

# Semi-hyperbolicity of entire functions

Masashi KISAKA (木坂 正史)

Department of Mathematical Sciences,  
Graduate School of Human and Environmental Studies,  
Kyoto University, Yoshida Nihonmatsu-Cho, Kyoto 606-8501, Japan  
(京都大学大学院 人間・環境学研究科 数理科学講座)

e-mail : kisaka@math.h.kyoto-u.ac.jp

## Abstract

In this paper, we investigate a condition for semi-hyperbolicity of (transcendental) entire functions (Theorem A). As an application of the main theorem, we show a result on a measure theoretical property for the dynamics of entire functions (Theorem B). In particular, we give a sufficient condition which guarantees that  $\{\infty\}$  is a metric global attractor (Corollary C).

## 1 Preliminaries

Let  $f$  be an entire function and  $f^n$  denote the  $n$ -th iterate of  $f$ . Recall that the *Fatou set*  $F_f$  and the *Julia set*  $J_f$  of  $f$  are defined as follows:

$$F_f := \{z \in \mathbb{C} \mid \{f^n\}_{n=1}^\infty \text{ is a normal family in a neighborhood of } z\},$$
$$J_f := \mathbb{C} \setminus F_f.$$

By definition,  $F_f$  is open and  $J_f$  is closed in  $\mathbb{C}$ . Also  $J_f$  is compact if  $f$  is a polynomial, while it is non-compact if  $f$  is transcendental. This is due to the fact that  $\infty$  is an essential singularity of  $f$ . A connected component  $U$  of  $F_f$  is called a *Fatou component* of  $f$ .  $U$  is called a *wandering domain* if  $f^m(U) \cap f^n(U) = \emptyset$  for every  $m, n \in \mathbb{N}$  ( $m \neq n$ ). If there exists an  $n_0 \in \mathbb{N}$  with  $f^{n_0}(U) \subseteq U$ ,  $U$  is called a *periodic component of period  $n_0$*  and it is well known that there are four possibilities, namely, an *attracting basin*, a *parabolic basin*, a *Siegel disk* and a *Baker domain*.

A *critical value* is a point  $p := f(c)$  for a point  $c$  with  $f'(c) = 0$ . This is a singularity of  $f^{-1}$ . For polynomials we have only to consider this type of singularities but there can be another type of singularities called an *asymptotic value* for transcendental entire functions. A point  $p$  is called an *asymptotic value* if there exists a continuous curve  $L(t)$  ( $0 \leq t < 1$ ) (which is called an *asymptotic path*) with

$$\lim_{t \rightarrow 1} L(t) = \infty \quad \text{and} \quad \lim_{t \rightarrow 1} f(L(t)) = p.$$

A point  $p$  is called a *singular value* if it is either a critical or an asymptotic value and we denote the set of all singular values by  $\text{sing}(f^{-1})$ . Also we define

$$P(f) := \overline{\bigcup_{n=0}^{\infty} f^n(\text{sing}(f^{-1}))}$$

and call it the *post-singular set* of  $f$ .

The following are some basic concepts from dynamical system theory:

**Definition 1.1.** Let  $f : \mathbb{C} \rightarrow \mathbb{C}$  be an entire function and  $z \in \mathbb{C}$ .

(1) The *forward orbit* of a point  $z$  is the set

$$O^+(z) := \{z, f(z), \dots, f^n(z), \dots\}.$$

(2) We define

$$\omega(z) := \{w \mid w = \lim_{n_i \nearrow \infty} f^{n_i}(z), \exists n_1 < n_2 < \dots\}$$

and call it the  $\omega$ -*limit set* of  $z$ .

(3) A point  $z$  is called *recurrent* if  $z \in \omega(z)$ , that is, the forward orbit of  $z$  passes through an arbitrary small neighborhood of  $z$  infinitely often. Otherwise, it is called *non-recurrent*.

(4)  $f$  is called *ergodic* if any measurable set  $A$  satisfying  $f^{-1}(A) = A$  has zero or full measure in  $\mathbb{C}$ .

## 2 The Mañé's Theorem —Semi-hyperbolicity—

The following is a part of the Mañé's theorem, which was proved in 1993.

