N})}{2N}\right)^{-K-\frac{1}{2}-nN} \Gamma\left(K + \frac{1}{2} + nN\right) \left(1 + O\left(\frac{1}{n}\right)\right), \quad (n \rightarrow \infty).$$

Here $B(x, y)$ denotes the Beta function.

For $N = 1$, this result was shown by Bender and Wu (Phys.Rev.D,7(1973)) in a heuristic manner. By making use of exact WKB analysis we verify the result rigorously.