**Theorem 2.1** (Mañé, [M]). *Let  $f$  be a rational function and  $x \in J_f$ . Suppose that*

- (i)  *$x$  is not a parabolic periodic point and*
- (ii)  *$x \notin \bigcup_{c \in \text{Rec} \cap J_f} \omega(c)$ ,*

*where*

$$\text{Rec} = \{\text{recurrent critical points of } f\}.$$

*Then for every  $\varepsilon > 0$ , there exists a neighborhood  $U$  of  $x$  which satisfies the following:*

- (1) *For every  $n \in \mathbb{N}$  and every connected component  $V$  of  $f^{-n}(U)$ ,*

$$\text{diam}_{\text{sph}}(V) \leq \varepsilon$$

*holds, where  $\text{diam}_{\text{sph}}$  denotes the spherical diameter on  $\hat{\mathbb{C}}$ .*

- (2) *There exists an  $N \in \mathbb{N}$  such that for any connected component  $V$  of  $f^{-n}(U)$  ( $\forall n$ ),  $f^n|_V : V \rightarrow U$  satisfies*

$$\deg(f^n|_V : V \rightarrow U) \leq N.$$

Taking this result into account, we define the semi-hyperbolicity of  $f$  at a point  $x_0 \in J_f$  as follows:

**Definition 2.2.**  *$f$  is semi-hyperbolic at  $x \in J_f$  if there exists a neighborhood  $U$  of  $x$  such that the condition (2) in Theorem 2.1 holds. In the case that  $f$  is transcendental, we add the following property:*

$$f^n|_V : V \rightarrow U \text{ is proper for every } V.$$

Recall that  $f : X \rightarrow Y$  is called *proper* if  $f^{-1}(K) \subset X$  is compact for every compact subset  $K \subset Y$ . Note that this property is automatically satisfied when  $f$  is a polynomial or rational. We say  *$f$  is semi-hyperbolic* if  $f$  is semi-hyperbolic at any point  $x_0 \in J_f$ .

The converse of Theorem 2.1 is also true. That is, if  $x$  is a parabolic periodic point or  $x \notin \bigcup_{c \in \text{Rec} \cap J_f} \omega(c)$ , then  $f$  is not semi-hyperbolic at  $x \in J_f$ . In this paper we investigate a condition for semi-hyperbolicity for transcendental entire functions. In transcendental case, a new phenomena can occur. For example, Bergweiler and Morosawa ([BM]) constructed an example of  $f$  with no parabolic periodic point and no recurrent critical point, but has a point  $x_0 \in J_f$  at which  $f$  is not semi-hyperbolic.

### 3 Main Result

Define the sets Rec, Non-Rec and AV as follows:

$$\begin{aligned}\text{Rec} &:= \{c \mid c \text{ is a recurrent critical point of } f\} \\ \text{Non-Rec} &:= \{c \mid c \text{ is a non-recurrent critical point of } f\} \\ \text{AV} &:= \{c \mid c \text{ is an asymptotic value of } f\}.\end{aligned}$$

Then the main result of this paper is the following:

**Theorem A (Mañé's Theorem for entire functions).** *Let  $f$  be a (transcendental) entire function and  $z_0 \in J_f$ . Then  $f$  is semi-hyperbolic at  $z_0$  if and only if  $z_0 \notin Z$ , where the set  $Z$  is defined as follows:*

$$Z = \overline{\left(\bigcup_{i=1}^3 X_i\right) \cup \left(\bigcup_{j=1}^5 Y_j\right)},$$

where

$$\begin{aligned}X_1 &= \overline{\{p \mid p \text{ is a parabolic periodic point of } f\}}, \\ X_2 &= \text{derived set of } \{p \mid p \text{ is a attracting periodic point of } f\}, \\ X_3 &= \{p \mid f^{n_i}|_W \rightarrow p \text{ (} n_i \rightarrow \infty \text{) for some wandering domain } W\}, \\ Y_1 &= \overline{\bigcup_{c \in \text{Rec} \cap J_f} \omega(c)}, \quad Y_2 = \overline{\bigcup_{n=0}^{\infty} f^n(\text{AV}) \cap J_f}, \\ Y_3 &= \{p \mid p = \lim_{i \rightarrow \infty} f^{n_i}(c_i), \text{ } c_i \in \text{Non-Rec} \cap J_f \text{ (} i \in \mathbb{N} \text{) are mutually} \\ &\quad \text{different and order of } c_i \rightarrow \infty \text{ (} i \rightarrow \infty \text{)}\}, \\ Y_4 &= \left\{p \mid p = \lim_{i \rightarrow \infty} f^{n_i}(c_i), \text{ } c_i \in \text{Non-Rec} \cap J_f \text{ (} i \in \mathbb{N} \text{) are mutually} \right. \\ &\quad \text{different with } \sup_i (\text{order of } c_i) < \infty \text{ and for any } \varepsilon > 0 \\ &\quad \text{let } N_i(\varepsilon) := \#\{c \mid c : \text{critical point, } O^+(c_i) \cap U_\varepsilon(c) \neq \emptyset\} \\ &\quad \left. \text{then } \sup_i N_i(\varepsilon) = \infty\right\}, \\ Y_5 &= \left\{p \mid p = \lim_{i \rightarrow \infty} f^{n_i}(c_i), \text{ } c_i \in \text{Non-Rec} \cap J_f \text{ (} i \in \mathbb{N} \text{) are mutually} \right. \\ &\quad \text{different with } \sup_i (\text{order of } c_i) < \infty \text{ and let} \\ &\quad \delta_i(n) := \sup\{\delta \mid \#\{O^+(c_i) \cap (U_\delta(c_i) \setminus \{c_i\})\} \leq n\} \\ &\quad \left. \text{then } \inf_i \delta_i(n) = 0 \text{ for } \forall n\right\}.\end{aligned}$$

## 4 Outline of the proof of Theorem A

Suppose  $z_0 \in J_f$ ,  $z_0 \notin Z$ , then take a neighborhood  $U$  of  $z_0$  with  $\overline{U} \cap Z = \emptyset$ .

**Definition 4.1.** For  $z \in U$  let  $S(z, \varepsilon)$  be a square centered at  $z$  with side length  $2\varepsilon$  and with sides parallel to coordinate axes. We say  $S(z, \varepsilon)$  is *admissible* if  $S(z, 3\varepsilon) \subset U$ .

**Lemma 4.2.** *For a given  $\varepsilon > 0$  and an  $N \in \mathbb{N}$ , there exists a  $\delta > 0$  which satisfies the following: If  $S(z, \delta)$  is an admissible square and  $S_n$  is a connected component of  $f^{-n}(S(z, \delta))$  such that  $\deg(f^n|_{S_n}) \leq N$ , then*

$$\text{diam}(f^{-n}(S(z, \frac{\delta}{2}))) \leq \varepsilon$$

*holds for the same branch of  $f^{-n}$ .*

**(Proof of Lemma 4.2)** : Suppose not, then there exist a  $z_l \in U$  and admissible squares  $S^l := S(z_l, 2^{-l})$  such that for some component  $V_l$  of  $f^{-n_l}(S(z_l, 2^{-(l+1)}))$  it holds that  $\text{diam} V_l \geq \varepsilon > 0$  and  $\deg(f^{n_l}|_{S(z_l, 2^{-l})}) \leq N$ .

Now suppose there exist a subsequence  $l_k \nearrow \infty$  and a disk  $D_{l_k} \subset V_{l_k}$  with (spherical) radius  $r > 0$  which is independent of  $l_k$ . Taking subsequence, if necessary, we have

$$D_{l_k} \rightarrow \exists D \quad (k \rightarrow \infty).$$

Then  $\{f^{n_{l_k}}|_D\}_{k=1}^\infty$  is bounded, since  $f^{n_{l_k}}(D) \subset U$ . Hence  $\{f^{n_{l_k}}|_D\}_{k=1}^\infty$  is normal. So we have  $D \subset F_f$  and let  $D_{F_f} \supset D$  be the Fatou component containing  $D$ . On the other hand, taking subsequence, if necessary, we have

$$S^{l_k} \rightarrow \exists z_\infty \in U \quad (k \rightarrow \infty).$$

Then

$$f^{n_{l_k}}|_D \rightarrow z_\infty.$$

Such a  $z_\infty$  is either one of the following:

- (i) attracting periodic point,
- (ii) parabolic periodic point,
- (iii) finite constant limit function on a wandering domain.

In other words,  $D_{F_f}$  is not a Siegel disk or a Baker domain. This is a contradiction by the assumption. Hence let  $D_l$  be the maximal disk in  $V_l$ , then it follows that  $\text{diam}(D_l) \rightarrow 0$ . This again contradicts the following

**Lemma 4.3** (cf. Carleson-Jones-Yoccoz, [CJY]). *Let  $W \subset \mathbb{C}$  be a simply connected domain and let  $g : W \rightarrow \mathbb{D}$ ,  $g(\partial W) \subset \partial \mathbb{D}$  be degree  $N$ . Then there exists a constant  $C > 0$  depending only on  $N$  such that*

$$B_{\mathbb{D}}(g(z), Cr) \subset g(B_W(z, r)) \subset B_{\mathbb{D}}(g(z), r).$$

□

Now since  $z_0 \notin Z$ , there is a neighborhood  $U$  of  $z_0$  satisfying

(0)  $U$  does not contain attracting periodic points, parabolic periodic points, wandering domains, points in orbits of recurrent critical points or asymptotic values.

Moreover,  $U$  satisfies either one of the following:

(1) The number of critical points with  $O^+(c) \cap U \neq \emptyset$  is finite (let us denote them by  $c_1, c_2, \dots, c_{N_0}$ ) and all of them are non-recurrent. Then for some  $\varepsilon_0 > 0$  we have

$$(O^+(c_i) \setminus \{c_i\}) \cap U_{\varepsilon_0}(c_i) = \emptyset.$$

(2) The number of critical points with  $O^+(c) \cap U \neq \emptyset$  is infinite (let us denote them by  $c_1, c_2, \dots$ ) and all of them are non-recurrent. There exists an  $M_0 > 0$  such that

$$\text{order of } c_i \leq M_0, \text{ for } \forall i \in \mathbb{N}.$$

Also there exists an  $\varepsilon_1 > 0$  and an  $N_0 \in \mathbb{N}$  such that

$$\#\{c \mid c : \text{critical point, } O^+(c) \cap U_{\varepsilon_1}(c) \neq \emptyset\} \leq N_0 < \infty$$

holds for every  $i \in \mathbb{N}$ . Furthermore there exists a  $\delta_1 > 0$  and an  $n_1 \in \mathbb{N}$  such that

$$\#\{O^+(c_i) \cap (U_{\delta_1}(c_i) \setminus \{c_i\})\} \leq n_1, \forall i \in \mathbb{N}.$$

In this case, we put  $\varepsilon_0 := \min(\varepsilon_1, \delta_1)$

Now let  $N := (M_0 + 1)^{N_0(n_1+1)}$  and take  $\varepsilon > 0$  with  $\varepsilon < \varepsilon_0/36N$ . Then there is a  $\delta > 0$  which is determined by the previous Lemma 4.2.

**Lemma 4.4.** *For any  $\eta$  with  $0 < \eta \leq \delta$  and  $n \in \mathbb{N}$ , we have*

$$\text{diam}(f^{-n}(S(z_0, \frac{1}{2}\eta))) \leq \varepsilon.$$

That is, the conclusion of Lemma 4.2 holds without the assumption on degree.  $\square$

Hence for any  $\varepsilon > 0$  with  $\varepsilon < \varepsilon_0/36N$  by taking  $\sigma > 0$  sufficiently small, we have

$$\text{diam}(f^{-n}(S(z_0, \sigma))) \leq \varepsilon, \forall n.$$

With a little more argument, we can conclude

$$\deg(f^n|_{S(z_0, \sigma)}) < N = (M_0 + 1)^{N_0(n_1+1)}.$$

For the opposite implication, it is rather easy to check that  $z_0 \in Z$  implies that  $f$  is not semi-hyperbolic at  $z_0$ .  $\square$

**Remark.** (1) Comparing Theorem A with the original Mañé's Theorem, in the case that  $f$  is rational, we have

$$Z = X_1 \cup Y_1$$

i.e.  $X_2, X_3, Y_2, Y_3, Y_4, Y_5$  are all empty.

(2) Theorem A includes the following result:

**Theorem 4.5 (Bergweiler-Morosawa (2002)).** *Let  $f$  be entire. If  $f$  is semi-hyperbolic at  $a \in \mathbb{C}$ , then  $a$  is not a limit function of  $\{f^n\}_{n=1}^\infty$  in any component of  $F_f$ .*

(3) Consider the following question:

**Question :** For each  $X_i$  ( $i = 1 \sim 3$ ) and  $Y_j$  ( $j = 1 \sim 5$ ), is there an  $f$  with  $X_i \neq \emptyset$  or  $Y_j \neq \emptyset$ ?

First, there are a lot of  $f$  with  $X_1 \neq \emptyset$ . But I do not know whether parabolic periodic points can accumulate to a finite point in  $\mathbb{C}$ . It is somehow surprising that there is an  $f$  with  $X_2 \neq \emptyset$ . We can construct such an example by using the similar method in [KS]. We omit the details. For  $X_3$ , Eremenko and Lyubich ([EL]) constructed an  $f$  with  $X_3 \neq \emptyset$ , that is,  $f$  has a wandering domain with (infinitely many) finite constant limit functions.

There are a lot of  $f$  with  $Y_1 \neq \emptyset$  or  $Y_2 \neq \emptyset$ . It is not difficult to construct an  $f$  with  $Y_3 \neq \emptyset$ . For  $Y_4$ , Bergweiler and Morosawa ([BM]) showed the

following example: Consider

$$f(z) = \frac{z}{2} - \frac{1}{2\pi} \sin \pi z + c(\cos \pi z - 1),$$

where  $c = 0.467763 \dots$  is a solution of

$$\pi + 2 \cos 2c\pi - 4c\pi \sin 2c\pi = 0.$$

Then,  $f$  has no asymptotic values, no parabolic periodic point and no recurrent critical point, but  $f$  is not semi-hyperbolic at  $1 \in J_f$ . This  $f$  has a sequence of critical points  $\{c_i\}_{i=1}^{\infty}$  with

$$f(c_i) = c_{i-1} \quad (i = 2, 3, \dots), \quad f(c_1) = 1$$

and  $f(1)$  is a repelling fixed point of  $f$  so  $1 \in J_f$ . Hence  $1 \in Y_4$  in this case. Finally we do not know an example of  $f$  with  $Y_5 \neq \emptyset$ .

## 5 Some applications of the main theorem

As an application of Theorem A, we can show the following result on a measure theoretical property for the dynamics of entire functions. This is a refinement of the result by Bock ([B]).

**Theorem B.** *Either one of the following (AT $\hat{Z}$ ) or (ERG) holds for an entire function  $f$ :*

(AT $\hat{Z}$ ) *Almost every point  $z \in J_f$  is attracted to the set  $\hat{Z}$ , that is,*

$$\lim_{n \rightarrow \infty} \text{dist}_{\text{sph}}(f^n(z), \hat{Z}) = 0, \quad (\text{i.e. } \omega(z) \subset \hat{Z})$$

*holds for a.e.  $z \in J_f$ , where  $\hat{Z} := Z \cup \{\infty\}$ .*

(ERG)  *$J_f = \mathbb{C}$  and  $f$  is ergodic.*

*Furthermore, (ERG) can be replaced by the following (IR) or (FOD):*

(IR)  *$J_f = \mathbb{C}$  and  $f$  is infinitely recurrent, i.e. for every  $X \subset \mathbb{C}$  with  $\text{Leb}(X) > 0$  and every  $z \in \mathbb{C}$ ,*

$$\#\{n \in \mathbb{N} \mid f^n(z) \in X\} = \infty$$

*holds, where  $\text{Leb}(\cdot)$  denotes the Lebesgue measure on  $\mathbb{C}$ .*

(FOD)  *$J_f = \mathbb{C}$  and for a.e.  $z \in \mathbb{C}$ , the forward orbit  $O^+(z) \subset \mathbb{C}$  is dense.*

**Corollary C.** *Let  $f$  be an entire function with the following properties:*

- (i) *Every critical point  $c$  of  $f$  is either preperiodic or satisfies  $f^n(c) \rightarrow \infty$  ( $n \rightarrow \infty$ ).*
- (ii) *Every asymptotic value is eventually periodic.*
- (iii) *The post-singular set  $P(f)$  is discrete in  $\mathbb{C}$ .*

*Then either one of the following holds:*

**(MGA)**  $\{\infty\}$  *is a metric global attractor, that is,  $f^n(z) \rightarrow \infty$  ( $n \rightarrow \infty$ ) for a.e.  $z \in \mathbb{C}$  (i.e.  $\omega(z) = \{\infty\}$ ).*

**(FOD)**  $J_f = \mathbb{C}$  *and  $O^+(z) \subset \mathbb{C}$  is dense for a.e.  $z \in \mathbb{C}$  (i.e.  $\omega(z) = \widehat{\mathbb{C}}$ ).*

*In particular, if  $f$  satisfies the conditions (i)  $\sim$  (iii) and  $J_f \neq \mathbb{C}$ , then  $\{\infty\}$  is a metric global attractor for  $f$ .*

**(Proof):** It follows from the assumptions (i)  $\sim$  (iii) that every singular value  $p$  satisfies either  $f^n(p) \rightarrow \infty$  or eventually lands on a repelling periodic point. If  $F_f \neq \emptyset$ , then only possible Fatou components are either Baker domains (or their preimages) or wandering domains. If there is a wandering domain  $U$ , then we have  $f^n|_U \rightarrow \infty$ , because in general a finite limit function on a wandering domain is a constant which belongs to the derived set of  $P(f)$  (see [BHKMT]), which is empty by (iii) in our case.

Then either **(AT $\widehat{Z}$ )** or **(FOD)** holds by Theorem A. In the case of **(AT $\widehat{Z}$ )**, it follows that

$$\omega(z) \subset \widehat{Z} = Y_2 \cup \{\infty\}, \text{ for a.e. } z \in J_f.$$

On the other hand,  $Y_2$  consists of repelling periodic points only and hence  $O^+(z)$  cannot accumulate on  $Y_2$ . Therefore

$$\omega(z) = \widehat{Z} = \{\infty\}, \text{ i.e. } f^n(z) \rightarrow \infty \text{ for a.e. } z \in J_f,$$

which implies that  $\{\infty\}$  is a metric global attractor.

In the case of **(FOD)**, it follows that  $J_f = \mathbb{C}$  and  $O^+(z) \subset \mathbb{C}$  is dense for a.e.  $z \in \mathbb{C}$ , which means that  $\omega(z) = \widehat{\mathbb{C}}$ . This completes the proof of Corollary C.  $\square$

**Corollary D.** *Let  $f$  be a semi-hyperbolic (transcendental) entire function with  $J_f \neq \mathbb{C}$ . Then,*

- (1)  $\text{Leb}(J_f) = 0 \iff \text{Leb}(J_f \cap I_f) = 0$ , where  $I_f := \{z \mid f^n(z) \rightarrow \infty\}$ .  
 (2)  $\text{Leb}(J_f) > 0 \implies f^n(z) \rightarrow \infty$  ( $n \rightarrow \infty$ ) for a.e.  $z \in J_f$

**(Proof):** Since  $f$  is semi-hyperbolic, we have  $Z = \emptyset$  by Theorem A. Also  $(\widehat{\text{ATZ}})$  holds from Theorem B, because we assume that  $J_f \neq \mathbb{C}$ . This means that  $f^n(z) \rightarrow \infty$  for a.e.  $z \in J_f$ . Now it is obvious to see that (1) and (2) hold.  $\square$

## References

- [B] H. Bock, *On the dynamics of entire functions on the Julia set*, Results Math. **30** No. 1-2 (1996), 16–20.
- [BHKMT] W. Bergweiler, M. Haruta, H. Kriete, H.-G. Meier and N. Terglane, *On the limit functions of iterates in wandering domains*, Ann. Acad. Sci. Fenn. Series A. I. Math. **18** (1993), 369–375.
- [BM] W. Bergweiler and S. Morosawa, *Semihyperbolic entire functions*, Nonlinearity **15** (2002), 1673–1684.
- [CJY] L. Carleson, P.W. Jones and J-C. Yoccoz, *Julia and John*. Bol. Soc. Brasil. Mat. (N.S.) **25** (1994), no. 1, 1–30.
- [EL] A. E. Eremenko and M. Ju. Lyubich, *Examples of entire functions with pathological dynamics*, J. London Math. Soc. (2) **36**No. 3 (1987), 458–468.
- [KS] M. Kisaka and M. Shishikura, *On multiply connected wandering domains of entire functions*, to appear in “Transcendental Dynamics and Complex Analysis”, Cambridge University Press, (2006).
- [M] R. Mañé, *On a theorem of Fatou*, Bol. Soc. Brasil. Mat. (N.S.) **24** No.1 (1993), 1–11.